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Charles University Prague

Essays on Finance and Risk

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Dissertation

Prague, August 2015

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Abstract

This dissertation consists of three chapters that empirically investigate questions of increasing relevance in the banking risk and financial economics literature. The first chapter studies bank risk in the context of its joint determination with bank liquidity and capital in the Eurozone. The second chapter examines the banks' appetite for risk using the comprehensive credit register of the Czech National Bank. Finally, the last chapter refers to model risk and analyzes the ability of the selected term structure models to value the interest rate swaps in the Polish market.

The first chapter analyzes the coordination of bank risk, liquidity and capital in the presence of securitization. Its outcome contributes to the debate on the effectiveness of the banking regulations. My findings with regard to the simultaneity of capital and risk decisions are consistent with previous empirical studies. Incorporation of bank liquidity permits me to establish the presence of the coordination of risk and liquidity decisions. At the same time, I find no evidence of the direct joint determination of capital and liquidity. Finally, the first chapter partially confirms the theoretical implications of Repullo (2005).

The second chapter, coauthored with Adam Geršl, Petr Jakubík, Steven Ongena and José-Luis Peydró, addresses the question of banks' appetite for risk. In particular, we examine the impact of monetary conditions on the risk-taking behaviour of banks in the Czech Republic. Our duration analysis indicates that expansionary monetary conditions promote risk-taking among banks. At the same time, a lower interest rate during the life of a loan reduces its riskiness.

The third chapter refers to model risk and analyzes the performance of the selected term structure models when valuing interest rate swaps in the Polish market. The Nelson-Siegel, cubic interpolation and CIR models generate adequate fit and transaction values similar to the realized contract values. The ample performance of the Cox-Ingersoll-Ross model suggests the rate-reliant nature of the interest rate volatility. The underperformance of Vasicek's emphasizes the role of the cross section of interest rates, and thus the importance of a no-arbitrage argument. Finally, the ex-post accuracy of the Nelson-Siegel and the cubic spline models indicates that a current cross-section of the yield curve is highly informative for the future.

Abstrakt

Tato dizertace obsahuje tři kapitoly, ve kterých jsou empiricky zkoumány klasické otázky bankovní rizika a finanční ekonomie. V první kapitole zkoumám jestli bankovní likvidita, kapitál a riziko jsou společně určeny v bankovním sektoru eurozóny. Ve druhé kapitole, spolu s Adamom Geršlem, Petrem Jakubíkem, Stevnem Ongena a José-Luis Peydró, zkoumáme roli měnových podmínek pro bankovní rizika v sektoru v ČR. Používáme komplexní úvěrového registru České národní banky. Třetí kapitola se vztahuje na modeloveho rizika a analyzuje výkonnost vybraného modelu vynosove krivky při oceňování úrokové swapy na polském trhu. Cox-Ingersoll-Ross navrhuje ze volatilita souvisi z velikosti úrokových sazeb. Ex-post Nelson-Siegel model ukazuje, že prurez výnosove křivky je velmi informativni pro budoucnost.

Acknowledgments

I wish to extend my particular gratitude to Professors Petr Zemčík, Jan Hanousek, Štěpán Jurajda and Sergey Slobodyan for their precious guidance and support. I am especially grateful to Professors Randy Filer, Ron Anderson, Jan Kmenta, Michal Kejak, Jan Večer, Paul Wachtel and Xavier Freixas for their constructive critiques. Professor Wachtel has actually provided an in-depth review of my other paper coauthored with Tamara Kochubey and not included in this thesis, but closely related to its first chapter. I acknowledge my gratitude to Dana Hájková and Tigran Poghosyan for refereeing my work and to Piotr Kowalczyk, AMIMA, for his assistance.

The first chapter of this thesis resulted from my project completed for the Financial Stability Division of the European Central Bank. I am highly indebted to John Fell and Olli Castrén for offering me an opportunity to join the Division as a PhD intern and develop my research idea in its stimulating atmosphere. I thank Dr. Stéphanie M. Stolz, Dr. Michael Wedow and Dr. Dawid Żochowski for their helpful suggestions, Ula Kochańska, Anna Heinrich, Martin Beck, Tatjana Zidulina and my other colleagues from the ECB for their support and time. The second chapter of this thesis is a joint work with Adam Geršl, Petr Jakubík, Steven Ongena and José-Luis Peydró, whose efforts made it possible to publish this study. Initially, the work was supported by the Czech National Bank Research Project No. C4/2009[1]. I am indebted to my coauthors and the Czech National Bank for the grant. Writing the third chapter of my thesis was possible thanks to the generosity one of the Big Four companies. I am thankful to Grzegorz Krawiec, FRM, and my other colleagues from PricewaterhouseCoopers.

I gratefully acknowledge the financial support I have received from CERGE-EI and the CERGE-EI Foundation, besides regular avenues, in the form of the teaching fellowships in Russia. Several cohorts of students I taught in Russia have been a source of invigoration to me. When conducting this research I gave presentations, seminars and lectures on the subject of my research and I am thankful to its participants for their comments. Finally, I am grateful to my friends at CERGE-EI for being a source of inspiration and support one could only dream about.

[1]The role of the PhD candidate involved adapting, implementing and developing research ideas of other coauthors, running probit and survival estimations as well as diagnostic and robustness tests, summarizing the results, preparing drafts of the working paper and working with the comprehensive dataset.

To Irek and Danusia

Introduction

This dissertation consists of three chapters which empirically examine questions of increasing relevance in the banking risk and financial economics literature. The first chapter studies bank risk in the context of its joint determination with bank liquidity and capital in the Eurozone banking sector. The second chapter examines the banks' appetite for risk using the comprehensive credit register of the Czech National Bank. Finally, the last chapter refers to model risk and analyzes the ability of the selected term structure models to value interest rate swaps in the Polish market.

The recent financial turmoil and developments leading to its emergence have altered the key sources of banks' risks. Financial innovations, deregulation and competition from non-bank financial intermediaries encouraged banks to seek higher returns and securitize their loans. The new banking model and banks' greater reliance on wholesale creditors have emphasized the importance of bank liquidity buffers. At the same time, the "originate-to-distribute" model and securitization might have resulted in an increased interdependence of bank capital, liquidity and risk. This process has revitalized a need for a proper recognition of risk, on balance and off-balance sheet, and led to revisions of banking regulations. This dissertation contributes to the discussion on banking regulations by drawing conclusions from the analysis of risk, liquidity and capital coordination in the Eurozone banking sector. The crisis has also fueled the discussion about the complexity of valuation of contemporary financial instruments. Following the crisis, practitioners and regulators have raised concerns regarding opaque financial products and their pricing. This dissertation poses a more basic question, namely it discusses the usefulness of advanced term structure models in valuation of plain vanilla interest rate sensitive derivatives. Finally, the relaxed monetary policy of major central banks has been listed among the causes of the recent financial turbulence. Existing theoretical work shows how changes in short-term interest rates may affect risk-taking by financial institutions, and empirical investigations that followed, largely confirmed the theoretical concepts. This dissertation complements the existing empirical studies with an analysis of the impact of monetary conditions on the risk-taking behaviour of banks in the Czech Republic, a small open economy with independent monetary policy and a banking sector dominated by the foreign ownership.

The first chapter investigates bank risk in the context of its joint determination with bank liquidity and capital. The study analyzes the coordination of bank liquidity, capital and risk in the presence of securitization. Its outcome contributes to the debate on the effectiveness of the banking regulations. The empirical strategy relies on the system of simultaneous equations and the partial adjustment approach, introduced by Shrieves and Dahl (1992) and advanced by Heid, Porath, and Stolz (2003). The estimation results for the securitization show that higher risk in the previous period implies greater securitization in the next period. My findings with regard to the simultaneity of capital and risk decisions are consistent with the previous empirical studies. Incorporation of bank liquidity permits me to establish a presence of the coordination of risk and liquidity decisions. At the same time, I find no evidence of the direct joint determination of capital and liquidity. Finally, the first chapter partially confirms the theoretical implications of Repullo (2005).

The second chapter, coauthored with Adam Geršl, Petr Jakubík, Steven Ongena and José-Luis Peydró, addresses the question of banks' appetite for risk. In particular, we examine the impact of monetary conditions on the risk-taking behaviour of banks in the Czech Republic. Our duration analysis indicates that expansionary monetary conditions promote risk-taking among banks. At the same time, a lower interest rate during the life of a loan reduces its riskiness. While seeking to assess the association between banks' appetite for risk and the short-term interest rate we answer a set of questions related to the difference between higher liquidity versus credit risk and the effect of a policy rate conditioned on bank and borrower characteristics.

The third chapter refers to model risk and analyzes five term structure models in order to compare their ability to capture the interest rate dynamics and value the interest rate swaps in the Polish market. Although model risk is typically associated with complex derivatives, the choice of a plain interest rate derivative allows for examination of model risk in the case of a dynamically growing OTC market, such as the Polish one, and a wider selection of interest rate models, which includes the frameworks of Nelson and Siegel (1987), Vasicek (1977), Cox, Ingersoll, and Ross (1985), Heath et al. (1992) and the cubic spline curves. The performance and predictive accuracy of the term structure models are assessed based on the realized contract values. The Nelson-Siegel, cubic interpolation and CIR models generate adequate fit and transaction values similar to the realized contract values. The Vasicek's approach gives contract values statistically different from the amounts actually swapped. The ample performance of the Cox-Ingersoll-Ross model suggests the rate-reliant nature of the interest rate volatility. The underperformance of Vasicek's emphasizes the role of the cross section of interest rates, and thus the importance of a no-arbitrage argument. Finally, the ex-post accuracy of the Nelson-Siegel and cubic spline indicates that a current cross-section of the yield curve is highly informative for the future.

Chapter 1

Capital, Liquidity and Risk Allocation in the Eurozone's Banking Sector

Capital, Liquidity and Risk Allocation in the Eurozone's Banking Sector*

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Abstract

This chapter investigates the capital, risk and liquidity decisions of the European banks in the period from 2001 to 2007. We examine the coordination of bank liquidity, capital and risk in the presence of securitization. The empirical strategy relies on the system of simultaneous equations and the partial adjustment approach, introduced by Shrieves and Dahl (1992) and advanced by Heid, Porath, and Stolz (2003). The estimation results for the securitization show that higher risk in the previous period implies greater securitization in the next period. Our findings with regard to the simultaneity of capital and risk decisions are consistent with the previous empirical studies. Incorporation of bank liquidity enables us to establish the presence of the coordination of risk and liquidity decisions. At the same time, we find no evidence of the direct joint determination of capital and liquidity. This outcome contributes to the debate on the effectiveness of the banking regulations. Finally, this study partially confirms the theoretical implications of Repullo (2005).

JEL Classification: G21, G28

Key Words: bank regulation, risk taking, bank capital, bank liquidity

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*The first draft of this paper was the outcome of my project completed within the European Central Bank, Frankfurt am Main, Germany. I wish to acknowledge and extend my particular gratitude to Professor Petr Zemčík for his precious guidance. I would like to thank Professors Randy Filer and Štěpán Jurajda for their suggestions. I am highly indebted to John Fell and Olli Castrén for offering me an opportunity to join the Division as a PhD intern and develop my research idea in its stimulating atmosphere. I thank Dr. Stéphanie M. Stolz, Dr. Michael Wedow and Dr. Dawid Żochowski for their helpful suggestions, Ula Kochańska, Anna Heinrich, Marin Beck, Tatjana Zidulina and my numerous other colleagues from the ECB for their support. Finally, I am grateful and the participants of the International Risk Management Conference on Financial Instability for their comments. All errors remaining in this text are the responsibility of the author.

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1.1 Introduction

Financial supervision authorities impose regulations on banks to ensure the safety and soundness of the banking system. Unregulated banks are believed to maintain too little capital and liquidity to absorb losses. Furthermore, it has been established that the resilient banking sector facilitates proper financial intermediation and enhances capital allocation in the economy. The key role of financial intermediation to the performance of the real sector has been empirically established, for instance by Rousseau and Rousseau and Wachtel (1998) or Dell'Ariccia, Detragiache, and Rajan (2008). Therefore, achieving and maintaining financial stability has been one of the main concerns of policy makers and has gained attention from researchers, as has the ongoing reform process of the banking industry launched in response to the recent financial crisis. Until recently, it had been believed that a bank's access to funding liquidity vitally depends on its assets' quality. Due to this commonly shared belief liquidity regulations were absent. The capital requirements were to assert the proper quality of bank assets and, in addition, the bank's sufficient liquidity. The recent crisis revealed the collective over-confidence in this respect.

The recent financial turmoil and developments leading to its emergence have altered the traditional roles performed by banking firms and the key sources of their risks. Financial innovations, deregulation and competition from the non-bank financial intermediaries encouraged banks to seek higher returns and securitize their loans. The new "originate and distribute" banking model, and banks' greater reliance on wholesale creditors, emphasized the importance of liquidity requirements. At the same time, the "originate-to-distribute" model and securitization might have resulted in an increased interdependence of bank capital, liquidity and risk. This process has revitalized a need for a proper recognition of risk, on balance and off-balance sheet, and led to revisions of banking regulations. One of the previously neglected determinants of bank risk is securitization activity. Furthermore, should the joint reshuffling of the two financial buffers and risk be confirmed by the banks' behavior, the design of banking regulations would need to account for this coordination effect. Any pairwise analysis overlooking the interplay between the two buffers and the asset quality may lead to inadequate regulatory liquidity and capital provisions.

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This paper tests whether banks coordinate their decisions on credit risk, capital and liquidity. In a sense, the main idea of this study was formulated years ago in banks' annual reports, where one may find descriptions of integrated approaches to managing capital, liquidity and balance sheet risk exposure, and the role of securitization in releasing capital and liquidity designed to fuel the banks' business growth[1]. Finding evidence for joint allocation of capital, liquidity and risk and for the role of securitization could shed some light on the way banks have relaxed the constraints of existing regulations and may have important implications for a potential revision of the banking regulations. While investigating the European banks' coordination of the quality of assets, capital and liquidity we - to some extent - test the predictions of Repullo (2005). Although Repullo (2005) focuses on the implications of the presence of the lender of last resort for bank liquidity, it also establishes that higher capital and liquidity induces lower risk. Our empirical investigation tests the latter theoretical relationship for the EMU banks. Repullo's conclusions regarding the reverse relations are more ambiguous. Nevertheless Repullo (2005), unlike many previous theoretical studies, does not ignore banks' liquidity buffers. Notably, it is the first theoretical paper to jointly model banks liquidity, capital and risk decisions. Therefore, we refer to its findings when discussing our empirical results.

This chapter is organized as follows: the following section discusses the theoretical and empirical literature, Section 3 outlines the methodology and model specification, while Section 4 describes the dataset. The estimation results are presented in Section 5 and Section 6 summarizes and concludes.

1.2 Literature Overview

Financial intermediation enhances capital allocation in the economy. Among essential functions performed by banks, banking theory identifies asset transformation, which in turn involves risk associated with financing illiquid loans with short-term deposits. This mismatch causes banks' vulnerability to depositors' confidence. Sufficient bank solvency

[1]For instance, Deutsche Bank states already in its report for the year 2000 that it carried out securitization transactions, which allowed for "growth by substantially reducing tied capital". And then it describes the success of its Global ALCO that managed "all strategic decisions on financial resources, including the allocation of capital, liquidity and balance sheet to the Group Divisions. This integrated approach enabled the Bank to release [...] regulatory capital through asset securitizations."

and liquidity are tools to maintain confidence in banking sectors. The academic literature on bank capital and capital regulations in the banking system has by now grown plentiful. Liquidity, on the contrary, is a more complex concept and has only recently emerged in banking firm theory. Baltensperger (1980) is the first to draw attention to a bank liquidity buffer. He analyzes the liquidity buffer from a perspective of the inventory theory. Baltensperger (1980) argues that, on one hand, it is costly for banks to keep a stock of liquid assets. However, it is at the same time beneficiary, since liquidity buffers reduce the probability of being 'out of stock' in case of deposit withdrawals. His study predicts that the size of liquidity buffer should reflect the cost of forgone return from holding liquid assets rather than loans, and the cost of raising funds at a short notice. Prisman, Slovin, and Sushka (1986) introduce liquidity risk into Monti-Klein model and show that the expected cost of liquidity shortage augments the cost of bank's resources.

Until Repullo (2005), little if any attention has been paid to modeling liquidity buffers. His paper investigates a strategic interaction between a bank and the lender of last resort, and concludes that the introduction of the latter reduces the size of bank's liquidity buffer. Furthermore, to the best of our knowledge it is the first theoretical study that addresses the question of banks' decisions about their level of capital, risk as well as liquidity. Repullo (2005) studies optimal liquidity, capital and risk choice with and without capital requirement, penalty rates and collateral lending. Crucial to our analysis is the result obtained under the capital requirement where the bank is obliged to maintain the amount of equity no lower than regulatory κ portion of its investment in the risky asset $(1-\lambda)$. Appendix $A.2$ features derivations of Repullo (2005) equilibrium under the capital requirement. In this equilibrium, the first-order condition (A.2.20) characterizing the bank's choice of risk suggests that higher capital and liquidity buffers imply lower risk. This outcome is obtained regardless of the type of the distribution function of the liquidity shock.

The conclusions of Repullo (2005) regarding the reverse relations are less straightforward. The bank's optimal level of capital is derived for a particular density function of liquidity shocks. Specifically, Repullo employs a simple case of a beta distribution, which ensures that larger liquidity shocks are less likely than small ones. For this choice, the equilibrum level of capital becomes the corner solution: $k^* = \kappa(1 - \lambda^*)$. In this case, the optimal level of capital depends depends inversely on the optimal level of liquidity. The results for the optimal level of liquidity are obtained solely numerically. In our investigation we focus on the general outcome, that is the derivations describing the bank's choice of optimal risk.

An insightful overview of theoretical approaches to bank capital is presented in VanHoose (2007), who discusses the efficiency of deposit insurance and solvency ratio as disciplining tools in the frameworks ranging from pure portfolio choice to moral hazard and incentive models. The stream of literature regarding banks as portfolio managers indicates that the imposition of a solvency ratio is likely to yield efficient and less risky asset allocation, providing that the risk weights are market based. On the other hand, the strand of literature viewing banks mostly as monitors for moral hazard argues that capital requirements may increase banks' risk appetites. The underlying rationale is that banks would seek to compensate for the costs of maintaining a capital cushion by incurring higher risk and increasing expected returns. Under the portfolio approach, first presented by Kahane (1977) and later advanced by Koehn and Santomero (1980) and Kim and Santomero (1988), a binding capital constraint changes the optimal composition of the bank's portfolio. The way in which the asset allocation is altered depends critically on the risk weights used in the solvency ratio. Koehn and Santomero (1980) employ a fixed capital-to-asset ratio and find that more stringent restriction on leveraging induces banks to augment their holdings of risky assets, which from the supervisory point of view is definitely an unintended outcome.

With mounting discussions on deficiencies of the Basel Capital Accord (1998), including the flat rate, the idea of uniform solvency ratio has been superseded by a risk-based approach. In their seminal paper, Kim and Santomero (1988) formally contrast the two approaches and establish that stricter uniform capital ratio regulation eliminates some leveraged parts of the bank's opportunity set. However, the optimal reduction in the insolvency risk is obtained under the risk-based plan. The most comprehensive study of economic theory implications for solvency restrictions and deposit insurance in various analytical banking models is due to Rochet (1992). Rochet shows that in the complete markets setup capital requirements prove to be a very inapt tool for limiting the risk taken by banks. In this case an increase in solvency ratio triggers portfolio reallocation leading to specialization in risky assets. Most importantly, Rochet proves that capital requirements attain the desired outcome in the portfolio model if and only if the risk weights are proportional to the market betas of respective assets. Other-than-market-based risk weights cause excessive investment in riskier assets, which corroborates with Kim and Santomero (1988) result. When Rochet extends his model to account for the limited liability of banks even market-based capital ratio does not prevent undercapitalized banks from specializing in riskier assets. This plethora of theoretical conflicting recommendations has motivated researchers to empirically examine the bank capital and capital regulations issues.

One of the most recognized empirical studies of capital buffers is due to Shrieves and Dahl (1992), who investigate changes in banks' capital and risk levels in order to determine which of the theoretical arguments are supported by the US data. They identify one theoretical rationale for a negative risk and capital dependence and four arguments predicting a positive relation between risk and leverage. In their opinion, a negative link is likely to characterize banks seeking to exploit deposit insurance subsidy, while those under regulatory pressure, facing high bankruptcy or regulatory costs, as well as banks exhibiting managerial risk aversion tend to adjust risk and leverage levels likewise. Shrieves and Dahl test the capital-risk relation using a simultaneous equations model with partial adjustment framework and find support for the effectiveness of regulatory policies on banks' capital and risk decisions. Heid, Porath, and Stolz (2003) build on Shrieves and Dahl's framework and examine German banks' risk-capital decisions. In addition to pooled regressions, Heid, Porath, and Stolz employ dynamic panel data techniques, subsample and rolling window approach. They test whether banks approaching the regulatory minimum adjust their leverage, risk or both and verify the impact of minimal capital ratio on well-capitalized banks. Their study indicates that banks adjust leverage faster than risk. However the speed does not depend on the level of capital buffers. Moreover, Heid, Porath, and Stolz establish that low capitalized banks tend to rebuild their capital cushions, while banks with substantial buffers tend to maintain their leverage levels and alter solely their allocation of risky assets.

The first broad investigation of determinants of banks' liquidity buffers is due to Aspachs, Nier, and Tiesset (2005), who build on theoretical implications of Repullo (2005). Aspachs, Nier and Tiesset study the liquidity policy of the UK banks and find that the higher the bank's expectations of receiving assistance from the lender of last resort the lower the liquidity buffers maintained. Their study also suggests that liquidity buffers are counter-cyclical as a result of the financial constraints on the banks' lending policy. Aspachs, Nier, and Tiesset (2005), however, focuse mainly on the liquidity moral hazard in the UK banking system and the interaction between the macroeconomic situation and the banks' liquidity buffers. In contrast, we employ the comprehensive theoretical framework of Repullo (2005) to investigate its implications for the banks' coordination of liquidity, capital and risk decisions. The first study to investigate the relation between banks' capital, credit risk and securitization was by Dionne and Harchaoui (2008). They find that securitization activity negatively affects banks' capital ratios and positively affects their credit risk. In other words, banks that are involved in securitization tend to be more risky. Moreover, banks constrained by the solvency ratio increase their securitization activity. In their opinion, a high risk level prevailing together with high total capital adequacy ratios suggests that BIS weights may inadequately capture the riskiness of banking activities.

1.3 Methodology and Model Specification

1.3.1 Determinants of Securitization

This paper tests whether banks coordinate their decisions on credit risk, capital and liquidity. We first focus on securitization as this activity is likely to provide an additional link for the interplay of risk, capital and liquidity. The immediate effect of securitization is a reduction in the risk-weighted assets and untying of regulatory capital due to a removal of the securitized loans from the bank's balance sheet. Whether or not it decreases the overall risk exposure depends on the bank's lending and investment strategies and the competitiveness of the financial sector. Financing new assets with the released liquidity should result in an increased diversification and should lower the bank risk. While Instefjord (2005) recognizes the benefits of risk sharing, he additionally shows that securitization encourages more risk-taking. Increased competition in the financial markets strengthens the impact of the latter effect (Instefjord (2005)). Moreover, Greenbaum and Thakor (1987) argue that banks tend to withhold poorer quality assets. Given the benign macroeconomic conditions and the search for yield observed in the analyzed period, we expect a positive dependancy between the asset quality, measured by credit risk, and the securitization activity. The predictions about the interaction between the liquidity and securitization and bank capital and securitization are even less evident. Therefore, we simply expect to obtain a significant relation. The research hypotheses regarding the securitization can be summarized as:

(H_A 1.1) Higher risk in the previous period implies greater securitization in the next period.

(H_A 1.2) There is a significant impact of the liquidity and capital on the securitization in the next period.
To verify the validity of these predictions we estimate the equation for the securitization activity given by:

$$\Delta SEC_{i,t} = \xi CAP_{i,t-1} + \psi RISK_{i,t-1} + \zeta LIQ_{i,t-1} + CONTROLS + \omega_{i,t} \tag{1.1}$$

where $CAP_{i,t-1}$, $RISK_{i,t-1}$ and $LIQ_{i,t-1}$ are the previous period levels of bank capital, risk and liquidity defined as in the next section. Following Altunbas, Gambacorta, and Marques-Ibanez (2009), $SEC_{i,t}$ relates the flow of the securitized lending in the current year to the total assets in the previous year. Using equation (1.1) we test whether ψ is positive and ψ, ξ and ζ are significant. All variable definitions are also provided in Table A1.1. The bank controls are bank-level variables affecting the decision to sell loans. Due to the economies of scale, the bank size is a good candidate for a control variable. Additionally, the alternative cost of funding new assets is likely to impact the securitization activity. Banks able to attract "cheap" deposits (low interest on deposits) or with a lot of "cheap" capital (low return on capital) should be less prone to finance new assets with securitization (for a detailed discussion refer to Han, Park, and Pennacchi (2010)). In our case, the size effect proved to significantly influence the securitization activity.

1.3.2 Coordination of Risk, Capital and Liquidity

As already mentioned, the main goal of this paper is to test whether banks coordinate their decisions on credit risk, capital and liquidity. By doing so, to some extent we verify the predictions of Repullo (2005). Repullo (2005) studies the implications of the presence of the lender of last resort for the bank liquidity. However Repullo also derives optimal liquidity, capital and risk choice with and without capital requirement, penalty rates and collateral lending. The outcome obtained under the capital requirement, where the bank is obliged to maintain the amount of equity no lower than regulatory κ portion of its investment in the risky asset (1 - λ), is vital to our analysis. A simplified exposition of the banking model of Repullo (2005) is provided in Appendix A.2. In short, Repullo argues that higher capital and liquidity induces lower risk. Conclusions regarding the reverse relations are more ambiguous. The bank's optimal level of capital is derived for a simple case of a beta distribution of liquidity shocks. Such a density function ensures that larger liquidity shocks are less likely than small ones. For this choice, the equilibrum level of capital becomes the corner solution: $k^* = \kappa(1 - \lambda^*)$. In this case, the optimal level of capital depends inversely on the optimal level of liquidity. The results for the optimal level of liquidity are obtained solely numerically. In our investigation we focus on the general outcome, that is the derivations describing the bank's choice of optimal risk.

While testing the implications of economic theory for the relationship between capital, risk and liquidity we employ a simultaneous equations estimation with partial adjustments. An important aspect of this approach is that it recognizes the simultaneity of leverage and risk decision-making, which is suggested by the theory and emphasized in the work of Shrieves and Dahl (1992) and Heid, Porath, and Stolz (2003), among others. The observed changes in banks' leverage, risk and liquidity levels are caused not solely by banks' discretionary behavior, but also as a result of unanticipated shocks. This argument has been emphasized by Hart and Jaffee (1974) and incorporated in most previous empirical studies. Accordingly, we model observed changes in capital, liquidity and risk as the sum of a discretionary component and a random shock. The fact that we obtain solely the estimates for the discretionary part of the observed changes is one of the justifications for the use of a partial adjustment framework. An even stronger rationale for using partial adjustment stems from rigidities and adjustment costs assumed in a number of theoretical banking models. This framework presumes that banks aim at establishing optimal capital, risk and liquidity and, when driven away from those targets by exogenous shocks, adjust their actual levels gradually. Full adjustments might be simply too costly or unfeasible. The partial adjustment can be generally expressed as:

$$\Delta CAP_{it} = \alpha \Delta CAP_{it}^{D} \quad + \quad \epsilon_{it} \tag{1.2}$$

$$\Delta RISK_{it} = \beta \Delta RISK_{it}^{D} \quad + \quad \nu_{it} \tag{1.3}$$

$$\Delta LIQ_{it} = \gamma \Delta LIQ_{it}^{D} \quad + \quad \eta_{it} \tag{1.4}$$

where ΔCAP_{it}, $\Delta RISK_{it}$ and ΔLIQ_{it} are the observed changes, while ΔCAP_{it}^{D}, $\Delta RISK_{it}^{D}$ and ΔLIQ_{it}^{D} are the endogenously determined changes in bank's capital, risk and liquidity respectively. The coefficients α, β and γ capture the speed of adjustment for capital, risk and liquidity. Following the conventional approach in the field, we estimate equations for the changes and not the absolute values of capital, risk and liquidity. In fact, in the absence of the theory on optimal leverage, modeling deviations from banks' internal targets within the partial adjustment framework becomes a very convenient strategy for conducting empirical studies. As a result, we obtain the following equations:

$$\Delta CAP_{it} = \alpha(CAP_{i,t}^{*} - CAP_{i,t-1}) \quad + \quad \epsilon_{it} \tag{1.5}$$

$$\Delta RISK_{it} = \beta(RISK_{i,t}^{*} - RISK_{i,t-1}) \quad + \quad \nu_{it} \tag{1.6}$$

$$\Delta LIQ_{it} = \gamma(LIQ_{i,t}^{*} - LIQ_{i,t-1}) \quad + \quad \eta_{it} \tag{1.7}$$

The unobservable internal targets are usually captured by proxies, which are suggested by banking theory, such as bank's size or return on assets. First, however, we discuss the measures of capital, risk and liquidity and only afterwards do we turn to their unobservable targets. The simultaneity of capital, risk and liquidity decisions implies that the system of equations (1.5 -1.7) becomes:

$$\Delta CAP_{it} = \alpha(CAP_{i,t}^{*} - CAP_{i,t-1}) + \phi_1 \Delta RISK_{i,t} + \varphi_1 \Delta LIQ_{i,t} + \epsilon_{it} \tag{1.8}$$

$$\Delta RISK_{it} = \beta(RISK_{i,t}^{*} - RISK_{i,t-1}) + \tau_1 \Delta CAP_{i,t} + \varphi_2 \Delta LIQ_{i,t} + \nu_{it} \tag{1.9}$$

$$\Delta LIQ_{it} = \gamma(LIQ_{i,t}^{*} - LIQ_{i,t-1}) + \phi_2 \Delta RISK_{i,t} + \tau_2 \Delta CAP_{i,t} + \eta_{it} \tag{1.10}$$

Finally, we can formulate our research hypotheses regarding the coordination effect and the test of Repullo's predictions as follows:

(H_A 2) There is a significant impact of the risk, capital and liquidity on each other.

(H_A 3) Higher capital and liquidity imply lower risk.

1.3.3 Empirical Model and Variable Measures

Empirical studies commonly use one of the following measures of capital, either the leverage ratio or the risk-based capital ratio. The leverage ratio is defined as total capital over total assets and the risk-based capital ratio is total capital to risk-weighted assets. The latter measure has become more popular after the introduction of risk-based regulation (see for example Jacques and Nigro (1997), Ediz, Michael, and Perraudin (1998), Aggarwal and Jacques (2001) and Rime (2001)). Given our definition of $RISK$, we measure capital as the leverage ratio[2] (CAP). The definition of risk causes even more problems. The empirical investigations mostly rely on the ratio of risk-weighted assets to total assets (*RWATA*) as the risk measure. Such a choice stems from the belief that, by reflecting the allocation of bank's assets among risk categories, *RWATA* is the true determinant of a bank's risk. However, Shrieves and Dahl (1992) argue that *RWATA* neglects the impact of loans quality and add a third equation, with nonperforming loans as an additional measure of risk. Contrary to them, Jacques and Nigro (1997), Rime (2001) and Heid, Porath, and Stolz (2003) claim that *RWATA* captures both the allocation and quality of portfolio risk and rely solely on this risk measure. We also employ the ratio of risk-weighted assets to total assets ($RISK$), since together with our capital measure they constitute the BIS capital adequacy ratio imposed on the EU banks to monitor their soundness. Finally, liquidity (LIQ) is measured by the relation of liquid assets to total assets (see for instance Aspachs, Nier, and Tiesset (2005)). The liquid assets comprise cash, reverse repos, bills and commercial papers. All variable definitions are summarized in Table A1.1.

The partial adjustment models account for the unobservable targets with the help of variables describing the nature of the bank's business and its current financial stance. For the sake of comparability we rely on variables typically chosen in the related empirical literature. As a rule, the bank size ($SIZE$) is considered to affect the target leverage, liquidity and risk. $SIZE$ is measured as the logarithm of a bank's total assets. Among others, the size effect matters for relative access to capital and liquidity, for the investment possibilities and diversification of bank activities. Additionally, due to the economies of scale in screening and monitoring, bigger banks might have less risky loan portfolios. Still, the sign of the size effect on the risk, capital and liquidity is undetermined. Loan losses

[2]The same measure is used for instance in Shrieves and Dahl (1992) and Heid, Porath, and Stolz

lower the amount of risk-weighted assets and thus may affect the risk target. Additionally to the size effect, we proxy the capital target by the ratio of net income to total assets (ROA). We expect a positive coefficient on the return on assets. Earnings, if retained, increase the capital. The bank size and loan losses are assumed to influence the target level of risk. We approximate loan losses ($LLOSS$) with the ratio of new loan loss provisions over the sum of banks' loans net loan loss reserves. $LLOSS$ is included in the risk equation with an expected negative sign. Among the idiosyncratic factors influencing the liquidity target we include the bank size and the loan growth rate ($\uparrow LOAN$). The loan growth rate is defined as new lending volume over the loan portfolio in the previous year. Finally, we include the year dummies. Thus, the system of equations to be estimated takes the following form:

$$\Delta CAP_{it} = \alpha_0 + \alpha_1 SIZE_{i,t} + \alpha_2 ROA_{i,t} - \alpha_3 CAP_{i,t-1} + \alpha_4 \Delta RISK_{i,t}$$
$$+ \alpha_5 \Delta LIQ_{i,t} + \alpha_6 \Delta SEC_{i,t} + DUMMIES_{YEAR} + \epsilon_{it} \qquad (1.11)$$

$$\Delta RISK_{it} = \beta_0 + \beta_1 SIZE_{i,t} + \beta_2 LLOSS_{i,t} - \beta_3 RISK_{i,t-1} + \beta_4 \Delta CAP_{i,t}$$
$$+ \beta_5 \Delta LIQ_{i,t} + \beta_6 \Delta SEC_{i,t} + DUMMIES_{YEAR} + \nu_{it} \qquad (1.12)$$

$$\Delta LIQ_{it} = \gamma_0 + \gamma_1 SIZE_{i,t} + \gamma_2 \uparrow LOAN_{i,t} - \gamma_3 LIQ_{i,t-1} + \gamma_4 \Delta CAP_{i,t}$$
$$+ \gamma_5 \Delta RISK_{i,t} + \gamma_6 \Delta SEC_{i,t} + DUMMIES_{YEAR} + \eta_{it} \qquad (1.13)$$

1.3.4 Estimation Strategy

Given the simultaneous nature of the model, the estimation strategy has to account for the endogeneity of ΔCAP_{it}, ΔLIQ_{it} and ΔLIQ_{it}. The two-stage least squares and three-stage least squares (3SLS) take into account the endogeneity of the regressors and yield consistent estimates. As we wish to recognize the contemporaneous correlation between the error terms in the three equations, we use 3SLS procedure. In such cases 3SLS produces asymptotically more efficient estimates[3]. Typically, the empirical studies in the

(2003).

[3]For a description of the procedure refer to econometric textbooks, e.g. (Baltagi 2008), pp. 131-132.

field rely on the pooled 3SLS methodology. Therefore, for the sake of comparability and as a robustness check, we estimate and report the results for the three-stage least squares. In the absence of the unobserved heterogeneity, the 3SLS procedure produces unbiased estimates. However, when the "left-aside" bank-specific effects are not negligible, the 3SLS outcome is biased. To control for bank-specific heterogeneity we employ dynamic panel data technique of Blundell and Bond (1999). The two-step Blundell-Bond system GMM estimator uses lagged variables as instruments in the first difference equations and lagged first differences in the levels equations. In both estimation techniques we use dummy variables for the years studied. In addition to the system of the capital, risk and liquidity equations, we model the bank securitization activity as a function of the three key variables and the bank size. The securitization is also estimated with the help of the two-step Blundell-Bond system GMM estimator. The fitted values serve as instruments for the securitization in the system of equations (1.11) - (1.13).

1.4 Data Description

This study uses annual bank financial data gathered from Bankscope, Bloomberg and AMC Dealogic (former Bondware). Bankscope is a commercial database maintained by International Bank Credit Analysis Ltd. and Bureau van Dijk, which contains financial results for banks together with such additional metrics as Fitch ratings. Dealogic is an independent data distributor, which provides information on various over-the-counter structural finance transactions and syndicated loans. We use the extended dataset of ABS securities and cash flow CDOs from Dealogic employed in Altunbas, Gambacorta, and Marques-Ibanez (2009). When it comes to the financial statements data, if both consolidated and unconsolidated figures are available, the consolidated ones are utilized. We analyze annual data since this frequency accounts for more discretionary behavior by capturing long-term trends. The dataset covers the period 2000 to 2007. However, due to the use of first differences and lagged values, the reported results refer to 2002-2007. The number of analyzed banks differs across years from 201 to 443 (see Table A1.4). We consider solely the banks operating in the Eurozone. The number of bank records is limited by the data availability. Table 1.1 displays main descriptive statistics for the obtained data, while Table A1.2 and Table A1.3 present the bivariate dependencies in our sample. The rationale for the variable selection is discussed in the sections: Empirical Model and Variable Measures and Determinants of Securitization. All variable definitions are summarized in Table A1.1.

Table 1.1: Data Descriptive Statistics

Variable	Type	Freq.	Obs.	Mean	Std. Dev.	Min.	Max.
$RISK$	#	year	$2,477$	0.605	0.313	0.011	2.693
CAP	#	year	$2,477$	0.090	0.061	0.002	0.632
LIQ	#	year	$2,458$	0.148	0.165	0	0.988
SEC	#	year	$2,477$	3.969	26.37	0	532
$SIZE$	#	year	$2,477$	15.21	2.44	9.47	21.67
ROA	%	year	$2,477$	0.895	0.976	-11.13	20.25
$LLOSS$	#	year	$2,366$	0.004	0.026	-0.733	0.407
$\uparrow LOAN$	%	year	$2,097$	15.63	29.28	-96	256

1.5 Estimation Results

The estimation results for Equation (1.1) are presented in Table 1.2. We estimate the securitization activity using the dynamic panel data procedure discussed in the section Estimation Strategy. This approach allows us to account for the possible bank-specific effects. To capture the two-way relation of risk, capital, liquidity and securitization, we use GMM-type instruments for $CAP_{i,t-1}$, $RISK_{i,t-1}$ and $LIQ_{i,t-1}$. The Hansen test indicates the validity of the instruments. The condition for the GMM estimator consistency is also met. The test reports serial correlation in the first-differenced residuals and the lack of the serial correlation for the second-order differences. The results show that the size effect play a significant role in the determination of the level of securitization (0.488^{**}). More importantly, the estimate for $RISK_{i,t-1}$ is significant and amounts to 4.695^{**}. Therefore, we find evidence to support our hypothesis (H_A **1.1**). Higher risk in the previous period implies greater securitization in the next period. The outcome suggests no significant influence of liquidity and capital on securitization (H_0 **1.2**). The fitted values obtained for securitization serve as instruments in the estimation of the coordination of the risk, capital and liquidity bank decisions.

