

Endogenous Banks' Networks, Cascades and Systemic Risk*

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Abstract

We develop a dynamic network model with heterogeneous banks which undertake optimizing portfolio decisions subject to liquidity and capital constraints and trade in the interbank market whose equilibrium is governed by a tatonnement process. Due to the micro-funded structure of the decisional process as well as the iterative dynamic adjustment taking place in the market, the links in the network structures are endogenous and evolve dynamically. We use the model to assess the diffusion of systemic risk, the contribution of each bank to it as well as the evolution of the network in response to financial shocks and across different prudential policy regimes.

Keywords: networks, complexity, tatonnement, contagion, marked to market.

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1 Introduction

Interconnections in the banking system, as fostered by fast developments in financial innovation, increased degree of complexity in modern financial systems and the diffusion of over the counter derivatives, made systemic risk endemic and epidemic at crises times. Interconnections, initially set-up to facilitate risk sharing, have created channels whereby financial distress is quickly spread onto the entire system. Not surprisingly the rationale behind government intervention and bank bail out programs in the aftermath of the recent financial crisis was to be found not in the too-big-to-fail argument but in the too-interconnected-to-fail argument. The dangers associated with highly interconnected systems come from the possibility that the financial distress, experienced by one bank, might turn through cascading effects into full-fledged systemic risk, whose monitor, assessment and prevention has become paramount. Indeed one of the most important legacies of the 2007-2008 crisis has been the creation and development of a number of institutions whose mission is that of measuring systemic risk, monitoring financial vulnerabilities and safeguarding the financial system¹.

Against this background the literature offered no concrete paradigm to account for network externalities in combination with micro-founded decisional rules and financial (mis)-incentives, to quantify systemic risk and to forecast the development of financial contagion. We do a step in that direction by constructing a dynamic network model with *heterogenous* and *micro-founded* banks, whose links emerge endogenously from the interaction of intermediaries' optimizing decisions and an iterative *tatonnement* process which determines market prices. The financial system featured by our model consists of a *network* with a finite number of financial institutions which solve an optimal portfolio allocation taking into account liq-

¹In the U.S. the Dodd–Frank Wall Street Reform and Consumer Protection Act (See Financial Stability Oversight Council [12]) had created the Financial Stability Oversight Council, whose statute states in Title 1 that the primary objective of this institute is that of monitoring systemic risk. The main mission of the European Systemic Risk Board, established 16 December 2010, is the prevention or mitigation of systemic risks to financial stability in the Union that arise from developments within the financial system. The Financial Stability Board (FSB) has been established to coordinate, at the international level, the work of national financial authorities in addressing vulnerabilities and to develop and implement strong regulatory and supervisory policies.

uidity and capital constraints and for given market prices. Banks hold liquid assets, such as cash and deposits, lend to each other in the interbank market and invest in non-liquid assets, such as bonds or collateralized debt obligations. Banks differ at time zero for the returns on non liquid assets due to different information and administrative cost. Such differences in returns gives rise, at time zero, to heterogenous optimal portfolio allocation, hence to excess demand or supply of bank borrowing and lending. Banks' links are given by the cross-lending and borrowing that takes place in the interbank market. A crucial feature of our model is that the links in the adjacency matrix characterizing the network are not assigned randomly as in random network models but emerge endogenously from the combination of the optimal banks' decision and the tatonnement processes taking place in both, the interbank market and the market for non liquid assets. Furthermore dynamic adjustment in our model emerges as an intrinsic feature of the market adjustment even in absence of an initial shock impulse. Network externalities thus emerge as a manifestation of individual optimizing behavior and market adjustment. Since non-liquid assets are marked-to-market, the model also features pecuniary externalities via the occurrence of fire-sales.

Contagion in this model occurs through the transmission of shocks to non-liquid assets. Shocks are generated from a multivariate lognormal distribution and are randomly drawn for a certain number of periods. Contagion manifests itself through direct and indirect effects. The direct effects comprise *common exposure to risky assets* and *local network externalities*. First, if banks invest in the same financial products their balance sheets are correlated due to the multinomial nature of the shocks. Second, as banks are interlinked through counterpart exposure in the interbank market, a defaulting bank transmits losses to creditor banks. Indirect contagion effects manifest through fire-sales (*pecuniary externalities*). A negative shock in the value of non liquid assets induces several banks to de-leverage, a credit event that produces a fall in the market price and a *cascade* of losses in marked-to-market balance sheet of all other banks.

We simulate our model in response to adverse shocks to non-liquid assets, interpreted

as a credit event, and analyze the evolution of the banking network and the contribution of each bank to systemic risk in response to changes in the prudential policy parameters. Systemic risk is computed through the Shapley value² and refers to the probability default for the whole system. We also compute the contribution to systemic risk of each individual bank in the system: the latter depends crucially on the banks' asset position and on the inter-linkages in the network. The prudential policies considered are changes in the liquidity requirements, changes in the capital requirements and changes in the assets' risk weights as outlined by the Basel III agreements. Generally speaking changes in policy and regulations affect the strength of the cascade in response to shocks and the extent of both, the network and pecuniary externalities. We find that an increase in the capital requirement, as well as an increase in the risk weights, induce a bell shaped dynamic of overall systemic risk. At low levels of capital requirements, for instance, banks endowed with high return investment tend to leverage up, therefore increasing the demand for liquidity as well as the lending rates in the interbank market. The market clusters the connections around the high leveraged banks, which end up contributing heavily to systemic risk. As the requirement raises (say beyond 0.1), the capital constraint becomes binding and banks start to hoard liquidity: the banking network becomes sparse and systemic risk decreases. Increases in liquidity requirement instead tend to decrease overall systemic risk: robustness tend to prevail on fragility and the network becomes safer.

The rest of the paper is structured as follows. Section 2 compares our model to the recent literature on systemic risk. Section 3 describes the model, the equilibrium formation process, the shock transmission and the measure of systemic risk. Section 4 describes the numerical results and comments on the ability of the model to replicate stylized facts characterizing financial contagion. Section 5 analyzes the policy designs. Section 6 concludes. Appendices describe the optimal portfolio problem and the algorithm used to solve the model. Tables and figures follow.

²See Bluhm and Krahnert [17] and Borio, C., N. Tarashev and K. Tsatsaronis [7].

2 Relation to the Literature

This paper is related to two main strands of the literature. It is related to the literature on models of economic networks and to an emerging literature on systemic risk, part of which also makes use of network models.

Over the last decade network models have emerged as an alternative paradigm to analyze a variety of economic and social problems ranging from the formation of contacts and links in labour, financial and product markets to the formation and evolution of research networks (see Jackson [16]). The recent financial crisis has conveyed increased attention toward models featuring pecuniary and network externalities. The first model to exploit network externalities in banking systems is Allen and Gale [2]. Recently Gai, Haldane and Kapadia [14] have developed a random network model for the inter-bank market and have analyzed the effects of complexity and concentration onto financial fragility. In their model inter-linkages are driven by Poisson distributions and evolve in response to shocks: in contrast to them our model allows for micro-founded optimizing decisions of agents and for an endogenous formation of the network links. Most importantly, and contrary to most of the models featuring random networks, dynamic adjustment arises in our model as an intrinsic outcome of the tatonnement equilibrium process without the need to resort upon an impulse and propagation logic. Unexpected shocks can occur in our model, but they are not essential to induce dynamic adjustment³. Caballero and Simsek [9] focus on the role of complexity in network models: given the intricate structure of inter-linkages, banks face ambiguity when trading in the interbank market. This might amplify fire-sale when rumors of financial vulnerabilities are released. Krahen and Bluhm [17] analyze the formation of systemic risk, through Shapley values, in a model with three interconnected banks. In their model tipping points for the diffusion of systemic risk are determined by exogenously given heuristics, hence contrary to us they do not analyze optimizing banks decisions. Finally

³This feature also distinguishes our model from the traditional macro models on business cycle dynamics, which mainly appealed onto the Frisch-Slutsky impulse and propagation approach.

Anand, Gai and Marsili [5] analyze the effects of rollover risk in a model combining features from the global game theory and from the random networks.

A number of other papers have dealt with the analysis of systemic risk: see for instance Lagunof and Schreft [19], Rochet and Tirole [23], Freixas, Parigi and Rochet [13], Leitner [20], Eisenberg and Noe [11], Cifuentes, Ferucci and Shin [10], Billio, Getmansky, Lo and Pelizon [20], Geanakoplos [15]. Allen and Babus [3] provide an excellent recent survey. Finally our paper is related to the literature studying the design of regulations aimed at abating systemic risk (see for instance Allen and Gale [4]).

3 The Model

The financial system is made up with a population of N banks. Let $N \in \{1, \dots, n\}$ represent a finite set of individual banks, each of whom is identified with a node of the network.. We define ex-ante for this population a network $g \in \mathcal{G}$ as the set of links among heterogenous banks N , with \mathcal{G} being the set of all possible networks. An arc or a link between two banks i and j is denoted by $g_{i,j}$ where $g_{i,j} \in \mathbb{R}$. Here $g_{i,j} \neq 0$ reflects the presence of a link (directed network), while $g_{i,j} = 0$ reflects the absence of it. Links bear the actual economic meaning of banks' cross borrowing and lending. A crucial aspect of our analysis lies in the fact that those cross investment positions (hence the network links) result endogenously from the banks' optimizing decision and the markets' tatonnement processes. An important dimension in the diffusion of risk concerns the number of direct links held by each bank: a loss of value in the balance sheet of bank i will affect immediately all banks directly connected with bank i . For this reason it is instructive to define $N^d(i; g) = \{k \in N \mid g_{i,k} \neq 0\}$ as the set of banks with whom bank i has a direct link in the network. The cardinality of this set is given by $\mu_i^d(g) = |N^d(i; g)|$, namely the number of banks with whom i is directly linked in the network g . The $n - square$ adjacency matrix $\mathbf{G}^{(t)}$ of the network g describes the connections which arise after (t) iterations of the tatonnement process. Given that our model features a directed weighted network, banks i and j are directly connected if $g_{ij} \neq 0$. Also given