With respect to the coordination of capital, risk and liquidity adjustments, the pooled 3SLS and the dynamic panel estimations agree on the simultaneity of the risk and capital decisions as well as the impact of the risk adjustment on the liquidity correction. None of the methodology finds evidence of the simultaneity between capital and liquidity. Therefore, our hypothesis (H_A **2**) is solely partially supported. In each equation, the effect of the fitted securitization proves to be insignificant. Its effect is likely to be already captured by the levels of endogenous variables themselves. As each equation includes corresponding lagged variables, $CAP_{i,t-1}$, $RISK_{i,t-1}$ and $LIQ_{i,t-1}$, each of them is identified. The validity of the overidentifying restrictions is confirmed by the Hansen test (see the estimation output in Tables 1.3 - 1.5). To capture the simultaneity of risk, capital and liquidity in the dynamic approach, we use GMM-type instruments for $CAP_{i,t-1}$, $RISK_{i,t-1}$ and $LIQ_{i,t-1}$. The Hansen test indicates the validity of the instruments. The condition for the GMM estimator consistency is also met. The test reports serial correlation in the first-differenced residuals and the lack of the serial correlation for the second-order differences (for details refer to Tables 1.3 -1.5). The outcome for the capital equation are provided in Table 1.3. The 3SLS and Blundell-Bond GMM estimates indicate a positive coordination of capital and risk. The estimated coefficients for the risk are 0.165^{***} and 0.078^{***} respectively. In addition, both procedures find the significant impact of the return on assets on the capital adjustment. The sign is as expected and corroborates with the empirical evidence (e.g. Heid, Porath, and Stolz (2003)). The size effect and the coefficient on CAP_{t-1} become significant under the dynamic panel treatment. Since this approach accounts for any possible unaccounted bank-specific effects, the dynamic panel estimators are regarded as more reliable in this context. The negative adjustment coefficient (-0.145^{***}) supports the validity of the partial adjustment framework. The size effect indicates the inverse relation between the bank size and the capital adjustment (-0.001^{***}).

Table 1.4 summarizes the results for the risk equation. The 3SLS and Blundell-Bond GMM estimates indicate a positive coordination of capital and risk. The estimated coefficients for the capital are 3.829^{***} and 2.629^{**} respectively. Moreover, both procedures find a significant impact of $RISK_{t-1}$, which corroborates with the partial adjustment framework. The GMM estimates for the coordination of the liquidity and risk adjustments is highly significant and amounts to -0.268^{***}. Due to the possible unaccounted bank-specific effects, the dynamic panel estimators are regarded as more reliable. Yet, the insignificant pooled 3SLS estimate signals the result is not robust. The size effect becomes marginally significant under the dynamic treatment (-0.006^{*}). As in the case of the capital equation, the size effect indicates the inverse relation between the bank size and the risk adjustment. The impact of the loan losses on the risk adjustment proves to be insignificant.

Table 1.2: Determinants of Securitization, 2003-2007

Dep. variable SEC	**Coefficient** (Robust Std. Err).
SEC_{t-1}	0.401^{*}
	(0.230)
SEC_{t-2}	-0.225
	(0.204)
$RISK_{t-1}$	4.695^{**}
	(2.072)
CAP_{t-1}	-5.343
	(4.094)
LIQ_{t-1}	-4.883
	(4.091)
$SIZE$	0.488^{**}
	(0.243)
Intercept	-8.396^{*}
	(4.572)
Year dummies	Yes
N	$1,844$
$\chi^2_{(11)}$	43.874
Sargan test	0.384
AR(1) test	0.113
AR(2) test	0.979

Notes: The dependent variable, SEC, is defined as securitization activity (deal values) in the current year over total assets at the end of the previous year. SEC_{t-1} and SEC_{t-2} are one- and two-period lags respectively. The fitted values of securitization are used in subsequent estimations. $CAP_{i,t}$ is total capital over total assets, $RISK_{i,t}$ are risk-weighted assets over total assets and $LIQ_{i,t}$ are liquid assets over total assets. $SIZE$ is defined as the natural log of total assets. Significance levels at 10%, 5%, 1% in a two-tailed t-test are shown as $*$, $**$ and $***$ respectively. Estimation with the Blundell-Bond Two-Step GMM procedure. To account for simultaneity of risk, capital, liquidity and securitization adjustments I use GMM-type instruments for $CAP_{i,t-1}$, $RISK_{i,t-1}$ and $LIQ_{i,t-1}$. The Hansen test reports a p-value for the Hansen test of overidentifying restrictions. AR(1) and AR(2) tests report p-values for the test of no first-order and second-order autocorrelation in the first-differences residuals.

Table 1.3: Simultaneous Estimation for the Capital Equation, 2002-2007

Dep. variable Δ **CAP**	**(I) Coefficient** (Robust Std. Err.)	**(II) Coefficient** (Robust Std. Err.)
CAP_{t-1}	0.007	-0.145^{***}
	(0.010)	(0.046)
$\Delta RISK$	0.165^{***}	0.078^{***}
	(0.019)	(0.011)
ΔLIQ	0.019	0.002
	(0.020)	(0.008)
$\Delta S\hat{E}C$	0.000	0.000
	(0.000)	(0.000)
$SIZE$	0.000	-0.001^{**}
	(0.000)	(0.001)
ROA	0.002^{***}	0.004^{***}
	(0.001)	(0.001)
Intercept	-0.009^{*}	0.026^{**}
	(0.005)	(0.012)
Year dummies	Yes	Yes
N	866	866
Log-likelihood	5191.150	
$\chi^2_{(10)}$		128.934
Hansen test	0.127	0.131
AR(1) test	-	0.002
AR(2) test	-	0.861

Notes: The dependent variable, $\Delta CAP_{i,t}$, is defined as a change in total capital over total assets. $\Delta RISK_{i,t}$ is a change in risk-weighted assets over total assets and $\Delta LIQ_{i,t}$ is a change in liquid assets over total assets. SEC is a fitted value of securitization activity (see Table 1.2). Δ indicates the first difference. SIZE is defined as the natural log of total assets. ROA is return on assets. Significance levels at 10%, 5%, 1% in a two-tailed t-test are shown as $*$, $**$ and $***$ respectively. **(I)** The capital equation, the risk equation and the liquidity equation are estimated simultaneously using the three-stage least squares procedure. The Hansen test reports a p-value for the Hansen-Sargan test of overidentifying restrictions. **(II)** Estimation with the Blundell-Bond Two-Step GMM procedure. To account for simultaneity of risk, capital and liquidity adjustments I use GMM-type instruments for $\Delta RISK_{i,t}$ and $\Delta LIQ_{i,t}$. The Hansen test reports a p-value for the Hansen test of overidentifying restrictions. AR(1) and AR(2) tests report p-values for the test of no first-order and second-order autocorrelation in the first-differences residuals.

Table 1.4: Simultaneous Estimation for the Risk Equation, 2002-2007

Dep. variable Δ **Risk**	**(I) Coefficient** (Robust Std. Err.)	**(II) Coefficient** (Robust Std. Err.)
$RISK_{t-1}$	-0.037^{***}	-0.108^{*}
	(0.012)	(0.061)
ΔCAP	3.829^{***}	2.629^{**}
	(0.002)	(1.193)
ΔLIQ	0.020	-0.268^{***}
	(0.111)	(0.083)
$\Delta \hat{SEC}$	0.000	0.000
	(0.000)	(0.001)
$SIZE$	-0.001	-0.006^{*}
	(0.002)	(0.003)
$LLOSS$	0.394	0.638
	(0.249)	(0.590)
Intercept	0.062^{**}	0.168^{*}
	(0.028)	(0.088)
Year dummies	Yes	Yes
N	866	866
Log-likelihood	5191.150	
$\chi^2_{(9)}$		51.691
Hansen test	0.127	0.299
AR(1) test	-	0.001
AR(2) test	-	0.100

Notes: The dependent variable $\Delta RISK_{i,t}$ is defined as a change in risk-weighted assets over total assets $\Delta LIQ_{i,t}$ is a change in liquid assets over total assets and $\Delta CAP_{i,t}$ is a change in total capital over total assets. $\hat{SEC}$ is a fitted value of securitization activity (see Table 2). Δ indicates the first difference. $SIZE$ is defined as the natural log of total assets. $LLOAN$ are loan loss provisions over the sum of banks loans net loan loss reserves. Significance levels at 10%, 5%, 1% in a two-tailed t-test are shown as $*$, $**$ and $***$ respectively. (I) The capital equation, the risk equation and the liquidity equation are estimated simultaneously using the three-stage least squares procedure. The Hansen test reports a p-value for the Hansen-Sargan test of overidentifying restrictions. (II) Estimation with the Blundell-Bond Two-Step GMM procedure. To account for simultaneity of risk, capital and liquidity adjustments I use GMM-type instruments for $\Delta CAP_{i,t}$ and $\Delta LIQ_{i,t}$. The Hansen test reports a p-value for the Hansen test of overidentifying restrictions. AR(1) and AR(2) tests report p-values for the test of no first-order and second-order autocorrelation in the first-differences residuals.

Table 1.5: Simultaneous Estimation for the Liquidity Equation, 2002-2007

Dep. variable **Δ LIQ**	**(I) Coefficient** (Robust Std. Err.)	**(II) Coefficient** (Robust Std. Err.)
LIQ_{t-1}	-0.168^{***}	-0.366^{***}
	(0.019)	(0.121)
$\Delta RISK$	0.543^{***}	-0.101^{***}
	(0.209)	(0.026)
ΔCAP	-1.811^{*}	-0.010
	(0.962)	(0.195)
$\Delta \hat{SEC}$	0.000	0.000
	(0.001)	(0.001)
$SIZE$	-0.003^{***}	-0.003^{*}
	(0.001)	(0.002)
$\uparrow LOAN$	0.0001^{***}	-0.0001^{***}
	(0.000)	(0.000)
Intercept	0.023	0.077^{**}
	(0.018)	(0.035)
Year dummies	Yes	Yes
N	866	866
Log-likelihood	5191.150	
$\chi^2_{(9)}$		408.791
Hansen test	0.127	0.148
AR(1) test	-	0.000
AR(2) test	-	0.268

Notes: The dependent variable is $\Delta LIQ_{i,t}$ defined as a change in liquid assets over total assets. $\Delta RISK_{i,t}$ is a change in risk-weighted assets over total assets and $\Delta CAP_{i,t}$ is a change in total capital over total assets. $\hat{SEC}$ is a fitted value of securitization activity (see Table 2). Δ indicates the first difference. $SIZE$ is defined as the natural log of total assets. $\uparrow LOAN$ is a loan portfolio growth defined as new volume to previous period loan amount. Significance levels at 10%, 5%, 1% in a two-tailed t-test are shown as $*$, $**$ and $***$ respectively. (I) The capital equation, the risk equation and the liquidity equation are estimated simultaneously using the three-stage least squares procedure. The Hansen test reports a p-value for the Hansen-Sargan test of overidentifying restrictions. (II) Estimation with the Blundell-Bond Two-Step GMM procedure. To account for simultaneity of risk, capital and liquidity adjustments I use GMM-type instruments for $\Delta RISK_{i,t}$ and $\Delta CAP_{i,t}$. The Hansen test reports a p-value for the Hansen test of overidentifying restrictions. AR(1) and AR(2) tests report p-values for the test of no first-order and second-order autocorrelation in the first-differences residuals.

Table 1.5 presents the estimation outcome for the liquidity equation. The 3SLS and Blundell-Bond GMM estimates indicate a significant impact of the risk adjustment on the liquidity adjustment. Yet, the procedures do not agree on the direction of this interaction. Given the concerns about the unaccounted bank-specific effects, we rely on the GMM dynamic estimation and conclude that there is coordination of the risk and liquidity adjustments. As the estimate for the impact of the liquidity adjustment in the risk equation, the corresponding coefficient for the risk adjustment in the liquidity equation is negative (-0.101^{***}). Both procedures find a significant impact of LIQ_{t-1}, which provides support the partial adjustment framework (-0.168^{***} and -0.366^{***} for the pooled 3SLS and dynamic GMM respectively). In addition, the effect of the loan growth on the liquidity adjustment proves to be negative and highly significant under both methodologies (0.0001^{***}). The size is significant and negative under the two estimation procedures, however only marginally significant in the GMM estimation (-0.003^{***} and -0.003^{*}).

Finally, all estimation results indicate the presence of the coordination of capital and risk. In addition, the dynamic approach suggests the coordination of liquidity and risk decisions. Therefore, our hypothesis (H_A **2**) is partially supported. In particular, we find no evidence for the coordination of the capital and liquidity decisions. Still, the reshuffling between the risk and liquidity as well as risk and capital calls for further investigation, especially in the context of the efficiency of the new capital and liquidity banking regulations. Furthermore, the estimation results only partially confirm the predictions of Repullo (2005). The corresponding hypothesis (H_A **3**) states that higher capital and liquidity imply lower risk. Our findings suggest that higher capital induces higher risk, while higher liquidity indeed yields to lower risk. Moreover, we confirm that the partial adjustment approach to modeling deviations from the internal target levels of risk, capital and liquidity is the appropriate one. The coefficients on the three lagged variables are significant, and of the expected size in each equation respectively. By expected sign we mean the range between -1 and 0, which suggest that after a shock occurs our model returns to the target equilibrium.

1.6 Conclusion

This study examines the capital, risk and liquidity decisions of the European banks in the period leading to the recent crisis. In particular, we investigate to what extent banks coordinate capital, risk and liquidity adjustments in the presence of securitization. We employ the partial adjustment approach, introduced by Shrieves and Dahl (1992) and advanced by Heid, Porath, and Stolz (2003). To account for the coordination effect, we estimate the system of simultaneous equations. The empirical strategy relies on the dynamic panel estimation and, additionally, includes the pooled 3SLS procedure. Our research contributes to the debate on the global banking reform process.

To the best of our knowledge, we are the first to jointly examine capital, risk and liquidity decisions of the European banks. In line with the previous empirical evidence, we find support for the simultaneity of capital and risk decisions. In addition, our results suggest a coordination of risk and liquidity decisions. At the same time, we find no evidence of the direct coordination of capital and liquidity.

Since securitization is one of the previously neglected determinants of the bank risk and a possible strengthening link for the capital, risk and liquidity coordination, we include the securitization in our investigations. The estimation results for the securitization show that higher risk in the previous period implies greater securitization in the next period. This study only partially confirms the theoretical implications of Repullo (2005). Our findings regarding the joint allocation of liquidity and risk suggest how banks could have relaxed the constraints resulting from the banking regulations. The issue of how the existing capital requirements proved ineffective is of critical importance to the reform of the banking regulatory framework.

Chapter 2

Monetary Conditions and Banks' Behaviour in the Czech Republic

Monetary Conditions and Banks' Behaviour in the Czech Republic*

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Abstract

This chapter examines the impact of monetary conditions on the risk-taking behaviour of banks in the Czech Republic by analysing the comprehensive credit register of the Czech National Bank. Our duration analysis indicates that expansionary monetary conditions promote risk-taking among banks. At the same time, a lower interest rate during the life of a loan reduces its riskiness. While seeking to assess the association between banks' appetite for risk and the short-term interest rate, we answer a set of questions related to the difference between higher liquidity versus credit risk and the effect of the policy rate conditioned on bank and borrower characteristics.

JEL Classification: E5, E44, G21

Key Words: business cycle, credit risk, financial stability, lending standards, liquidity risk, monetary policy, policy interest rate, risk-taking

*This work was supported by Czech National Bank Research Project No. C4/2009. The authors thank Dana Hájková, Štěpán Jurajda and Xavier Freixas for useful comments. The opinions expressed in this paper are only those of the authors and do not represent the official views of the institutions with which the authors are affiliated.

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2.1 Introduction

One of the factors often mentioned as a cause of the recent financial turbulence has been the relaxed monetary policy of major central banks, which might have increased financial institutions' appetite for risk. Monetary policy influences bank behaviour and the supply of loans via several channels (Bernanke and Gertler 1995). Because of imperfect information, incomplete contracts and imperfect bank competition, monetary policy may affect loan supply. In particular, expansive monetary policy may increase bank loan supply either directly (the bank lending channel) or indirectly by improving borrower net worth and, hence, by reducing the agency costs of lending (the balance sheet channel). In the "balance sheet channel", higher interest rates, by reducing borrower net worth, may induce a flight to quality from financiers (Bernanke and Gilchrist 1996) or more lending to borrowers with more pledgeable assets (Matsuyama 2007). On the other hand, when there is a reduction of overnight rates, financiers start lending more to borrowers that previously had a too-low net worth (hence, too-high agency costs of lending), because thanks to the lower rates their net worth rises enough to make lending possible. However, in this case, the potential softening of credit standards is not regarded as greater bank appetite for risk induced by low rates.

Recent theoretical work shows how changes in short-term interest rates may affect risk-taking by financial institutions. This effect has been labelled the "risk-taking channel" of monetary policy following Borio and Zhu (2012) and can be considered a part of the credit channel (Diamond and Rajan 2006), and Stiglitz and Greenwald (2003). Borio and Zhu (2012) advocate that the policy rate may affect the risk tolerance of banks due to increased wealth or the presence of "sticky" targets for rates of return. The latter transmission mechanism is quite self-explanatory. Banks targeting rigid rates of returns would reach out to riskier borrowers to recoup their drop in profits at times of monetary expansion. The former argument rests upon the conjecture that, in general, the risk tolerance of any economic agent increases with wealth. Such an effect can be found, for instance, in the mean-variance portfolio framework, where investors become less risk-averse during economic expansions because their consumption increases relative to its normal level (Campbell and Cochrane 1999). If risk aversion decreases with wealth, lower interest rates may in turn induce more risk-taking among banks by augmenting asset and collateral values.

Furthermore, lower interest rates may reduce the threat of deposit withdrawals (Diamond and Rajan 2006), reduce adverse selection problems in credit markets (Dell'Ariccia and Marquez 2006), improve bank net worth (Stiglitz and Greenwald 2003), or lead to a search for yield (Rajan 2006), allowing banks to relax their credit standards. This softening happens not only for riskier loans, which have an adjusted net present value (NPV) close to zero, but also for average loans. On the other hand, higher interest rates increase the opportunity cost of holding cash for banks, thus making risky alternatives more attractive (Smith 2002). Higher interest rates could also reduce bank net worth down to a point where a "gambling for resurrection" strategy becomes attractive (Kane 1989), and Hellman and Stiglitz (2000). Given the conflicting theoretical implications, the impact of short-term interest rates on risk-taking is ultimately a critical empirical question.

Theoretical advancements in the field of monetary policy and bank risk interaction, together with recent economic developments, have invigorated the related empirical work. Altunbas, Gambacorta, and Marques-Ibanez (2009) re-examine the monetary policy transmission mechanism in the euro area and, contrary to previous studies, accounts for the role of bank risk. However, Altunbas, Gambacorta, and Marques-Ibanez (2009) concentrate on the influence of bank risk on the credit supply and not risk tolerance as such. In contrast, Altunbas, Gambacorta, and Marques-Ibanez (2014) examine banks' risk responses to changes in the monetary policy indicator. The study concludes that low interest rates increase bank risk, but employs solely bank-level and macroeconomic data. The renewed interest has also fuelled research of bank lending standards. Lown and Morgan (2006) estimate a VAR model for credit standards, lending volumes and output fluctuations in order to examine the role of lending frictions on the two latter quantities. The authors find that fluctuations in commercial credit standards significantly explain changes in bank loan supply and real GDP. Maddaloni and Scope (2009), on the other hand, assess the impact of monetary policy on bank lending standards and establish that lower interest rates lead to softening of bank credit standards.

To the best of our knowledge, the first empirical investigations of the impact of monetary policy on bank risk-taking behaviour were by Ioannidou and Peydró-Alcalde (2007) and Jiménez and Saurina (2008). The latter tests the effect of interest rates on banks' appetite for credit risk on Spanish data, while the former explores this question using the credit register from Bolivia. Both papers find that in the short run a lower short-term interest rate augments banks' appetite for risk, while the medium-term effect is a decrease in credit risk for existing bank portfolios. In the longer term, both effects yield a net increase in the risk incurred. The analysis of Bolivian banks' appetite for risk is further advanced in Ioannidou and Peydró-Alcalde (2009), where the authors additionally explore the pricing of credit risk. We draw upon the methodology of Jiménez and Saurina (2008) and answer many of their questions in the Czech context. The Czech banking sector has undergone tremendous changes with respect to regulatory policy and banks' attitude towards corporate lending and credit risk assessment. The Czech Republic is an example of an economy that has paved a way from central planning to a small open economy with a banking sector dominated by foreign ownership. Meanwhile, and in addition to the transition experience, EU accession and Basel II implementation have taken place. Clearly, the Czech banking sector is an appealing one to investigate.

Estimating the impact of short-term interest rates on banks' attitude to liquidity and credit risk should enhance the understanding of the link between monetary policy and financial stability in the Czech Republic. This link has been explored using macroeconomic modelling, VAR methodology and bank-by-bank stress testing (e.g. Babouček and Jančar (2005), Čihák and Heřmánek (2005), and Jakubík and Schmieder (2008)) as well as validation of credit risk (rating) models on a simulated corporate loan portfolio of the Czech banking sector Kadlčáková and Keplinger (2004). However, our study is the first to apply panel data analysis on macroeconomic, bank, loan and borrower data to study the Czech monetary conditions and financial stability relation from the perspective of banks' attitude to risk and its sensitivity to the short-term interest rate. In contrast to other studies, which investigate the link between asset quality and macroeconomic indicators for a panel of countries (e.g. Nkusu (2011), or Glen and Mondragón-Vélez (2011)) we employ a unique microlevel dataset obtained from the Czech Credit Registry. Moreover, most studies focus on the advanced economies, while we explore these linkages for a transition economy.

This chapter is organized as follows. The following section outlines the methodology and model specification, Section 3 describes the dataset, while Section 4 presents the estimation results and provides robustness checks. Section 5 summarizes and concludes.

2.2 Methodology and Model Specification

This study poses two main and distinct research questions that relate monetary policy stance and bank risk-taking. First, we examine whether lower interest rates promote more lending to borrowers with a riskier past ($H1.1$). Such an effect is likely to be attributed to higher current net worth of borrowers. Next, we investigate whether lower interest rates encourage banks to incur more risk by accepting borrowers with a higher probability of default ($H1.2$). Default is defined as failure to pay a loan instalment and/or interest 90 or more days past the due date. Risky past stands for other overdue loans prior to the origination of a new loan. In addition to these main questions, we test whether all types of banks are equally affected by the monetary policy stance. In this vein, we also study the impact of the interest rate conditioned on bank liquidity ($H2.1$), capital ($H2.2$) and lending strategy diversification ($H2.3$).

Most studies exploring the theoretical mechanisms that could be directly or indirectly linked to the "risk-taking channel" suggest that banks should be more reluctant to grant risky loans at times of monetary contraction. Thus, we state hypotheses $H1.1$ and $H1.2$ in the spirit of opposite movements: lower interest rates imply more credit risk-taking. Naturally, in the econometric analysis we expect a negative sign on the estimated coefficient on the interest rate prior to loan origination. This negative relation can be attributed to weaker incentives to screen borrowers when interest rates that determine banks' financing costs are low (Dell'Ariccia and Marquez 2006). Lower interest rates decrease financing costs, thus banks' motivation to screen borrowers declines, which in turn may result in them accepting riskier applicants. Another reason could be a reduced threat of deposit withdrawals at times of excess liquidity, as in Diamond and Rajan (2006). Lower interest rates generate more liquidity in the banking sector, which provides less of an incentive for depositors to withdraw and more of an incentive for banks to finance risky projects.

It is reasonable to assume that a bank's risk tolerance might vary with its economic profile. Typically, the theoretical banking literature links a bank's riskiness with its level of capital and, as in Keeley (1990), predicts a negative relation between the two. Note, however, that the theory concentrates on bank capital and default risk, not risk tolerance. Moreover, in a banking sector shared between few banks, a highly capitalized bank might easily become "too big to fail". Due to this moral hazard problem, banks rich in capital may engage in riskier lending at times of monetary expansion. On the other hand, the Czech banking sector is not only concentrated, but also dominated by foreign capital, and foreign capital usually induces more monitoring effort. In short, the effect of bank capital is not easily foreseeable and we expect any outcome, albeit an insignificant one ($H2.1$). Bank liquidity is another characteristic likely to differentiate a bank's attitude to risk in low and high interest rate regimes. Diamond and Rajan (2006) develop a model of the "liquidity channel", as a modification of the "lending channel", and obtain that banks accumulating liquid assets tend to grant less risky loans. In our hypothesis $H2.2$ we test their implications. Finally, economic theory provides us with contradicting suggestions about the optimal strategy and, thus, loan portfolio composition. The literature on intermediation following Diamond (1984) promotes diversification as a way of minimizing the risk of failure. In doing so, such authors use the argument of uncorrelated returns in line with Markowitz's (1952) portfolio theory. On the other hand, the corporate finance literature argues that specializing may lead to improvement in a bank's monitoring effectiveness and incentives, and thus is likely to reduce credit risk Stomper (2006). Nevertheless, we formulate hypothesis $H2.3$ based on studies on financial intermediation, and expect less risk-taking among more diversified banks. Therefore, our main research hypotheses can be summarized as follows:

H 1 The monetary policy stance affects credit risk, in particular:

H 1.1 Lower interest rates lead to more lending to borrowers with a riskier past.

H 1.2 Lower interest rates encourage banks to incur more risk by accepting not only borrowers who are riskier ex ante, but also those with a higher probability of default per time period.

H 2 Not all types of banks are equally affected by the monetary policy stance; in particular:

H 2.1 Banks with a poorer liquidity profile tend to take more risk in lower-interest-rate periods.

H 2.2 Banks' capital significantly influences and differentiates their risk-taking behaviour in response to monetary and macroeconomic changes.

H 2.3 A lending strategy based on diversification, *ceteris paribus*, limits banks' risk appetite.

This study considers two different measures of credit risk-taking. First, we estimate the likelihood that a borrower with observable past non-performance obtains a new loan. We treat all firms with overdue loans six months prior to new loan origination as borrowers with a bad credit history and, thus, ex-ante riskier. The dependant variable in our probit model[1], *Bad history*, equals one for the ex-ante riskier borrowers. We explain the probability that a borrower with a "bad history" receives a loan, conditioning on selected bank, loan, firm and macroeconomic variables. Among those explanatory variables, the interest rate prior to loan origination is of primary interest to us. Consequently, within the probit framework we explore whether lower interest rates lead to more lending to borrowers with a riskier past ($H1.1$) and estimate the following model:

$$P(Bad\ history = 1|X) = \Phi(X\beta + e) \tag{2.1}$$

where:
$Bad\ history = 1$ if a borrower had overdue loans 6 months prior to new loan initiation
$\Phi(\ )$ – the standard normal cumulative distribution function
X – a set of macroeconomic, bank, borrower and loan-related regressors

The other measure of credit risk-taking employed in this paper is the time-specific likelihood of loan default. Default is defined as failure to pay a loan instalment and/or interest 90 or more days past the due date. By time-specific likelihood we mean the probability that loan default occurs within a specific time-span. Such a treatment emphasizes that there is a dynamic element to loan performance and that defaults differ at different points of the loan "life". After all, the loan survival time, i.e. the time for which the borrower has managed to pay regularly, affects the risk of default in the following period. By incorporating duration dependence we do not ignore the data on regular loans that eventually become nonperforming. On the contrary, all the available information helps us to determine the credit default risk at each point in the loan "life" (see Kiefer (1988)). Our methodology follows Shumway (2001), Chava and Jarrow (2004) and Duffie and Wang (2007), who strongly advocate the importance of duration in bankruptcy pre-

[1]A situation of a binary choice – a borrower with or without a bad history – calls for a discrete choice model such as probit.

dictions. Moreover, including duration dependence enables us to differentiate between the effects of monetary policy on new and outstanding loans. Finally, Matsuyama (2007) and Dell'Ariccia and Marquez (2006) show that monetary policy influences risk-taking and also lending standards and, thus, maturity. Ideally, to disentangle credit risk from liquidity risk, or the maturity effect, one should employ a measure of default probability normalized per period of time. The duration model offers such a dynamic measure of risk, namely the hazard rate. The same treatment of time-specific credit risk-taking is employed in Jiménez and Saurina (2007) and Ioannidou and Peydró-Alcalde (2009), making the results of all three studies comparable.

The hazard function is the limiting probability of default in a given interval conditional on the loan having survived until this period, divided by the width of the period. Duration, i.e. the length of time a loan is performing, is also referred to as spell length (t). In general, the hazard function depends on the survival probability and the density function associated with the distribution of the spells, $f(t)$. When estimating hazard functions, it is convenient to assume a proportional hazard specification with the baseline hazard $\lambda_0(t)$ a function of t alone. This paper follows the Cox semi-parametric approach, which specifies no shape for the baseline hazard function Cox (1972). Therefore, we model the time to loan default, T, using a set of macroeconomic, bank, borrower and loan-related regressors (X) within the following framework:

$$\lambda(t) = \lambda_0(t) exp\left(f\left(X, X(\tau); \beta, \beta^{\tau}\right)\right) \tag{2.2}$$

where:
X – characteristics constant over time
$X(\tau)$ – time-varying covariates
β and β^{τ} – parameters (including time-varying variables)
T – duration of a spell
t – loan spell
τ – calendar time

The regressors are described in the data section. As we use flow sampling and consider only new loans, our data does not suffer from left censoring. The right censoring problem is alleviated in a standard way, that is by expressing the log-likelihood function as a weighted average of the sample density of completed duration spells and the survivor function of uncompleted spells. We estimate four duration models and contrast their outcomes. The survival models differ in line with the shifting focus of our analysis. Each formulation contains the core covariates, namely a set of macroeconomic variables to control for major economic developments in the Czech Republic. First of all, we explore how risk-taking varies with bank characteristics. The role of banks' balance sheets (Matsuyama 2007) and moral hazard problems (Rajan 2006) in determining the sensitivity of bank risk-taking to monetary policy is well-established in the theory. Initially, we account for banks' heterogeneity[2] by applying shared frailty duration analysis ($Model\ I$). The shared frailty effect is estimated along with the other model parameters, and the random effects are common among groups of loan spells of the same bank. A comprehensive introduction to frailty and shared frailty duration analysis is provided in Gutierrez (2002). In the next formulation ($Model\ II$), we incorporate bank characteristics and thus capture the variety across banks in their risk-taking reactions to changing monetary conditions. Naturally, banks tend to differ in their lending strategies and thus their loan portfolio diversification may impact on their risk behaviour in different interest rate regimes. Therefore, the specification for $Model\ IV$ incorporates additionally the Hirschman-Herfindahl Index (hereof: HHI) as a measure of bank loan portfolio diversification.

By introducing firm and loan characteristics in $Model\ III$ we control for changes in the loan and borrower pools throughout the time span of our study. More importantly, we hope to separate credit supply and demand effects. As we examine bank risk-taking, we need to identify whether the observed increases in riskier loans are supply-driven. On the other hand, bad borrowers seeking more credit when rates are low could also cause higher loan hazard rates. The difference is that with a demand-driven increase in hazardous loans the risk premiums should also rise, while the supply effect should cause a drop in the risk premiums. Ideally, we would test how risk "pricing" reacts to changes in monetary conditions in the Czech banking sector and identify either the supply or demand effect. However, that requires data on loan pricing, specifically each loan contract interest rate, and the Central Credit Register maintained by the Czech National

[2]Generally, when controlling for unobserved heterogeneity we follow the flexible approach of Heckmann and Singer (1984).

Bank does not record such data. The second-best empirical strategy is to control for the quality of borrowers throughout the time span and for those loan characteristics which are regarded by financial intermediation theory as screening devices. The role of loan size and collateral as intermediary screening devices is grounded in the theory. Loan maturity also plays some role in disentangling supply and demand effects, as banks taking more risk will not mind engaging in long-term financing. This is no longer true for a demand-driven rise in loan riskiness.

Finally, we note that this study examines two distinct research questions relating bank risk-taking to the monetary policy stance, uses two different measures of risk-taking (the likelihood of financing an ex-ante riskier borrower and the time-specific loan default risk) and subsequently estimates two different models – a probit model and a duration model. Obviously, the outcomes of the two examinations are not comparable. Still, one would expect to see low interest rates promoting either more risk-taking in both cases or less risk-taking in both cases. However, this is not what our results for the Czech banking sector suggest. We come back to this issue when discussing the outcome of our estimations.

2.3 Data Description

2.3.1 Data Sources

The dataset used in this study contains 207 356 loan-period observations (N; loan spells in the duration analysis). The data on loans is combined with information from bank financial reports and, where available, from the financial statements of borrowers. We consider solely corporate loans for non-financial firms. In addition, we complete the dataset with macroeconomic variables describing the performance of the Czech and euro area economies. Prior to any analyses our dataset was anonymized.

The loan data comes from the Czech National Bank's Central Credit Register (CCR). Out of all the borrowers issued with loans between October 2002 and January 2010 we select a random sample amounting to 3% of all companies granted new loans in this period.[3] The CCR was launched in October 2002, so this is the first available month for

[3] We consider solely loans and overdrafts granted by the bank, and exclude unauthorized debits and

the loan data. The information on borrowers is obtained from two sources: the CCR and the Magnus database maintained by CEKIA. The time span for the firms' financials is also limited by data availability and covers the period from January 2000 to December 2009. We discuss the two data sources in greater detail below. The bank covariates originate from the Czech National Bank's (CNB) internal database. Clearly, the scope of the central bank's knowledge about the economic situation of each "supervised" bank is quite broad. In our analysis we limit ourselves to the key bank performance variables and the bank ownership type, foreign or local. Finally, the macroeconomic variables are collected from the Statistical Data Warehouse of the European Central Bank (SDW), the Czech Statistical Office (CZSO) and the CNB's public database ARAD. ARAD contains time series of monetary indicators, aggregated financial markets data, balance of payments statistics and fiscal statistics. ARAD data is processed directly by the CNB, but also comes from external sources such as the CZSO. The macrofinancial variables include overnight money market rates (CZEONIA and EONIA), GDP growth rates and consumer price indices (CPI) for the Czech and euro area economies as well as the exchange rate between the Czech koruna and the euro.

The Central Credit Register of the Czech National Bank contains monthly information on clients' loans, overdrafts, current account debit balances, guarantees, undrawn lending arrangements and standby credits. Our study focuses solely on the first three categories. The CCR data includes the loan identification number, NACE code[4], type, purpose, currency and classification. In accordance with CNB amending Regulation No. 193/1998, Czech banks classify loans according to a five-tier scheme and assign each loan a "standard", "watch", "substandard", "doubtful" or "loss" grade. In the case of nonperforming loans, the dataset provides information on the loan's principal, interest, fees and days overdue. Moreover, the CCR records the loan amounts granted and remaining as well as the dates of loan origination, maturity and, if applicable, write-off.

The firm-related covariates are obtained from two sources: the CCR and the Magnus database maintained by CEKIA. The Magnus data is mostly available at a yearly frequency. CEKIA supplies business information about Czech companies and their financial

loans bought from other banks.

[4]NACE is the European industry standard classification system (Statistical Classification of Economic Activities in the European Community).

statements, namely balance sheets and profit and loss accounts. The corporate characteristics cover the firm's identification number, NACE code[5], legal form, ownership type, amount of registered capital, number of employees, turnover and state of operation. The Magnus dataset also carries information on the dates when the company was launched and, where applicable, ceased to operate. Additionally, it contains the firm's position among the top 100 Czech companies and its rating, if provided by the Czech Rating Agency. The accounting variables are numerous and include, among others, the value of assets (total, fixed, current and other), equity, liabilities (total, other), sales, costs, operating income and net and pre-tax profits.

2.3.2 Data Description and Construction of Variables

In the paper, we use several money market rates to represent the monetary conditions in which Czech banks operate. Given that in the Czech Republic most traditional banking business is done in local currency (Czech koruna), the koruna money market rates (such as the PRIBOR reference rates or the overnight CZEONIA index) are the relevant variables to which banks react. The central bank of the Czech Republic, the Czech National Bank (CNB), pursues an independent monetary policy within its inflation targeting regime and a floating exchange rate.

The Czech banking market is not euroized – the share of foreign currency loans in total loans to households is virtually zero. This contrasts with the situation in many other Central and Eastern European countries, where FX loans to households are much more common. The main reason for the total dominance of local currency loans is the very low and sometimes even negative spread between koruna and FX interest rates, so that households have not had any incentive to demand FX loans in order to benefit from better interest rate conditions. In the non-financial corporations segment, FX loans exist, but only on a relatively minor scale (roughly 20% of loans to non-financial corporations are denominated in foreign currencies, mainly euro). This instrument is used mainly by export-oriented companies and commercial real estate developers for hedging purposes, as these two types of corporations have large revenues in euro.

[5]The same classification system as in the case of loans (the European industry standard classification system), although this time the code applies to the company's industry.

Nevertheless, given the deep economic integration of the Czech Republic into the rest of the EU via foreign trade, the Czech business cycle is to a large extent synchronized with that of the Eurozone and especially Germany. Therefore, Czech monetary policy rates – and thus also money market rates, which follow monetary policy rates quite closely – co-move with ECB monetary policy rates. The relationship works via two channels – directly, i.e. via the exchange rate transmission channel (a decrease in ECB rates and thus euro area money market rates leads to appreciation of the Czech koruna vis-a-vis the euro, contributing to lower inflation pressures and thus lower CNB rates), and indirectly, via common movement of the Eurozone and Czech economies in the cycle.