the nature of the connections, which materialize in the form of borrowing and lending it is always true that $g_{ij} = -g_{ji}$, thus \mathbf{G} is a *anti-symmetric* matrix with elements in the upper triangle carrying an opposite sign with respect to elements in the lower triangle.

Our network features optimizing banks which undertake an optimal portfolio allocation by maximizing profits subject to liquidity and capital requirement constraints and a non zero non liquid asset constraint. Banks decides about the optimal amount of liquid assets (cash), the optimal amount of lending and borrowing in the interbank market, and the optimal investment in non-liquid assets (bonds or collateralized debt obligations). Network externalities materialize through the cross-lending and borrowing taking place in the interbank market, while pecuniary externalities materialize since non-liquid assets are marked-to-market.

Banks differ at time zero for their allocation of non-liquid assets, which results, after optimization has taken place, in heterogenous optimal portfolio allocations. The optimizing decision together with the dynamic adjustment taking place in the various asset markets determines the final portfolio allocations and the final cross-borrowing and lending positions: the latter represent the entry of the adjacency matrix \mathbf{G} characterizing the interbank network. Sequential tatonnement processes⁴ take place in the interbank market and in the market for non liquid assets. The sequence of events can be described as follows. At time zero banks' optimization leads to heterogenous portfolio allocations in terms of both, interbank lending and investment in non-liquid assets. In the subsequent period banks enter the interbank market to search for the closest possible counterpart match: if the latter is not found an aggregate excess demand (or supply) of liquidity will materialize and will determine a change in the price of lending (or borrowing). At the new price banks re-optimize, re-enter the market with a new demand for borrowing (or lending) and start the search process once again. The described sequence of iterative steps converges to an equilibrium when the rela-

⁴See MasColell [22] and Mas Colell [21].

tive excess demand (or supply) of interbank liquidity is below a certain tolerance level⁵. A similar iterative process takes place in the market for non-liquid assets **and is outlined in the following sub-section**.

Contagion occurs when the financial system is subject to shocks to non-liquid assets. Initial shocks to non liquid assets are distributed according to a multivariate lognormal distribution and are transmitted through the changes in balance sheet values as triggered by changes in the market price (*fire-sale externalities*) and through the direct lending interlinkages (*network externalities*) and

3.1 Banks' Optimization

Banks' portfolio allocations are determined through an optimization process. As banks are heterogenous, individual asset allocations carry an index i . Aggregate variables or market prices are instead denoted without the index. As explained above the iterative market adjustment process is intrinsically dynamic⁶. For this reason we also equip our variables with a specific time index t .

Banks in the model start at time $t = 0$ with a certain amount of cash, c_0^i , deposits, d_0^i , and wealth, nw_0^i . At every generic period t of the iterative process and given the prevailing market prices, banks choose the optimal amounts of liquid assets, lending in the interbank market, bl_t^i , borrowing in the interbank market, bb_t^i , and investment in non liquid asset, nla_t^i . Aggregate excess demand (or supply) in the interbank market is defined as $z_t^1(p_t) = (bl_t - bb_t) = \sum_{i=1}^N (bl_t^i - bb_t^i)$. The links in the adjacency matrix \mathbf{G} representing the network links will be given by the final allocation of cross-borrowing and lending in the interbank market after optimization and the iterative market adjustment have come to convergence.

It is assumed that cash and deposits are risk-less assets which pay no interest, so that

⁵The crucial condition for convergence is that the rate at which the price (vector) approaches the equilibrium value behaves as a *Liapunov* function namely it is a real-valued function which takes decreasing values along the dynamic trajectory and a value of zero at the stationary point.

⁶Time here is not meant to be the actual real time but rather to represent the intervals occurring during and trial and error procedure that conveys the banks' counterpart search in the interbank market to actual matches.

their prices in the market are normalized to one⁷. Since now on individual variables will be indexed by i , while aggregate prices will not carry such index. Bank lending yields an interest rate r_t^{bl} , which will adjust in the iterative process to equilibrate aggregate excess demand and supply. Bank borrowings on the other side requires the payment of an additional premium so that the interest rate is given by $r_t^{bl} + \Delta_t^{bl}$, where Δ_t^{bl} is the spread between borrowing and lending. The spread can be rationalized in several ways, one of those is that borrowing entails risks linked to limited commitment to repayment, moral hazard and/or asymmetric information on the side of the borrower. Finally, non-liquid assets yield an interest of r_t^{nla} . We assume that at time zero banks receive different interest rates on non-liquid assets, reflecting different information costs and efficiency. The interest rate on non-liquid assets is $r_2^{nla,i}$. Heterogeneity in the asset returns implies that banks will differ at time zero for their optimal allocation of non liquid assets. The ensuing difference in the equity to liquidity ratios implies that banks will enter the interbank market with heterogenous excess demands (or supplies) for liquidity. Since interest rates on non-liquid assets, $r_t^{nla,i}$, do not depend upon the equilibrium in the interbank market, they can be set exogenously⁸. Finally notice that non-liquid assets are traded at a market price, p_t^{nla} : the latter is taken as given by atomistic banks ex ante and is determined ex post as result of the market equilibrium (see next section).

A summary of bank's balance sheet is depicted in Table 1.

Assets	Liabilities
Cash	Deposits
Bank lendings	Bank borrowings
Non-liquid assets	Equity

Table 1: Banks' Balance Sheets

Banks' optimization problem is detailed as follows. Banks' objective function is given

⁷Once the equilibrium is reached in the remaining markets (the interbank and the market for non liquid assets), equilibrium in the market for liquid asset is implied by Walras' law.

⁸It is also implicitly assumed that banks are atomistic so that their optimal allocation of non liquid assets cannot influence the returns.

by:

$$\pi_t^i = r_t^{bl} \cdot bl_t^i - (r_t^{bl} + \Delta_t^{bl}) \cdot bb_t^i + r_t^{nla,i} \cdot \frac{nla_t^i}{p_t^{nla}} \quad (1)$$

Banks face a *liquidity constraint*, of the type envisaged in Basel III agreements, due to which they have to hold at least a percentage, α , of their deposits in cash⁹:

$$c_t^i \geq \alpha \cdot d_t^i \quad (2)$$

Furthermore banks face a *capital requirement constraint*, as they must maintain an equity ratio, er_t^i , of at least $\gamma + \tau^i$:

$$er_t^i = \frac{c_t^i + p_t^{nla} \cdot nla_t^i + bl_t^i - d_t^i - bb_t^i}{\chi_1 \cdot p_t^{nla} \cdot nla_t^i + \chi_2 bl_t^i} \geq \gamma + \tau^i \quad (3)$$

where χ_1 and χ_2 are risk weights assigned respectively to the two risky assets, namely non liquid investment and bank lending. The parameter γ identifies the regulatory requirement, while the parameter τ^i reflect individual banks capital buffer. The risk coefficients are set exogenously as part of the regulatory system. Realistically we assume that banks need to hold less capital for bank lending than for investments in non-liquid assets, i.e. $\chi_1 \succ \chi_2$. If banks' equity ratio, er_t^i , is lower than the minimum capital requirement, τ , banks begin to reduce their exposure into bank lending (or in non-liquid assets): effectively this results in a reduction of the denominator of equation 3, relatively to the numerator, until the required ratio is achieved. This implies for instance, as we shall see later on, that any change in the regulatory capital requirement, τ , will result in a change of the demand (or supply) of bank lending in the interbank market, hence in a change of the cross-exposure of the network. Changes in the regulatory levels of the risk weights parameter χ_1 and χ_2 will also trigger an adjustment in the interbank market. The higher are those weights, the larger is the extent to which banks have to re-adjust their non-liquid asset and bank lending positions in order to satisfy the capital requirement. We further assume that if a bank cannot fulfill the capital

⁹For simplicity this fraction is assumed constant.

requirement it defaults: this event obviously will also result in an ex post change of the structure of the adjacency matrix, $\mathbf{G}^{(t)}$.

Two further observations are worth noticing. First, note that liquid assets do not appear in the denominator of equation 3; this is so since banks do not have to hold capital for their liquid asset holdings. Second, note that non-liquid assets are marked to market, which gives the potential for fire-sale spirals in the model.

At last, banks face a *no-short sales constraint*:

$$nla_t^i \geq 0. \tag{4}$$

The latter is needed for the problem to be well-behaved: this indeed rules out the possibility of negative prices for non-liquid assets.

3.2 Equilibrium in the Market for Non-Liquid Assets: Iterative Procedure

In the model, the market price of the non-liquid asset is found via a tâtonnement process between supply and demand. Following Shin, Cifuentes and Ferrucci, the inverse demand function is assumed to follow Equation 5

$$p = \exp(-\xi \sum_i s_i), \tag{5}$$

where ξ is a positive constant to scale the price responsiveness with respect to non-liquid assets sold, and s_i is the amount of bank i 's non-liquid assets sold in the market.

Solving Equation 3 for the amount of non-liquid assets sold by bank i to fulfill the capital requirement, yields the banks' supply of non liquid assets to the market, s_i . Since each s_i is decreasing in p , the aggregate sales function, $S(p)$, is also decreasing in p .