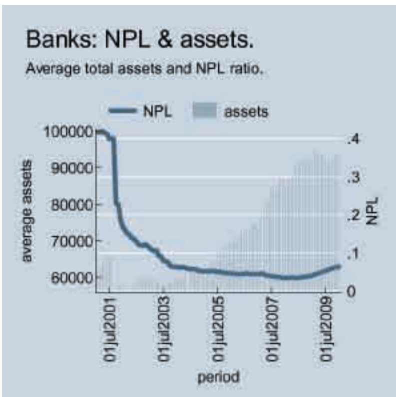
A natural candidate for capturing the monetary conditions in the Czech Republic is CZEONIA. CZEONIA is a weighted average of O/N rates on trades executed in a given day and, as such, it reflects real trading in the money market among Czech banks. Moreover, the O/N segment is the most liquid part of money market trading (CNB, 2010). We could also employ the PRIBOR rate. However, PRIBOR rates are solely reference rates and do not reflect real trading. In order to properly capture the effect of the monetary conditions on credit risk both on the date of loan origination and during the life of individual loans, we have to control for potential reverse causality and endogeneity of the monetary conditions represented by CZEONIA. CZEONIA, mirroring the official $2W$ repo rate of the CNB, may itself strongly depend on the level of credit risk in the banking system, as the central bank would react to worsening economic conditions and an increase in bad loans in banks' portfolios by decreasing the official CNB repo rates. Furthermore, if we happen to ignore controls correlated with both the Czech monetary stance and Czech banks' risk-taking, our analysis would suffer from omitted variable inconsistency. Thus, we use EONIA as an instrument, or alternatively a proxy, for CZEONIA. The tests applied confirmed that EONIA is a valid instrument for CZEONIA, reflecting strong correlation between these two rates as discussed above. Therefore, throughout our analysis we rely upon the monthly average of euro area money market overnight rates to capture the existing monetary policy conditions in the Czech Republic.

Apart from interest rates, each duration or probit model contains a set of macroeconomic variables to control for major economic developments in the Czech Republic.

The set includes Czech inflation[6] (CPI_t) and the spread[7] between Czech and European Monetary Union 10-year maturity government bond yields (*Country risk*$_t$) dated at loan origination. We also add a time trend and time trend squared, which are functions of calendar time. In the duration models we also incorporate two GDP growth rates, one dated prior to loan origination and the other prior to loan default or maturity. The probit analysis, which lacks the dynamic loan-life perspective, contains solely the GDP growth rate prior to loan origination. GDP growth is the seasonally adjusted quarterly rate of change of gross domestic product in the Czech Republic.

Banks tend to differ in their lending strategies and thus also in their credit risk behaviour. In order to account for differences in credit risk profiles across banks, and for the reasons discussed in the methodology section, we introduce bank characteristics stemming from the CCR as well as the banks' financial statements reported to the CNB. We include bank size, bank type and risk appetite as well as the liquidity and own funds to total assets ratios. Typically, bank size is given as the logarithm of total assets. Bank type is a dummy variable equal to one if the loan is granted by a foreign-owned bank. *Liquidity ratio*$_{t-1}$ and *Own funds/total assets*$_{t-1}$ are, respectively, the bank's liquid assets over its total assets and its equity over total assets. The difference between the bank's and other banks' non-performing loan ratios, *Bank NPL*$_b$ - *NPL*$_{t-1}$, depicted in Figure 2.1, measures the credit risk already on the books.

Figure 2.1: The Average NPL in the Czech Banking Sector



The methodology section contains a discussion of the identification challenges faced in our econometric investigation. It points out that the second-best empirical strategy for

[6]Inflation is measured by monthly consumer price indices (CPI).

[7]Monthly averages.

the troublesome separation of the loan supply and demand sides is to control for changes in the quality of borrowers and loan characteristics. As the borrower-related controls we employ the firm's turnover and employment categories as well as the firm's regional and industry dummies. In addition, we construct measures of the firm's age and its number of bank relations. The turnover and number of employees categories are obtained based on the classes recorded in the CCR. The regional and industry dummies are also derived from CCR data. Following Jiménez and Saurina (2007) we proxy the firm's age by its age as a borrower, that is the time since the origination of the first loan taken by the firm. *Bank relations*$_{t-1}$ is the logarithm of the number of bank relationships of the borrower plus one measured prior to loan origination. By the same token, *Bank debt*$_{t-1}$ is the logarithm of the borrower's total amount of bank debt augmented by one. We account for the changing pool of loans by controlling for their size, purpose, maturity and currency and the way they are collateralized. The methodology section outlines the rationale for the inclusion of loan size, collateral and maturity. What is left to describe is the construction of the variables. As typically done in the literature, we calculate the loan size as the logarithm of the amount granted. The effect of loan maturity is captured by three dummy variables accounting for terms of up to three, six and twelve months. Dummy variables are also employed to allow for difference in the riskiness of loans with collateral and granted[8] in euros, dollars or pounds as opposed to other currencies. The CCR dataset contains ten possible variables accounting for the type of collateral and fifteen possible types. We coarsely classify each type based on its riskiness and pool those with a similar likelihood of default. As a result we obtain ten collateral dummy variables displayed together with their statistical characteristics in Table 2.1.

Table 2.1: Collateral Type: Data Descriptive Statistics

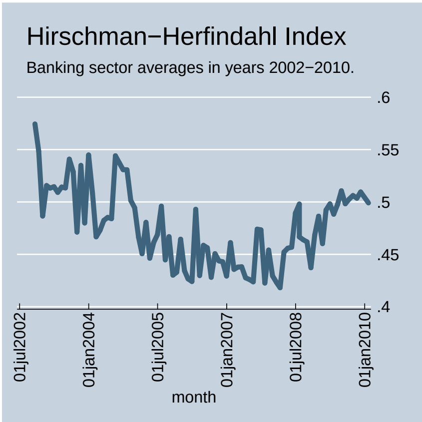
Variable	Unit	Mean	Std Dev	Max	Min
No collateral	0|1	0.34	0.47	1.00	0.00
Pledge on own real estate	0|1	0.15	0.36	1.00	0.00
Pledge on third party's real estate	0|1	0.02	0.14	1.00	0.00
Pledge on movable property without transfer	0|1	0.02	0.15	1.00	0.00
Ensuring note	0|1	0.25	0.43	1.00	0.00
Guarantee deposit	0|1	0.02	0.14	1.00	0.00
Guarantee	0|1	0.05	0.21	1.00	0.00
Pledged assets	0|1	0.07	0.26	1.00	0.00
Blockage of premium	0|1	0.00	0.07	1.00	0.00
Other collateral	0|1	0.05	0.22	1.00	0.00

While investigating banks' risk-taking behaviour in the Czech banking system, we

[8] Loan currency$_t$ = 1 if the loan is granted in euros, dollars or pounds.

also examine whether or not it depends on the type of bank lending strategy – focused or diversified. We measure the banks' loan portfolio diversification using the Hirschman-Herfindahl Index. The Hirschman-Herfindahl Index (HHI) is a commonly accepted measure of concentration, which we employ to measure each bank's relative credit exposure to a particular industry prior to new loan origination. The index is the sum of the squares of banks' relative credit exposures to each industry. Figure 2.2 depicts the evolution of the Hirschman-Herfindahl index for the Czech banking sector. On average, Czech banks moderately increased their loan portfolio diversification until mid-2008, when a slight decline can be observed.

Figure 2.2: The Average Herfindahl-Hirschman Index in the Czech Banking Sector



2.4 Estimation Results

2.4.1 Ex-ante Riskier Borrowers

In this section we explore Czech banks' appetite for ex-ante riskier borrowers at times of monetary easing. In particular, we examine whether lower interest rates promote more lending to corporate clients with overdue loans prior to new loan origination. This question is addressed by estimating the probability that a new loan is granted to a borrower with a recent bad credit history. Those recently "bad" borrowers, or – more accurately – borrowers with overdue loans six months prior to new loan origination, are considered to be "ex-ante riskier". We estimate a probit model using the bank, firm, loan and macroeconomic variables described in the data section, and primarily focus on the impact of the interest rate present in the money market one month prior to loan origination. The estimation results are given in Table 2.2.

Due to the presence of the endogeneity problem, we instrument the Czech money market interest rate (*CZEONIA*) by the EONIA rate reported by the ECB. The instrumental variable probit regression shows that expansive monetary policy encourages Czech banks to grant fewer loans to borrowers who exhibited a recent bad credit history prior to loan origination. This means that lower interest rates imply less credit risk incurred by Czech banks. Consequently, our data do not support hypothesis $H.1.1$ and contradict the findings of Ioannidou and Peydró-Alcalde (2007) and Jiménez and Saurina (2008). However, the probit results of our study and the other two are not completely comparable due to differences in defining the dependent variables. In Ioannidou and Peydró-Alcalde (2007), bad credit history refers to borrower past default and not to non-performance. Prudential regulations prevent Czech banks from financing previously defaulted firms. Jiménez and Saurina (2008) classifies a borrower as ex-ante riskier when it is overdue on another loan, as in our study, but contrary to us checks any time before the new loan is granted. As the CCR was launched in 2002 and our analysis spans to the year 2010, we consider solely the six-month period preceding new loan origination.[9] The other coefficients are mostly as expected. Larger banks, *ceteris paribus*, are less prone to lend to firms with a recent bad credit history (*-0.025****). By the same token, banks holding more liquid assets are likely to accept fewer risky borrowers (*-1.910****). Moreover, banks with

[9]We also experiment with one year prior to new loan origination and obtain the same positive depen-

higher than average non-performing loan ratios are less inclined to tolerate additional risk and finance companies with overdue loans in the previous six months (*-0.721****). Surprisingly, the estimation output suggests that less leveraged banks are likely to grant loans to borrowers with a risky past (*0.190***), while more indebted borrowers are less likely to have a recent bad credit history (*-0.016****). Table 2.3 presents the riskiness of industries obtained within the instrumental probit framework. We note that lower interest rates imply, *ceteris paribus*, a lower likelihood of default on loans granted to manufacturers (*0.120****), and higher defaults on loans provided to construction companies (*-1.185****). Finally, we observe recent default or bad history less frequently in the case of younger firms (*0.166****) with fewer bank relationships (*0.757****).

Table 2.2: Estimation Results: Instrumental Probit

Variable	Coefficient	Std. Err.
Interest rate$_{t-1}$	0.152***	0.012
Bank size$_{t-1}$	-0.025***	0.005
Liquidity ratio$_{t-1}$	-1.910***	0.036
Bank NPL$_b$ - NPL$_{t-1}$	-0.721***	0.070
Own funds/total assets$_{t-1}$	0.190**	0.083
Bank type$_{t-1}$	0.139***	0.015
ln(2+ age as borrower)$_{t-1}$	0.166***	0.004
Bank relations$_{t-1}$	0.757***	0.015
Bank debt$_{t-1}$	-0.016***	0.001
Loan size$_t$	0.020***	0.002
Loan currency$_t$	0.235***	0.015
Maturity 0–3 months$_t$	0.345***	0.017
Maturity 3–6 months$_t$	0.251***	0.018
Maturity 6–12 months$_t$	0.252***	0.012
Loan purpose$_t$	-0.085***	0.008
GDPCR$_{t-1}$	-0.018***	0.002
CPI$_t$	-0.009**	0.004
Country risk$_t$	0.048***	0.015
Time trend	0.015***	0.001
Time trend sq.	0.000***	0.000
Intercept	-1.632***	0.072
Collateral dummies	yes	
Firm turnover categories	yes	
Firm employment categories	yes	
Firm regional dummies	yes	
Firm industry dummies	yes	
N	205,270	
$\chi^2_{(49)}$	21,841.344	
Wald $\chi^2_{(1)}$	410.84	

The endogeneity problem is detected both by the Wald statistic, reported in Table 2.2, and the tests robust to weak instruments. The test outcome obtained in the presence

dence.

of potentially weak instruments, an approach due to Finlay and Magnusson (2009), is provided in Table $B1.3$. We rely on IV probit estimates rather than on the coefficients of the regular probit regression, but the two approaches yield similar results with respect to the monetary policy impact. To be precise, we refer to the probit model as the one estimated on the loan-level clusters. We also perform probit analysis on clusters of borrowers. Since the outcome corrected for firm-level clustering remains almost unaltered, we refrain from reporting it. The probit estimates corrected for loan clustering and the corresponding robust standard errors are reported in Table $B1.4$ in Appendix $B.1$. One final remark concerning endogeneity is that its presence strengthens the main points and concerns underlying our previous discussion of the potential reverse causality issue.

Table 2.3: Estimation Results: Instrumental Probit by Industries

Variable	Coefficient	Std. Err.
Manufacturing	0.120***	0.014
Other	-0.091***	0.018
Repair & related	-0.586***	0.082
Electricity, gas & heat	0.079	0.057
Water distribution & related	0.137***	0.028
Construction	-1.185***	0.117
Motor vehicle trade	-0.022	0.018
Transport	0.038***	0.015
Accommodation	0.073***	0.025
Broadcasting	-0.166***	0.025
Information activities	0.115***	0.034
Financial intermediation	0.044	0.044
R&D, advertising & market research	-0.155***	0.030
Scientific & technical activities	-0.068***	0.019
Security & investigation	-0.578***	0.098
Education	-0.222***	0.042
Artistic & entertainment activities	-0.579***	0.074
Gambling	0.385***	0.042
Sport & recreation	-1.416***	0.172
N	204,757	
$\chi^2_{(65)}$	22,304.536	

We fit the probit model to assess the influence of the monetary policy stance on banks' willingness to accept ex-ante riskier borrowers. If Czech banks were more prone to grant loans to ex-ante riskier firms at times of monetary expansion, we could claim that banks believed economic fundamentals were strong enough to reduce the default probability. One reason for that could have been higher net worth of borrowers in periods of lower interest rates. However, our data do not confirm that. One possible explanation of the link between low interest rates and lower probability of granting loans to borrowers with a riskier past might be the specific time period for which the analysis is done, which was marked by several structural changes. As banks were privatized before 2002, the banking sector experienced no state interventions and was relatively competitive in the period 2002–2010. Nevertheless, the rises and falls of money market rates (mirroring the CNB repo rate) between 2002 and 2009 happened under different conditions. There were two pronounced sub-periods of monetary policy expansion (2002–2004 and 2007–2009) and one pronounced sub-period of monetary policy tightening (2005–2007). In the first expansionary period of 2002–2004, the major domestic banks had just been cleared of nonperforming assets dating from the 1990s, as a part of a balance sheet consolidation process before privatization, and started to refocus their business on household loans. In this sub-period, referred to in the literature as a "credit crunch" in the corporate segment Geršl and Hlaváček (2007), corporate loans were declining and banks were not keen on providing new loans to corporations with a bad credit history despite the monetary expansion, effectively decreasing their risk-taking. The second monetary expansion, in 2007–2009, was a reaction to the global economic crisis and the economic recession in the euro area, again a period when banks were not keen on financing risky borrowers. On the contrary, anecdotal evidence shows that in this period, banks decreased their risk-taking, got rid of risky borrowers and maintained their loan relationships with rather less risky ones. In the period of monetary tightening, 2005–2007, which was itself a reaction to accumulating inflation pressures due to the strong economic and credit boom in those years, the banks strengthened their risk-taking owing to both competitive pressures and overall optimism in the economy, relaxed their lending standards and fuelled the credit boom even further, despite increases in money market rates. These structural factors are likely to have produced the puzzling positive relation between interest rate levels and banks' appetite for risk.

We conducted several robustness checks on the probit estimations. We test our hypotheses on models developed according to the guidelines of Hosmer & Lemeshow (1999, pp. 158–180) and Hosmer & Lemeshow (2000, pp. 92–116) for the probit regressions. Both suggest an approach to building a model with covariates chosen optimally. Generally, our choice of covariates is grounded in economic reasoning, supported, to some extent, by the findings of the previous studies. When constructing the specifications for the robustness checks, we greatly emphasize another important variable selection criterion, namely statistical significance. We employ the fractional polynomials methodology as a tool to validate the significance of the variables. The methodology of fractional polynomials is presented in Appendix B.2. When necessary, we also use fractional polynomials to suggest transformations of the continuous variables. All the steps involved in building the statistically desirable probit models for our data are also discussed in Appendix B.2. There are cases where the methodology suggested the inclusion of additional predictors, some transformation of continuous covariates or different grouping of selected categorical variables. Therefore, Table B2.1 contains the definitions of the optimally chosen covariates which differ from those employed in the main part of our analysis. The reasoning provided above also applies to the survival analysis (see next section). The descriptive statistics of the alternative predictors are summarized in Tables B2.2–B2.4.

We begin with our first measure of risk-taking, namely the likelihood of financing an ex-ante riskier borrower. Similarly to our regular analysis, the estimates obtained for the robust probit suggest that a relaxed monetary policy encourages Czech banks to finance fewer borrowers with a recent bad credit history (*0.471****). Therefore, the model with optimally selected covariates also does not support hypothesis H.1.1, which says that lower interest rates lead to more lending to borrowers with a riskier past. As in the case of our main probit regressions, we reject the null hypothesis of no endogeneity and rely on IV probit estimates. Still, the two approaches produce comparable outcomes, which for the probit model with observations clustered on the loan level are displayed in Tables B2.5–B2.9 in Appendix B.2. Additionally, we perform the analysis on borrower clusters and obtain almost unaltered coefficients.

Following the optimal variable selection strategy for the probit regressions results in the inclusion of an additional bank characteristic ($Deposit\ ratio_{t-1}$), a different bank type measure ($Bank\ unit_{t-1}$) and an altered grouping of loan maturity, purpose and collateral. Bank unit is a dummy variable taking the value of one if the loan is granted by a branch in the Czech Republic (as opposed to a headquarters in the Czech Republic or a branch abroad). Additionally, $Bank\ debt_{t-1}$ and loan size are excluded from the alternative probit specification. Thus, we solely compare the other estimated parameters for bank and firm covariates. We observe a reverse sign of the bank capital measure. Contrary to our main analysis findings, here banks holding more own funds are likely to accept fewer risky borrowers (-*0.466****). The other coefficients in the robust and regular probit analysis are alike. Larger and more liquid banks are less prone to lend to firms with a recent bad credit history (-*0.028**** and -*1.123****). Moreover, banks with higher than average non-

Table 2.4: Estimation Results: Robust Instrumental Probit

Variable	Coefficient	Std. Err.
Interest rate$_{t-1}$	0.471***	0.020
Bank size$_{t-1}$	-0.028***	0.007
Liquidity ratio$_{t-1}$	-1.123***	0.058
Bank NPL$_b$ - NPL$_{t-1}$	-1.402***	0.173
Own funds/total assets$_{t-1}$	-0.466***	0.152
Deposit ratio$_{t-1}$	1.228***	0.069
Bank unit$_{t-1}$	-0.224***	0.014
ln(2+ age as borrower)$_{t-1}$	0.411***	0.006
Bank relations$_{t-1}$	1.412***	0.016
Loan currency$_t$	-0.159***	0.022
Maturity 2–3.5 years$_t$	-0.074***	0.015
Maturity 4–8 years$_t$	-0.311***	0.014
Maturity 5.5 years$_t$	0.342***	0.032
Maturity 8.5–10 years$_t$	-0.181***	0.026
GDPCR$_{t-1}$	0.021***	0.003
CPI$_t$	-0.138***	0.006
Country risk$_t$	0.195***	0.025
Time trend	0.015***	0.001
Time trend sq.	0.000***	0.000
Intercept	-4.067***	0.102
Loan collateral: 1^{st} - 3^{rd}	yes	
Loan purpose: [1]-[5]	yes	
Firm turnover categories	yes	
Firm employment categories	yes	
Firm regional dummies	yes	
Firm industry dummies	yes	
N	207,352	
$\chi^2_{(67)}$	24,849.675	
Wald $\chi^2_{(67)}$	286.23	

performing loan ratios are less likely to tolerate additional risk and finance companies that were late with loan payments in the previous six months (*-1.402****). Finally, we observe recent default or bad history less frequently in the case of younger firms (*0.411****) with fewer bank relationships (*1.412****).

2.4.2 Dynamic Riskiness of Loans

Duration models of loan default consider not only the default itself, but also its timing. As in the probit regression, we still account for observed and unobserved loan quality by origination date. However, in addition, survival analysis enables us to capture the changing conditions over the loan life. Thus, we may investigate bank risk-taking in a broader, dynamic, context. This richer approach also allows for a richer set of covariates. Duration analysis enables us to examine the impact of the monetary policy stance on the riskiness of new loans as well as its effect on the existing loan portfolio. Therefore, our hazard rate models comprise not only the interest rate measured prior to loan origination, but also the interest rate prior to loan default or maturity. The latter allows us to test how monetary policy affects the performance of loans already on the books. We also incorporate two GDP growth rates, one dated prior to loan origination and the other prior to loan default or maturity.

We fit four duration models and contrast their outcomes. The rationale for each specification is laid out in the empirical strategy section. The survival models differ in line with the shifting focus of our analysis. Nevertheless, each formulation contains the core covariates, namely a set of macroeconomic variables to control for major economic developments in the Czech Republic. The first two models, Model I and Model II, control for diverse lending strategies across banks. The former is the estimated shared frailty survival model, with frailties common to loans of the same bank. The latter analyses a duration model with bank characteristics incorporated in an explicit manner. Model III accounts for the changes over time in the pool of borrowers and loans, and includes the firm and loan covariates. Model IV further enriches our analysis with the loan portfolio concentration measure (HHI).

The coefficient on the short interest rate preceding loan origination is negative and significant in all the estimated formulations. The models with bank unobserved heterogeneity (Model I) and loan portfolio diversification (Model IV) yield coefficients significant solely at the 10 per cent level and equal respectively to *-0.214** and *-0.289**. The two other models render even more significant negative results. The estimated impact of the interest rate prior to loan origination in the model with bank characteristics (Model II) amounts to *-0.312*** and that in the model with bank, loan and borrower covariates equals *-0.298*** (Model III). Therefore, all cases indicate that at times of lower interest rates banks tend to grant loans with higher hazard rates. In other words, a more relaxed monetary conditions policy encourages banks to take on more credit risk. This finding gives support to hypothesis $H.1.2$ and corroborates the outcomes of Ioannidou and Peydró-Alcalde (2007) and Jiménez and Saurina (2008).

All four formulations produce highly significant and positive estimated coefficients on the interest rate prevailing during the loan life. The impact of the interest rate prior to loan maturity ranges from *0.278**** to *0.296**** for the model with bank characteristics (Model II). The lowest impact is obtained for the case of bank, loan and borrower characteristics (Model II). The formulation with bank unobserved heterogeneity yields an only slightly higher estimate (*0.279****, Model I). The outcome for the case with the incorporated measure of loan portfolio diversification is also not much different (*0.282****, Model IV). The positive dependence in all four cases implies that the higher the interest rate prior to loan maturity, the greater the probability of loan default per time period. This result is as expected and can be attributed to lower refinancing costs or a reduced loan repayment burden at times of low interest rates. Thus, relaxed monetary conditions give rise to fewer loan defaults or lower riskiness of the outstanding portfolio.

The results for the GDP growth rate offer limited scope for interpretation. Out of the two rates, solely the GDP growth rate during the loan life proves to be statistically significant. Moreover, it is significant only when borrower characteristics are accounted for. We obtain a significant and negative coefficient on the GDP growth rate during the loan life for the specification with bank, loan and borrower covariates without and with the measure of loan portfolio diversification – *-0.066*** and *-0.067*** respectively for Model III and Model IV. The direction of the effect of GDP on the riskiness of the outstanding portfolio is as expected. At times of higher economic growth, loan defaults are less frequent.

The parameters for inflation remain positive and highly significant for all four models (*0.215****, *0.210****, *0.194**** and *0.191****). They indicate that higher inflation at origination increases the loan hazard rate. Finally, the negative and highly significant estimated coefficients on the time trend indicate an overall decrease in new credit volume observed over (calendar) time in the Czech banking sector. Indeed, since 2002 Czech banks have substantially changed their lending strategies and credit risk assessment. This observed general improvement is revealed on top of the effects captured by bank characteristics and the change in the pool of loans and borrowers.

Next, we focus on the results for bank characteristics. The sole bank covariates that prove to be statistically significant in all three model specifications[10] are bank liquidity and type. We find that more liquid banks, *ceteris paribus*, are likely to grant loans with lower hazard rates. The estimated parameters amount to -*3.083****, -*3.437**** and -*3.758**** for the model with bank characteristics (Model II), the model with bank, loan and borrower covariates (Model III) and the model with loan portfolio diversification (Model IV) respectively. The inverse influence of the bank's liquidity on its loan hazard rate supports hypothesis H.2.1 and suggests that banks accumulating liquid assets tend to grant less risky loans, thus confirming one of the implications of Diamond and Rajan (2006). The bank size effect proves to be positive in the specification with bank covariates. Such an outcome indicates that larger banks are willing to accept more credit risk (*0.181***). One might argue that in a banking sector dominated by few banks, as in the Czech Republic, the positive bank size could be attributable to a "too big to fail" effect. In doing so, we would employ the same line of argument as Boyd and Runkle (1993) and Ioannidou and Peydró-Alcalde (2007), who obtained similarly puzzling estimates for their data. In our study, Model II is the only case where the size effect is significant. Finally, we obtain that foreign banks tend to extend more hazardous loans (*0.470**, *0.840**** and *0.831****). The impact of all other bank characteristics is statistically insignificant. Therefore, we find no support for hypothesis H.2.2, which relates bank leverage and bank credit risk appetite.

[10]In Model I differences between banks are captured by the "frailty effect". Given the standard error of θ and the likelihood-ratio test statistic ($\bar{\chi}^2_{(01)} = 47.25$), we find a significant frailty effect, meaning that the correlation across loans grouped by banks cannot be ignored.

In the Czech Republic, the association between high liquidity and low risk appetite (low hazard rates) may be explained by a preference of most large banks to attract and keep depositors. Domestic banks apply a very conservative banking model, hardly engage in risky investments and focus on collecting deposits and granting loans. Moreover, compared to their European counterparts, Czech banks are very prudent in their lending activities and prefer to maintain low credit risk profiles. At the same time, they prefer to hold large liquidity buffers, mainly for two reasons: first, when relying on a large pool of (mainly sight) deposits, the banks need liquid assets to be able to saturate potential demand for liquidity should deposit withdrawals increase in stress times; second, a large liquidity buffer is an important signal – together with low credit risk indicators – to existing and potential depositors. Moreover, given their conservative banking model and overhang of deposits, most domestic banks invest in Czech government bonds, which constitute an important part of their liquid assets CNB (2011).

Furthermore, we examine the estimated impact of loan and borrower covariates. Not surprisingly, we obtain that hazardous borrowers are more likely to default in the future (*1.129****). To measure the firm's riskiness we look for previous overdue loans in its recent credit history. As in the probit analysis, younger firms are safer. In other words, loans to younger firms tend to survive longer (*0.241****). Table 2.6 presents the hazard rates for firm industries. Interestingly, all significant industry effects are solely positive. Such results indicate that lower interest rates imply, *ceteris paribus*, a lower likelihood of default or no significant effect of loans granted to all but agricultural producers. By introducing credit size, purpose, currency and maturity we wish to control for modifications in the loan pool over the time span of our study. We find that modest-sized loans tend to be more risky (-*0.202****). The estimated effect of loan purpose, captured by a dummy for overdrafts, suggests that overdrafts and current account debits exhibit a lower hazard rate (-*0.553****). Additionally, loans granted in euros, dollars or pounds are more hazardous than the others, which are mostly granted in Czech korunas (*0.997****). The influence of each loan maturity dummy is highly significant and positive. All the same, the magnitude of the estimated maturity parameters decreases with the loan maturity and amounts to *1.889****, *1.132**** and *0.729**** respectively. In other words, the shorter the loan term, the greater the probability of default.

Adding the diversification measure of banks' corporate loan portfolio only slightly modifies the magnitude of the effect of bank, firm and loan covariates on the hazard rate. In all but one case we note no change of sign or significance level. The sole exception is the bank capital coefficient, which remains insignificant, but changes sign. Finally, we obtain that in the analysed period the type of lending strategy, diversified or focal, has no explanatory power for Czech banks' risk appetite. We present solely the results for the Herfindahl-Hirschman index. However, we find that using neither the Gini coefficient nor the Shannon entropy as a concentration measure speaks in support of hypothesis $H2.3$.

The results of our analysis offer important policy lessons for the macroprudential policy of central banks, which – ideally – need to take into account the consequences of the monetary policy stance regarding bank risk-taking. The estimated parameters for interest rates, both prior to loan origination and during the life of the loan, enable us to quantify the potential effect of different interest rate paths on credit risk. The coefficient on the interest rate prior to loan origination varies between -0.2 and -0.3, with a standard error of around 0.15. For macroprudential purposes, it is recommended to be rather conservative. Assuming the highest (in absolute terms) coefficient plus two standard deviations implies that an interest rate decline of one percentage point increases the hazard rate by 0.6 percentage points. Thus, a substantial easing of monetary policy which would bring interest rates down from 5% (as in 2001–2002) quickly to 2% (as in 2004) could increase the hazard rates by almost 2 percentage points. The increase in the hazard rate would, however, happen under two conditions: (a) a worsening of the economic environment, such as an economic decline and an increase in retail interest rates, which would make it more difficult for borrowers to repay loans, (b) a worsening of the economic environment happening after a time of, say, at least one or two years, in order to "enable" new borrowers who took out loans in the period of rapid monetary easing to default on their obligations. Assuming that the hazard rates were in line with the default rates, which remained between 2% and 3% in 2007, just the risk-taking behaviour could increase the default rates by some 2 percentage points[11] in addition to the effect of the economic decline and a possible increase in interest rates (i.e. debt servicing costs).

As to the robustness checks, we used the same approach as in the probit analysis.

[11] To be precise, the figure would be 1.8 percentage points given a 3 percentage point drop in interest rates and a conservative change in the hazard rate of 0.6 percentage points.

Table 2.5: Estimation Results: Duration Models

Variable	Model I	Model II	Model III	Model IV
Interest rate$_{t-1}$	-0.214*	-0.312**	-0.298**	-0.289*
	(0.129)	(0.131)	(0.149)	(0.151)
Interest rate$_{T-t-1}$	0.279***	0.296***	0.278***	0.282***
	(0.069)	(0.066)	(0.077)	(0.077)
GDPCR$_{t-1}$	-0.018	0.006	-0.018	-0.013
	(0.037)	(0.038)	(0.042)	(0.042)
GDPCR$_{T-t-1}$	-0.019	-0.024	-0.066**	-0.067**
	(0.028)	(0.027)	(0.032)	(0.032)
Bank size$_{t-1}$		0.181**	0.155	0.056
		(0.085)	(0.105)	(0.142)
Liquidity ratio$_{t-1}$		-3.083***	-3.437***	-3.758***
		(0.676)	(0.836)	(0.900)
Own funds/total assets$_{t-1}$		-1.942	0.037	-0.061
		(1.494)	(2.074)	(2.119)
Bank NPL$_b$ - NPL$_{t-1}$		0.048	-0.624	-1.616
		(0.429)	(2.100)	(2.786)
Bank type$_{t-1}$		0.470*	0.840***	0.831***
		(0.282)	(0.312)	(0.316)
ln(2+ age as borrower)$_{t-1}$			0.241***	0.249***
			(0.085)	(0.086)
Bad history$_{t-1}$			1.129***	1.129***
			(0.151)	(0.151)
Bank relations$_{t-1}$			-0.151	-0.170
			(0.227)	(0.228)
Loan size$_t$			-0.202***	-0.205***
			(0.021)	(0.021)
Loan currency$_t$			0.997***	0.989***
			(0.228)	(0.229)
Maturity 0–3 months$_t$			1.889***	1.915***
			(0.443)	(0.454)
Maturity 3–6 months$_t$			1.132***	1.134***
			(0.411)	(0.413)
Maturity 6–12 months$_t$			0.729***	0.710***
			(0.232)	(0.230)
Loan purpose$_t$			-0.553***	-0.557***
			(0.153)	(0.153)
HHI$_{t-1}$				0.085
				(0.076)
CPI$_t$	0.215***	0.210***	0.194***	0.191***
	(0.056)	(0.056)	(0.064)	(0.064)
Country risk$_t$	-0.410	-0.429	-0.563	-0.542
	(0.290)	(0.297)	(0.343)	(0.344)
Time trend	-0.097***	-0.098***	-0.095***	-0.095***
	(0.020)	(0.020)	(0.020)	(0.020)
Time trend sq.	0.001***	0.001***	0.001***	0.001***
	(0.000)	(0.000)	(0.000)	(0.000)
Collateral dummies	no	no	yes	yes
Firm regional/industry dummies	no	no	yes	yes
N	154,372	154,368	152,316	152,316
Log-likelihood	-2,092.978	-2,103.071	-1,564.971	-1,564.218
χ^2	108.63(8)	169.781(13)	563.224(40)	570.244(40)

We proceed with modelling the time to loan default, our other measure of risk-taking. We consider the statistically robust survival model with bank characteristics. The choice of bank-level controls used here is described in Appendix $B.2$. In the duration model we include bank size, risk appetite and profitability, bank unit, and the ratios of liquidity

Table 2.6: Estimation Results by Industries

Variable	Coefficient	Robust Std. Err.
Manufacturing	1.539**	0.719
Other	1.630**	0.735
Repair & related	-41.827	0.000
Electricity, gas & heat	-43.143	0.000
Water distribution & related	2.323***	0.779
Construction	-41.215	0.000
Motor vehicle trade	1.262*	0.758
Transport	1.551**	0.714
Accommodation	1.952**	0.780
Broadcasting	1.956**	0.775
Information activities	0.274	1.246
Financial intermediation	-42.773	0.000
R&D, advertising & market research	0.606	0.916
Scientific & technical activities	1.191	0.760
Security & investigation	2.404**	1.226
Education	-43.602	0.000
Artistic & entertainment activities	-42.248	0.000
Gambling	-42.981	0.000
Sport & recreation	-42.560	0.000
N	152,316	
Log-likelihood	-1,552.528	
$\chi^2_{(48)}$	594.003	

and own funds to total assets. Bank size and the liquidity and leverage ratios are defined as in the regular survival analysis. The bank unit is designed as in the robust probit model and equals one if the loan is granted by a bank branch in the Czech Republic. As a measure of the credit risk already on the books the fractional polynomials method suggested the inverse of the capital adequacy ratio (${CAR^{-1}}_{t-1}$). Finally, we add to the model bank profits scaled down by millions of Czech korunas. We also use another measure of bank profitability, namely the return on equity (ROE). Since the outcome with ROE instead of scaled profits leaves the main results almost unaltered, we refrain from reporting it here. The estimation output is displayed in Table 2.7.

Consistently with our core analysis, the coefficient on the interest rate prior to loan origination is negative and significant (*-0.463****). This negative relationship indicates that an expansionary monetary policy encourages more credit risk-taking among banks. Moreover, the alternative specification with bank characteristics produces a positive and highly significant coefficient on the interest rate during the loan life (*0.290****). This positive dependence indicates that a higher interest rate prior to loan maturity raises the probability of loan default per time period and confirms our previous results. Thus, once again we find evidence to support hypothesis $H.1.2$.

Table 2.7: Estimation Results: Robust Model with Bank Characteristics

Variable	Coefficient	Robust Std. Err.
Interest rate$_{t-1}$	-0.463***	0.141
Interest rate$_{T-t-1}$	0.290***	0.068
GDPCR$_{t-1}$	0.074*	0.043
GDPCR$_{T-t-1}$	-0.021	0.027
Bank size$_{t-1}$	0.347***	0.105
Liquidity ratio$_{t-1}$	-3.559***	0.913
Own funds/total assets$_{t-1}$	-11.994***	2.721
$\text{CAR}^{-1}{}_{t-1}$	-23.354***	5.471
Bank unit$_{t-1}$	-0.523***	0.162
Bank profit$_{t-1}$	-0.406***	0.076
CPI$_t$	0.247***	0.061
Country risk$_t$	-0.540*	0.307
Time trend	-0.092***	0.021
Time trend sq.	0.001***	0.000
N	136,680	
Log-likelihood	-1,974.24	
$\chi^2_{(14)}$	210.815	

As previously, higher inflation at origination tends to augment the loan hazard rate (*0.247****). In addition, the optimally derived bank-level specification confirms that banks with higher liquidity ratios are likely to grant loans with lower hazard rates (*-3.559****). Not surprisingly, emphasizing statistical significance in variable selection produces a model with numerous significant characteristics. Therefore, in contrast to our main model with bank characteristics, here the impact of all bank characteristics matters. More capitalized and profitable banks are likely to grant loans with lower hazard rates (*-0.406**** and *-11.994**** respectively). The negative coefficient on own funds to total assets corroborates the theoretical findings of Keeley (1990), where banks with more capital exhibit a lower default risk. The negative coefficient on the inverse of the capital adequacy ratio suggests that banks persist in their hazardous lending (*-23.354****).

Finally, we compare the two survival models with bank, loan and borrower characteristics. The estimation output for the survival model with robust borrower and loan covariates is provided in Table 2.8. Consistently with our core analysis, we observe that adding the firm and loan variables does not alter our key findings. A lower interest rate prior to loan origination increases the hazard rate of new loans (*-0.383***). Once again, we find evidence in support of hypothesis $H.1.2$, which relates increases in bank riskiness to expansionary monetary conditions. At the same time, a lower short rate during the loan life decreases non-payment of outstanding loans (*0.349****). Contrary to the main model, the robust specification also produces a significant and negative coefficient on the GDP growth rate during the loan life (-*0.080****). Therefore, the robust model suggests that more dynamic economic growth reduces the riskiness of the outstanding loan portfolio. The parameter for inflation remains positive and highly significant, which indicates that higher inflation at origination increases the loan hazard rate (*0.186****).

The bank, firm and loan covariates employed in the optimally built survival model are defined as in the corresponding main specification. However, the core survival analysis contains more bank, loan and borrower characteristics. Implementing the optimal variable selection strategy results in the exclusion of bank type, loan currency and the measure of bank relations maintained by the borrowers prior to new loan origination. In addition, the fractional polynomials method suggested capturing the credit risk already on the books by the capital adequacy ratio (CAR_{t-1}) instead of the non-performing loan ratio ($Bank\ NPL_b$ - NPL_{t-1}).