Tatonnement on the market for non-liquid assets can be described by the following process. Prior to any shock, the market price for non liquid assets equals 1, which is the initial price when all banks fulfill their regulatory requirements, and sales of the non-liquid asset are zero. A shock to bank i , say a certain loss of assets, shifts the supply curve upwards,

resulting in $S(1) = s_i \succ 0$ because bank i starts selling non-liquid assets to fulfill its capital ratio. However, for $S(1)$ the bid price, given by an exogenous demand function equals only $p(S(1))^{bid}$, while the offer price is one. The resulting market price is $p(S(1))^{mid}$, the midprice between bid and offer prices. Since the market price thus decreases and banks have to mark their non-liquid assets to market, additional non-liquid asset sales may be needed to fulfill the capital requirement. The stepwise adjustment process continues the demand and the supply curves intersect at p^* . Note that the supply curve may become horizontal from some value of non-liquid assets sold onwards, as the total amount of non-liquid assets on the banks' balance sheets is limited. Since a shock to a bank will always result in an upward shift of the supply curve, and the maximum price of the non-liquid asset being equal to 1, while the initial equilibrium prior to the shock equals zero, a market price $p \in (0, 1)$ always exists. The tatonnement process on the market for non liquid assets is given on figure 1.

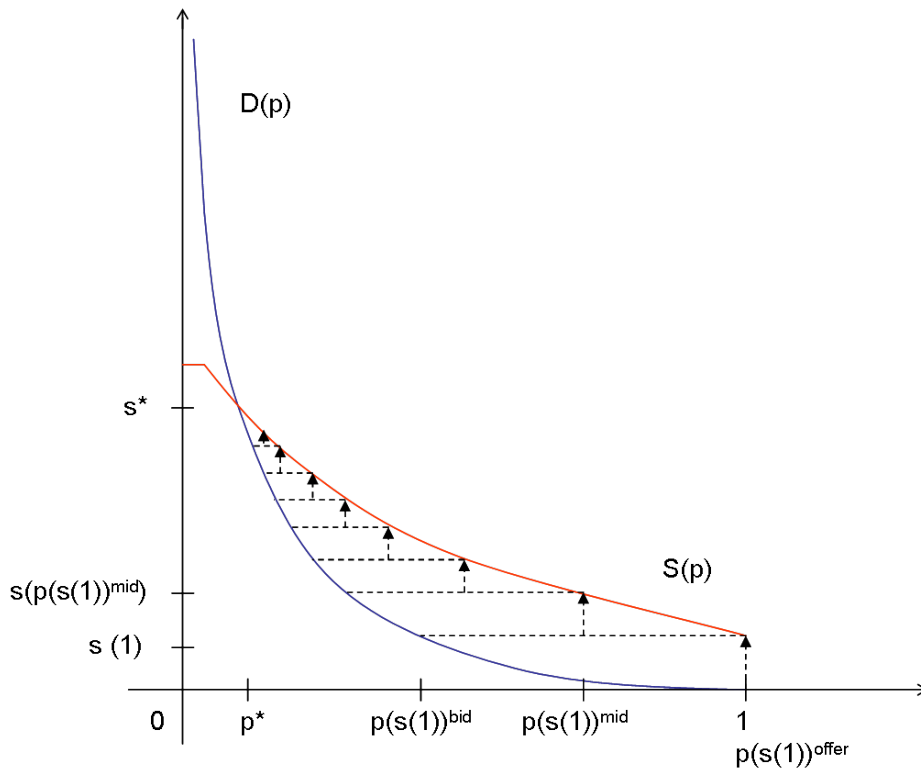


Figure 1: Tatonnement Process on the Market for Non Liquid Assets

As banks differ in terms of their initial equity allocation, the individual optimization gives rise to heterogeneous portfolio allocations. The next section shows how individual portfolio positions are allotted in the financial market giving rise to an equilibrium price and an aggregate asset allocation.

3.3 Equilibrium in the Interbank Market: Iterative Procedure

Once individual asset positions are determined we obtain the equilibrium in the interbank through an iterative trading process on bank lending and borrowing. At time zero banks start with different optimal portfolio allocations which also imply heterogeneous excess demand and supply of lending and borrowing. Banks enter the market with their optimal demand of borrowing and lending and search for the closest match. If a close match is not found, the price of bank lending, r_t^{bl} , adjusts in response to the aggregate excess demand and supply. Given the new prices banks will re-optimize and start a new search. Convergence is achieved when the relative matching error is below a certain tolerance level.