All characteristics included in both the main and robust model yield similar results. As in the core part of our survival study, more modest loans tend to be more risky (-*0.246****). In both regression outputs, overdrafts have a lower hazard rate (-*0.662**** in the robust model versus -*0.553**** in the main model). Moreover, we obtain the same effect of loan maturity as in the main model, namely each coefficient on the maturity dummy is highly significant and positive. In addition, we observe that the shorter the loan term, the greater the probability of default (*2.341****, *1.319**** and *0.979**** respectively).

Table 2.8: Estimation Results: Robust Model with Bank, Loan and Borrower Characteristics

Variable	**Coefficient**	**Robust Std. Err.**
Interest rate$_{t-1}$	-0.383**	0.152
Interest rate$_{T-t-1}$	0.349***	0.077
GDPCR$_{t-1}$	0.011	0.043
GDPCR$_{T-t-1}$	-0.080***	0.031
Bank size$_{t-1}$	0.193**	0.085
Liquidity ratio$_{t-1}$	-4.978***	0.770
Own funds/total assets$_{t-1}$	-4.202**	2.126
CAR$_{t-1}$	0.035***	0.008
ln(2+ age as borrower)$_{t-1}$	0.180**	0.070
Bad history$_{t-1}$	0.939***	0.195
Loan size$_t$	-0.246***	0.020
Maturity 0–3 months$_t$	2.341***	0.447
Maturity 3–6 months$_t$	1.319***	0.348
Maturity 6–12 months$_t$	0.979***	0.239
Loan purpose$_t$	-0.662***	0.158
CPI$_t$	0.186***	0.064
Country risk$_t$	-0.570	0.347
Time trend	-0.097***	0.020
Time trend sq.	0.001***	0.000
Firm regional dummies	yes	
Firm industry dummies	yes	
N	152,316	
Log-likelihood	-1,594.287	
$\chi^2_{(27)}$	556.858	

2.5 Conclusions

This paper contributes to the debate on the impact of monetary conditions on banks' appetite for risk by investigating the case of the Czech Republic. The mechanism of bank risk-taking coined by Borio and Zhu (2012) can be identified in studies on the credit channel, for instance Diamond and Rajan (2006) and Stiglitz and Greenwald (2003). Generally speaking, higher tolerance to risk implies that at times of low interest rates banks will seek to finance riskier borrowers. We focus on two aspects of the discussion, namely whether a monetary easing leads to more lending to borrowers with a riskier past and whether it encourages banks to extend new loans that default sooner. The two questions are vital both for macroprudential authorities and for academics due to their contradictory theoretical implications and their consequences for monetary policy design. We use Czech National Bank Credit Register data to model the probability of accepting borrowers with a bad credit history and the time to loan failure in association with a set of macroeconomic, firm, loan and bank characteristics. We ask two distinct research questions, employ two different measures of risk, and thus use two different econometric methodologies – a probit model and a duration model. Therefore, our results are not directly comparable.

The outcome of our probit analysis suggests that at times of monetary expansion Czech banks do not necessarily believe that the economic fundamentals are strong enough to reduce the default probability of borrowers with a recent bad credit history and are less likely to finance them. We provide a possible explanation for this – at first glance – puzzling result. The estimated influence of bank characteristics shows that larger and more liquid banks tend to extend fewer loans to firms with a recent bad credit history. Additionally, banks with a worse relative credit risk track record tend to finance fewer companies with a riskier past. Interestingly, we find that less leveraged banks are less likely to incur credit risk.

The result of our survival analysis indicates that relaxed monetary conditions promote risk-taking among banks. This outcome is confirmed irrespective of the way we address differences in bank profiles. Specifically, we obtain a positive association between low interest rates prior to loan origination and the loan hazard rate both when bank covariates are explicitly accounted for and when the effect of unobserved bank heterogeneity is estimated. Controlling for loan and borrower characteristics confirms that banks tend to extend loans with a higher hazard rate at times of monetary easing. We separate demand and supply for hazardous loans by using a set of borrower and loan characteristics, and proxy the Czech money market rate by the euro area overnight rate. Estimation of the hazard model, which in its essence works with the failure rate normalized per time period, enables us to disentangle credit risk from the effect of the overnight rate on loan maturity and, thus, liquidity risk. The survival model allows us to examine the effect of the monetary policy stance on the outstanding loan portfolio. Conditional on the loan being extended, a lower interest rate during the loan life reduces its hazard rate. This result can be attributed to lower refinancing costs or, simply, a reduced loan repayment burden.

At the same time, we find hardly any support for an impact of the real cycle in determining the risk of new loans and the outstanding portfolio. The specification with bank, borrower and loan covariates yields a negative impact of the GDP growth rate on existing loans. Other estimated effects of the real cycle turn out to be statistically insignificant in the survival data for Czech loans. The impact of monetary policy on risk-taking varies with bank characteristics. More liquid banks tend to grant loans with lower hazard rates. The negative association between loan riskiness and bank liquidity shows that banks accumulating liquid assets tend to grant less risky loans and confirms one of the implications of Diamond and Rajan (2006). Finally, we find that foreign-owned banks are willing to accept more credit risk than local banks or foreign branches.

Chapter 3

Evaluation of Swap Contracts Using Various Term Structure Models in the Polish Market

Evaluation of Swap Contracts Using Various Term Structure Models in the Polish Market *

Dorota Kowalczyk†

Abstract

A growing role of derivatives reinforces a need for their consistent valuation and thus for reliable term structure models. In this paper I analyze five term structure models in order to compare their ability to capture the interest rate dynamics and value the interest rate swaps in the Polish market. A choice of the plain interest rate derivative allows for a wider selection of interest rate models, which includes the frameworks of Nelson and Siegel (1987), Vasicek (1977), Cox, Ingersoll, and Ross (1985), Heath et al. (1992) and the cubic spline curves. The performance and predictive accuracy of the term structure models are assessed based on the realized contract values. The Nelson-Siegel, cubic interpolation and CIR models generate adequate fit and transaction values similar to the realized contract values. A special case of the Heath-Jarrow-Morton model with the volatility corroborated by the observable market data produces mostly unreliable fitted curves, while Vasicek's approach gives contract values statistically different from the actually swapped amounts. The ample performance of the Cox-Ingersoll-Ross model suggests the rate-reliant nature of the interest rate volatility. The underperformance of the Vasicek model emphasizes the role of the cross section of interest rates, and thus the importance of a no-arbitrage argument. Finally, the ex-post accuracy of the Nelson-Siegel and the cubic spline models indicates that a current cross-section of the yield curve is highly informative for the future.

JEL Classification: G13, C53, C52

Key Words: term structure, interest rate models, calibration, interest rate derivatives, swap pricing

*I wish to acknowledge and extend my special gratitude to Professor Petr Zemčík for his precious guidance. I would like to thank Professors Randy Filer, Štěpán Jurajda, Jan Večer and Sergey Slobodyan for their constructive comments. Finally, I thank Piotr Kowalczyk, AMIMA, for his assistance. All errors remaining in this text are the responsibility of the author.

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3.1 Introduction

The financial turmoil, which took its first toll in 2007, but seemed to unveil its truly disastrous nature only in 2008, has fueled a discussion about the complexity of contemporary financial instruments' valuation. Following the crisis, practitioners and regulators have raised concerns regarding opaque financial products and their pricing. This study poses a more basic question, namely it discusses the usefulness of advanced term structure models in the valuation of plain vanilla interest rate sensitive derivatives. As it investigates the value added of theoretically-grounded term structure models in valuation of real life swap contracts, this chapter also refers to model risk. Model risk "occurs when the investment strategy relies on valuation or risk models that are flawed" (Jorion (2009), p. 419). Although model risk is typically associated with complex derivatives, the choice of a plain interest rate derivative allows for examination of model risk in the case of a dynamically growing OTC market, such as the Polish one, and a wider selection of interest rate models.

However, the motivation of my investigation does not stem solely from the recent developments. The relationship of yields on risk-free assets to maturity and its determinants has already gained a great deal of attention in the economic literature over previous decades. Economists have attempted to derive, model and understand the term structure for reasons ranging from extracting expectations to valuing future contingent claims. With the growing role of derivatives a quest for pricing, hedging and, in general, managing risk associated with their portfolios has become one of the priorities in finance. Since valuing instruments contingent on future developments calls for deriving a yield curve, a need for a commonly accepted term structure model has followed. While for stock options Black and Scholes (1973) have successfully established the major model, interest contingent claims and term structure modeling continue to be addressed by a vast number of methods used among scholars and practitioners.

The lack of a universally employed term structure model could be attributed to the complex stochastic behavior exhibited by interest rates and their non-tradability. Unquestionably, the efforts to explain the behavior of the yield curve, which depicts the relation between maturities and interest rates, have been intensified since the introduction of option trading on bonds and interest rate swaps (IRS). Roughly speaking, under an interest rate swap agreement, a fixed and agreed upon rate is exchanged for a floating interest rate. Thus, in the case of interest rate swaps as well as other interest contingent derivatives the term structure affects not only the discounting future but, more importantly, it drives the underlying 'asset' value. Despite all the effort undertaken, understanding and modeling the term structure remains one of the challenging topics in financial research.

This study empirically examines whether the five models of the yield curve differ significantly in terms of the implications for valuing interest rate swaps. Selecting a plain interest rate derivative allows me to investigate interest rate models that are applicable in a broader context and may be used to extract expectations about the economy, conduct monetary policy, assess and manage financial risk[1]. Consequently, this investigation centers on the valuation for a longer horizon. Translated to a real-life setting, the choice of the term structure model is considered from the perspective of financial institutions, such as banks, and their portfolio of the interest sensitive claims with a longer holding period. Thus, the performance of the term structure models is essentially assessed by the predictive accuracy of the fitted yield curves and fitted swap contract values. I estimate the term structure models of Nelson and Siegel (1987), Vasicek (1977), Cox, Ingersoll, and Ross (1985), Heath et al.(1992) and construct the cubic spline curves. By comparing the outcomes of these techniques, I also investigate the value added of a complex and computationally expensive versus simplistic approach to interest rate modeling. Pricing of interest rate derivatives depends vitally on the term structure and it is of great interest to verify the extent to which the discrepancies in modeling the term structure impact the valuation of these contingent claims.

This chapter is organized as follows: Section 2 describes the term structure models, while Section 3 focuses on the interest rate swaps and their valuation. Section 4 discusses the data and the estimation methodology. Section 5 summarizes the findings, and Section 6 concludes.

[1]This argument is discussed in more detail in Section 3.

3.2 Term Structure Models

The term structure of interest rates describes the relationship between the yield on a default free debt instrument and its time to maturity. Strictly speaking, this term is reserved for more formal descriptions of the yield-to-maturity relation, while the graphical relationship is often called the yield curve. Nonetheless, both constructs tend to be employed interchangeably. Term structure modeling refers to two related, but separate, questions in finance. The first question focuses merely on curve construction, whereas the second - and much broader problem - involves a specification of factors driving the term structure dynamics over time. Within the latter approach two distinct research strands have evolved. Simply put, researchers either develop empirical models and estimation methods for the yield curve or propose theories about the nature of the stochastic process that governs interest rates. Loosely speaking, the empirical methods seek to design mathematical functions that reproduce the typical yield curve shapes. Thus, clearly they lack any theoretical underpinnings. On the contrary, the theoretical studies derive interest rate models from the theory of asset pricing and economic fundamentals. The model building blocks are state variables, which constitute the source of uncertainty in the economy, and processes characterizing their intertemporal dynamics. These dynamics successively determine the stochastic behavior of interest rates. Many term structure frameworks assume that the instantaneous interest rate itself is a source of uncertainty in the economy.

Essentially, two approaches to the theoretical modeling of interest rates dominate in the finance literature[2]. Earlier works focus on the evolution of the short rate, while studies following Heath, Jarrow, and Morton (1992) start directly with the movement of the entire term structure. From the economic perspective the two classes of stochastic interest rate models are arbitrage-free or equilibrium models. Strictly speaking, the vast majority of equilibrium frameworks are built under the partial equilibrium condition. Even in their case, however, the no-arbitrage argument is crucial for obtaining a bond pricing formula[3]. Such an approach has been pioneered by the celebrated paper of Va-

[2]For a detailed discussion of various classifications of interest rate models and the models' exposition refer to Brigo and Mercurio (2006), Cairns (2004), James and Webber (2009) or Wu (2009) to name just a few.

[3]Therefore, even the seminal Vasicek (1977) paper can be classified as the no-arbitrage model (see a critical evaluation of Vasicek (1977) provided in Heath, Jarrow, and Morton (1992), pp. 77-78).

sicek (1977), which is discussed in the following section. The general equilibrium models achieve the specification of the term structure within a representative agent endowment economy from a set of steady-state conditions. Their prominent example, due to Cox, Ingersoll, and Ross (1985), is also presented below. Finally, the arbitrage pricing models impose an exogenous stochastic process on spot or forward rates in a way that excludes unexploited price differences in the market of interest rate contingent claims. The far reaching paper of Heath, Jarrow, and Morton (1992) with its unifying framework is an excellent illustration of the arbitrage-free studies.

3.2.1 The Vasicek Model

In his seminal paper, Vasicek starts with a certain stochastic behavior of interest rates and the market price for risk and then derives the price of all contingent claims assuming the absence of arbitrage opportunities. He models the instantaneous interest rate as an Ornstein-Uhlenbeck process of the following form:

$$dr(t) = \kappa(\theta - r(t))dt + \sigma dW(t), \tag{3.1}$$

where κ, θ and σ are positive constants and $W(t)$ denotes a standard Wiener process. The reasoning is similar to that of Black and Scholes (1973). The solution to the stochastic differential equation (3.1) is given by:

$$r(t) = \theta + (r(s) - \theta)\, e^{-\kappa(t-s)} + \sigma \int_s^t e^{-\kappa(t-u)} dW(u) \tag{3.2}$$

The interest rate $r(t)$ implied by this structure is a Gaussian random variable with the following conditional moments:

$$\mathrm{E}\left(r(t)|\mathcal{F}_0\right) = \theta + (r_0 - \theta)\, e^{-\kappa t}$$

$$\mathrm{Var}\left(r(t)|\mathcal{F}_0\right) = \frac{\sigma^2}{2\kappa}\left(1 - \mathrm{e}^{-2\kappa t}\right)$$

where $\mathcal{F}_0$ is the filtration for $W(0)$ and, more generally, $\mathcal{F}_t$ is the filtration for the Wiener process $W(t)$. The derivation of equation (3.2) and the conditional variance of $r(t)$ are provided in the Appendix. It is easy to see that $\lim_{t\to\infty} \mathrm{E}\left(r(t)\right) = \theta$. Therefore, the parameter θ represents the average long-run interest rate. Parameter σ is the volatility of the short rate. Moreover, whenever $r(t)$ exceeds its long-run mean (θ), the drift term becomes negative, which pushes $r(t)$ down towards θ. Clearly, the speed of this adjustment is captured by the parameter κ. Likewise, when the interest rate is lower than its long-run mean ($r(t) < \theta$), the positive drift term pushes $r(t)$ up towards θ. Thus, the Vasicek model exhibits mean reversion, an appealing feature from the economic point of view. Roughly speaking, in times of an economic downturn and rising interest rates the demand for loans decreases, which eventually pulls the rates down. By the same token, the opposite can be argued. The mean reversion process prevents the expected value and variance of the instantaneous rate from exploding, which in turn limits the occurrence of unreasonably high and low rates. However, negative yields for distant maturities may still appear.

Generally, the term structure of interest rates is characterized by yields to maturity. The continuously compounded yield at time t of bond maturing at T is defined by:

$$R(t,T) = -\frac{\ln P(t,T)}{T-t} \tag{3.3}$$

where $P(t,T)$ is the price in period t of a discount bond maturing at T, for all $0 \leq t \leq T \leq T^*$. We fix a time horizon, T^*, before which all bonds will mature. To ensure that the bond prices satisfy the no-arbitrage condition one needs to apply the Girsanov theorem and redefine the diffusion process for the instantaneous interest rate under the risk neutral probability measure[4]. The Appendix features a detailed presentation of the risk neutral determination of bond prices and yields within the Vasicek framework. The obtained risk-adjusted diffusion process for the short rate is characterized by:

$$dr(t) = \kappa \left(\theta - \frac{\lambda\sigma}{\kappa} - r(t)\right)\, dt + \sigma\, d\widetilde{W}(t) \tag{3.4}$$

In a nutshell, the above given equation reformulates the short rate equation (3.1) using the following equivalence:

$$dW(t) = d\widetilde{W}(t) - \lambda\, dt \tag{3.5}$$

where $W(t)$ is the original Wiener process and $\widetilde{W}(t)$ is the Wiener process under a risk-

[4]Rigorously speaking, it is required that the diffusion process for the bond prices has the martingale property. For the full treatment of risk-neutral pricing refer to Shreve (2004).

adjusted measure Q. The parameter λ denotes a market price of risk and is assumed to be constant in the Vasicek model. Finally, given the character of the spot rate process, $r(t)$, Vasicek (1977) derives prices for discount bonds $(P(t,T))$ under the risk neutral probability measure Q and conditional on the information available at time t. The price of a discount bond at time t with maturity $T \leq T^*$ is defined as:

$$P(t,T) = E_Q\left(\exp\left(-\int_t^T r(u)\,du\right)\middle|\mathcal{F}_t\right) \tag{3.6}$$

In the Vasicek model the time-t price of the discount bond maturing at T for any $0 \leq t \leq T \leq T^*$ is described by:

$$\begin{aligned} P(t,T) = \exp\Bigg[&\left(\theta - \frac{\lambda\sigma}{\kappa} - r(t)\right)\frac{1-\mathrm{e}^{-\kappa(T-t)}}{\kappa} - (\theta - \frac{\lambda\sigma}{\kappa})(T-t) \\ &+\frac{\sigma^2}{4\kappa^3}\left(4\mathrm{e}^{-\kappa(T-t)} - \mathrm{e}^{-2\kappa(T-t)} + 2\kappa(T-t) - 3\right)\Bigg] \end{aligned} \tag{3.7}$$

Once inverted, the bond prices serve to determine the term structure of interest rates, which Vasicek (1977) expresses as:

$$\begin{aligned} R(t,T) \; = \; &-\frac{1}{T-t}\Bigg[\frac{1}{\kappa}\left(e^{-(T-t)\kappa} - 1\right)r(t) + \frac{\sigma^2}{4\kappa^3}\left(1 - e^{-2(T-t)\kappa}\right) \\ &+\frac{1}{\kappa}\left(\theta - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{\kappa^2}\right)\left(1 - e^{-(T-t)\kappa}\right) - \left(\theta - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{\kappa^2}\right)(T-t)\Bigg] \end{aligned} \tag{3.8}$$

where κ, θ and σ are the same positive constants as in equation (3.1), the parameter λ is the risk premium introduced by the risk-neutral pricing and $r(t)$ is the current period instantaneous interest rate driven by the stochastic differential equation (3.1). The derivations are included in the Appendix. In addition, the term $\theta - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{2\kappa^2}$ represents the infinite maturity interest rate $R(t,\infty)$. Depending on the relation between $r(t)$, $R(t,\infty)$ and $\frac{\sigma^2}{4\kappa^2}$ Vasicek (1977) generates upward sloping, downward sloping and humped yield curves. All three types of shapes corroborate the stylized facts about yields curves.

3.2.2 The Cox, Ingersoll and Ross Model

Cox, Ingersoll, and Ross (1985) adapt an equilibrium approach to interest rate modeling. The paper develops a single-good continuous time homogeneous economy in which interest rates are derived from the supply and demand of investors with a constant relative risk aversion. In each period a representative agent faces production opportunities determined by the state of technology, which evolves according to a first-order stochastic differential equation. The equilibrium risk-free rate and the expected return on contingent claims are such that "all wealth is invested in the physical production"[5]. Therefore, the endogenized interest rate depends on the production opportunities and the rate dynamics moves in line with the evolution of the stochastic state of the production technology. Using the assumption about the production dynamics, Cox, Ingersoll, and Ross (1985) arrive at the following diffusion process for the risk-free rate of interest:

$$dr(t) = \kappa\left(\theta - r(t)\right)dt + \sigma\sqrt{r(t)}dW(t) \tag{3.9}$$

where κ, θ and σ are positive constants and $W(t)$ denotes a standard Wiener process associated with filtration $\mathcal{F}_t$. Generally, a closed-form solution cannot be derived, however a unique positive solution takes the form of:

$$r(t) = \theta + \left(r(s) - \theta\right)e^{-\kappa(t-s)} + \sigma e^{-\kappa(t-s)}\int_s^t e^{\kappa(u-s)}\sqrt{r(u)}dW(u) \tag{3.10}$$

Additionally, in order to preserve a positive short rate the condition $2\kappa\theta > \sigma^2$ is imposed. Derivations are provided in the Appendix. The endogenously determined process characterized by equation (3.10) is similar to the stochastic process for the interest rate in Vasicek (1977). However, the volatility of the CIR process for the short rate increases with the rate level. Feller (1951) shows that processes as described by equation (3.10) have a noncentral chi-squared distribution. Specifically, conditioned on filtration $\mathcal{F}_s$ for $s < t$ the short rate $r(t)$ follows a noncentral chi-squared distribution[6]:

$$r(t)|\mathcal{F}_s \sim \chi^2\left[2cr(t); 2q+2, 2u\right]$$

with $2q+2$ degrees of freedom and parameter of noncentrality $2u$ proportional to $r(s)$ and where:

$$c \equiv \frac{2\kappa}{\sigma^2\left(1 - e^{-\kappa(t-s)}\right)}$$

$$u \equiv cr(s)e^{-\kappa(t-s)}$$

[5]Cox, Ingersoll, and Ross (1985), p. 387.

[6]For more details refer to Cox, Ingersoll, and Ross (1985), pp. 391-392.

$$q \equiv \frac{2\kappa\theta}{\sigma^2} - 1$$

The interest rate $r(t)$ implied by the square-root diffusion process has the following conditional moments:

$$E(r(t)|\mathcal{F}_s) = \theta + (r(s) - \theta)e^{-\kappa(t-s)}$$

$$V\left(r(t)|\mathcal{F}_s\right) = r(s)\frac{\sigma^2}{\kappa}\left(e^{-\kappa(t-s)} - e^{-2\kappa(t-s)}\right) + \theta\frac{\sigma^2}{2\kappa}\left(1 - e^{-\kappa(t-s)}\right)^2$$

The Appendix presents derivations of the conditional moments in the Cox-Ingersoll-Ross framework. As in the case of the instantaneous short-rate process proposed by Vasicek (1977), in the CIR model the parameter θ represents the average long-run interest rate and the parameter κ captures the speed of mean-reverting dynamics. Whenever $r(t)$ moves away from its long-run mean, the drift term pulls it back towards θ. However, the two models differ with respect to the volatility function of the short rate, which for the CIR process becomes $\sigma\sqrt{r(t)}$ and moves with the spot rate level.

Similar to Vasicek (1977), Cox, Ingersoll, and Ross (1985) use the market efficiency to derive the price of discount bonds and the yield-to-maturity. The no-arbitrage price of discount bonds is implied by the existence of a risk-neutral measure Q. Changing the probability measure from the actual to the risk-neutral modifies the drift term exactly as in the Vasicek model. In the CIR model the mean-reverting risk-neutral dynamics becomes:

$$dr(t) = \kappa\left(\theta - \frac{\kappa + \lambda\sigma}{\kappa}r(t)\right)\ dt + \sigma\ \sqrt{r(t)}d\widetilde{W}(t) \tag{3.11}$$

At the Cox-Ingresoll-Ross equilibrium the risk premium, $\lambda(t)$, assumes the form of $\lambda\sqrt{r(t)}$ and corresponds to the constant λ in Vasicek's equation (3.5). In both cases the market price for risk preserves the same structure under the actual and the risk-neutral probability measure. The Appendix outlines steps involved in the derivation of bond prices in the Cox-Ingresoll-Ross framework. Given the character of the square-root process for the spot rate, $r(t)$, the price of a discount bond at time t with maturity $T \leq T^*$ is given as:

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)} \tag{3.12}$$

where

$$\begin{aligned} A(t,T) &= \left[\frac{2\gamma e^{(\kappa+\gamma+\lambda)(T-t)/2}}{(\gamma+\kappa+\lambda)(e^{\gamma(T-t)}-1)+2\gamma}\right]^{2\kappa\theta/\sigma^2} \\ B(t,T) &= \frac{2(e^{\gamma(T-t)}-1)}{(\gamma+\kappa+\lambda)(e^{\gamma(T-t)}-1)+2\gamma} \\ \gamma &= \sqrt{(\kappa+\lambda)^2+2\sigma^2} \end{aligned}$$

where κ, θ and σ are the same positive constants as in equation (3.9), the parameter λ is introduced by the risk-neutral pricing and $r(t)$ is the current period instantaneous interest rate driven by the stochastic differential equation (3.11). Using equation (3.3) the continuously compounded yield for discount bonds in the CIR model can be expressed as:

$$R(t,T) = \frac{-\ln A(t,T) + B(t,T)r(t)}{T-t}$$

where $A(t,T)$ and $B(t,T)$ are defined as in formula (3.12). It can be shown that the infinite maturity yield in the Cox-Ingresoll-Ross model amounts to:

$$R(t,\infty) = \frac{2\kappa\theta}{\gamma+\kappa+\lambda}$$

As in Vasicek (1977), all three types of yield curves shapes are admissible. The CIR term structure becomes upward sloping for instantaneous rates lower than the long-term yield, downward sloping for rates exceeding $\frac{\kappa\theta}{\kappa+\lambda}$ and exhibits humps for intermediate values of the short interest rate.

The great appeal of both interest rate models is their analytical tractability. On the other hand, one of the drawbacks of the Vasicek and the Cox- Ingersoll-Ross model stems from the fact that they do not take the whole yield curve as an input in the price structure. Instead, the yield curve is what those models produce. In particular, Heath, Jarrow, and Morton (1992) criticizes the models of Cox, Ingersoll, and Ross (1985) and Vasicek (1977) for introducing arbitrary specification of the market price for risk. Both models derive the term structure from the prices for risk-free claims, which originate from the value of the discount bond bearing some risk. Such reverse pricing may lead to inconsistencies and arbitrage opportunities. To exclude arbitrage, the price structure should take into account the entire curve observed in the market. This consideration has fueled the evolution of the arbitrage pricing models.

3.2.3 The Heath-Jarrow-Morton Model

Heath, Jarrow, and Morton (1992) propose a unifying framework for all arbitrage pricing models. Any specific arbitrage-free model can be expressed as a special case of their model. Amongst others, their work is a generalized formulation of such classical cases as Vasicek (1977), Hull and White (1987) (Extended Vasicek) or Ho and Lee (1986), which is the first arbitrage-free term structure model calibrated to the term structure data. As already mentioned, a basic feature of arbitrage models is taking the entire curve as state variables. Heath-Jarrow-Morton's methodology imposes stochastic structure on the forward rates and its state variable is the forward rate curve. The evolution of the term structure of the forward rates is described by[7]:

$$df(t,T) = \alpha(t,T)dt + \sigma(t,T)dW(t), \quad 0 \leq t \leq T \leq T^{*} \tag{3.13}$$

or in an integral form:

$$f(t,T) = f(0,T) + \int_{0}^{t} \alpha(u,T)du + \int_{0}^{t} \sigma(u,T)dW(u) \tag{3.14}$$

where $\sigma(u,T)$ denotes a family of volatilities of the forward rate for maturity T and $W(u)$ is a standard Wiener process. For any maturity T, the volatilities $\sigma(u,T)$ and the drifts $\alpha(u,T)$ are adapted processes in the u variable. The model also postulates that

$$\int_{0}^{T} |\alpha(u,T)|du + \int_{0}^{T} |\sigma(u,T)|^{2}du$$

is almost surely finite. In general, the volatilities $\sigma(u,T)$ and the drifts $\alpha(u,T)$ depend on the history of the Wiener process and on the rates up to time t. For any time t, $0 \leq t \leq T \leq T^{*}$, the model assumes that the initial forward rate curve $f(0,T)$ is known. Thus, by construction, the model fits the initial observed term structure.

A major contribution of Heath, Jarrow, and Morton (1992) is that it recognizes the implication of the arbitrage-free assumption for the relationship between the drift and the volatility of the forward rate. In the no-arbitrage setting, the drifts of the forward

[7] For the simplicity of the exposition I present a one factor specification

rates, $\alpha(u,\cdot)$ are uniquely determined by the volatilities, $\sigma(u,\cdot)$, and the market price for risk. Formally, the condition reads:

$$\alpha(t,T) = \sigma(t,T)\left[\int_t^T \sigma(t,u)du + \lambda(t)\right], \forall t \in [0,T], T \in [0,T^*] \tag{3.15}$$

The Appendix features derivation of equation (3.15). Using the no-arbitrage drift restriction, the forward-rate process in (3.13) can be formulated under the risk-neutral probability measure Q as:

$$df\left(t,T\right) = \left(\sigma\left(t,T\right)\int_t^T \sigma\left(t,u\right)\ du\right)dt + \sigma\left(t,T\right)d\widetilde{W}(t) \tag{3.16}$$

where $\widetilde{W}(t)$ is the Wiener process under the martingale measure Q, while $\lambda(t)$ denotes a market price of risk and $0 \leq t \leq T \leq T^*$. The Heath-Jarrow-Morton methodology offers a broad range of models, each characterized by specific functional forms of the drifts and the volatilities of the forward rates. Due to the drift condition (3.15), the choice of any particular model reduces to the choice of a functional form of the volatility.

A simple way to postulate the dynamics for the short rate under the Heath-Jarrow-Morton framework is to consider the spot rate as a limit of the instantaneous forward rate:

$$\begin{aligned} r(t) &= \lim_{s\to t^-} f(s,t) \\ &= f(0,t) + \int_0^t \sigma(u,t)\left(\int_u^t \sigma(u,\nu)d\nu\right)du + \int_0^t \sigma(u,t)d\widetilde{W}(u) \end{aligned} \tag{3.17}$$

In general, the HJM forward rate framework results in a path-dependant evolution (3.17) for the spot rates. Since the Markovian models are numerically easier to handle, a condition additionally imposed on the Heath-Jarrow-Morton specification is that the short rate needs to be Markovian. Equation (3.17) implies the following differential form for the spot interest rate process:

$$dr(t) = df(t,T)\Big|_{T=t} = \frac{\partial f(t,T)}{\partial T}\Big|_{T=t} dt + \sigma(t,t)d\widetilde{W}(t) \tag{3.18}$$

Given the regularity conditions and under the risk-neutral probability, the yields on contingent claims are determined by the initial term structure and the volatility function of the forward rates in the following way:

$$\begin{aligned} R(t,T) &= R(0,T) + \frac{\frac{1}{2}\int_0^t \left(\int_u^T \sigma(u,s)ds\right)^2 du}{T-t} - \frac{\int_0^t r(u)du}{T-t} \\ &+ \frac{\int_0^t \int_u^T \sigma(u,s)ds\ d\widetilde{W}(u)}{T-t} \end{aligned} \tag{3.19}$$

The Appendix outlines the derivations of equations (3.16) and (3.19). It is worth noting that Heath, Jarrow, and Morton (1992) obtain the value of the discount bonds directly from a finite number of state variables under the assumption of the absence of arbitrage. Thus, by construct, their model consistently prices all contingent claims on the term structure. Given the generality of the Heath-Jarrow-Morton methodology and its flexibility regarding the functional forms of $\sigma(t,\cdot)$, there have been numerous attempts to extend and validate the HJM model by assuming and testing a certain functional form for the forward-rate volatility.

3.2.4 Empirical Yield Curve Models

Early attempts within the empirical approach focused on advancing the polynomial splines models. This methodology was introduced by McCulloch (1971) and extended by McCulloch (1975) and Langetieg and Smoot (1981). Polynomials, however, fail to conform to the variety of yield curve shapes observed in reality and tend to produce unstable forward rates with unacceptable asymptotic properties. A pioneering work in the exponential approximation of the yield curve is due to Vasicek and Fong (1982). The paper suggests an exponential spline formulation, well-fitting the term structures observed in reality. Vasicek and Fong (1982) avoid a tedious nonlinear estimation of the discount factor coefficients due to a logarithmic transform of the discount function's arguments. As a result, an ordinary least squares estimation can be employed. When tested on the

US Treasury bills, the methodology generates stable forward rates that are a continuous function of time, exhibit desirable asymptotic characteristics for long maturities and conform to a wide range of shapes.

Building on Vasicek and Fong's results, Nelson and Siegel (1987) proposed an exponential model which is simple, parsimonious and at the same time flexible enough to generate monotonic, humped and S-shaped curves. Their model is capable of replicating important stylized facts of the yield curve. Nelson and Siegel (1987) develop the term structure model starting with a function for the instantaneous forward rate. The framework includes three specifications of the instantaneous forward rate functions, namely a second-order exponential equation with unequal roots, with equal roots and, finally, a simple exponential specification. Given that the simple specification performs unsatisfactorily, whereas the case of unequal roots tends to cause overparametrization, they are both excluded from my investigation. Nelson and Siegel (1987) report that the formulation with equal roots failed to accurately approximate solely the extreme cases. In particular, the model with equal roots overestimated the long-term discount rates and underestimated their short term counterparts in their in- and out-of-sample estimations for the U.S. Treasury bills. The second-order exponential equation with equal roots characterizing the instantaneous forward rate $r(m)$ for maturity m has the form[8]:

$$r(m) = \beta_0 + \beta_1 \exp \frac{-m}{\tau} + \beta_2 \left(\frac{m}{\tau} \cdot \exp \frac{-m}{\tau} \right) \tag{3.20}$$

where τ is a time constant that determines the rate at which the regressors decay to zero and $\beta_0, \beta_1, \beta_2$ are the coefficients to be estimated. The determinant of the speed of decay needs to be calibrated to the data. Small τ conforms to low maturities, whereas high values of τ generate slow decay and thus fit long maturities best. Using equation (3.20), the evolution for the yield to maturity m can be expressed as:

$$R(m) = \beta_0 + \beta_1^* \frac{1 - e^{-\frac{m}{\tau}}}{\frac{m}{\tau}} + \beta_2 \left(-e^{-\frac{m}{\tau}} \right) \tag{3.21}$$

where $R(m)$ is the yield for maturity m and $\beta_1^* = \beta_1 + \beta_2$. Parameters τ, β_0, β_1 and β_2 are defined as in expression (3.20). By construct, β_0 is the limiting value of the yield for very large maturities. It can be also shown that for maturities close to 0 the yield

[8] I follow the notation of Nelson and Siegel (1987).

approaches ($\beta_0 + \beta_1$). The derivations are provided in the Appendix. The factors of the Nelson-Siegel model have appealing interpretations, namely they can be regarded as the yield curve level, slope and curvature. The long-term factor β_0 governs the level of the term structure of interest rates, while β_1 and β_2 are responsible for monotonic and humped shapes of the curves.

The Nelson-Siegel methodology has become very popular and gave rise to an entire class of models. The approach is extensively used by practitioners, in particular central banks (ECB (2008) and BIS (2005)). The most recognized extensions of the Nelson-Siegel approach are by Diebold and Li (2006) and Dahlquis and Svensson (1996). The Svensson model adds even more flexibility to the Nelson-Siegel framework by including a second hump factor, which produces a much broader range of term structure shapes. The real value added of Diebold and Li (2006) stems from modeling the coefficients dynamically with the use of the autoregressive process of order one. By doing so, the study enriches the Nelson-Siegel model with a time-series dimension. An out-of-sample forecasting based on Diebold and Li estimates for US Treasury bonds exhibits an improvement in precision of prediction as time horizon lengthens. This result could be attributed to the dynamics imposed in the estimated parameters. The Diebold and Li variation of the Nelson-Siegel model proves the robustness of the latter one to modifications and restrictions. It also supports the choice for this study of the second-order exponential model with equal roots.

Finally, a frequently employed algorithm for the curve construction is a cubic spline interpolation. Hagan and West (2006) features a comprehensive survey of the yield curve construction. The interpolation methods use the set of available observations to recover the in-between "missing" yields. A cubic spline[9] is a spline constructed of piecewise third-order polynomials, which pass through a set of control points. The second derivative of each polynomial is commonly set to zero at the endpoints, which provides a boundary condition that completes the system of equations and allows for solving for the system's coefficients.

[9] `http://mathworld.wolfram.com/CubicSpline.html`

3.3 Interest Rate Swaps

A plain-vanilla interest-rate swap is an agreement between two parties to exchange fixed for floating payments in the future[10]. The interest rate swaps are an appropriate choice to empirically compare the performance of the term structure models for several reasons. Since their introduction in the early 1980s, swaps have dominated the over-the-counter derivative market. Additionally, the interest rate swap market is liquid, deep and practically hardly regulated. What's more, selecting a plain interest rate derivative allows the investigation of interest rate models applicable in a broader context. If instead the interest-rate options were selected, the choice of models would be naturally confined to the interest-rate market models. As noticed by Brigo and Mercurio (2006), the unquestionable popularity of the market models stems from the fact that they have been tailored to a specific interest rate option's type and its market segment[11]. Unlike options, the interest rate swap is equivalent to a portfolio of bonds or a series of forward rate agreements. As shown below in this section, the swap can be valued as a portfolio of either of the two instruments. The term structure models that characterize evolution of bond prices, the same that serve to extract expectations about the economy, conduct monetary policy, assess and manage financial risk, may also be used to price plain interest rate derivatives such as swaps. In his acknowledged text on term structure modeling, Rebonato (1998) emphasizes that "any yield curve model capable of pricing discount bonds exactly must recover the market swap rates correctly for any choice of the model volatility"[12]. Contrary to this traditionally established approach, Duffie and Singleton (1997) develop a new framework for pricing caps, floors swaptions and swaps using a default and liquidity-adjusted instantaneous short rate. Their risk-adjusted model is a natural consequence of the empirical studies of the default spreads in the swap market (Sun, Sundaresan, and Wang (1993) and Malhotra (1997)). To mitigate the problem of default risk premia, I analyze the contracts between banks of similar credit quality. None

[10]Rebonato (1998), p. 8

[11]Brigo and Mercurio (2006), p. 195. In particular, the Brace-Gatarek-Musiela (BGM) model is compatible with Black's pricing formula for caps, floors or caplets, while the lognormal forward-swap model (LSM) is compatible with pricing swaptions. Their calibration to each specific instrument's market in developed countries is relatively straightforward. A calibration to the markets in transition economies is examined in Vojtek (2004). An overview of the market models can be found in Brigo and Mercurio (2006) or Cairns (2004).