The steps in the numerical implementation of the iterative procedure can be described as follows. At the beginning banks set three reference points: an upper interest bound, \bar{r}_0^{bl} , a lower interest bound, r_0^{bl} , and the actual lending rate, r_0^{bl} . It is assumed that $r_0^{bl} \leq r_0^{bl} \leq \bar{r}_0^{bl}$. Given those bounds and the initial level of the returns banks optimization might result in excess demand or supply of lending. To fix ideas let's assume that it results in an excess supply of bank lending. In this case the lending rate will adjust downwards to re-equilibrate bank lending. The new lending rate is adjusted by $r_1^{bl} = \frac{r_0^{bl} + r_1^{bl}}{2}$. Given this new lending rate, banks re-optimize their portfolio allocation, which result in a new bank lending position. Gradually the excess supply of bank lending is absorbed through a sequential adjustment of the lending rate. The opposite adjustment takes place if demand for liquidity exceeds supply. The process converges when the relative matching error, defined as $\frac{|z_t^1(p_t)|}{(bl_t - bb_t)}$ is smaller than some specified tolerance value.

3.4 Price Tatonnement Stability Conditions

Since the price of liquid assets has been normalized to one, a dynamic equilibrium adjustment only takes place in the interbank market and in the market for non-liquid assets. We assume that in both markets the equilibrium takes place through a tatonnement process, namely a trial and error process taking place in fictional time and run by an abstract agent bent on finding and restoring the equilibrium after any perturbation¹⁰. A crucial assumption of tatonnement processes is that the actual trading never takes place until the dynamic price adjustment has reached convergence.

In this section we outline some general conditions under which global and local stability, namely the convergence of any price trajectory to the equilibrium level, is guaranteed for both markets. Stability of equilibrium is important for two reasons, First the stability conditions implicitly define the requirements that the numerical analysis would need to satisfy to guarantee that the system, perturbed by a shock, can return to an equilibrium. Second, since inter-bank lending determines the entry of the matrix describing network inter-linkages, equilibrium stability is a sufficient condition for the existence of an ergodic adjacency matrix.

The price vector in our model is given by $p_t = [p_t^1, p_t^2] = [r_t^{bl}, p_t^{nla}]$. Furthermore the excess demand function can be defined as follows: $z_t(p_t) = [z_t^1(p_t), z_t^2(p_t)] = [(bl_t - bb_t), nla_t]$, where $z_t^1(p_t) = (bl_t - bb_t) = \sum_{i=1}^N (bl_t^i - bb_t^i)$ represents aggregate excess demand in the interbank market and $z_t^2(p_t) = \sum_{i=1}^N nla_t^i$ represents aggregate excess demand in the market for non liquid asset. If we start with an initial price vector p_0 which is not an equilibrium, namely $z_t(p_0) \neq 0$, the demand and price adjustment will take place according to the following differential equation:

$$\frac{dp_t^j}{dt} = \gamma^j z_t^j(p_t) \quad (6)$$

where j indicates the reference market and $\gamma^j > 0$ is a speed adjustment factor. Global stability implies that prices, moving along the above dynamic trajectory, converge toward

¹⁰See Mas-Colell and Whinston [22], and Mas-Colell [21].

the equilibrium. One possibility for this to happen is that in presence of an aggregate excess demand $z_t(p_t)$ prices in the market adjust so as to cause a proportional decrease in the magnitude of all excess demand and supply. In vectorial notation this implies that the Jacobian of the excess demand functions $Dz_t(p_t)(\frac{dp}{dt}) = -\lambda z_t(p_t)$, where λ represents the factor of proportionality. Rearranging the last system of differential equations one obtains the following solution for the price trajectory:

$$\frac{dp}{dt} = -\lambda [Dz_t(p_t)]^{-1} z_t(p_t) \quad (7)$$

A sufficient condition for restoring stability after small shocks is the existence of the inverse of $Dz_t(p_t)$.

3.5 Calibration

The model parameters are chosen to match values observed in the financial system and/or imposed by supervisory policy. The parameter α , the amount of liquid assets banks have to hold as a function of the amount of deposits, is set to 0.1, thus being equivalent to the cash reserve ratio in the U.S. The parameter χ_1 , the risk weight for non liquid assets, is set to 1: this value reflects the risk weight applied in Basel II to commercial bank loans. The parameter χ_2 , the weight for interbank lending, is set to 0.2, which is also the risk weight actually applied to interbank deposits between banks in OECD countries. The amount of equities and deposits that banks have initially on their balance sheets is set to 60 billions (mean with variance 10) and 600 billions which as a ratio is closely to figures actually found on banks' balance sheets. Following federal reserve bank regulatory agency definitions, banks must hold a capital ratio of at least 8%. Finally, banks return on non-liquid assets is uniformly distributed on the interval between 0% to 10%.

The parameters settings which will subsequently be investigated in a comparative static analysis are displayed on Table 2.

α	χ_1	χ_2	γ	Deposits	ς	Equity	Yield on NLA
0.1	1	0.2	0.08	500	0.01	$N(60, 10)$	$U(0, 0.1)$

Table 2: Parameter Values in the Baseline Setting

The table displays the parameter values in the baseline setting. α is banks' liquidity requirement, χ_1 is the risk weight for non-liquid asset investments, χ_2 is the risk weight for interbank lending, γ is the capital requirement ratio, and ς is the amount by which banks overfulfill regulatory requirements. N and U designate normal and uniform distributions, respectively.