[12]Rebonato (1998), p. 12

of the examined deals incorporates a spread over the money market reference rate[13]. The symmetric nature of the interest rate swaps implies no default risk premium (Sorensen and Bollier (1994)) and Duffie and Huang (1996)). Therefore, the swap pricing theory and common yield curve models may be applied.

The series of swap fixed payments are often referred to as the fixed legs, and the floating payments as the floating legs of a swap. Both legs are calculated on the same principal and for the same period. The fixed rate and the future dates of exchanges are know at the outset of the swap. The floating rate is reset at each payment time. The widely known reference rate of interest employed in standardized quotations and financial derivatives valuation is the London Interbank Offered Rate. LIBOR is the reference in the Eurocurrency market which, though centered in London, is the global money market for wholesale deposits and loans in all easily convertible currencies. More precisely, LIBOR is the rate of interest offered by banks on deposits from other banks in the Eurocurrency market. However, the use of interbank interest rates is not confined to London. Naturally, most financial centers quote their own interbank offered rates. These rates include, among many others, the Frankfurt Interbank Offered Rate (FIBOR), the Paris Interbank Offered Rate (PIBOR) or the Warsaw Interbank Offered Rate (WIBOR), which is charged on bank deposits in the Warsaw money market. As previously mentioned, the reference rates are key to derivatives valuation. In this study I examine the interest rate swap agreements signed in the Polish market, in the Polish zloty, and where fixed interest payments were exchanged for the WIBOR-linked payments. Therefore, the reference rate for the sake of this investigation is the Warsaw Interbank Offered Rate (WIBOR).

The value of the interest rate swap is by default zero or close to zero at the contract outset. Under IRS, the principal amount is not exchanged and thus it is solely a notional principal. The swap fixed rate is determined by equalizing the net present value of both fixed- and floating-rate cash flows. Throughout the swap's life the contract gains or loses in value. The swap pricing theory suggests two approaches to interest rate swap valuation. One approach determines the swap value in terms of bond prices, the other by treating an IRS as a portfolio of forward rate agreements (FRAs). Valuing the interest

[13]Unlike in many other studies, my dataset contains very detailed information on the interest rate swaps including all contractual provisions.

rate swap as bonds reduces to finding a difference between the prices of a floating- and a fixed-rate bond. From the point of view of the fixed-rate payer, an interest rate swap is a long position in a floating-rate bond combined with a short position in a fixed-rate bond. Thus, the fixed-rate payer calculates the value of the IRS by subtracting the price of the fixed-rate bond from the price of the floating-rate bond. The opposite is true for the floating-rate payer. At any time of the IRS's life its value to the fixed-rate payer is equal to[14]:

$$V_{swap} = B_{fl} - B_{fix} \tag{3.22}$$

$$B_{fix} = \sum_{i=1}^{n} k e^{-r_i t_i} + L e^{-r_n t_n} \tag{3.23}$$

$$B_{fl} = (L + k^*) e^{-r^* t^*} \tag{3.24}$$

where t_i is a time interval until the i-th payments are exchanged, r_i is the WIBOR[15] zero rate corresponding to maturity t_i and k is a fixed payment made on each payment date. Expression (3.24) presumes that the next exchange date is t^*. Then, r^* is the corresponding WIBOR zero rate and k^* is the floating-rate payment. To understand formula (3.24) one needs to recall that immediately after each exchange, the floating-rate bond is worth just the notional amount (L). Therefore, in the period between the payment date t^* and a preceding payment date, the bond's value equals the notional principal augmented by the corresponding discounted floating payment.

Alternatively, any IRS could be valued in terms of FRAs. A FRA or a forward rate agreement is a contract that determines the rate of interest to be paid or received on an obligation beginning at some future date[16]. In order to price an interest rate swap as a portfolio of FRAs one needs to determine the implied forward rate for each future floating payment specified under the contract. The forward interest rate $r^F_{i,i+1}$ for the period between time t_i and t_{i+1} is given by the formula[17]:

$$r^F_{i,i+1} = \frac{r_{i+1} t_{i+1} - r_i t_i}{t_{i+1} - t_i} \tag{3.25}$$

[14] Hull (2002), pp. 136-137.

[15] More generally, the LIBOR zero rate corresponding to maturity t_i. In what follows, I will refrain from recalling that for the Eurocurrency market the appropriate LIBOR rate would be used instead of WIBOR.

[16] Based on Hull (2008), p. 85.

[17] Hull (2008), p. 83.

where r_i and r_{i+1} are the WIBOR zero rates for maturities t_{i+1} and t_i respectively. Clearly, further steps assume that the forward rates are realized. Under this assumption, the future swap floating rates equal the implied forward rates. As a result, the swap cash flows can be calculated for each exchange date t_{i+1} as the net present value[18] of the swap payments:

$$\textbf{NPV}\, CF_{i+1} = (t_{i+1} - t_i)L(r_{fix} - r^F_{i,i+1})e^{-r_{i+1}\frac{i+1}{12}} \tag{3.26}$$

where t_{i+1} and t_i is a time interval until respectively the $i+1$ and i-th payments are exchanged, r_{i+1} is the WIBOR zero rate corresponding to maturity t_{i+1} and r_{fix} is the fixed interest rate. Finally, $r^F_{i,i+1}$ is the implied forward interest rate for the period between dates t_i and t_{i+1}. Eventually, the IRS value is simply equal to the present value of all cash flows:

$$V_{swap} = \sum_{i=1}^{n} \textbf{NPV}\, CF_i \tag{3.27}$$

It can be seen from the valuation of interest rate swaps to what extent IRS are sensitive to changes in interest rates. This sensitivity makes IRS a good candidate to test the impact of differences in modeling term structure on derivatives valuation. The choice of the interest rate swap contracts is further motivated by the fact, that the swap market is liquid, deep and practically hardly regulated. Not all interest rate contingent claims are traded in the markets, which share these characteristics. The swap market liquidity follows from the amount of deals concluded. The size of the IRS market constitutes a most significant portion of the over-the-counter (OTC) interest rate derivatives both in terms of outstanding notional amounts and gross market values of the contracts. The IRS dominance in the OTC interest rate market is reflected in market data surveys, a good example of which are the Bank for International Settlements (BIS) quarterly reviews listing outstanding total derivatives notional amounts and market values in the G10 countries. The statistics are provided in Tables 3.1 – 3.4. BIS regularly gathers data from national regulators and extracts from it the worldwide consolidated derivatives exposure of major banks and dealers in the developed countries to assess the OTC market size. Table 3.1 shows the IRS constitutes the most significant position out of all OTC interest rate derivatives for the reported periods.

[18]The NPV of a contract under which the fixed-rate interest is received and the interest linked to the floating rate is paid (Hull (2002), p. 138).

[19]Source: www.bis.org/statistics/dt21a21b.pdf

Table 3.1: Notional Amounts of OTC Interest Rate Derivatives by Instruments

	Notional Amounts (billions of US dollars)								
	Dec 2005	Dec 2006	Dec 2007	Dec 2008	Dec 2009	Dec 2010	Dec 2011	Dec 2012	Dec 2013
Total	211,970	291,582	393,138	432,657	449,875	465,260	504,117	489,706	584,364
FRA	14,269	18,668	26,599	41,561	51,779	51,587	50,596	71,353	73,819
IRS	169,106	229,693	309,588	341,128	349,288	364,378	402,611	370,002	461,281
Options	28,596	43,221	56,951	49,968	48,808	49,295	50,911	48,351	49,396

Source: BIS statistics. The data are available at the BIS website (for details consult the references). IRS: interest rate swaps, FRA: forward rate agreements. The figures have been adjusted for double- counting. Under the interest rate category BIS reveals solely the single currency swaps, no currency interest rate swaps (CIRs) enter this position. [19]

IRS dominance is even more visible in Table 3.2, which presents the shares of each type of derivatives. In all analyzed periods the interest rate derivatives amount to at least 76% of the OTC interest rate market in terms of outstanding notional amounts.

Table 3.2: Share in Notionals of Outstanding OTC Interest Rate Derivatives

	Share in Notional Amounts								
	Dec 2005	Dec 2006	Dec 2007	Dec 2008	Dec 2009	Dec 2010	Dec 2011	Dec 2012	Dec 2013
Total	100%	100%	100%	100%	100%	100%	100%	100%	100%
FRA	7%	6%	7%	10%	12%	11%	10%	14%	13%
IRS	80%	79%	79%	79%	78%	78%	80%	76%	79%
Options	13%	15%	14%	11%	10%	11%	10%	10%	8%

Source: Own calculations based on BIS statistics available at the BIS website (for details consult the references). IRS: interest rate swaps, FRA: forward rate agreements. The figures have been adjusted for double-counting. Under the interest rate category BIS reveals solely the single currency swaps, no currency interest rate swaps (CIRs) enter this position.

In addition, BIS International Financial Statistics contain data on gross market value of the OTC derivatives. Gross market value represents the cost of replacing all open contracts at the prevailing market prices. Also in this category interest rate swaps outclass other OTC interest rate derivatives, which is reflected both in the absolute and percentage values shown respectively in Tables 3.3 and 3.4.

The presented statistics confirm the interest rate swaps' primacy among the OTC interest rate derivatives which together with the lack of regulations makes IRS a suitable candidate to test the impact on interest rate contingent claims of the difference in the modeling the term structure.

[20]Source: www.bis.org/statistics/dt21a21b.pdf.

Definition cited from: www.bis.org/statistics/rqa1406notes.pdf

[21]Source: www.bis.org/statistics/dt21a21b.pdf.

Table 3.3: Gross Market Value of OTC Interest Rate Derivatives by Instruments

	Gross Market Value								
	Dec 2005	Dec 2006	Dec 2007	Dec 2008	Dec 2009	Dec 2010	Dec 2011	Dec 2012	Dec 2013
Total	5,397	4,826	7,177	20,087	14,020	14,608	20,001	18,833	14,039
FRA	22	32	41	165	80	206	67	47	108
IRS	4,778	4,163	6,183	18,158	12,576	13,001	18,046	17,080	12,758
Options	597	631	953	1,764	1,364	1,401	1,888	1,706	1,174

Source: BIS statistics available at the BIS website (see the references). IRS: interest rate swaps, FRA: forward rate agreements. Gross market value represents the cost of replacing all open contracts at the prevailing market prices and is defined as "the sum [. . .] of the positive market value of all reporters' contracts and the negative market value of their contracts with non-reporting counterparties"[20]

Table 3.4: Share in Gross Market Value of Outstanding OTC Interest Rate Derivatives

	Share in Gross Market Values								
	Dec 2005	Dec 2006	Dec 2007	Dec 2008	Dec 2009	Dec 2010	Dec 2011	Dec 2012	Dec 2013
Total	100%	100%	100%	100%	100%	100%	100%	100%	100%
FRA	0%	1%	1%	1%	1%	1%	0%	0%	1%
IRS	89%	86%	86%	90%	90%	89%	90%	91%	91%
Options	11%	13%	13%	9%	9%	10%	10%	9%	8%

Source: Own calculations based on BIS statistics available at the BIS website (see the references). IRS: interest rate swaps, FRA: forward rate agreements. Gross market value represents the cost of replacing all open contracts at the prevailing market prices and is defined as "the sum [. . .] of the positive market value of all reporters' contracts and the negative market value of their contracts with non-reporting counterparties"[21]

3.4 Estimation and Valuation Methodology

3.4.1 Data Description

This study uses a dataset consisting of yields quoted by Reuters and parameters for 369 interest rate swap agreements (IRS) signed in Polish zlotys. The contracts are real life transactions concluded in the Polish interbank market between the 1st August 2001 and the 27th June 2007 by banks of similar credit quality. None of the examined deals incorporates a spread over the money market reference rate. None of the parties involved were dependent in terms of capital or any other relation. The IRS parameters include trade date, beginning of contract's life date, its maturity, principal, currency, floating and corresponding fix rate. Tables 3.5 – 3.7 and Figures 3.2 – 3.7 present an overview of the contracts' characteristics. Figure 3.2 displays the distribution of grouped notional amounts with the intervals of 5 million euros. Clearly the majority of the notional values amount to 25 million euros[22] and most of them do not exceed 50 million euros. It also

Definition cited from: www.bis.org/statistics/rqa1406notes.pdf

[22]The exchange rate PLN/EUR is 3.3460 (the average exchange rate quoted by the National Bank of Poland on the 29th August 2008)

proves that contracts with principal values oscillating around 15 and 30 million euros dominate the sample, which is also reflected in the basic descriptive statistics of the contracts' notional values shown in Table 3.5.

Table 3.5: Notional Amounts of Sample Interest Rate Swap Contracts

Measure	*million EUR*
Mean	27.4
Median	29.0
Mode	30.0
Standard deviation	21.3
Minimum	1.2
Maximum	224.1

Figure 3.1: Box Plots of Observed Yields from August 2000 to November 2007

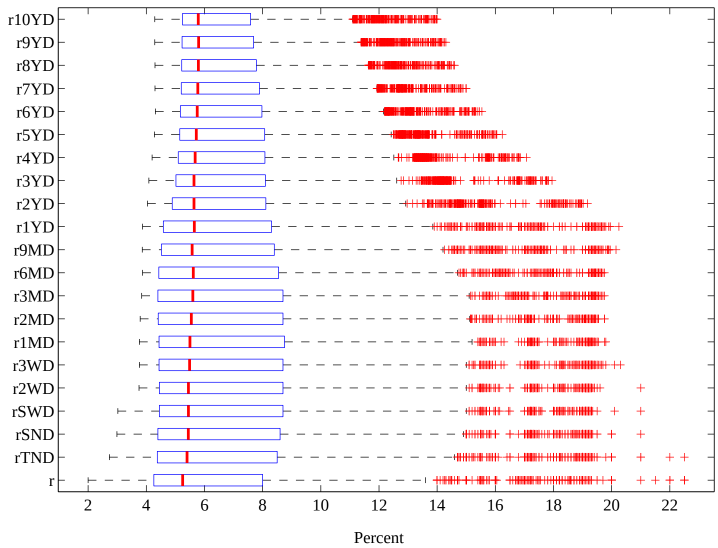


Table 3.6 contains descriptive statistics for sample maturities and indicates that the examined interest rate swaps were most frequently concluded for a year, but on average lasted 2 years. The longest contracts in the sample span over 6 years, while the shortest were signed for 3 months. OTC agreements for 3 and 6 moths as well as 1 and 2 years are standard in the Polish interbank market.

The frequent occurrence of one-year deals and mean maturity of 2 years are also visible on Figure 3.3, which depicts the distribution of principals by contracts' lengths. The chart shows that one-year contracts span over all range of notional values, but generally amount to 100 million euros. Shorter deals as well as those over 2 years never surpass 25 million euros, while two-year agreements come to 100 million euros.

Figure 3.2: Distribution of Notional Amounts

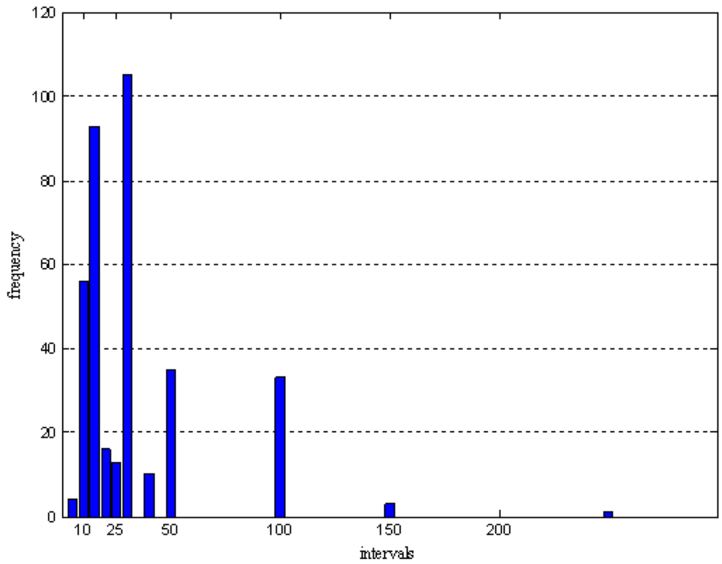


Table 3.6: Sample Interest Rate Swap Contracts: Descriptive Statistics for Maturities

Measure	*Maturity, months*
Mean	23
Median	24
Mode	12
Standard deviation	12
Minimum	3
Maximum	72

Issuance date matters as much as maturity for valuation of claims. The investigated transactions were signed between years 2001 and 2007 and expired in years 2007 – 2008. Such a time structure results from the sampling procedure. Contracts that entered the sample had to be alive at the data collection time that is in the middle of 2007 year and be due by mid 2008, which allowed for ex post actual value calculation. The overview of issuance and due dates indicates that the deals commonly originated in 2005, 2006 and 2007, which corroborate with the most frequent one-year and average two-year maturities.

The payments to be exchanged under interest rate swap agreements are denominated in Polish zlotys and are calculated based on Warsaw Interbank Offer Rates (WIBOR). Roughly speaking, the WIBOR rate is a Polish equivalent of the LIBOR rate (for more details on the reference rates employed in interest rate swaps refer to Section 3). In the examined sample, the WIBOR rate for 6 months determines payments of 59% of the sample contracts. It is clear from Figure 3.4 that the next most commonly employed rate

Figure 3.3: Notional Amounts by Contract Maturities

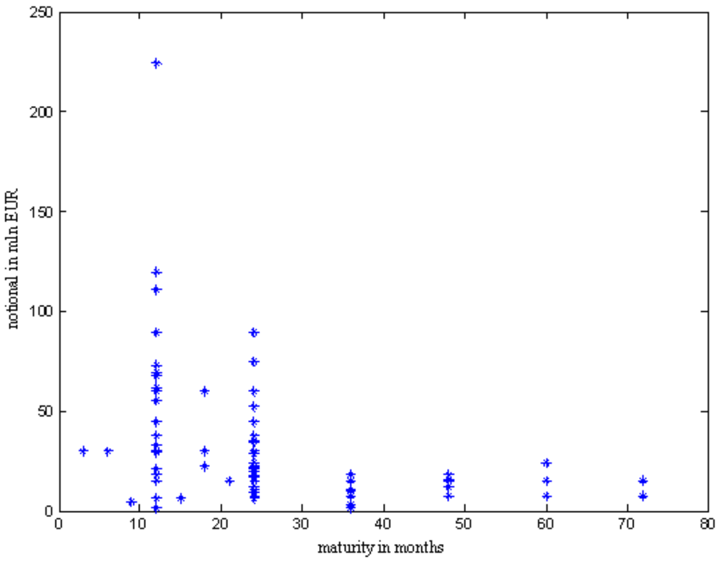


Table 3.7: Sample Interest Rate Swap Contracts: Issuance and Expiration Dates

Year	*Issued contracts*	*Expired contracts*
2001	1%	–
2002	1%	–
2003	5%	–
2004	5%	–
2005	25%	–
2006	44%	–
2007	19%	50%
2008	–	50%
Total	100%	100%

(40% of deals) is the 3-month WIBOR. Together the two rates cover 99% of the sample transactions.

Knowing the sample composition with respect to the transaction dates and reference rates, it is illustrative to verify the ex post evolution of the two base rates for the period when the sample contracts were originated. Figures 3.6 and 3.5 depict the development of 3-month and 6-month WIBOR rates for the contracts' inception times. Figure 3.5 indicates that the 6-month rate experienced a plunge of 1.500 bps in years 2001-2003 and thereafter remained within the brackets of 5 to 7 percent. Given that over 88 percent of the 6-month contracts were signed from 2004 to 2006, the vast majority of them originated in a relatively stable period. Figure 3.6 shows a stable performance of 3-month rate except from the last three months, when it slightly rose by around 35 basis points.

Figure 3.4: Sample Data: Composition of Interest Rates

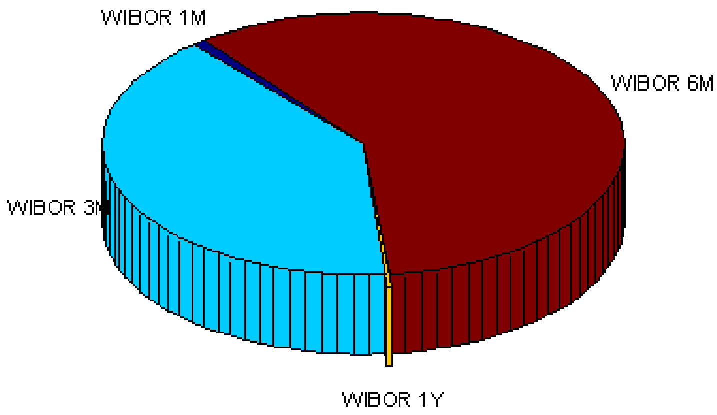
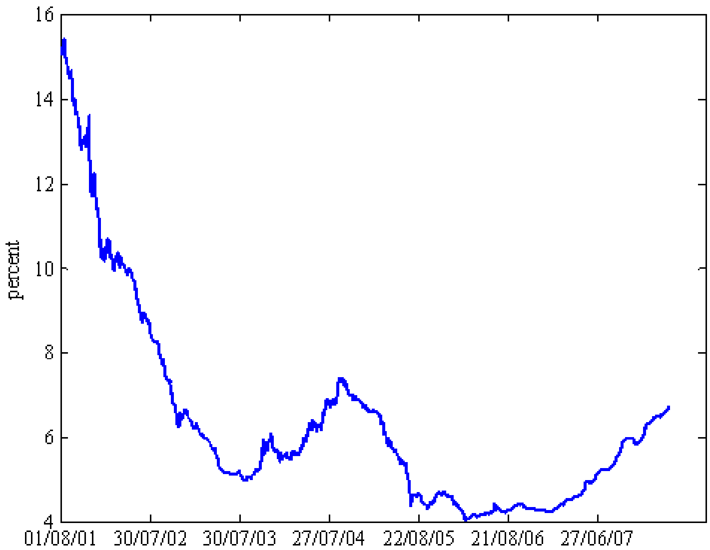


Figure 3.5: 6-month WIBOR Rates from August 2001 to June 2007



Finally, the presentation of the ex-post evolution of the IRS reference rates is completed with the development of the term structures prevailing in the market at each transaction date. Figure 3.7 depicts the average input rates for each maturity together with their standard deviation bands. The daily frequency of observations makes the short end of the term structure much more volatile than the long end, and thus the bands are wider for the shorter rates. Figure 3.8 presents the full scope of rate variation for the entire examined period and all maturities. In the case of the 3- and 6-month WIBOR, the period 2001 – 2003 of higher rate levels is followed by times of relative rate stabilization. The augmented rate levels in the initial years should not create a concern, given that each contract is valued based on the cross-sectional set of rates for available maturities prevailing in the market at the swap transaction dates.

Figure 3.6: 3-month WIBOR Rates from May 2006 to June 2007

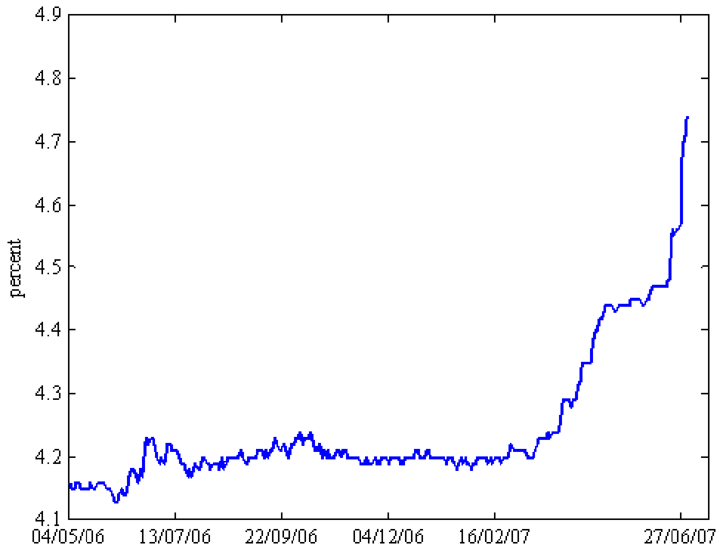


Figure 3.7: Evolution of Average Input Rates from August 2000 to November 2007

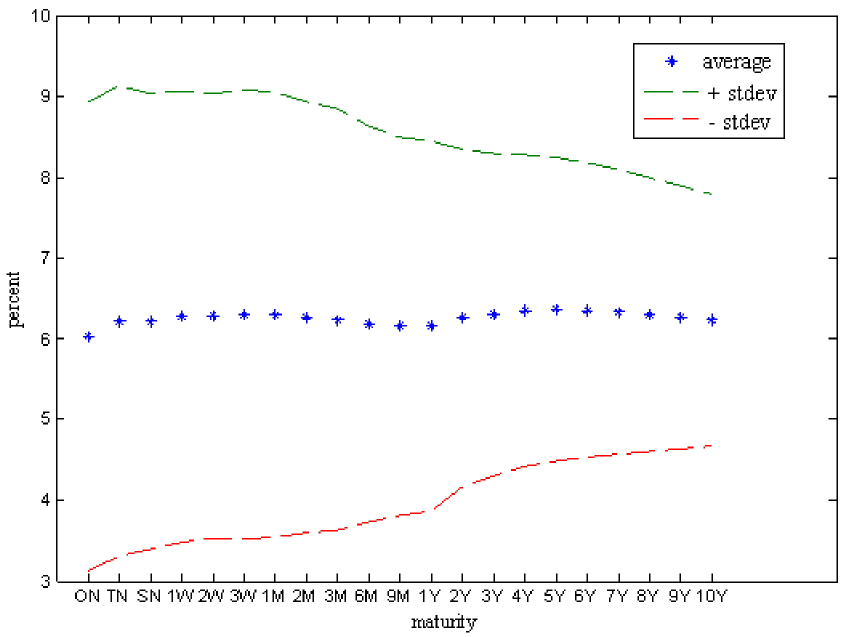
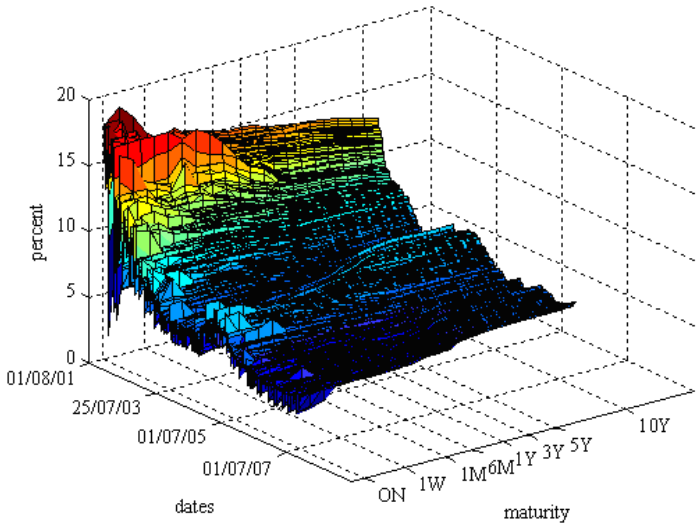


Figure 3.8: Evolution of Input Rates from August 2000 to November 2007



3.4.2 Methodology and Models Specification

This study empirically examines whether the five models of the yield curve differ significantly in terms of the implications for valuing plain interest rate contingent claims. Roughly speaking, the idea is to express the model accuracy in terms of how much one could have gained or lose on average on the set of real life interest rate swaps. The research strategy can be summarized by the following steps: (1) estimation of the term structure model, (2) valuation of the set of interest rate swaps using the fitted yields, and (3) comparison of the performance and predictive accuracy of the fitted yield curves and "fitted" contract values. The term structure estimations are performed for every transaction date in order to generate the fitted curves to value each contract in the sample. The key measures to assess the ex post forecasts of the term structure models herein are based on contrasting the 'fitted' and realized contract values. The realized deal values are computed based on the interest rate that are quoted by the market at the date of payment exchange. Those realized rates determine the amounts to be swapped and thus the effective contract value. The IRS valuation involves calculating the implied forward rates, swap cash flows, and finally, the swap total values as shown in formulas (3.25) to (3.27). All computations are done in MATLAB. The Kalman filter algorithm is implemented using Dynare[23] for MATLAB.

[23]Dynare is a user oriented general program for the simulation of deterministic or stochastic models that translates into a GAUSS or a MATLAB program. Barillas et al. (2007) provide a good overview of Dynare implementations.

The parameters for the Nelson-Siegel framework are determined by running the Ordinary Least Squares (OLS) with the fixed decay parameter, $\tau = 50$, on a slightly altered version of equation (3.21). The equation to be estimated is:

$$R(m) = \beta_0 + \beta_1 \frac{1 - e^{-\frac{m}{\tau}}}{\frac{m}{\tau}} + \beta_2 \left(\frac{1 - e^{-\frac{m}{\tau}}}{\frac{m}{\tau}} - e^{-\frac{m}{\tau}} \right) \tag{3.28}$$

The interpolated curve is fitted using the cubic spline function. The parameters for the interest rate dynamics proposed by Vasicek (1977) and Cox, Ingersoll, and Ross (1985) are obtained using the Kalman filter. The algorithm of the Kalman filter and the estimated state-space formulation for the models are discussed in the following section. In this study there are as many datasets to be filtered by the Kalman algorithm as contracts to be evaluated. Each dataset consists of time series of yields with daily frequency and 21 maturities ranging from overnight to 10 years. The maturities create the cross-sectional dimension of each dataset. A standard approach in the literature is to employ a window of 100 to 200 daily observations to determine the coefficients. Initially, sets with 100, 150 and 200 observations preceding each transaction date were used. Given that all three cases yield similar output, only the results for the window of 100 observations are reported. The initial starting values chosen for the parameters of the Vasicek and the CIR models are $\kappa = 0.15$, $\sigma = 0.05$, $\lambda = -0.1$ and $\theta = 0.05$.

Given the generality of the Heath-Jarrow-Morton model, its implementation requires a specification of the volatility, $\sigma(t,T)$, for $0 \leq t \leq T \leq T^*$. As the same diffusion process appears in the rate dynamics under the actual and the risk-neutral probability measures, the historical data may be utilized to determine the volatilities. Next, $\sigma(t,T)$ serves to calculate integral $\sigma^*(t,T)$, and to parametrize formula (3.16). The adopted construction of the HJM model with parametric components follows Wu (2009) and Avellaneda & Laurence (1999) (see Wu (2009) pp. 94-106, and Avellaneda & Laurence (1999) pp. 239-247). The HJM model is built using the distributional properties of the time series of forward rates obtained from the spot yields. When the covariance among the rates is estimated, the Principal Component Analysis (PCA) is carried out to detect the random factors that drive the evolution of the rates. Essentially, PCA reduces the number of factors to the most informative ones. The empirical covariance[24] between $\Delta f_{n,i}$ and

[24]The method's presentation and the notations refer to (Wu 2009) p. 100. For more details, refer to

$\Delta f_{n,j}$ is given by:

$$\hat{c}_{i,j} = \frac{1}{K}\sum_{k=0}^{K}(\Delta f_{k,i} - \Delta^{-} f_i)(\Delta f_{k,j} - \Delta^{-} f_j)$$

where $\Delta f_{k,i}$ is a daily change of the forward rates for each maturity T_i, K is the number of daily observation, here 100, and $\Delta^{-} f_i$ is the sample mean:

$$\Delta^{-} f_i = \frac{1}{K}\sum_{k=0}^{K}(\Delta f_{k,i})$$

With the help of eigenvalue decomposition of the covariance matrix, $\hat{C}$, the change of the forward rates can be further expressed as:

$$\Delta f_{k,i} = \Delta^{-} f_i + \sum_{p=1}^{21} \sqrt{\lambda_p} v_k^{(p)} \xi_{p,k}$$

where λ_p are the covariance matrix eigenvalues, $v_k^{(p)}$ are the covariance matrix eigenvectors, $\xi_{p,k}$ are independently distributed with mean 0 and variance equal to 1. The most informative factors are those associated with eigenvalues that - when ranked - explain together about 99% of the empirical covariance. The uncovered key factors and their fitted functional forms are used to express the volatility of the forward rates. The next steps involve estimating the HJM curves with the Kalman procedure and valuation of the interest rate swap contracts.

3.4.3 The Kalman Filter

The Kalman filter is an algorithm that makes inferences about the values of unobserved state variables based on the observed data. As such, it necessitates a state-space representation of the dynamic system describing the model in question. State-space is a minimal space of states, which fully describes the system at any point in time. In a nutshell, the state-space formulation is characterized by the measurement and transition equations. The transition equation specifies how the state changes over time, while the measurement equation relates the observable output, possibly a control input at time t, and the unobservable state. Both equations include error terms. Generally speaking, the

the whole section ((Wu 2009), pp. 94-106).

state or transition equation takes the form[25]:

$$X_{t+1} = F\ X_t + M\ W_t + \epsilon_t \tag{3.29}$$

where X_{t+1} is a $k \times 1$ vector of the next period state, X_t is the current period state, F is a $k \times k$ transition matrix, W_t is a $r \times 1$ vector of exogenous variables, M is a $k \times r$ coefficient matrix and ϵ_{t+1} is a $k \times 1$ vector of the transition noise. The measurement or observation equation can be written as:

$$Y_t = H\ X_t + N\ W_t + \nu_t \tag{3.30}$$

where Y_t is a $n \times 1$ vector of the observed variables, H is the $n \times k$ system matrix, W_t is a $r \times 1$ vector of exogenous variables, N is a $n \times r$ coefficient matrix, ν_t is a $n \times 1$ vector of the measurement noise, and X_t is the unobserved state from equation (3.29). The residuals ϵ_t and ν_t have mean zero and covariance matrix Q and R respectively.

$$Cov(\epsilon_t, \epsilon_t) = Q\ ,\ Cov(\nu_t, \nu_t) = R \text{ and } Cov(\epsilon_t, \nu_t) = 0$$

A comprehensive description of the Kalman filter can be found in Harvey (1989). This section features solely the gist of how the the Kalman filter recursion works. In the Kalman methodology each observation adds to the information set at each run of the algorithm. While iterating for each observation data, three steps are performed: prediction, innovation and updating. The prediction stage involves obtaining the estimate for the next period state, $X_{t|t-1}$, based on the current period state value. First, the time 0 value of underlying state process, $X_{0|0}$, and the covariance matrix of the state vector, $P_{0|0}$, are guessed. Using the predicted state value, the predicted output value, $Y_{1|0}$, is computed and it serves to calculate the prediction error or innovation. In period t this stage produces:

$$X_{t|t-1} = F\ X_{t-1|t-1} + M\ W_{t-1}$$

$$P_{t|t-1} = F\ P_{t-1|t-1}\ F' + Q$$

$$Y_{t|t-1} = H\ X_{t|t-1} + N\ W_{t|t-1}$$

[25]The presentation draws upon Wang (2009), pp. 151-154. The application of the Kalman filter to the affine term structure models relies on Geyer and Pichler (1999).

where $P_{t|t-1} \equiv Var(X_t|F_{t-1})$, while $P_{t|t} \equiv Var(X_t|F_t)$. Thus, the time 1 state value X_1 and output value Y_1 are predicted with the help of state and observation equations respectively. The difference between the observation Y_1 and its prediction, $Y_{1|0}$, determines the so-called one-step prediction error. The goal is to minimize this prediction error based on the previous observations. The next step consists of updating the prediction for X_1 in the light of the observation Y_1. The update phase yields adjusted-predicted state value ($X_{t|t}$) and Kalman gain (K_t). Kalman gain is determined by taking the partial derivative of the adjusted predicted state variance with respect to the gain so as to minimize the state value variance based on the Kalman gain. The updating stage in time t produces:

$$K_t = P_{t|t-1}H'(HP_{t|t-1}H' + R)^{-1}$$

$$X_{t|t} = X_{t|t-1} + K_t(Y_t - Y_{t|t-1})$$

$$P_{t|t} = (I - K_tH)P_{t|t-1}$$

The adjusted predicted state value, $X_{1|1}$, serves as an input in the transition equation in the iteration for the periods from 2 to T. The Kalman filter propagates throughout the time. A joint likelihood function for all observations can be computed assuming that observable variables are serially independent and normally distributed. The parameters to be estimated are chosen such that they maximize the value of the score function. Maximum likelihood estimation of parameters is performed for each iteration. The parameters for the next period (iteration) are updated based on the parameters in the previous period.