3.6 Systemic Risk Measure

Generally speaking systemic risk occurs in the event in which a shock to a single institution spread to the system in a way that determines the collapse of the entire system, rather than simply the default of individual banks or of a limited group of financial intermediaries. A prerequisite for the emergence of systemic risk is the presence of inter-linkages and interdependencies in the market, so that the default (or a run) on a single intermediary or on a cluster of them leads to a cascade of failures, which could potentially undermine the functioning of the entire financial system. Generally, systemic risk is defined as the risk that large parts of the financial system default leading to negative repercussions in the real economy because of a subsequent lack of financial services provision and credit. In our paper we define systemic risk as the proportion of the financial system in default subsequent to a shock which hits the banks. Say, for example, the financial system consists of three banks of which each bank holds one third of the assets held systemwide. If subsequent to a shock two banks default, systemic risk conditional on the shock equals $1/3$.

Since we are also interested in how much each bank contributes to systemic risk, we need a metric to measure their impact. While there is much agreement about the general definition of systemic risk, there is much less agreement upon quantitative measures for individual contributions. The traditional analysis for measuring contribution to systemic risk was based upon the judgement of whether the defaulting bank or group of intermediary was *too big to fail*: such an assessment is based on indicators such as the institution's size relative to the system, market share concentration, based for instance on the Herfindahl-Hirschman Index, the oligopolistic structure of the market and the presence of barriers to entries. Recently and due to the emergence of complex financial relations, the focus of

contribution to systemic risk measures has been shifted toward an assessment of the *too interconnected to fail*. It is on this last concept that we focus. One measure which has been recently proposed to determine the link between systemic risk and interconnection is the Shapley value¹¹, an indicator which allows us to determine the contribution of individual banks to aggregate risk. In game theory this value is used to find the fair allocation of gains obtained by cooperation among players. For a game consisting of I players the Shapley value is defined as:

$$\xi_i(v) = \frac{1}{j!} \sum_{K \ni i; K \subset I} v(K) - (v(K) - \{i\}) \quad (8)$$

where I is the set of all players, $v(K)$ is the value obtained by coalition K , including player i and $(v(K) - \{i\})$ is the value of coalition K without player, and j is the number of coalitions.

The Shapley value for player i is the average contribution to the gain of the coalition over all permutations in which players can form a coalition. The Shapley value has the following properties:

1. *Pareto efficiency*. The total gain of a coalition is distributed.
2. *Symmetry*. Players with equivalent marginal contributions obtain the same Shapley value.
3. *Additivity*. If one coalition can be split into two sub-coalitions then the pay-off of each player in the composite game is equal to the sum of the sub-coalition games.
4. *Zero player*. A player that has no marginal contribution to any coalition has a Shapley value of zero.

Since the number of permutations involved in calculating the Shapley value increases strongly with the number of banks, the analysis is subject to the curse of dimensionality. The Shapley value can be approximated by the average contribution of banks to systemic

¹¹See Shapley [24]. See also Tarashev, Borio, and Tsatsaronis [7] and Bluhm and Krahen [17]. Alternative measures of systemic risks are proposed for instance in Adrian and Brunnermeier [1] through a CoVaR methodology.

risk over l randomly sampled permutations as displayed in Equation 9:

$$\hat{\phi}_i(v) = \frac{1}{l} \sum_{K_l \ni i; K_l \subset I} v(K) - v(K - \{i\}), \quad (9)$$

The parameter l determines the discrepancy between the real Shapley value and its estimate, that is, the error. It can be shown that this estimator is unbiased and efficient.¹²

Generally speaking the Shapley value is affected by the degree of bank interconnections. In our model interconnection occurs through both, direct and indirect links. Direct links are given by the correlations of shocks to non liquid assets and the exposure to others' banks balance sheet. Indirect links are given by the effects that a fall in the market price of non-liquid assets has on the balance sheet of the entire system. Generally speaking the overall degree of interconnections in our model is affected by the parameters characterizing the optimizing decision: we will return on this point later on. The link between interconnections and systemic risk implies that any parameter change which affects the inter-connection in the network structure will have an impact on systemic risk as well.

3.7 Shock Transmission

In the model shocks take the form of a loss in banks' non-liquid asset holdings¹³.

If subsequent to a shock realization, a bank cannot fulfill its capital requirement, it will sell non-liquid assets, thereby indirectly transmitting the shock to other banks, via downward pressure on the market prices of non-liquid assets. If it still cannot fulfill the capital requirement, the bank will default. The clearing algorithm for shock transmission is an iterative process displayed on Figure 2.¹⁴

Banks' assets are diminished by the initial shock (step A on Figure 2). Banks that cannot fulfill the capital requirement start selling non-liquid assets in the market (step B

¹²See, for example, [25].