Following the work of Duan & Simonato (1995), Geyer and Pichler (1999) and Babbs & Nowman (1999), the technique of (Kalman 1960) gained popularity in the affine term-structure literature. Geyer and Pichler (1999) obtains a quasi-optimal filter for the non-Gaussian state-space models. As discussed in Section 2, the building blocks of the theoretical interest rate models are state variables, which constitute the source of uncertainty in the economy, and processes characterizing their evolution. Thus, the state equation (3.29) corresponds to a discrete time version of the stochastic differential equations capturing the instantaneous interest rate dynamics. For the Vasicek framework the transition equation is a discretized equation (3.2), while for the CIR methodology a discretized equation (3.10). In the affine term structure models bond prices are of the affine form. The frameworks of Vasicek (1977) and Cox, Ingersoll, and Ross (1985) belong to the affine term structure models. Furthermore, the yields for this class of model can be expressed as:

$$R(t,T) = \frac{-\ln A(t,T) + B(t,T)r(t)}{T-t}$$

where $A(t,T)$ and $B(t,T)$ depend on the interest rate dynamics postulated by each model. Using this affine relationship, the term structure model can be recast in the state-space form. After the addition of a measurement error, the yield-to-maturity is characterized by:

$$R_t(X_t;\psi,T) = -\frac{1}{T-t}\ln(A(\psi,t,T)) + \frac{1}{T-t}B(\psi,t,T)X_t + \nu_t$$

where X_t is for both models the instantaneous interest rate, ψ denotes the model parameters and ν_t is an error term with 0 mean and standard deviation σ_{ν_t}. This is equation (3.30) formulated in the context of the affine term structure models and their notation. Given that claims with N maturities are traded in the market, the N corresponding yields give the following representation:

$$\begin{bmatrix} R_t(X_t,\psi,T_1) \\ R_t(X_t,\psi,T_2) \\ \vdots \\ R_t(X_t,\psi,T_N) \end{bmatrix} = \begin{bmatrix} -\ln(A(\psi,t,T_1))/(T_1-t) \\ -\ln(A(\psi,t,T_2))/(T_2-t) \\ \vdots \\ -\ln(A(\psi,t,T_N))/(T_N-t) \end{bmatrix} + \begin{bmatrix} B(\psi,t,T_1)/(T_1-t) \\ B(\psi,t,T_2)/(T_2-t) \\ \vdots \\ B(\psi,t,T_N)/(T_N-t) \end{bmatrix} X_t + \begin{bmatrix} \nu_{t,1} \\ \nu_{t,2} \\ \vdots \\ \nu_{t,N} \end{bmatrix}$$

In terms of the Vasicek model, the functional forms for $A(\psi,t,T)$ and $B(\psi,t,T)$ follow from equation (3.9) and are given by:

$$A(\psi,t,T) = \exp\left(\left(\theta + \frac{\sigma\lambda}{\kappa} - \frac{\sigma^2}{2\kappa^2}\right)(B(\psi,t,T) - (T-t)) - \frac{\sigma^2 B^2(\psi,t,T)}{4\kappa}\right)$$

$$B(\psi,t,T) = \frac{1}{\kappa}\left[1 - e^{-\kappa(T-t)}\right]$$

For the Cox-Ingersoll-Ross model, $A(\psi,t,T)$ and $B(\psi,t,T)$ are stipulated directly in the equation (C.2.3) as:

$$A(\psi,t,T) = \left[\frac{2\gamma e^{(\kappa+\gamma+\lambda)(T-t)/2}}{(\gamma+\kappa+\lambda)(e^{\gamma(T-t)}-1)+2\gamma}\right]^{2\kappa\theta/\sigma^2}$$

$$B(\psi,t,T) = \frac{2(e^{\gamma(T-t)}-1)}{(\gamma+\kappa+\lambda)(e^{\gamma(T-t)}-1)+2\gamma}$$

$$\gamma = \sqrt{(\kappa + \lambda)^2 + 2\sigma^2}$$

Given that both models propose the same form for the interest rate drift, for both cases F and M in the equation (3.29) can be represented as:

$$\begin{aligned} M(\psi, \Delta t) &= \theta(1 - e^{-\kappa \Delta t}) \\ F(\psi, \Delta t) &= e^{-\kappa \Delta t} \end{aligned}$$

where Δt is the length of the discrete time interval. With daily observations and 20 maturities traded, the time interval equals 1 and N is 20. The variance of the transition residuals is given by the discretized conditional variance of the instantaneous rate. Therefore, the variance takes the following forms:

$$\begin{aligned} Vasicek: \quad Q(X_t, \psi, \Delta t) &= \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa \Delta t}) \\ CIR: \quad Q(X_t, \psi, \Delta t) &= X_t \frac{\sigma^2}{\kappa}(e^{-\kappa \Delta t} - e^{-2\kappa \Delta t}) + \theta \frac{\sigma^2}{2\kappa}(1 - e^{-\kappa \Delta t})^2 \end{aligned}$$

Derivations of the conditional moments in the Vasicek and the Cox-Ingersoll- Ross framework are provided in the Appendix. As discussed in Section 2, Cox, Ingersoll, and Ross (1985) propose a non-Gaussian model. Geyer and Pichler (1999) show how to apply the Kalman filter to the non-Gaussian statespace representation and obtains the quasi-maximum likelihood estimates of the model parameters.

3.5 Empirical Results

The estimated term structures are depicted in Figure 3.9 to Figure 3.12. A visual inspection of the fitted curves suggests a resemblance of the Nelson- Siegel and cubic spline interpolation outcomes. At the same time, the term structures modeled by the Vasicek and the Cox-Ingersoll-Ross methodologies differ significantly from the others and assume a greater variety of shapes through time.

The variation in the level, the slope and the curvature of the examined term structures, visible in Figure 3.8, is reflected in the descriptive statistics in Table 3.8 for the estimated level, slope and curvature parameters. The long-term interest rate implied by the Nelson-Siegel model is always positive and on average equals 7.13%. However, the median and mode indicate a lower level of the long end of the term structure, 5.73% and 4.92%

Figure 3.9: Nelson-Siegel Model: Fitted Yield Curves from August 2000 to November 2007

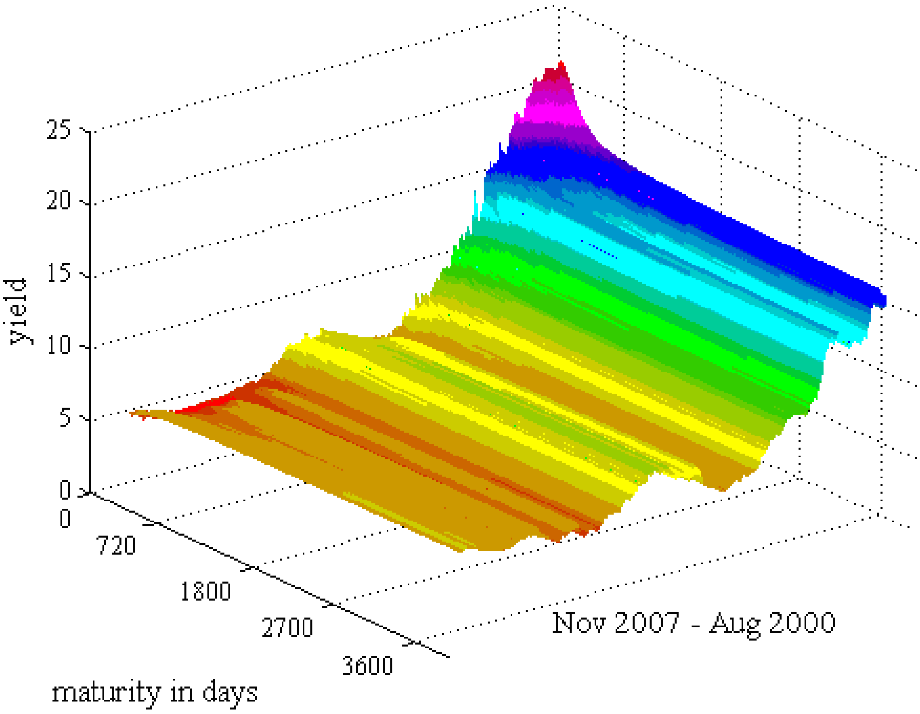


Figure 3.10: Cubic Interpolation: Fitted Yield Curves from August 2000 to November 2007

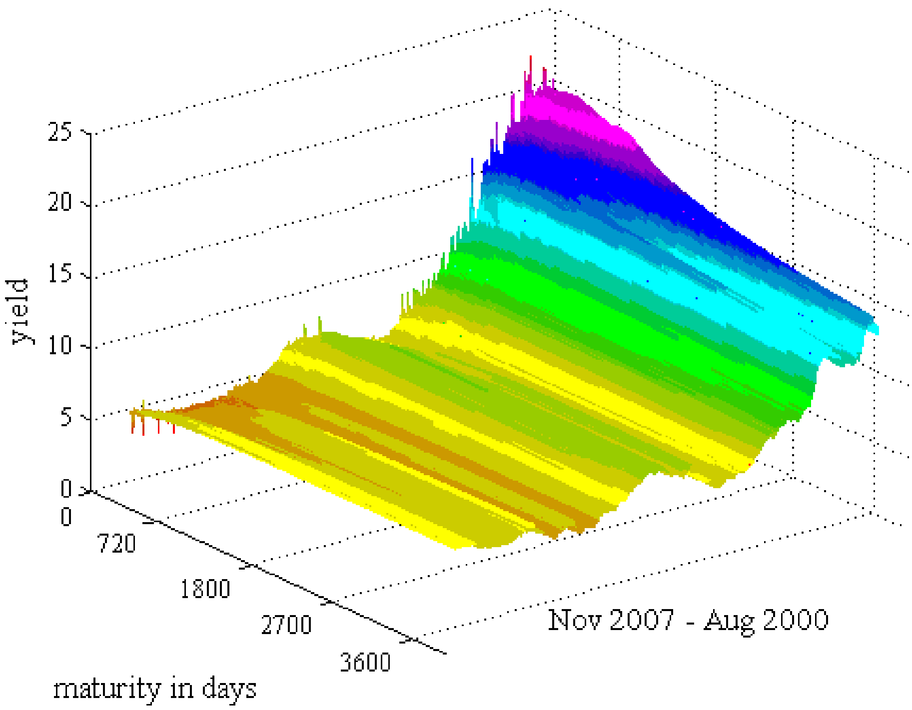


Figure 3.11: Vasicek Model: Fitted Yield Curves from August 2000 to November 2007

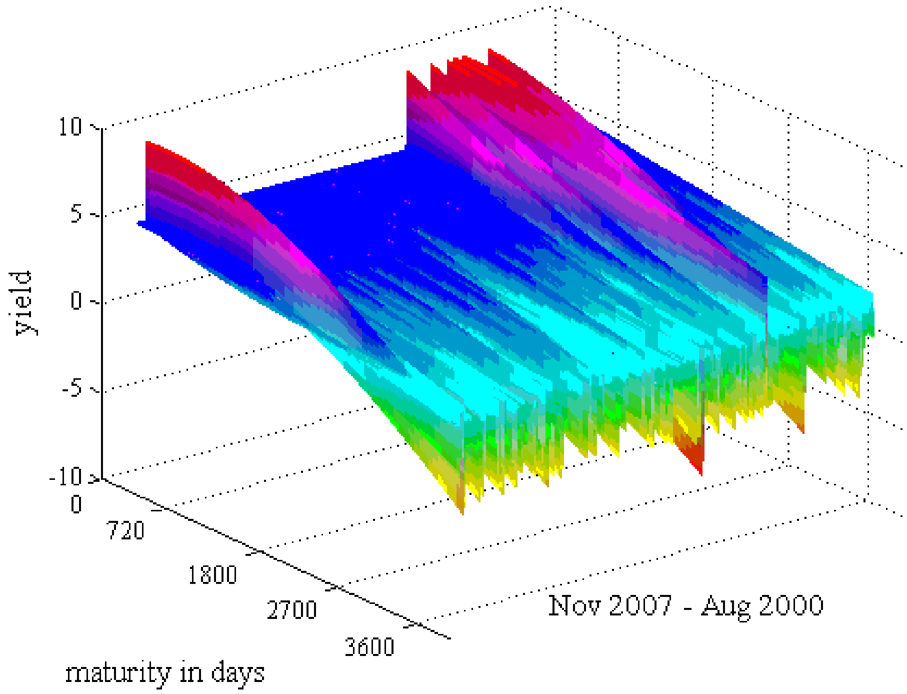


Figure 3.12: Cox-Ingersoll-Ross Model: Fitted Yield Curves from August 2000 to November 2007

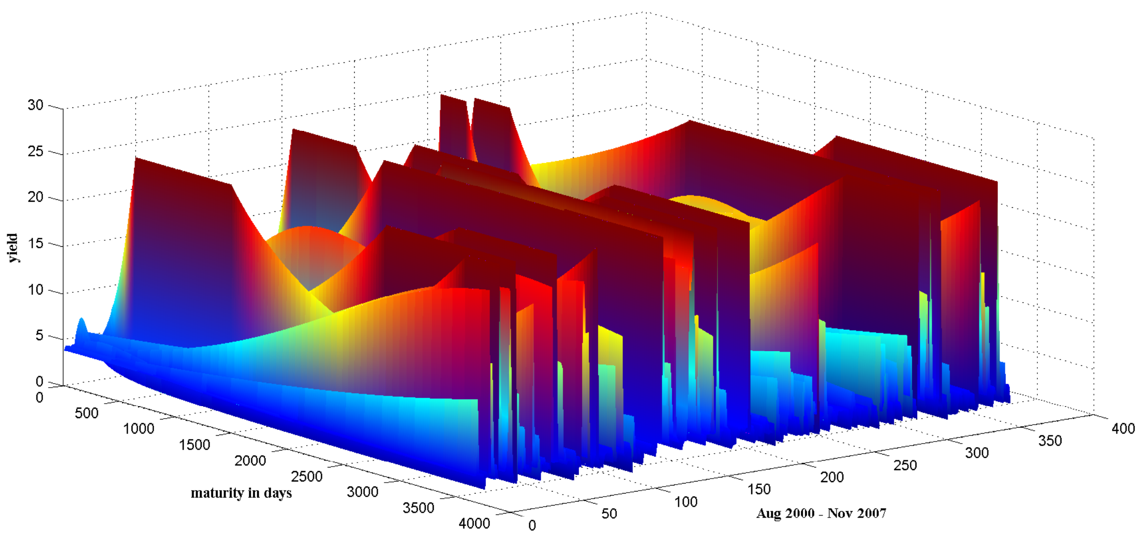


Table 3.8: Summary Statistics for the Nelson-Siegel Estimated Parameters

Measure	β_0	β_1	β_2	β_{short}
Mean	0.0713	0.0036	0.0137	0.0749
Median	0.0573	-0.0012	-0.0021	0.0547
Mode	0.0492	-0.0240	-0.0321	0.0369
Standard deviation	0.0286	0.0198	0.0493	0.0447
Minimum	0.0427	-0.0440	-0.0438	0.0319
Maximum	0.1581	0.0738	0.3288	0.2082

Note: Summary of the Nelson-Siegel model parameters estimated for each working date from August 29, 2000 to November, 1 2007. Measures computed across all estimations. β_{short} determined based on β_0 and β_1 results.

respectively. The average yield curve, that is the curve corresponding to the mean values of β_0, β_1 and β_2, is increasing (positive β_1), but inverted (positive β_2). Still, the typical upward sloping yield curve is implied by the median and mode values of the estimated parameters. The development of the curvature parameter in Figure 3.13 shows that the concave curve persisted in the second half of the sample interval. Another stylized fact about the term structure of interest rates, a higher volatility of the short end than the long end of the curve, is obtained within the Nelson-Siegel framework by construct. As shown in the Appendix, the long-term interest rate is captured by β_0, while the short rate by $(\beta_0 + \beta_1)$. Greater short-term volatility follows. What is more, for the examined sample the estimates of $(\beta_0 + \beta_1)$ are always positive, and so is the short rate. Finally, Figure 3.13 demonstrates that the estimated curves assume a variety of shapes through time. The discussion of the models' goodness of fit is deliberately postponed to the end of this section.

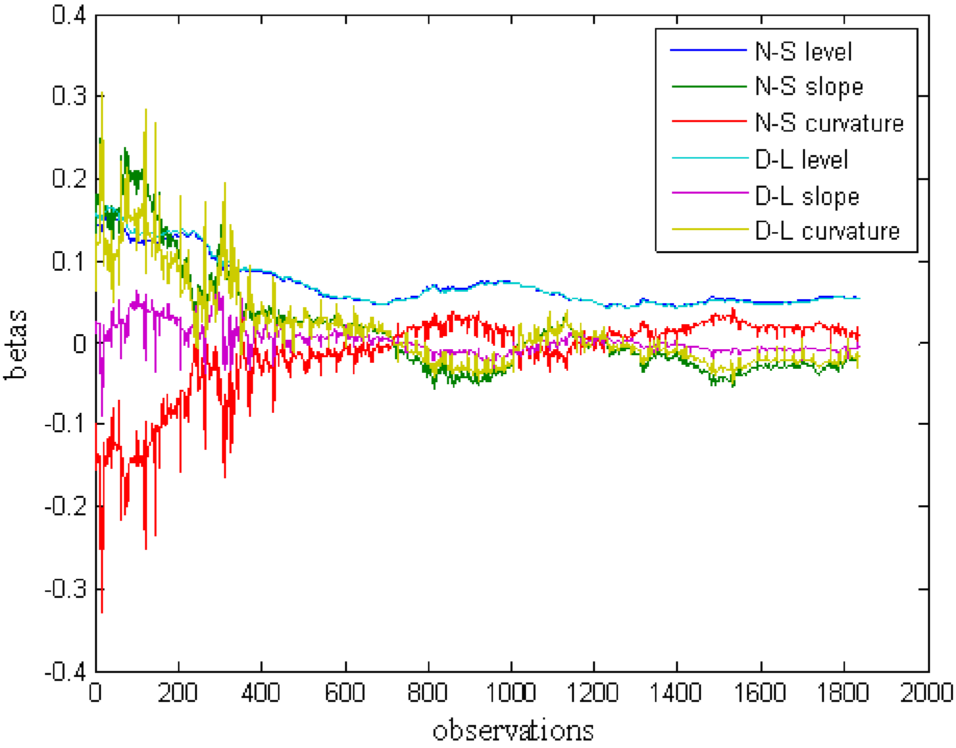
Table 3.9: Summary of the Vasicek & CIR Estimated Parameters

Model	θ	κ	σ	λ
Average Estimated Parameters				
Vasicek	0.053	0.005	0.086	-0.133
CIR	0.056	0.006	0.016	-0.009
Share % of Significant Parameters				
Vasicek	99.73	100.00	100.00	100.00
CIR	98.92	90.24	85.91	93.50

Note: Summary of the Vasicek & CIR model parameters estimated for each IRS deal time. The average of estimated parameters and the percent of cases the parameters are significant. The average across transactions.

The parameter estimates obtained with the Vasicek methodology are significant in all but one case for θ. The Cox-Ingersoll-Ross model also yields mostly significant estimated

Figure 3.13: Evolution of the Nelson-Siegel Model Parameters



parameters (in over 85% of the cases). The estimation results are summarized in Table 3.9. The average long-term interest rate implied by the Vasicek and the Cox-Ingersoll-Ross frameworks equal 5.3% and 5.6% respectively. Given that both models propose the same form for the drift of the interest rate process, those close results for θ are as expected. By the same token, the average estimates for κ should be and are similar (0.005 and 0.006 respectively). Those results are not encouraging and imply a very slow mean reversion process. The Vasicek and the CIR methods assume different volatility functions of the interest rate dynamics, σ versus $\sigma\sqrt{r}$. The obtained volatility parametric values are necessarily different (0.086 versus 0.016). Both estimates for volatilities are comparable in magnitude with the empirical evidence (see, for example, Duan & Simonato (1995) or Gibbons and Ramaswamy (1993)). Finally, (Vasicek 1977) and Cox, Ingersoll, and Ross (1985) deviate in the formulation of the market price of risk. The estimated λ parameters for the Vasicek and the CIR models are -0.133 and -0.009, which implies positive risk premia. Figures 3.14 and 3.15 display the evolution of the model parameters across transactions for the Vasicek and the Cox-Ingersoll-Ross method respectively.

Figure 3.14: Evolution of the Estimated Vasicek Model Parameters Across Transactions

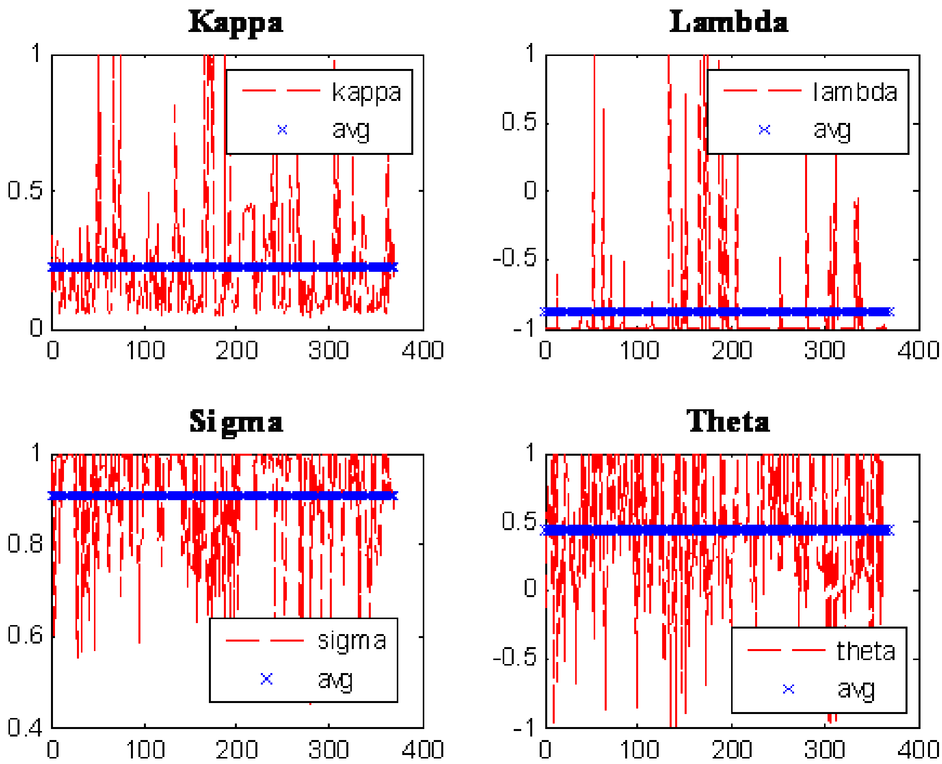
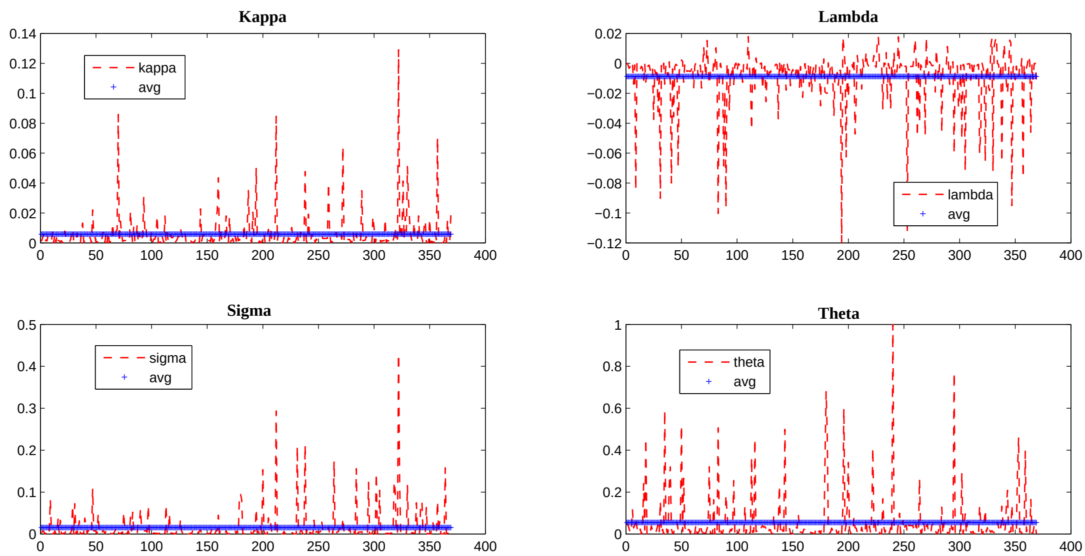


Figure 3.15: Evolution of the Estimated CIR Model Parameters Across Transactions



The Principal Component Analysis produced the following volatility components for the Heath-Jarrow-Morton framework:

$$\sigma_1(t,T) = \alpha_1 \left(1 - \exp\frac{-(T-t)}{\beta_1}\right)$$
$$\sigma_2(t,T) = \alpha_2 \left(1 - \beta_2 \exp\frac{-(T-t)}{\gamma_2}\right) + \delta_2$$

Their fit is presented in Figure 3.16 and 3.17, respectively. The estimated volatility parameters equal $\alpha_1 = -0.33$, $\beta_1 = 1.36$, $\alpha_2 = -0.33$, $\gamma_2 = -0.47$, $\delta_2 = 0.47$ and $\beta_1 = 1.36$. The fitted values serve as the initial starting values in the estimation of the HJM model with the Kalman filter.

Figures 3.18 and 3.19 present the evolution of the model parameters across transactions. The application of the Kalman filter to the two-factor Heath- Jarrow-Morton model produces statistically insignificant estimates in as many as 98% of the cases. Estimating the HJM framework with solely the first principal component driving the rate volatility yields insignificant parameters in 80% of the cases. The multi-factor model, which requires more parameter input and adds complexities, generates more noise and imprecision. Next, I experiment with the GARCH-family of models for the forward rates selected in Zhou (2002). The results are equally discouraging. Most probably the problem lies in the numerous approximations necessary to build and calibrate the HJM model. Other possible explanations and solutions will be considered in the extension of this work. At this stage, I refrain from calculating and presenting the contract values obtained for the model of Heath, Jarrow, and Morton (1992).

Table 3.10: Test for the Mean Contract Values Equality

Pairs of means	*t-statistic*	*Decision*
Cubic vs. N-S	-7.633	reject H_0
Cubic vs. Vasicek	8.627	reject H_0
Cubic vs. CIR	1.483	accept H_0
N-S vs. Vasicek	9.765	reject H_0
N-S vs. CIR	1.761	accept H_0
CIR vs. Vasicek	-0.088	accept H_0

Note: The null hypothesis: equal means of IRS contract value for the two respective yield curve models (paired samples). The test statistics is distributed with 368 degrees of freedom. The reported decision considers a 5% significance level.

The fitted curves for the Nelson-Siegel, the Vasicek, the Cox-Ingersoll- Ross and the cubic spline models are used to compute the values of IRS. The ultimate aim of this

Figure 3.16: Heath-Jarrow-Morton: Forward Rate Volatility Component

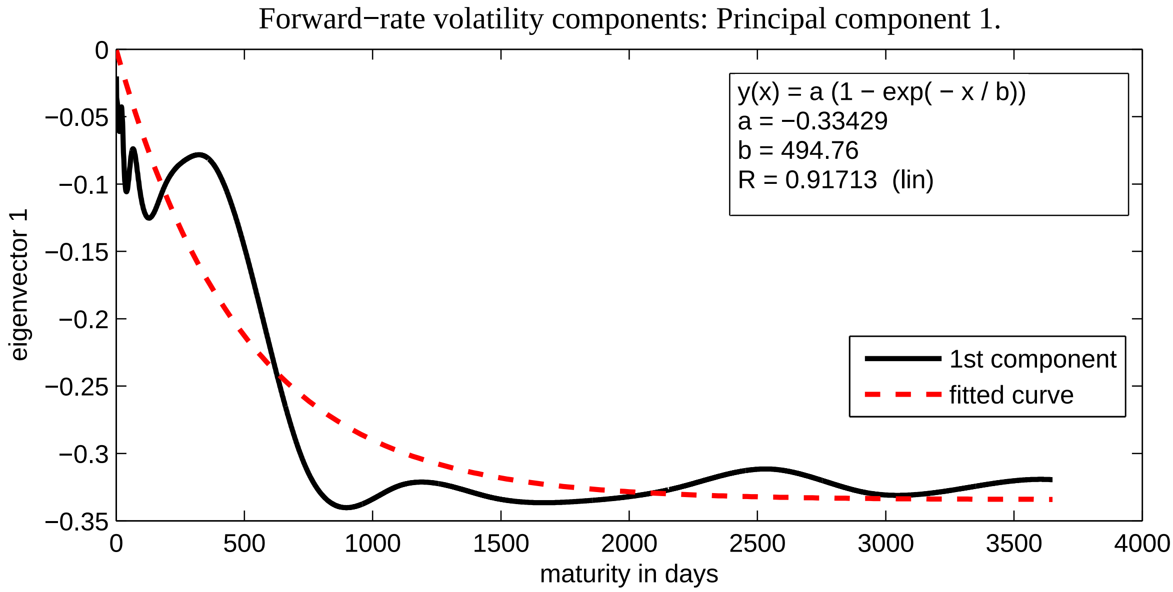


Figure 3.17: Heath-Jarrow-Morton: Forward Rate Volatility Component

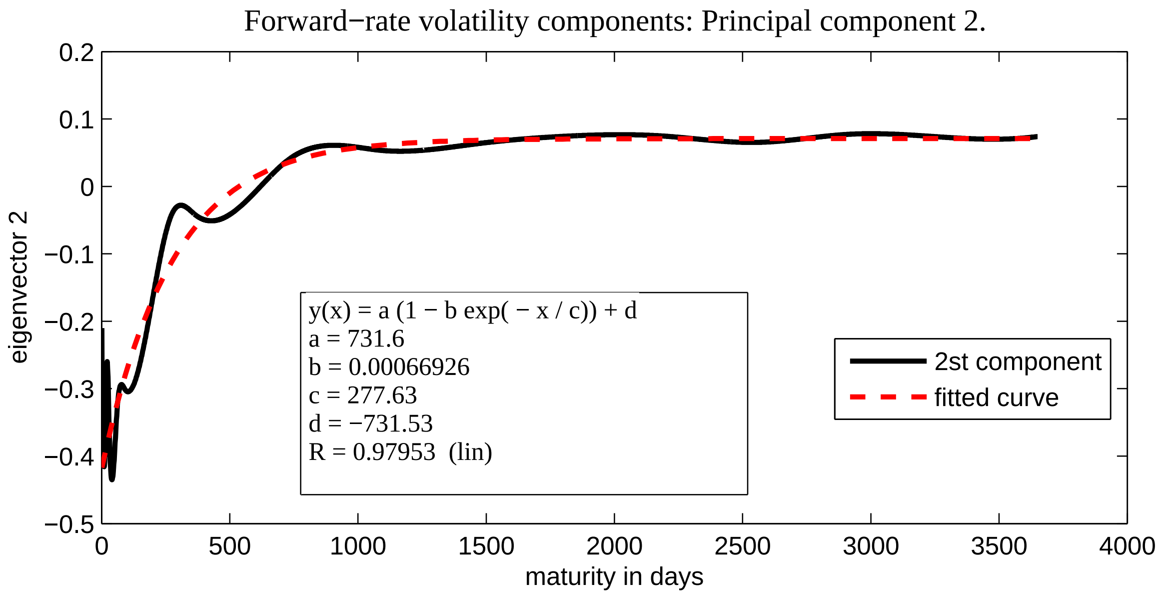


Figure 3.18: Evolution of the Estimated HJM Model Parameters Across Transactions

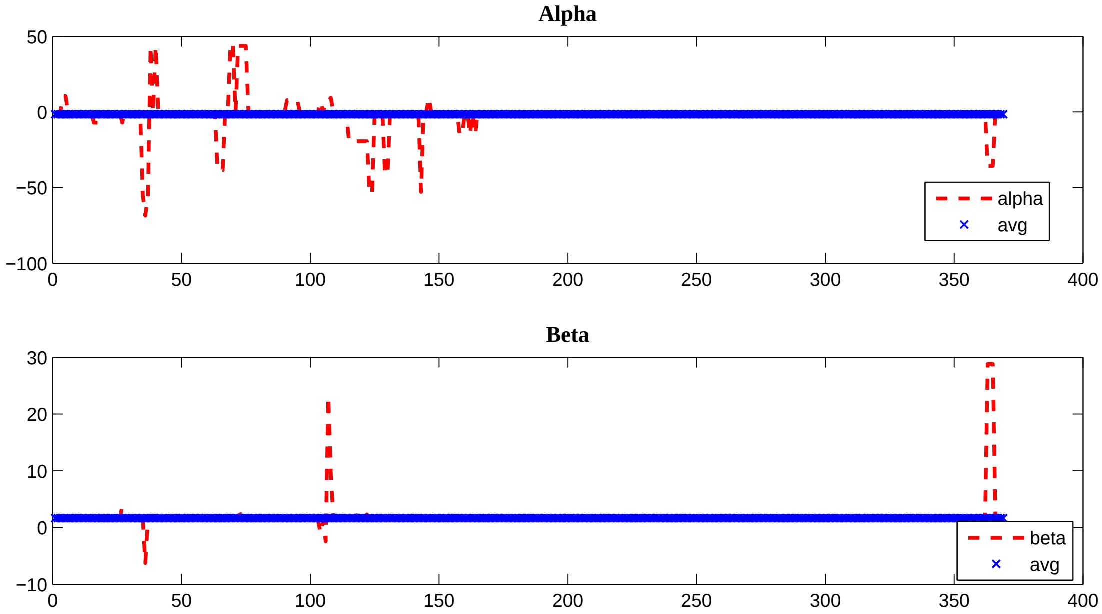
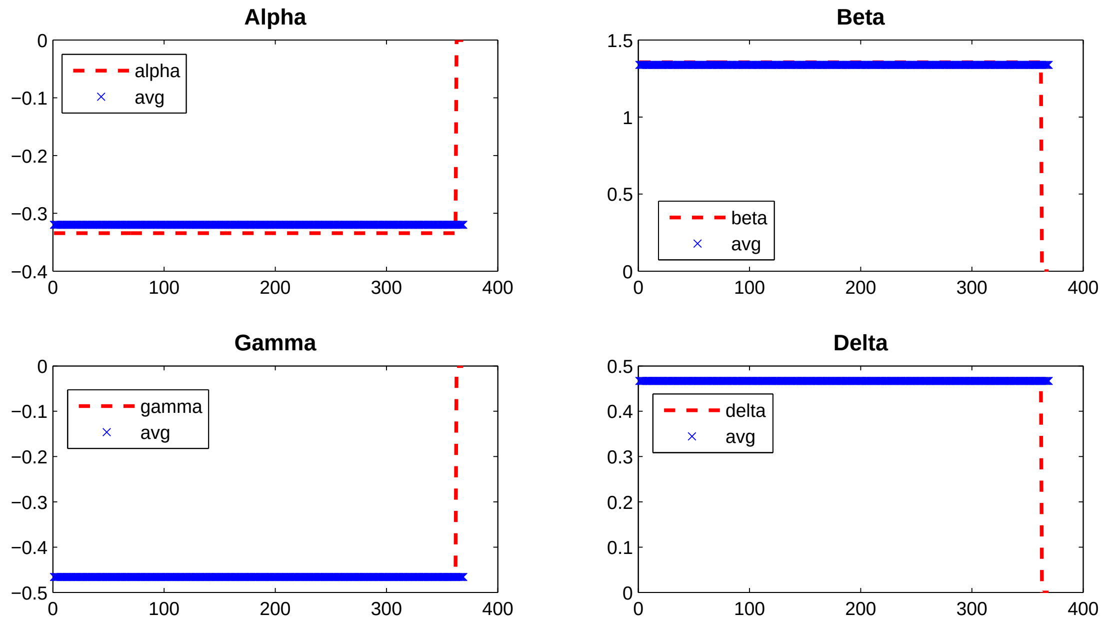


Figure 3.19: Evolution of the Estimated HJM Model Parameters Across Transactions



study is to evaluate the performance and predictive accuracy of the four interest rate models, when applied to the interest rate swap agreements. The ex post forecasts of the term structure models are assessed by contrasting the 'fitted' and realized contract values. First, I determine whether the four term structure methodologies deliver similar IRS evaluations. The t-test for the equality of mean contract values under the four different approaches indicates that the mean deal values do not statistically differ for the pairs of the Cox-Ingersoll-Ross and cubic interpolation, the Nelson-Siegel and the Vasicek fitted curves respectively. Thus, the CIR approach produces IRS values relatively close to all other methodologies. At the same time, the term structures generated by the Vasicek model, cubic interpolation and the Nelson-Siegel methodology yield on average net present values of swaps statistically different from each other. The corresponding t-statistics for the test are presented in Table 3.10. The null hypothesis assumes no difference in the means of IRS values for paired samples. The reported decision considers a 5% significance level.

Table 3.11: Descriptive Statistics for the Ex-post Accuracy of the Fitted Curves in Relative Terms

Measure	*Cubic*	*N-S*	*Vasicek*	*CIR*
Mean	-0.014	-0.07	0.365	0.127
Median	0.046	0.036	0.100	0.115
Mode	0.047	0.079	-0.242	0.064
Minimum	-11.618	-11.254	-13.698	-51.335
Maximum	2.238	2.095	5.738	14.552
Standard deviation	0.877	0.822	1.214	5.138
MAE	0.325	0.299	0.59	2.178
RMSE	0.876	0.824	1.266	5.132

Note: The measures calculated for the differences between the realized contract values and fitted contract values expressed as % of the contract notionals. The fitted contract values are determined based on the yields from the fitted term structures.

The remaining question is which of the fitted curves performs better when applied to the valuation of real-life contracts and whether a more elegant and precise solution for interest rate modeling proposed by Vasicek (1977) or Cox, Ingersoll, and Ross (1985) is worth the cost of considerably higher mathematical sophistication. To address this matter one needs to contrast the evaluation of the sample deals under the four methodologies with the realized transaction values. The realized values are computed based on the interest rate that are quoted by the market at the date of payment exchange. Those realized rates determine the amounts to be swapped and thus the effective contract value. By examining the differences between the realized IRS exchanges and the deal amounts computed under the four approaches, I establish the ex post predictive accuracy of the four interest rate models. Tables 3.11 and 3.12 show the descriptive statistics for the differences in the fitted and realized contract values expressed as a percent of notional amounts and in thousand of euros respectively.

Table 3.12: Descriptive Statistics for the Ex-post Accuracy of the Fitted Curves in Absolute Terms

Measure	*Cubic*	*N-S*	*Vasicek*	*CIR*
Mean	10.87	-0.39	72.38	68.96
Median	14.05	9.44	36.86	41.73
Mode	123.76	116.83	-42.03	-118.98
Standard deviation	384.93	350.72	182.01	810.07
Minimum	-1,783.15	-1,681.74	-2,046.87	-7,671.16
Maximum	384.93	350.72	994.83	3118.03
RMSE	124.07	116.67	195.65	811.90

Note: The measures calculated for the differences between the realized contract values and fitted contract values in '000 EUR. The fitted contract values are determined based on the yields from the fitted term structures.

The discrepancies in the fitted and realized contract values amount on average to less than 0.1 percent of the notional amounts for the cubic spline, Nelson-Siegel and CIR methodologies. The first two methods generate negative differences between the realized and fitted deal values and thus they tend to overestimate the fitted values. In the case of the Vasicek model the mean deal difference is almost 3 times higher than for the Cox-Ingersoll-Ross case. Both models result in the underestimated fitted contract values. On the other hand, the mean absolute error (MAE) indicates that, irrespective of the direction being positive or negative, the CIR model yields the most different IRS values from the effective deal values. Here the discrepancies do not cancel out and on average amount to over 2 percent of the notional amounts. This tendency is confirmed by the most pronounced root mean squared error (5.13%), standard deviation (5.13%), minimum (-51.33%) and maximum (14.55%) values obtained for the case of CIR versus realized contract valuation. The Vasicek fitted curves generate more significant minimum (-13.70%) and maximum (5.74%) differences in contract values than the cubic spline and the Nelson-Siegel methodology. The Vasicek model also produces more volatile differences in contract values. The higher volatility corroborates with the unstable evolution of the Vasicek fitted curves presented on Figure 3.14. Finally, all measures in Tables 3.11 and 3.12, but the mode, indicate that the deal values closest to the realized ones are obtained under the cubic spline and the Nelson-Siegel model. Tables 3.11 and 3.12 characterize the performance of the four term structure models for the set of interest rate swaps at hand. The true question is whether the sample mean differences between the realized and fitted interest rate swap values are statistically significant.

Table 3.13: Test for the Mean Equality: Realized vs. Fitted Contract Values

Pairs of sample means		*t-statistic*	*Decision*
Relative to Notional	Realized vs. cubic	-0.307	accept H_0
	Realized vs. N-S	-1.628	accept H_0
	Realized vs. Vasicek	5.770	reject H_0
	Realized vs. CIR	0.476	accept H_0
Absolute Terms	Realized vs. cubic	1.687	accept H_0
	Realized vs. N-S	-0.063	accept H_0
	Realized vs. Vasicek	7.639	reject H_0
	Realized vs. CIR	1.635	accept H_0

Note: The null hypothesis considers equal means of realized and fitted IRS contract values for the paired samples. The test statistics in the upper module refer to the deal values per notional amounts. The lower module simply presents the results for the paired mean contract values realized and fitted. The test statistics are distributed with 368 degrees of freedom. The reported decision relates to a 5% significance level.

Table 3.13 presents the results of the t-test for the equal means of the realized and fitted IRS contract values. The realized transaction values are calculated based on the interest rates that are quoted by the market at the date of payment exchange. Those realized rates determine the amounts to be swapped. The fitted IRS deal values are computed using the yields generated by each model. The t-statistics indicate that the fitted mean transaction values do not statistically differ from the mean realized values under three out of the four methodologies: the cubic interpolation, the Nelson-Siegel and the Cox-Ingersoll-Ross approach. The term structures generated by the Vasicek model yield the net present values of interest rate swaps statistically different from the realized deal amounts. The null hypothesis assumes no difference in the sample means of IRS fitted and realized deal values. The test statistics in the upper module refer to the deal values per notional amounts. The relative means show to what extent the values differ per 1 euro of notional and the t-statistics refer to such notional-weighted means. The lower module simply presents the results for the realized and fitted mean contract values. The reported decisions relate to a 5% significance level.

The poor results of the Vasicek model raise a question as to whether the outcome is not driven by a different number of estimated parameters. To check whether this might be the case, I compute the Akaike information criterion (AIC) for the curves obtained with the Nelson-Siegel, the Vasicek and Cox-Ingersoll- Ross models. Additionally, I provide the root mean square error (RMSE) for the estimated yields. The measures are presented in Table 3.14. Both measures suggests that the Nelson-Siegel outperforms the Vasicek model, and AIC suggests that the number of parameters is not the reason for the the Vasicek underperformance. Contrasting the low and high performance of the Vasicek and the Nelson-Siegel methodologies can be put in a broader perspective. Within the Nelson-Siegel exponential framework the entire yield curve is estimated for every time the interest rate swap contract is initiated. Such an approach centers on accurately fitting the cross section of interest rates at any given time. In the Vasicek model the yield curve is estimated at every contract initial time using a window of observation prior to the contract beginning. The model works with a structure of four parameters fixed for a window of observations. Therefore, Vasicek's framework focuses on a time-series dynamics and imposes a structure across time, which the Nelson- Siegel does not. As the Vasicek approach is more restrictive and focused on a time-series dynamics, one might hope that it will deliver better forecasts. However, the test results for the realized versus fitted contract values indicate the opposite. In this case a current yield curve is more informative than the historical time series of yields.

Table 3.14: Measures of Fitted Curves Accuracy

Measure	*N-S*	*Vasicek*	*CIR*
AIC	-25,145	-11,485	-14,807
RMSE	0.90	2.97	1.35

Note: The measures calculated for the yields observed in the market corresponding to the maturities of claims traded in the interbank market. By construct, the cubic spline interpolation perfectly fits the traded yields.

Finally, the Cox-Ingersoll-Ross model exhibits a significantly greater ability to capture the dynamics of the interest rate than the Vasicek framework. This outcome is consistent, among others, with (Chan et al. 1992), who document that the interest rate volatility is an increasing function of $r(t)$, as postulated by Cox, Ingersoll, and Ross (1985).

3.6 Conclusions

This chapter determines the usefulness of advanced term structure models in the valuation of plain vanilla interest rate sensitive derivatives. In particular, examines five term structure models in order to compare their ability to capture the interest rate dynamics and value the interest rate swaps in the Polish market. I analyze the methodologies proposed by Nelson and Siegel (1987), Vasicek (1977), Cox, Ingersoll, and Ross (1985) and Heath, Jarrow, and Morton (1992) as well as the cubic spline curves. A special case of the Heath-Jarrow-Morton model with the volatility corroborated by the observable market data produces mostly unreliable fitted curves that are not used for the valuation. The tentative explanations link the HJM insignificant outcome to the technical issues. The tests for the equality of mean contract values under the four other approaches indicate that the Cox-Ingersoll-Ross approach produces IRS values relatively close to all other methodologies. More importantly, the Nelson-Siegel, the cubic interpolation and the CIR models return fitted transaction values that on average do not differ from the realized contract values. The ex-post accuracy of the Nelson-Siegel and the cubic spline values suggests that a current yield curve is highly informative for the future yields. Furthermore, the Vasicek yield curves generate very different net present values of interest rate swaps which statistically differ from the actually swapped amounts. The solid performance of the Cox-Ingersoll-Ross methodology, especially as opposed to Vasicek's approach, supports the conjecture of the rate-reliant nature of the interest rate volatility. The Vasicek model's underperformance emphasizes the role of the cross section of interest rates, and thus the importance of no-arbitrage argument.

As this study investigates the value added of theoretically-grounded term structure models in valuation of real life swap contracts, it refers to model risk. Model risk is typically associated with complex derivatives, however, the choice of a plain interest rate derivative allows for examination of model risk in the case of a dynamically growing OTC market, such as the Polish one. In this context, I obtain that the less computationally demanding approaches to the term structure modeling perform at least as well the more advanced solutions. Thus, in practice the latter may not always be worth the cost of a considerably higher mathematical sophistication and should result in smaller exposures to model risk.

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Appendix

A Appendices to Chapter 1

A.1 Appendix A.I to Chapter 1

Table A1.1: Variables Definition

Variabls	**Definition**
RISK	the ratio of risk-weighted assets to total assets, *RWATA*, reported under the Basel Capital Accord
CAP	the ratio of the bank' total capital to total assets
LIQ	the ratio of liquid assets to total assets, liquid assets defined as a sum of the bank's cash, reverse repos, bills and commercial papers
SEC	a ratio of the sum of securitization deal values in current year to the total assets at the end of previous year; deals included asset backed securities and mortgage backed securities issued based on the loans granted by the bank
SIZE	the logarithm of the bank's total assets
ROA	the bank's return on assets
LLOSS	a ratio of loan loss provisions to the sum of bank's loans net loan loans reserves
$\uparrow$ *LOAN*	a ratio of new loans granted in the current year to the amount of loans on the bank's balance sheet at the end of the previous year; the new lending volume expressed as a difference between the gross loans at the end of the current and previous year
LTA	a ratio of net loans to total assets

Table A1.2: Correlations Between Variables in Levels

Variables	*RISK*	*CAP*	*LIQ*	*SEC*	*SIZE*	*ROA*	*LLOSS*	$\uparrow$ *LOAN*
RISK	1.00							
CAP	0.67	1.00						
LIQ	−0.21	−0.01	1.00					
SEC	−0.04	−0.06	−0.07	1.00				
SIZE	−0.53	−0.62	−0.19	0.11	1.00			
ROA	0.40	0.48	−0.03	−0.04	−0.29	1.00		
LLOSS	0.06	0.03	−0.11	−0.00	−0.03	−0.10	1.00	
$\uparrow$ *LOAN*	0.05	0.05	−0.06	0.03	−0.02	0.13	0.08	1.00

Table A1.3: Cross-Correlations Between First-Differenced Endogenous Variables

Variables	$\Delta RISK$	ΔCAP	ΔLIQ	SEC	$SIZE$	ROA	$LLOSS$	$\uparrow LOAN$
$\Delta RISK$	1.00							
ΔCAP	0.42	1.00						
ΔLIQ	−0.21	−0.06	1.00					
SEC	0.01	−0.01	0.03	1.00				
$SIZE$	−0.03	−0.01	−0.01	0.11	1.00			
ROA	0.08	0.17	−0.05	−0.04	−0.29	1.00		
$LLOSS$	0.01	−0.08	−0.03	−0.00	−0.03	−0.10	1.00	
$\uparrow LOAN$	−0.05	−0.13	−0.21	0.03	−0.02	0.13	0.08	1.00

Table A1.4: Distribution of Banks Across Years

Year	*No. of Banks*
2000	209
2001	213
2002	201
2003	212
2004	367
2005	423
2006	443
2007	409
Total	2,477

A.2 Appendix A.II to Chapter 1

Repullo (2005) models the behavior of a bank facing random withdrawal of deposits and supervised by the lender of last resort (LLR) that, if needed, decides about providing the support based on the information on the quality of banks assets. Both agents are risk neutral. The bank chooses the liquidity buffer that it wants to hold against deposit withdrawals (λ), a parameter $p \in [0,1]$ linked to the risk of its loan portfolio $(1-p)$ and level of capital (k) subject to capital requirements (κ). The bank's total assets are normalized to 1. The economy lasts from period 0 to 2. At $t = 0$ the bank attracts $(1-k)$ insured deposits and raises (k) equity capital. At the same time, it invests in a safe and perfectly liquid assets (λ) as well as illiquid and risky assets $(1-\lambda)$. It is important to emphasize that the risk of the illiquid assets (i.e. loan portfolio), denoted above by $1-p$, is chosen by the bank. The cost of deposits, the interest rate, is normalized to zero. Still, the bank's equity providers demand a return of $\delta \geq 0$. Since the bank is subject to capital requirements it must hold that:

$$k \geq \kappa(1-\lambda) \tag{A.2.1}$$

As mentioned above, the return on the illiquid asset depends on the level of its risk chosen by the bank. Specifically, the illiquid asset yields:

$$R = \begin{cases} R_1 = R(p), \text{ with probability p} \\ R_0 = 0, \quad\;\; \text{with probability } 1-\text{p} \end{cases} \tag{A.2.2}$$

It is assumed that $R'(p) < 0$ and $R"(p) \leq 0$. In addition, $R(1) \geq 1$ and $R(1) + R'(1) < 0$. At time $t = 1$ a fraction $\nu \in [0,1]$ of deposits is withdrawn. From the perspective of the initial date, at time $t = 0$, ν is a continuous random variable assuming values between 0 and 1 with the cumulative distribution function $F(\nu)$. Since the bank has $(1-k)$ deposits, $\nu(1-k)$ is withdrawn at date 1. If $\nu(1-k) \leq \lambda$, the bank can repay the deposits using its liquidity buffer λ. In such a case the bank keeps $\lambda - \nu(1-k)$ in the safe asset and its payoff in the high-return is given by:

$$\lambda - \nu \cdot (1-k) + (1-\lambda) \cdot R(p) - (1-\nu) \cdot (1-k) = (1-\lambda) \cdot [R(p) - 1] + k \tag{A.2.3}$$

When the opposite situation happens and $\nu \cdot (1-k) > \lambda$, the supervisory intervention is necessary. If the bank obtains the missing part of the funding, that is $\nu \cdot (1-k) - \lambda$, its payoff in the high-return state equals:

$$(1-\lambda) \cdot R(p) - (1-\nu) \cdot (1-k) - [\nu \cdot (1-k) - \lambda] = (1-\lambda) \cdot [R(p) - 1] + k \tag{A.2.4}$$

By limited liability, the bank's payoff in both low-return states is zero. The LLR decides about providing assistance based on the signal on the bank's asset quality s. The information is solely about the high or low return on asset and not about a particular realization of $R(p)$. Let s_1 stand for the good and s_0 for the bad supervisory signal. It is further assumed that the quality of the supervisory information is described by a parameter $q = P(s_0|R_0) = P(s_1|R_1) \in \left[\frac{1}{2}, 1\right]$. Using the fact that $P(R_1|s) = 1 - P(R_0|s)$ and the Bayes' rule, the probabilities of high return given the two types of supervisory signal can be expressed as:

$$P(R_1|s_0) = \frac{p \cdot (1-q)}{p \cdot (1-q) + q \cdot (1-p)} \tag{A.2.5}$$

$$P(R_1|s_1) = \frac{p \cdot q}{p \cdot q + (1-q) \cdot (1-p)} \tag{A.2.6}$$

If the LLR decides to provide emergency lending based on the signal s observed at time 1, then at date 2 the supported bank can prove solvent with probability $P(R_1|s_1)$ or insolvent with probability $P(R_0|s_1)$. When solvent, the bank repays $\nu \cdot (1-k) - \lambda$ to the LLR. Its failure, on the other hand, causes the LLR to incur a social cost c and the loss of the rented amount, $\nu \cdot (1-k) - \lambda$. The lack of funding assistance causes the bank's liquidation at time 1, which generates the cost c to the LLR. Therefore, the LLR provides the assistance as long as:

$$-[\nu \cdot (1-k) - \lambda + c]P(R_0|s) \geq -c \tag{A.2.7}$$

The LLR will support the bank if the liquidity shock, ν, meets:

$$\nu \leq \frac{c \cdot P(R_1|s) + \lambda \cdot P(R_0|s)}{P(R_0|s) \cdot (1-k)} \tag{A.2.8}$$

Repullo (2005) obtains critical values for the liquidity shocks in the model without the liquidity buffer and bank equity. Under such conditions, the relations (A.2.7) and (A.2.8) simplify to:

$$-[\nu + c] \cdot P(R_0|s) \geq -c \tag{A.2.9}$$

$$\nu \leq \frac{c \cdot P(R_1|s)}{P(R_0|s)} \tag{A.2.10}$$

Substituting (A.2.5) into (A.2.10) yields the critical value for the liquidity shocks under a bad signal:

$$\nu \leq \frac{c \cdot P(R_1|s_0)}{P(R_0|s_0)} = \frac{c \cdot p \cdot (1-q)}{(1-p) \cdot q} \equiv \nu_0 \tag{A.2.11}$$

Under a good signal, combining (A.2.6) and (A.2.10) produces:

$$\nu \leq \frac{c \cdot P(R_1|s_1)}{P(R_0|s_1)} = \frac{c \cdot p \cdot q}{(1-p) \cdot (1-q)} \equiv \nu_1 \tag{A.2.12}$$

Further, the critical values ν_0 and ν_1 serve to simplify the expression for the liquidity shortage triggering the financial assistance in the model with liquidity buffers and bank's equity. As a result, formula (A.2.10) in case of a bad signal becomes:

$$\nu \leq \frac{\nu_0 + \lambda}{1-k} \tag{A.2.13}$$

While under a good signal expression (A.2.10) translates into:

$$\nu \leq \frac{\nu_1 + \lambda}{1 - k} \tag{A.2.14}$$

Finally, the bank's objective is to maximize the expected shareholder value. Clearly, the bank's shareholders obtain zero payoff if the bank does not succeed. The only positive return is generated by the successful bank under the liquidity shortfall which satisfies condition (A.2.13) for the bad and (A.2.14) for the good supervisory signal. The bank's success yields the payoff derived in equations (A.2.3) and (A.2.4), that is $(1-\lambda)\cdot[R(p)-1]+k$. That happens with either of the following probabilities:

$$P\left(R_1, s_0, \nu \leq \frac{\nu_0 + \lambda}{1 - k}\right) = p \cdot (1-q) \cdot F\left(\frac{\nu_0 + \lambda}{1 - k}\right) \tag{A.2.15}$$

$$P\left(R_1, s_1, \nu \leq \frac{\nu_1 + \lambda}{1 - k}\right) = p \cdot q \cdot F\left(\frac{\nu_1 + \lambda}{1 - k}\right) \tag{A.2.16}$$

Consequently, the following expression for the bank's utility function is obtained:

$$U_B = p\left[(1-q)F\left(\frac{\nu_0 + \lambda}{1 - k}\right) + qF\left(\frac{\nu_1 + \lambda}{1 - k}\right)\right][(1-\lambda)(R(p)-1)+k] - (1+\delta)k \tag{A.2.17}$$

The game between the bank and the LLR has a Nash equilibrium characterized by (λ^*, k^*, p^*), which maximizes U_B given in (A.2.17) subject to the critical values for the liquidity shortage triggering the financial assistance under the bad and good signals respectively:

$$\nu_0^* = \frac{c \cdot p^* \cdot (1-q)}{(1-p^*) \cdot q} \quad \text{and} \quad \nu_1^* = \frac{c \cdot p^* \cdot q}{(1-p^*) \cdot (1-q)},$$

as well as the capital requirements:

$$k \geq \kappa(1-\lambda).$$

Using the equilibrium choice p^* of parameter p in the expression for the critical values for the liquidity shortage, ν^*, reflects the fact that the bank's choice of risk is unobservable for the LLR. What follows, these critical values only depend on the equilibrium risk. It further simplifies the bank's problem to a maximization of:

$$p \cdot [(1-\lambda)(R(p)-1)+k] \tag{A.2.18}$$

The first-order condition of this problem with respect to p is given by:

$$[(1-\lambda^*)(R(p^*)-1) + k^* + p^* \cdot (1-\lambda^*) \cdot R'(p^*)] = 0 \tag{A.2.19}$$

That yields:

$$R(p^*) + p^* \cdot R'(p^*) = 1 - \frac{k^*}{1-\lambda^*} \tag{A.2.20}$$

Given that $R(p^*) + p^*R'(p^*)$ is a decreasing function of p, higher capital and liquidity buffers imply lower risk. The first-order condition with respect to k can be obtained solely when a parametrization of the liquidity shock distribution is introduced. Repullo (2005) employs a simple type of a beta distribution, namely $F(\nu) = \nu^\eta$ with $\eta \in (0, 1)$. Such a specification of the distribution function yields the objective function (A.2.17) convex in k. Consequently, the optimal level of equity can be either 1 or $\kappa(1 - \lambda^*)$. For $k^* = 1$ the cumulative distribution functions $F\left(\frac{\nu_0+\lambda}{1-k}\right)$ and $F\left(\frac{\nu_1+\lambda}{1-k}\right)$ equal:

$$F\left(\frac{\nu_0^* + \lambda}{1 - k}\right) = F\left(\frac{\nu_1^* + \lambda}{1 - k}\right) = 1 \tag{A.2.21}$$

Substituting (A.2.21) into (A.2.17) reduces the bank's objective function to:

$$p[(1 - \lambda)(R(p) - 1) + k] - (1 + \delta)k \tag{A.2.22}$$

Differentiating $A.2.22$ with respect to k gives $p - (1 + \delta)$, which is less than zero. Thus, the optimal level of equity can be only $k^* = \kappa(1 - \lambda^*)$. Substituting this corner solution into $A.2.20$ yields:

$$R(p^*) + p^* \cdot R'(p^*) = 1 - \kappa \tag{A.2.23}$$

Following Repullo (2005), we refrain from deriving the analytical results for the optimal liquidity level.

B Appendices to Chapter 2

B.1 Appendix B.I to Chapter 2

Table B1.1: Definitions of Variables

Variable	Definition
Interest rate$_{t-1}$	Monthly average of euro overnight interest rate for month prior to loan origination
GDPCR$_{t-1}$	Rate of change of gross domestic product, chain-linked working day and seasonally adjusted, quarterly frequency
CPI$_t$	Monthly indices of consumer prices
Country risk$_t$	Long-term interest rate spread between Czech 10-year maturity government bond yield and EMU 10-year maturity T-bond yield, both yields expressed as monthly averages and in per cent
Bank size$_{t-1}$	Natural logarithm of bank total assets measured 1 month prior to loan origination
Liquidity ratio$_{t-1}$	Amount of bank liquid assets over total assets measured 1 month prior to loan origination
Bank NPL$_b$ - NPL$_{t-1}$	Difference between bank and other banks' level of NPLs measured 1 month prior to loan origination
Own funds/total assets$_{t-1}$	Bank's equity amount over bank's total assets measured 1 month prior to loan origination
Bank type$_{t-1}$	= 1 if bank is foreign owned
ln(2+ age as borrower)$_{t-1}$	Natural logarithm of number of years (augmented by 2 and measured 1 month prior to loan origination) that have elapsed since first time firm borrowed from bank
Bank relations$_{t-1}$	Natural logarithm of number of bank relationships of borrower plus 1 measured prior to loan origination
Bank debt$_{t-1}$	Natural logarithm of borrower bank debt plus 1 measured prior to loan origination
Loan size$_t$	Natural logarithm of loan amount
Maturity 0-6 months$_t$	= 1 if loan maturity is less than or equal to 6 months
Maturity 6-12 months$_t$	= 1 if loan maturity is between 6 and 12 months
Maturity 12-18 months$_t$	= 1 if loan maturity is between 1 and 1.5 year
Loan currency$_t$	= 1 if loan is granted in euros, dollars or pounds
Loan purpose$_t$	= 1 if overdrafts or current account debit
Firm turnover categories	Dummy variables created for CNB categories of firm turnover in CZK million
Firm employment categories	Dummy variables created for CNB categories of number of firm employees

Table B1.2: Correlations Between Variables

Variable	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	
Interest rate$_{t-1}$	1.00													
Interest rate$_{T-t-1}$	0.09	1.00												
GDPCR$_{t-1}$	0.25	0.35	1.00											
GDPCR$_{T-t-1}$	-0.24	0.64	0.23	1.00										
CPI$_t$	0.52	-0.11	0.06	-0.34	1.00									
Bank size$_{t-1}$	0.10	-0.06	0.02	-0.10	0.14	1.00								
Liquidity ratio$_{t-1}$	-0.24	0.06	0.11	0.18	-0.26	0.44	1.00							
Bank NPL$_b$ - NPL$_{t-1}$	-0.05	-0.00	0.05	0.01	-0.03	-0.04	0.07	1.00						
Own funds$_{t-1}$	-0.12	-0.00	0.03	0.07	-0.09	-0.19	0.13	0.16	1.00					
ln(2+ age as borrower)$_{t-1}$	0.00	-0.01	-0.02	0.02	0.06	0.05	-0.02	-0.01	-0.03	1.00				
Bad history$_{t-1}$	0.10	0.01	-0.05	-0.02	0.09	-0.07	-0.18	-0.05	-0.03	0.21	1.00			
Bank relations$_{t-1}$	-0.09	0.01	-0.04	0.10	-0.12	-0.06	0.06	-0.01	0.06	0.55	0.20	1.00		
Bank debt$_{t-1}$	-0.10	-0.02	-0.09	0.09	-0.14	-0.06	0.04	-0.02	0.05	0.48	0.16	0.84	1.00	
Loan size$_t$	-0.01	-0.01	0.01	0.01	-0.01	-0.05	-0.06	0.01	-0.07	0.27	0.08	0.22	0.26	1.00

Table B1.3: Weak Instrument Robust Tests for IV Probit

Test	Statistic	p-value
AR	$\chi^2_{(1)} = 165.66$	Prob > $\chi^2_{(1)} = 0.0000$
Wald	$\chi^2_{(1)} = 165.31$	Prob > $\chi^2_{(1)} = 0.0000$

Table B1.4: Estimation Results for Probit Model with Clustered Loans

Variable	Coefficient	Robust Std. Err.
Interest rate$_{t-1}$	0.079***	0.030
Bank size$_{t-1}$	-0.032	0.027
Liquidity ratio$_{t-1}$	-1.862***	0.199
Bank NPL$_b$ - NPL$_{t-1}$	-0.713*	0.377
Own funds/total assets$_{t-1}$	0.143	0.429
Bank type$_{t-1}$	0.150*	0.081
ln(2+ age as borrower)$_{t-1}$	0.163***	0.021
Bank relations$_{t-1}$	0.759***	0.076
Bank debt$_{t-1}$	-0.015***	0.004
Loan size$_t$	0.019**	0.009
Loan currency$_t$	0.236***	0.064
Maturity 0–3 months$_t$	0.331***	0.037
Maturity 3–6 months$_t$	0.241***	0.042
Maturity 6–12 months$_t$	0.246***	0.037
Loan purpose$_t$	-0.082**	0.038
GDPCR$_{t-1}$	-0.031***	0.008
CPI$_t$	0.006	0.013
Country risk$_t$	0.037	0.072
Time trend	0.014***	0.002
Time trend sq.	0.000***	0.000
Intercept	-1.342***	0.334
Collateral dummies	yes	
Firm turnover categories	yes	
Firm employment categories	yes	
Firm regional dummies	yes	
Firm industry dummies	yes	
N	205,270	
Log-likelihood	-98,985.748	
$\chi^2_{(67)}$	1,126.521	

B.2 Appendix B.II to Chapter 2

This section describes the steps involved in building the optimal survival and probit models developed as a robustness check for our probit and loan survival analysis. In the probit analysis we first evaluate the significance of each potential measure by considering its univariate probit fit. All covariates with p-values less than 25% along with all those of known economic importance are initially included in the multivariable model. Following the fit of the initial model we verify the significance of each variable in the model to identify those which can be removed. In order to nominate covariates that might be deleted from the model we use the p-values from the Wald tests of the individual coefficients, and then examine the p-value of the partial likelihood ratio test to confirm that the deleted covariate is indeed not significant. Having eliminated all insignificant measures at this stage, we coarsely classify the discrete characteristics overly rich in their categories, such as the 72 firm regional affiliations. We fit a hazard model for each category and group the characteristics with similar parameter estimates and significance levels. Thereafter, we employ the method of fractional polynomials to suggest transformations of the continuous variables. To ensure the economic validity of the transformed continuous covariates, we limit our search for proper functional forms to the natural logarithm and powers of plus and minus one. Moreover, we use the fractional polynomials procedure as a tool for validating the variables' significance once the optimal transformations have been incorporated. Finally, we determine whether our model necessitates interaction terms. We test the significance at the 5% level of all economically plausible interaction terms formed from the main effects in our model. As previously, we examine the p-values from the Wald test and the partial likelihood ratio test.

To select the covariates for the survival analysis we employ essentially the same methods as those used in the probit regression. We begin with the bivariate analysis of the association between all plausible variables and the loan survival time. For all potential predictors we compute the first, fifth, tenth, fifteenth and twentieth percentiles of the survival times. No estimates of higher survival quantiles are needed, as the loan data are typically characterized by low default occurrence. In our dataset the default ratio does not exceed 20% in specific sub-groups and is approximately 2% on average. For descriptive purposes, we break continuous variables into ten and twenty quantiles and compare the survivorship experience across the groups so defined. We examine the equality of the survivor functions using a set of available non-parametric tests, but we mostly rely on the log-rank test. Additionally, we consider the partial likelihood ratio test obtained in the estimation of each covariate's group-specific impact on the time to loan failure. Evidently, the same type of bivariate analysis is performed for categorical predictors. All variables with log-rank and partial likelihood ratio test p-values less than 20% along with all those that are economically vital are initially included in the multivariable model. Thereafter, we repeat all the steps already described for the probit variable selection. We fit the initial model, remove insignificant covariates, coarsely classify the discrete characteristics and apply the method of fractional polynomials to the multivariable proportional hazards regression model. Next, we determine whether any economically plausible interaction terms need to be added. Finally, we check the model's validity and its adherence to the proportionality assumption.

The methodology of fractional polynomials due to Royston and Altman (1994) offers an analytical way of determining the scale of the continuous predictors. Royston and Altman (1994) introduce a family of curves called fractional polynomials with power terms limited to a small predefined set of values and show how to find the best powers yielding the best-fitting and parsimonious model. In a single covariate case, a fractional polynomial of degree m is defined as:

$$\phi_m(X;\xi,p) = \xi_0 + \sum_{j=1}^{m} \xi_j X^{p_j} \tag{B.2.1}$$

where m is a positive integer, $p = (p_1,...,p_m)$ is a vector of powers with p_1 <...<p_m, ξ = (ξ_0, ξ_1,..,ξ_m) are coefficients and X^{p_j} signifies:

$$X^{p_j} = \begin{cases} X^{p_j} \ \ if \ \ p_j \neq 0 \\ \ln(X) \ \ if \ \ p_j = 0 \end{cases} \tag{B.2.2}$$

Expressions B.2.1 and B.2.2 combined and generalized can be rewritten into:

$$\phi_m(X;\xi,p) = \sum_{j=0}^{m} \xi_j H_j(X) \tag{B.2.3}$$

$$H_j(X) = \begin{cases} X^{p_j} \ \ if \ \ p_j \neq p_{j-1} \\ H_{j-1}(X)\ln(X) \ \ if \ \ p_j = p_{j-1} \end{cases} \tag{B.2.4}$$

Royston and Altman (1994) advocate that p={−2, −1, −0.5, 0, 0.5, 1, 2, 3} is a set of powers sufficiently rich to handle many practical cases. The best model is the one with the largest log likelihood. We use the fractional polynomials routine extended for multivariable specifications and implemented in STATA. An iterative search of scale within multivariable models involves checking for the scale of each covariate. To briefly illustrate the process, let's consider $m = 2$. For each variable the routine tests the best $J = 2$ model versus the linear model, the best $J = 2$ versus the best $J = 1$ fractional polynomial model and the linear model versus the model excluding the tested covariate. Having checked each predictor, the procedure repeats for each variable using the outcome of the first cycle for all covariates other than the one currently being tested in the second cycle. The reiteration aims to ascertain whether changing the functional form of one covariate alters the transformation of the other covariates. The routine runs until no further transformation is suggested. Table $B2.1$ contains the definitions of the optimally chosen covariates, while tables $B2.2$–$B2.4$ present their descriptive statistics.

Table B2.1: Robust Specification: Definitions of Variables

Variable	Definition
Deposit ratio$_{t-1}$	Amount of bank's deposits over bank's total assets measured 1 month prior to loan origination
Bank type$_{t-1}$	= 1 if bank is branch in CZ (as opposed to headquarters in CZ or branch abroad)
Probit: Maturity 2–3.5 year	= 1 if loan maturity is between 2 and 3.5 years
Probit: Maturity 4–8 years	= 1 if loan maturity is between 4 and 8 years but not 5.5 years
Probit: Maturity 5.5 year	= 1 if loan maturity is 5.5 years
Probit: Maturity 8.5–10 years	= 1 if loan maturity is between 8.5 and 10 years
1^{st} collateral: [1]	= 1 if none or 3^{rd} party real estate
1^{st} collateral: [2]	= 1 if guarantee deposits or real estate
1^{st} collateral: [3]	= 1 if movable property with ownership transfer
1^{st} collateral: [4]	= 1 if pledged securities
2^{nd} collateral: [1]	= 1 if real estate or movable property without ownership transfer
2^{nd} collateral: [2]	= 1 if movable property with ownership transfer
2^{nd} collateral: [3]	= 1 if pledged securities
2^{nd} collateral: [4]	= 1 if state guarantee
2^{nd} collateral: [5]	= 1 if other collateral
3^{rd} collateral: [1]	= 1 if real estate or movable property without ownership or guarantee deposit
3^{rd} collateral: [2]	= 1 if pledged assets or ensuring notes or other
3^{rd} collateral: [3]	= 1 if guarantee (incl. bank guarantee) or blockage of premium
Loan purpose: [1]	= 1 if temporary shortage of resources *or* residential property
Loan purpose: [2]	= 1 if residential property for business purposes, overdrafts *or* debit, other investment loans
Loan purpose: [3]	= 1 if residential property without state aid
Loan purpose: [4]	= 1 if purchase of securities
Loan purpose: [5]	= 1 if seasonal costs *or* subordinated loans
Loan currency	= 1 if loan granted in Czech or Slovak koruna or Japanese yen

Table B2.2: Robust Probit Model: Data Descriptive Statistics

Variable	Unit	Mean	Std. Dev.	Max	Min
Interest rate$_{t-1}$	%	2.74	0.88	4.30	0.35
GDPCR$_{t-1}$	%	4.77	2.58	7.70	-4.70
CPI$_t$	%	2.59	1.85	7.50	-0.40
Country risk$_t$	%	0.13	0.34	1.26	-0.38
Bank relations$_{t-1}$	#	0.32	0.41	1.80	0.00
ln(2+ age as borrower)$_{t-1}$	#	2.17	1.06	4.00	1.00
Bad history$_{t-1}$	0|1	0.08	0.27	1.00	0.00
Bank size$_{t-1}$	CZK	12.45	1.16	13.59	5.33
Liquidity ratio$_{t-1}$	%	0.32	0.13	0.71	0.00
Bank NPL$_b$ - NPL$_{t-1}$	%	0.02	0.07	6.40	-0.12
Own funds/total assets$_{t-1}$	%	0.09	0.05	0.61	-0.08
Deposit ratio$_{t-1}$	%	0.66	0.11	0.98	0.00
Bank type$_{t-1}$	0|1	0.67	0.47	1.00	0.00
Loan size$_t$	CZK	14.55	2.00	22.69	0.00
Maturity 2–3.5 year	0|1	0.17	0.38	1.00	0.00
Maturity 4–8 years	0|1	0.25	0.43	1.00	0.00
Maturity 5.5 year	0|1	0.02	0.13	1.00	0.00
Maturity 8.5–10 years	0|1	0.04	0.20	1.00	0.00
Loan currency$_t$	0|1	0.95	0.22	1.00	0.00

Table B2.3: Robust Survival Model: Data Descriptive Statistics

Variable	**Unit**	**Mean**	**Std. Dev.**	**Max**	**Min**
Interest rate$_{t-1}$	%	2.74	0.88	4.30	0.35
Interest rate$_{T-t-1}$	%	2.39	1.34	4.30	0.34
GDPCR$_{t-1}$	%	4.77	2.58	7.70	-4.70
GDPCR$_{T-t-1}$	%	2.55	3.86	7.70	-4.70
CPI$_t$	%	2.59	1.85	7.50	-0.40
Country risk$_t$	%	0.13	0.34	1.26	-0.38
Bank size$_{t-1}$	CZK	12.45	1.16	13.59	5.33
Liquidity ratio$_{t-1}$	%	0.32	0.13	0.71	0.00
Own funds to total assets$_{t-1}$	%	0.09	0.05	0.61	-0.08
CAR$_{t-1}$	%	12.36	7.56	147.14	0.00
CAR$^{-1}_{t-1}$	%	0.08	0.02	0.12	0.01
Bank profit$_{t-1}$	CZK	0.48	0.55	12.22	-4.68
Bank type$_{t-1}$	0|1	0.67	0.47	1.00	0.00
ln(2+ age as borrower)$_{t-1}$	#	2.17	1.06	4.00	1.00
Bad history$_{t-1}$	0|1	0.08	0.27	1.00	0.00
Loan size$_t$	CZK	14.55	2.00	22.69	0.00
Maturity 0–6 months$_t$	0|1	0.04	0.19	1.00	0.00
Maturity 6–12 months$_t$	0|1	0.06	0.23	1.00	0.00
Maturity 12–18 months$_t$	0|1	0.22	0.41	1.00	0.00
Loan purpose$_t$	0|1	0.30	0.46	1.00	0.00
Herfindahl-Hirschman index$_t$	#	0.48	1.07	6.40	0.00

Table B2.4: Robust Models: Correlations Between Variables

Variable	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
Interest rate$_{t-1}$	1.00													
Interest rate$_{T-t-1}$	0.07	1.00												
GDP$_{t-1}$	0.23	0.36	1.00											
GDP$_{T-t-1}$	-0.27	0.64	0.23	1.00										
CPI$_t$	0.52	-0.13	0.05	-0.37	1.00									
Country risk$_t$	-0.42	-0.20	-0.44	-0.09	0.28	1.00								
Bank size$_{t-1}$	0.09	-0.08	0.00	-0.15	0.17	0.01	1.00							
Liquidity ratio$_{t-1}$	-0.29	0.05	0.09	0.19	-0.29	-0.02	0.37	1.00						
Own funds$_{t-1}$	-0.12	-0.02	-0.01	0.05	-0.07	0.01	-0.46	0.05	1.00					
CAR$_{t-1}$	-0.15	-0.02	-0.14	0.08	-0.18	0.04	-0.47	0.21	0.78	1.00				
Loan size$_t$	-0.02	-0.01	0.01	0.01	-0.02	0.02	-0.01	-0.06	-0.04	-0.02	1.00			
Bad history$_{t-1}$	0.04	0.01	-0.01	0.03	0.00	-0.02	-0.02	-0.03	0.05	0.02	0.10	1.00		
Borrower age$_{t-1}$	0.00	-0.01	-0.01	0.02	0.06	0.04	0.10	-0.02	-0.00	-0.05	0.28	0.29	1.00	
Bank profit$_{t-1}$	0.00	-0.07	0.00	-0.07	0.11	0.04	0.46	0.24	-0.07	-0.10	-0.00	-0.00	0.09	1.00

Table B2.5: Estimation Results for Robust Probit Model with Clustered Loans

Variable	Coefficient	Robust Std. Err.
Interest rate$_{t-1}$	0.237***	0.048
ln(2+ age as borrower)$_{t-1}$	0.402***	0.032
Bank relations$_{t-1}$	1.438***	0.089
Bank size$_{t-1}$	-0.042	0.041
Liquidity ratio$_{t-1}$	-0.981***	0.291
Bank NPL$_b$ - NPL$_{t-1}$	-1.424	0.935
Own funds/total assets$_{t-1}$	-0.550	0.727
Deposit ratio$_{t-1}$	1.285***	0.387
Bank type$_{t-1}$	-0.237***	0.068
Maturity 2–3.5 years$_t$	-0.072	0.060
Maturity 4–8 years$_t$	-0.296***	0.074
Maturity 5.5 years$_t$	0.355*	0.200
Maturity 8.5–10 years$_t$	-0.166	0.154
Loan currency$_t$	-0.159*	0.093
GDPCR$_{t-1}$	-0.019	0.012
CPI$_t$	-0.089***	0.021
Country risk$_t$	0.161	0.116
Time trend	0.011***	0.004
Time trend sq.	0.000**	0.000
Intercept	-3.275***	0.416
Collateral	yes	
Loan purpose	yes	
Firm turnover categories	yes	
Firm employment categories	yes	
Firm regional dummies	yes	
Firm industry dummies	yes	
N	207,352	
Log-likelihood	-37,066.548	
$\chi^2_{(67)}$	1,475.642	

Table B2.6: Robust Probit Results for Firm Turnover Controls

Firm turnover in CZK millions	Coefficient	Robust Std. Err.
<0.2 or ≥1500	-0.911***	0.126
⟨0.2, 0.5),⟨10, 30),⟨200, 300)	-0.595***	0.098
⟨0.5, 1), ⟨30, 60)	-0.657***	0.121
⟨500, 1000)	-0.114	0.074
⟨100, 200), ⟨1000, 1500)	-0.074	0.077

Table B2.7: Robust Probit Results for Firm Employment Controls

Firm employment	Coefficient	Robust Std. Err.
⟨1500, 1999)	-1.186***	0.271
⟨6, 9), ⟨50, 99), ⟨250, 499)	-0.379***	0.093
⟨1, 5), ⟨10, 19), ⟨25, 49)	-0.260***	0.086
⟨20, 24), ⟨100, 199)	-0.354***	0.103
⟨500, 999)	0.284**	0.125
⟨1000, 499)	1.008**	0.507

Table B2.8: Robust Probit Estimation Results for Loan Collateral Types

Variable	Coefficient	Robust Std. Err.
1^{st} collateral		
None *or* 3^{rd} party real estate	-0.153***	0.057
Guarantee deposits *or* real estate	-0.412***	0.081
Movable property with ownership transfer	-0.960***	0.326
Securities	1.074**	0.493
2^{nd} collateral		
Real estate *or* movable property w/o ownership transfer	-0.255***	0.093
Movable property with ownership transfer	0.769**	0.312
Securities	-0.159**	0.062
State guarantee	-1.117***	0.405
Other collateral	-0.479***	0.141
3^{rd} collateral		
Real estate *or* movable property w/o ownership *or* deposit	0.495***	0.169
Assets *or* ensuring notes *or* other	0.197	0.133
Guarantee (incl. bank guarantee) *or* blockage of premium	0.226	0.199

Table B2.9: Robust Probit Estimation Results for Loan Purpose

Loan purpose	Coefficient	Robust Std. Err.
Temporary shortage of resources *or* residential property	-0.591***	0.201
Residential property for business purposes, overdrafts *or* debit, other investment loans	-0.104*	0.057
Residential property w/o state aid	0.331*	0.198
Purchase of securities	1.031**	0.434
Seasonal costs *or* subordinated loans	0.960***	0.285

C Appendices to Chapter 3

C.1 The Vasicek Model

Recall that Vasicek (1977) defines the short-term rate process as:

$$dr(t) = \kappa\left(\theta - r(t)\right)dt + \sigma dW(t) \qquad \text{(C.1.1)}$$

where κ, θ and σ are positive constants and $W(t)$ denotes a standard Wiener process associated with filtration $\mathcal{F}_t$. The solution to the stochastic differential equation (C.1.1) is given by:

$$r(t) = \theta + \left(r(s) - \theta\right)e^{-\kappa(t-s)} + \sigma\int_s^t e^{-\kappa(t-u)}dW(u) \qquad \text{(C.1.2)}$$

And for $s = 0$ we have:

$$r(t) = r_0 e^{-\kappa t} + \theta\left(1 - e^{-\kappa t}\right) + \sigma e^{-\kappa t}\int_0^t e^{\kappa s}dW(s)$$

To verify [26] the solution to equation (C.1.1), I first compute the term:

$$d\left(r(t)\cdot\exp\left(\int_0^t \kappa\ ds\right)\right) = \exp\left(\int_0^t \kappa\ ds\right)dr(t) + r(t)\cdot\exp\left(\int_0^t \kappa\ ds\right)\cdot\kappa\ dt =$$

$$= \exp\left(\int_0^t \kappa\ ds\right)\cdot\kappa\cdot\theta\ dt + \exp\left(\int_0^t \kappa\ ds\right)\cdot\sigma\ dW(t)$$

From the above one obtains:

$$r(t) - r(0) = \int_0^t e^{\kappa u}\kappa\theta\ du + \int_0^t e^{\kappa u}\sigma\ dW(u) = \theta\left(e^{\kappa t} - 1\right) + \int_0^t e^{\kappa u}\sigma\ dW(u)$$

Rearranging and using the fact that $r(0) = r_0$ gives:

$$r(t) = \theta + \left(r_0 - \theta\right)e^{-\kappa t} + \sigma\int_0^t e^{-\kappa(t-u)}\ dW(u)$$

The solution for $s = 0$ can be generalized for any $s < t$, which yields equation (C.1.2). Thus, the process $r(t)$ defined by equation (C.1.2) solves the stochastic differential equa-

[26] I follow the proof of Proposition 3.7 in Nielsen (1999) (Nielsen (1999), pp. 103-104).

tion given by (C.1.1). It follows that $r(t)$ is a Gaussian random variable with conditional moments equal to:

$$\mathrm{E}\left(r(t)|\mathcal{F}_0\right) = \theta + (r_0 - \theta)\, e^{-\kappa t}$$

$$\mathrm{Var}\left(r(t)|\mathcal{F}_0\right) = \mathrm{Var}\left(\sigma \int_0^t \mathrm{e}^{-\kappa(t-s)}\, dW(s)\right) = \sigma^2 \int_0^t \mathrm{e}^{-2\kappa(t-s)}\, ds = \frac{\sigma^2}{2\kappa}\left(1 - \mathrm{e}^{-2\kappa t}\right)$$

The term structure of interest rates is characterized by the yields to maturity. Generally, the continuously compounded yield at time s of bond maturing at t is defined by:

$$R(s,t) = -\frac{\ln\, P(s,t)}{t-s} \tag{C.1.3}$$

Vasicek (1977) determines the yields from the prices of discount bonds. To ensure that the bond prices satisfy the no-arbitrage condition one needs to redefine the diffusion process for the instantaneous interest rate under the risk neutral probability measure[27]. It follows from Girsanov's theorem that a new Brownian motion under a risk-adjusted measure Q is defined as:

$$\widetilde{W}(t) = W(t) + \int_0^t \lambda\ \mathrm{ds} = W(t) + \lambda t$$

In Vasicek's model λ denotes a market price of risk and is assumed to be constant. Equation (C.1.1) can be restated using process $\widetilde{W}(t)$ in the following manner:

$$dr(t) = \kappa\left(\theta - r(t)\right) dt + \sigma\left[d\widetilde{W}(t) - \lambda\ dt\right] = \kappa\left(\theta - \frac{\lambda\sigma}{\kappa} - r(t)\right)\ dt + \sigma\ d\widetilde{W}(t) =$$

$$= \kappa\left(\widetilde{\theta} - r(t)\right)\ dt + \sigma\ d\widetilde{W}(t)$$

As a result, we arrive at the risk-neutral diffusion process for the instantaneous interest rate:

$$dr(t) = \kappa\left(\widetilde{\theta} - r(t)\right)\ dt + \sigma\ d\widetilde{W}(t) \tag{C.1.4}$$

where $\widetilde{\theta} = \theta - \frac{\lambda\sigma}{\kappa}$. The solution to the above stochastic differential equation is given by:

$$r(t) = \widetilde{\theta} + \left(r(s) - \widetilde{\theta}\right) e^{-\kappa(t-s)} + \sigma \int_s^t e^{-\kappa(t-u)} d\widetilde{W}(u) \tag{C.1.5}$$

Given the character of the spot rate process $r(t)$ under the market efficiency assumption, Vasicek (1977) derives prices for discount bonds. Under the risk-neutral probability

[27]Rigorously speaking, it is required that the diffusion process for the bond prices has the martingale property. For the full treatment of risk-neutral pricing refer for example to Karatzas and Shreve (1998).

measure Q the price at time 0 of a discount bond maturing at time t is given by:

$$P(0,t) = E_Q\left(\exp\left(-\int_0^t r(u)\ du\right)|\mathcal{F}_0\right) \tag{C.1.6}$$

By the property of the log-normal random variable equation (C.1.6) can expressed as:

$$P(0,t) = \exp\left(E_Q\left(-\int_0^t r(u)\ du\right) + \frac{1}{2}\cdot \mathrm{Var}_Q\left(-\int_0^t r(u)\ du\right)\right)$$

Next, equation (C.1.5) is used to compute $\int_0^t r(u)du$:

$$\int_0^t r(u)\ du = \widetilde{\theta}\cdot t + \int_0^t \left(r_0 - \widetilde{\theta}\right)^{-\kappa\cdot u}\ du + \int_0^t\int_0^u \sigma\cdot \mathrm{e}^{-\kappa\cdot(u-s)} d\widetilde{W}(s)\ du$$

The order of integration can be changed by Fubnini's theorem, which yields:

$$\int_0^t r(u)\ du = \widetilde{\theta}\cdot t + \frac{1-\mathrm{e}^{-\kappa\cdot t}}{\kappa}\cdot\left(r_0-\widetilde{\theta}\right) + \sigma\cdot\int_0^t \mathrm{e}^{\kappa\cdot s}\left(\int_s^t \mathrm{e}^{-\kappa\cdot u}\ du\right) d\widetilde{W}(s) =$$

$$= \widetilde{\theta}\cdot t + \frac{1-e^{-\kappa\cdot t}}{\kappa}\cdot\left(r_0-\widetilde{\theta}\right) + \frac{\sigma}{\kappa}\cdot\int_0^t \left(1-\mathrm{e}^{-\kappa\cdot(t-s)}\right)\ d\widetilde{W}(s)$$

Then, the moments are given by:

$$E\left(-\int_0^t r(u)du\right) = \left(\widetilde{\theta}-r_0\right)\cdot\frac{1-\mathrm{e}^{-\kappa\cdot t}}{\kappa} - \widetilde{\theta}\cdot t$$

and

$$\mathrm{Var}\left(-\int_0^t r(u)\ du\right) = \frac{\sigma^2}{\kappa^2}\cdot\int_0^t \left(1-\mathrm{e}^{-\kappa\cdot(t-s)}\right)^2 ds = \frac{\sigma^2}{2\cdot\kappa^3}\left(4\cdot\mathrm{e}^{-\kappa\cdot t} - \mathrm{e}^{-2\cdot\kappa\cdot t} + 2\cdot\kappa\cdot t - 3\right)$$

Plugging the expressions for both moments into equation (C.1.6) produces:

$$P(0,t) = \exp\left[\left(\widetilde{\theta}-r_0\right)\cdot\frac{1-\mathrm{e}^{-\kappa\cdot t}}{\kappa} - \widetilde{\theta}\cdot t + \frac{\sigma^2}{4\cdot\kappa^3}\left(4\cdot\mathrm{e}^{-\kappa\cdot t} - \mathrm{e}^{-2\cdot\kappa\cdot t} + 2\cdot\kappa\cdot t - 3\right)\right]$$

More generally, for any $0 \leq s \leq t$ the Vasicek model postulates the following bond prices and yields:

$$P(s,t) = \exp\left[\left(\widetilde{\theta} - r(s)\right)\frac{1-\mathrm{e}^{-\kappa(t-s)}}{\kappa} - \widetilde{\theta}(t-s) + \frac{\sigma^2}{4\kappa^3}\left(4\mathrm{e}^{-\kappa(t-s)} - \mathrm{e}^{-2\kappa(t-s)} + 2\kappa(t-s) - 3\right)\right]$$

Finally, the yield is given by:

$$\begin{aligned} R(s,t) = -\frac{1}{t-s}\Bigg[\left(\theta - \frac{\lambda\sigma}{\kappa} - r(s)\right)\frac{1-\mathrm{e}^{-\kappa(t-s)}}{\kappa} - \left(\theta - \frac{\lambda\sigma}{\kappa}\right)(t-s) \\ + \frac{\sigma^2}{4\kappa^3}\left(4\mathrm{e}^{-\kappa(t-s)} - \mathrm{e}^{-2\kappa(t-s)} + 2\kappa(t-s) - 3\right)\Bigg] \end{aligned}$$

Rearranging results in:

$$\begin{aligned} R(s,t) &= -\frac{1}{t-s}\Bigg[\frac{1}{\kappa}\left(e^{-(t-s)\kappa} - 1\right)r(s) + \frac{\sigma^2}{4\kappa^3}\left(1 - e^{-2(t-s)\kappa}\right) \\ &+ \frac{1}{\kappa}\left(\theta - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{\kappa^2}\right)\left(1 - e^{-(t-s)\kappa}\right) - \left(\theta - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{\kappa^2}\right)(t-s)\Bigg] \end{aligned}$$

where κ, θ and σ are the same positive constants as in equation (C.1.1), the parameter λ is the risk premium introduced by the risk-neutral pricing and $r(s)$ is the current period instantaneous interest rate driven by the stochastic differential equation (C.1.1). In addition, the term $\theta - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{2\kappa^2}$ represents the infinite maturity interest rate:

$$R(s,\infty) \equiv \lim_{t\to\infty} R(s,t) = \theta - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{2\kappa^2}$$

C.2 The Cox-Ingresoll-Ross Model

Using the assumption about the factor and production dynamics Cox, Ingersoll, and Ross (1985) arrive at the following diffusion process for the risk-free rate of interest:

$$dr(t) = \kappa\left(\theta - r(t)\right)dt + \sigma\sqrt{r(t)}dW(t) \qquad \text{(C.2.1)}$$

where κ, θ and σ are positive constants and $W(t)$ denotes a standard Wiener process. A unique positive solution to equation (C.2.1) is given by:

$$r(t) = \theta + \left(r(s) - \theta\right)e^{-\kappa(t-s)} + \sigma e^{-\kappa(t-s)}\int_s^t e^{\kappa(u-s)}\sqrt{r(u)}dW(u) \qquad \text{(C.2.2)}$$

Let us also denote by $\bar{r}(t)$ the solution of (C.2.1) starting at $\bar{r}$. To derive[28] equation (C.2.2) we first employ Itô's formula to compute $d\left(e^{\kappa t}r(t)\right)$.

$$d\left(e^{\kappa t}r(t)\right) = \kappa e^{\kappa t}r(t)dt + e^{\kappa t}dr(t) = e^{\kappa t}\left[\kappa r(t)dt + \kappa(\theta - r(t))dt + \sigma\sqrt{r(t)}dW(t)\right] =$$

[28]The derivations follow an example in Karatzas and Shreve (1998), pp. 152-53.

$$= e^{\kappa t}\left(\kappa\theta dt + \sigma\sqrt{r(t)}dW(t)\right)$$

Next, we integrate both sides:

$$r(t)e^{\kappa t} = r(0) + \theta\int_0^t \kappa e^{\kappa u}du + \sigma\int_0^t e^{\kappa u}\sqrt{r(u)}dW(u)$$

Rearranging yields:

$$r(t) = r(0)e^{-\kappa t} + \theta(1 - e^{-\kappa t}) + \sigma\int_0^t e^{-\kappa(t-u)}\sqrt{r(u)}dW(u)$$

The solution for $s = 0$ can be generalized for any $s < t$, which gives equation (C.2.2). Since the expectation of an *Itô* integral is 0, the conditional mean of $r(t)$ has the same form as in the Vasicek model.

$$\mathrm{E}\left(r(t)|\mathcal{F}_0\right) = \mathrm{E}\left(r(0)e^{-\kappa t} + \theta\left(1 - e^{-\kappa t}\right)|\mathcal{F}_0\right) = \theta + (r_0 - \theta)e^{-\kappa t}$$

To compute the conditional variance of $r(t)$, the expression $\mathrm{E}\left(r^2(t)e^{2\kappa t}|\mathcal{F}_0\right)$ can be used to find $\mathrm{E}\left(r^2(t)|\mathcal{F}_0\right)$. By *Itô*'s lemma one has:

$$\begin{aligned} d\left(\left[e^{\kappa t}r(t)\right]^2\right) &= 2e^{\kappa t}r(t)d\left(e^{\kappa t}r(t)\right) + d\left(e^{\kappa t}r(t)\right)d\left(e^{\kappa t}r(t)\right) = \\ &= e^{2\kappa t}(2\kappa\theta + \sigma^2)r(t)dt + 2\sigma e^{2\kappa t}r^{\frac{3}{2}}(t)dW(t) \end{aligned}$$

Integrating both sides and taking expectations yields:

$$\begin{aligned} r^2(t)e^{2\kappa t} &= r^2(0) + (2\kappa\theta + \sigma^2)\int_0^t e^{2\kappa u}r(u)du + 2\sigma\int_0^t e^{2\kappa u}r^{\frac{3}{2}}(u)dW(u) \\ \mathrm{E}\left(r^2(t)e^{2\kappa t}\right) &= r^2(0) + (2\kappa\theta + \sigma^2)\int_0^t e^{2\kappa u}\mathrm{E}(r(u))du \end{aligned}$$

Next, the expected value of $r(u)$ is plugged to obtain:

$$\begin{aligned} \mathrm{E}\left(r^2(t)e^{2\kappa t}\right) &= r^2(0) + (2\kappa\theta + \sigma^2)\int_0^t e^{2\kappa u}\left(r(0)e^{\kappa u} + \theta(e^{2\kappa u} - e^{\kappa u})\right)du = \\ &= r^2(0) + r(0)\frac{2\kappa\theta + \sigma^2}{\kappa}\left(e^{\kappa t} - 1\right) + \frac{\theta(2\kappa\theta + \sigma^2)}{2\kappa}\left(e^{2\kappa t} - 2e^{\kappa t} + 1\right) \end{aligned}$$

Rearranging gives:

$$\mathrm{E}(r^2(t)) = r^2(0)e^{-2\kappa t} + \frac{2\kappa\theta + \sigma^2}{\kappa}r(0)\left(e^{-\kappa t} - e^{-2\kappa t}\right) + \frac{\theta(2\kappa\theta + \sigma^2)}{2\kappa}\left(1 - 2e^{-\kappa t} + e^{-2\kappa t}\right)$$

And finally, the conditional variance of $r(t)$ equals:

$$\text{Var}(r(t)) = \text{E}(r^2(t)) - [\text{E}(r(t))]^2 = \frac{\sigma^2}{\kappa} r(0)\left(e^{-\kappa t} - e^{-2\kappa t}\right) + \frac{\sigma^2\theta}{2\kappa}\left(1 - 2e^{-\kappa t} + e^{-2\kappa t}\right)$$

Key to any pricing within the Cox-Ingersoll-Ross model is the characterization of the law of the pair of the random variables $\left(\bar{r}(t), \int_0^t \bar{r}(u)du\right)$. Their joint law is characterized by the following theorem. Recall that by $\bar{r}(t)$ we denote the solution of (C.2.1) starting at $\bar{r}$.

Theorem[29] For any $\lambda > 0$ and $\mu > 0$, we have:

$$\text{E}\left(e^{-\lambda \bar{r}(t)} e^{-\mu \int_0^t \bar{r}(u)du}\right) = e^{-\kappa\theta\phi_{\lambda,\mu}(t)} e^{-\bar{r}\psi_{\lambda,\mu}(t)},$$

where

$$\begin{aligned}
\phi_{\lambda,\mu}(t) &= -\frac{2}{\sigma^2}\ln\left(\frac{2\gamma e^{t(\kappa+\gamma)/2}}{\sigma^2\lambda(e^{\gamma t}-1)+\gamma-\kappa+e^{\gamma t}(\gamma+\kappa)}\right), \\
\psi_{\lambda,\mu}(t) &= \frac{\lambda(\gamma+\kappa)+e^{\gamma t}(\gamma-\kappa)+2\mu(e^{\gamma t}-1)}{\sigma^2\lambda(e^{\gamma t}-1)+\gamma-\kappa+e^{\gamma t}(\gamma+\kappa)}, \\
\gamma &= \sqrt{\kappa^2+2\sigma^2\mu}\ .
\end{aligned}$$

Applying the theorem with $\mu = 0$, Lamberton and Lapeyre (2007) show how to obtain the price of a zero-coupon bond in the Cox-Ingersoll-Ross framework. The price of the bond with maturity T at time 0 is characterized by:

$$P(0,T) = \text{E}^*\left(e^{-\int_0^T r(s)ds}\right) = e^{-\kappa\theta\phi(T) - r(0)\psi(T)}$$

where the functions ϕ and ψ are given by:

$$\phi(t) = -\frac{2}{\sigma^2}\ln\left(\frac{2\gamma^* e^{\frac{t(\gamma^*+\kappa^*)}{2}}}{\gamma^*-\kappa^*+e^{\gamma^* t}(\gamma^*+\kappa^*)}\right)$$

and

$$\psi(t) = \frac{2\left(e^{\gamma^* t}-1\right)}{\gamma^*-\kappa^*+e^{\gamma^* t}(\gamma^*+\kappa^*)}$$

with $\kappa^* = \kappa + \lambda$ and $\gamma^* = \sqrt{(\kappa^*)^2 + 2\sigma^2}$. The price at time t becomes:

$$P(t,T) = \exp\left(-\kappa\theta\phi(T-t) - r(t)\psi(T-t)\right)$$

[29] This is Theorem 9.6.4 in Elliott & Kopp (1998) or Proposition 6.2.4 in Lamberton and Lapeyre (2007). Its proof and the derivations of the bond prices under the Cox-Ingersoll- Ross model are provided in Elliott & Kopp (1998), pp. 273-276, and in Lamberton and Lapeyre (2007) pp. 162-164.

or equivalently

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)} \tag{C.2.3}$$

where

$$\begin{aligned} A(t,T) &= \left[\frac{2\gamma e^{(\kappa+\gamma+\lambda)(T-t)/2}}{(\gamma+\kappa+\lambda)(e^{\gamma(T-t)}-1)+2\gamma}\right]^{2\kappa\theta/\sigma^2} \\ B(t,T) &= \frac{2\left(e^{\gamma(T-t)}-1\right)}{(\gamma+\kappa+\lambda)(e^{\gamma(T-t)}-1)+2\gamma} \\ \gamma &= \sqrt{(\kappa+\lambda)^2+2\sigma^2} \end{aligned}$$

where κ, θ and σ are the same positive constants as in equation (C.2.1), the parameter λ is introduced by the risk-neutral pricing and $r(t)$ is the current period instantaneous interest rate driven by the stochastic differential equation (3.11). Using the equation (3.3), the continuously compounded yield for discount bonds in the CIR model can be expressed as:

$$R(t,T) = \frac{-\ln A(T,t) + B(t,T)r(t)}{T-t}$$

where $A(t,T)$ and $B(t,T)$ are defined as in formula (C.2.3). Alternatively, the CIR bond price and yield may be obtained by solving the partial differential equation for $P(t,T) = f(t,r(t))$. Shreve (2004) propose a way to find the PDE for the function $f(t,r(t))$ that characterizes the bond prices[30]. First, they suggest a martingale

$$D(t)f(t,R(t)) = P(t,T)\exp\left(-\int_0^t r(u)du\right)$$

and differentiate it to get:

$$\begin{aligned} d(D(t)f(t,r(t))) &= f(t,r(t))dD(t) + D(t)df(t,r(t)) = \\ &= D(t)\left[-rfdt + f_t dt + f_r dr + \frac{1}{2}f_{rr}drdr\right] = \end{aligned}$$

$$= D(t)\left[f_t(t,r) + (\kappa\theta-\kappa r)f_r(t,r) + \frac{1}{2}\sigma^2 r f_{rr}(t,r) - rf(t,r)\right]dt + D(t)\sigma\sqrt{r}f_r d\widetilde{W}.$$

The partial differential equation for the function $f(t,r(t))$ is then obtained by setting the dt term equal to zero.

$$f_t(t,r) + (\kappa\theta-\kappa r)f_r(t,r) + \frac{1}{2}\sigma^2 r f_{rr}(t,r) = rf(t,r) \tag{C.2.4}$$

The terminal condition reads:

$$f(T,r) = 1 \text{ for all } r$$

[30]See Shreve (2004), pp. 273-276.

At this stage, the solution to the above PDE can be guessed to have a form typical for the affine term structure models, namely:

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)} = e^{-A^*(t,T)-B(t,T)r(t)}$$

In such a case, partial differential equation (C.2.4) becomes:

$$f(t,r)\left[\left(-B'(t,T) + \kappa B(t,T) + \frac{1}{2}\sigma^2 B^2(t,T) - 1\right) r - A^{*\prime}(t,T) - \kappa\theta B(t,T)\right] = 0$$

Since the term:

$$\left(-B'(t,T) + \kappa B(t,T) + \frac{1}{2}\sigma^2 B^2(t,T) - 1\right)$$

must be 0, the remaining term must also equal 0, which in turn implies the following ODEs:

$$\begin{aligned} B'(t,T) &= \kappa B(t,T) + \frac{1}{2}\sigma^2 B^2(t,T) - 1 \\ A^{*\prime}(t,T) &= -\kappa\theta B(t,T) \end{aligned}$$

Under the terminal conditions $A^*(T,T) = B(T,T) = 0$ the solutions are given by:

$$\begin{aligned} B(t,T) &= \frac{\sinh(\gamma(T-t))}{\gamma\cosh(\gamma(T-t)) + \frac{1}{2}\kappa\sinh(\gamma(T-t))}, \\ A(t,T) &= \left[\frac{\gamma e^{\frac{1}{2}\kappa(T-t)}}{\gamma\cosh(\gamma(T-t)) + \frac{1}{2}\kappa\sinh(\gamma(T-t))},\right]^{-\frac{2\kappa\theta}{\sigma^2}} \end{aligned}$$

where

$$\gamma = \frac{1}{2}\sqrt{\kappa^2 + 2\sigma^2}, \quad \sinh u = \frac{e^u - e^{-u}}{2} \quad \text{and} \quad \cosh u = \frac{e^u + e^{-u}}{2}$$

Rearranging and using relation (3.3) gives the expression for the yield in the CIR model:

$$R(t,T) = \frac{-\ln A(t,T) + B(t,T)r(t)}{T-t}$$

where $A(t,T)$ and $B(t,T)$ are defined as in formula (C.2.3).

C.3 The Heath-Jarrow-Morton Model

To exclude arbitrage from bond trading, the existence of a martingale measure for all maturities needs to be established. Shreve (2004) present a fairly short way to find such a martingale measure[31]. They examine the discounted bond price for $T \in [0, T^*]$ and seek the risk-adjusted probability measure, under which the discounted price is a martingale.

[31] See Shreve (2004), pp. 425-428

The differential of the discounted bond price is given by:

$$d\left(P(t,T)\exp\left(-\int_0^t r(u)du\right)\right)= \tag{C.3.1}$$

$$= \left(-r(t)P(t,T)dt + dP(t,T)\right)\exp\left(-\int_0^t r(u)du\right)$$

Given the character of the instantaneous and continuously compounded forward rate, $f(t,T)$, the price of a discount bond at time t with maturity $T \leq T^*$ is given as:

$$P(t,T) = \exp\left(-\int_t^T f(t,u)du\right) \tag{C.3.2}$$

Applying the Itô lemma to the bond price dynamics implies:

$$dP(t,T) = P(t,T)\left(d\left(-\int_t^T f(t,u)du\right) + \frac{1}{2}\left[d\left(-\int_t^T f(t,u)du\right)\right]^2\right) \tag{C.3.3}$$

The differential of the discounting term equals:

$$d\left(-\int_t^T f(t,u)du\right) = f(t,t)dt - \int_t^T df(t,u)du$$

Plugging the dynamics for the forward rate, $f(t,T)$, recalling that $f(t,t) = r(t)$, and then changing the order of integration yields:

$$\begin{aligned} d\left(-\int_t^T f(t,u)du\right) &= r(t)dt - \int_t^T \left[\alpha(t,u)dt + \sigma(t,u)dW(t)\right] du = \\ &= r(t)dt - \int_t^T \alpha(t,u)dtdu - \int_t^T \sigma(t,u)dW(t)du = \\ &= r(t)dt - \alpha^*(t,T)dt - \sigma^*(t,T)dW(t) \end{aligned}$$

where

$$\alpha^*(t,T)dt = \int_t^T \alpha(t,u)dudt \quad \text{and} \quad \sigma^*(t,T)dW(t) = \int_t^T \sigma(t,u)dudW(t)$$

Inserting the obtained expression for the differential of the discounting term into equation (C.3.1) produces:

$$dP(t,T)\exp\left(-\int_0^t r(u)du\right) = \tag{C.3.4}$$

$$= P(t,T)\exp\left(-\int_0^t r(u)du\right)\left[\left(-\alpha^*(t,T) + \frac{1}{2}(\sigma^*(t,T))^2\right)dt - \sigma^*(t,T)dW(t)\right]$$

For the discounted bond price to be a martingale relation (C.3.5) needs to be rewritten using a new Brownian motion under a risk-adjusted measure Q defined as:

$$\widetilde{W}(t) = W(t) + \int_0^t \lambda(s)ds = W(t) + \lambda(t)t$$

What follows, it must hold that:

$$\left[\left(-\alpha^*(t,T) + \frac{1}{2}(\sigma^*(t,T))^2\right)dt - \sigma^*(t,T)dW(t)\right] = -\sigma^*(t,T)\lambda(t)dt - \sigma^*(t,T)dW(t)$$

Clearly, it implies:

$$-\alpha^*(t,T) + \frac{1}{2}(\alpha^*(t,T))^2 = -\sigma^*(t,T)\lambda(t)$$

Differentiating both sides with respect to T gives:

$$\alpha(t,T) = \sigma(t,T)\left[\sigma^*(t,T) + \lambda(t)\right]$$

Since

$$\frac{\partial}{\partial T}\alpha^*(t,T) = \alpha(t,T) \quad \text{and} \quad \frac{\partial}{\partial T}\sigma^*(t,T) = \sigma(t,T)$$

Substituting for $\sigma^*(t,T)$ yields the no-arbitrage drift condition (3.15):

$$\alpha(t,T) = \sigma(t,T)\left[\int_t^T \sigma(t,u)du + \lambda(t)\right], \quad \forall t \in [0,T],\ T \in [0,T^*]$$

Using the no-arbitrage drift restriction, the forward-rate process in (3.13) can be formulated under the risk-neutral probability measure Q as:

$$df(t,T) = \alpha(t,T)dt + \sigma(t,T)dW(t) = \left(\sigma(t,T)\int_t^T \sigma(t,u)du\right)dt +$$

$$+\sigma(t,T)(\lambda(t) + dW(t)) = \left(\sigma(t,T)\int_t^T \sigma(t,u)du\right)dt + \sigma(t,T)d\widetilde{W}(t) \tag{C.3.5}$$

where $\widetilde{W}(t)$ is the Wiener process under the martingale measure Q, $\lambda(t)$ denotes a market price of risk and $0 \leq t \leq T \leq T^*$. In an integral form, the dynamics for the instantaneous forward rate (C.3.5) is given as:

$$f(t,T) = f(0,T) + \int_0^t \sigma(v,T)\sigma^*(v,T)dv + \int_0^t \sigma(v,T)d\widetilde{W}_v \tag{C.3.6}$$

From equation (C.3.6), the yields on contingent claims for every fixed maturity $T \leq T^*$ can be written as:

$$R(t,T) = \frac{\int_t^T f(t,u)du}{T-t} \tag{C.3.7}$$

Combining equations (C.3.6) and (C.3.7) gives formula (3.19) for the yield in the Heath-Jarrow-Morton framework[32]. To simplify, term $\bar{R}(t,T) = (T-t)R(t,T)$ is first considered. From equations (C.3.6) and (C.3.7), it follows that $\bar{R}(t,T)$ must satisfy:

$$\bar{R}(t,T) = \int_t^T f(0,u)du + \int_t^T \int_0^t \sigma(v,u)\sigma^*(v,u)dv\,du + \int_t^T \int_0^t \sigma(v,u)d\widetilde{W}_v\,du$$

Applying Fubini's theorem and rearranging gives:

$$\begin{aligned}\bar{R}(t,T) \quad &= \quad \int_0^T f(0,u)du + \int_0^t \int_v^T \sigma(v,u)\sigma^*(v,u)du\,dv + \int_0^t \int_v^T \sigma(v,u)du\,d\widetilde{W}_v \\ &- \quad \int_0^t f(0,u)du - \int_0^t \int_v^t \sigma(v,u)\sigma^*(v,u)du\,dv - \int_0^t \int_v^t \sigma(v,u)du\,d\widetilde{W}_v\end{aligned}$$

The instantaneous interest rate can be expresses as:

$$r_u = f(u,u) = f(0,u) + \int_0^u \sigma(v,u)\sigma^*(v,u)dv + \int_0^u \sigma(v,u)d\widetilde{W}_v$$

From the above it follows that:

$$\bar{R}(t,T) = \bar{R}_0 - \int_0^t r_u du + \int_0^t \int_u^T \sigma(u,v)\sigma^*(u,v)dv\,du + \int_0^t \int_u^T \sigma(u,v)dv\;d\widetilde{W}_u$$

or equivalently

$$\bar{R}(t,T) = \bar{R}_0 - \int_0^t r_u du + \frac{1}{2}\int_0^t (\sigma^*(u,T))^2 du + \int_0^t \sigma^*(u,T)d\widetilde{W}_u$$

[32]The derivations are loosely based on Musiela and Rutkowski (2005), pp. 420-421

Consequently, the yields on the contingent claims are charaterized by relation (3.19):

$$
\begin{aligned}
R(t,T) &= R(0,T) + \frac{\frac{1}{2}\int\limits_0^t \left(\int\limits_u^T \sigma(u,s)ds\right)^2 du}{T-t} - \frac{\int\limits_0^t r(u)du}{T-t} \\
&+ \frac{\int\limits_0^t \int\limits_u^T \sigma(u,s)ds\, d\widetilde{W}(u)}{T-t}
\end{aligned}
$$

C.4 The Nelson-Siegel Model

One of the specifications postulated in (Nelson and Siegel 1987) describes the instantaneous forward rate for maturity m by the following exponential equation with equal roots[33]:

$$r(m) = \beta_0 + \beta_1 \exp\frac{-m}{\tau} + \beta_2 \left(\frac{m}{\tau}\exp\frac{-m}{\tau}\right) \tag{C.4.1}$$

where τ is a time constant that determines the rate at which the regressors decay to zero and β_0, β_1, β_2 are the coefficients to be estimated. Generally, the yield to maturity m equals:

$$R(m) = \frac{1}{m}\int\limits_0^m r(x)dx \tag{C.4.2}$$

Integrating $r(m)$ over time from 0 to m, as in expression (C.4.2), gives:

$$
\begin{aligned}
\int\limits_0^m r(x)dx &= \int\limits_0^m \left(\beta_0 + \beta_1 \cdot e^{-\frac{x}{\tau}} + \beta_2 \cdot \frac{x}{\tau}e^{-\frac{x}{\tau}}\right)dx = \\
&= [\beta_0 \cdot x]_0^m + \beta_1 \cdot \int\limits_0^m e^{-\frac{x}{\tau}}dx + \beta_2 \cdot \int\limits_0^m \frac{x}{\tau}\cdot e^{-\frac{x}{\tau}}dx = \beta_0 \cdot m + \beta_1 \cdot I_1 + \beta_2 \cdot I_2
\end{aligned}
$$

$$I_1 = \int\limits_0^m e^{-\frac{x}{\tau}}dx = -\tau \int\limits_0^m -\frac{1}{\tau}e^{-\frac{x}{\tau}}dx = -\tau \cdot \int\limits_0^m d\left(e^{-\frac{x}{\tau}}\right) = -\tau \cdot \left[e^{-\frac{x}{\tau}}\right]_0^m = \tau \cdot \left(1 - e^{-\frac{m}{\tau}}\right)$$

$$
\begin{aligned}
I_2 &= \int\limits_0^m \frac{x}{\tau}\cdot e^{-\frac{x}{\tau}}dx = \left[-x \cdot e^{-\frac{x}{\tau}}\right]_0^m + \int\limits_0^m e^{-\frac{x}{\tau}}dx = -m \cdot e^{-\frac{m}{\tau}} + \int\limits_0^m e^{-\frac{x}{\tau}}dx = -m \cdot e^{-\frac{m}{\tau}} + I_1 = \\
&= -m \cdot e^{-\frac{m}{\tau}} + \tau \cdot \left(1 - e^{-\frac{m}{\tau}}\right)
\end{aligned}
$$

Substituting for I_1 and I_2 gives:

$$\int\limits_0^m r(x)dx = \beta_0 \cdot m + (\beta_1 + \beta_2)\cdot \tau \cdot \left(1 - e^{-\frac{m}{\tau}}\right) - \beta_2 \cdot m \cdot e^{-\frac{m}{\tau}}$$

[33] I follow the notation of (Nelson and Siegel 1987)

Next, plugging the above expression into (C.4.2) produces:

$$\frac{1}{m} \cdot \int_0^m r(x)dx = \beta_0 + (\beta_1 + \beta_2) \cdot \left(1 - e^{-\frac{m}{\tau}}\right) \cdot \frac{\tau}{m} - \beta_2 \cdot e^{-\frac{m}{\tau}}$$

Finally, rearranging yields the expression for the yield[34]:

$$R(m) = \beta_0 + \beta_1 \frac{1 - e^{-\frac{m}{\tau}}}{\frac{m}{\tau}} - \beta_2 e^{-\frac{m}{\tau}} \tag{C.4.3}$$

where parameters τ, β_0, β_1 and β_2 are defined as in expression (C.4.1). By construct, β_0 is the limiting value of the yield for large maturities. The limit of the yield for maturity approaching infinity equals:

$$\lim_{m\to\infty} R(m) = \beta_0 + (\beta_1 + \beta_2) \cdot \lim_{m\to\infty} \left(\frac{1 - e^{-\frac{m}{\tau}}}{m} \cdot \tau\right) - \beta_2 \cdot \lim_{m\to\infty} e^{-\frac{m}{\tau}} = \beta_0$$

It can be also shown that for maturities close to 0 the yield approaches $(\beta_0 + \beta_1)$.

$$\begin{aligned}\lim_{m\to 0} R(m) &= \beta_0 + (\beta_1 + \beta_2) \cdot \lim_{m\to 0} \left(\frac{1 - e^{-\frac{m}{\tau}}}{m} \cdot \tau\right) \cdot \frac{\tau}{m} - \beta_2 \cdot \lim_{m\to 0} e^{-\frac{m}{\tau}} = \\ &= \beta_0 - \beta_2 + (\beta_1 + \beta_2) \cdot \lim_{m\to 0} \frac{1 - e^{-\frac{m}{\tau}}}{\frac{m}{\tau}} = \beta_0 + \beta_1\end{aligned}$$

[34] It is a slightly altered formulation of the one presented in formula (2) in Nelson and Siegel (1987), p. 475.