¹³We follow Bluhm and Krahen [17] to model the shock transmission process. Other shocks are possible, for example a sudden drop in non-liquid asset prices or the default of a bank in the system.

¹⁴Note that we use a sequential clearing algorithm here. The clearing algorithm used in [11] is not necessary since in our setting no circular lending relations between banks can come up because banks are either lenders or borrowers.

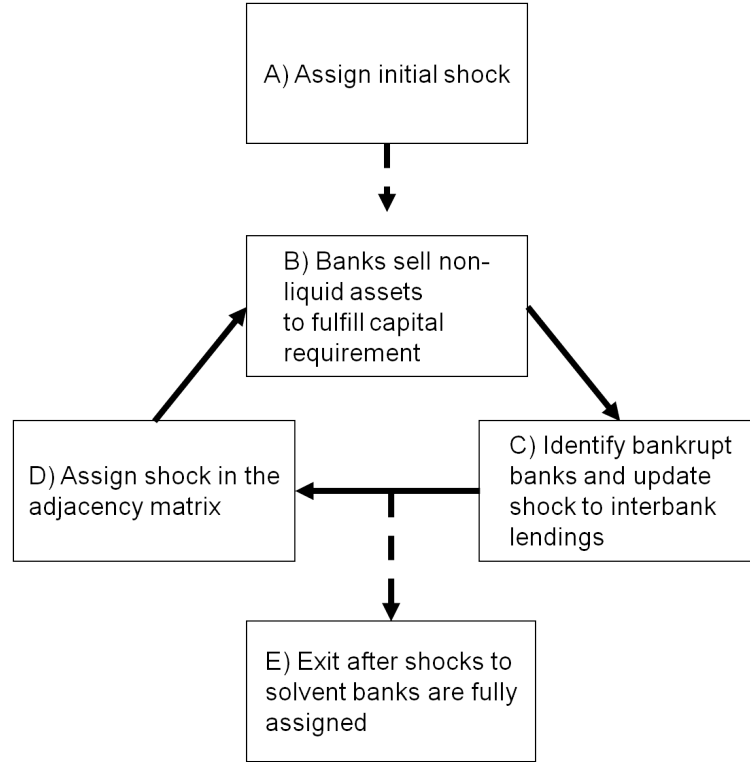


Figure 2: Algorithm for Shock Transmission

on Figure 2 modeled by the tâtonnement process outlined before). Banks that are not able to fulfill the capital requirement even after selling all their non-liquid assets default. Insolvent banks with negative equity-value transmit shocks to their creditors until they have an equity-value of zero. Thus, the overall shock to bank i 's creditors is computed as $-\sum_j a_j + p \cdot b_i + c_i - \sum_j l_j - d_i$ in case it defaults (step C on Figure 2). In case there are shocks via the interbank liability channel they are assigned proportionally to the insolvent bank's individual liabilities, respecting seniority of deposit holders (step D on Figure 2), and the iteration restarts (step B on Figure 2). If there are no shocks via the interbank liability channel the initial shock has been absorbed and systemic risk conditional on this shock is computed (step E on Figure 2).

4 Adverse Shocks and Prudential Policy: Effects on Network Evolution and Systemic Risk

In this section we analyze the effects of changes in the policy regulatory parameters and in response to a shock to non-liquid assets on the contribution of each individual bank to systemic risk. The contribution to systemic risk will be interpreted through the lenses of the evolution in the network structure: certain changes in the regulatory and policy parameters will determine certain optimal portfolio allocation, which through the evolution of the network structure, will affect the dynamic contribution to systemic risk. To fix ideas we will consider a system of $N = 15$ which we consider as representative of mildly concentrated banking systems. In our analyses we will change several of our baseline parameters outlined on Table 2 to investigate the impact on systemic risk and banks' contribution to it. Furthermore, we will investigate how risk charges affect banks' portfolio decisions and thus equilibria on the markets for interbank and non-liquid assets. Risk charges have been suggested as a Pigouvian instrument to achieve two goals at the same time, (i) incentivize banks to lower their contribution to systemic risk, and, (ii), charge banks according to their negative externality on the financial system. For example, with the introduction of the Restrukturierungsfondsgesetz in Germany banks are charged a levy which depends (i) on their interconnectedness with other banks and, (ii), the extent of their derivative investments. The proceeds of these levies are used to finance a resolution fund to stabilize the financial system. Note that all results reported as well as confidence intervals given are based on the mean outcomes from 1000 multivariate normally distributed random shocks which are centered at a value of 5% and have a variance of 25.

4.1 Analyses Without Central Bank Intervention

The financial system which will be used in the following analyses is displayed on Table 3. It is a random realization based on the baseline parameters outlined on Table 2. On the table, banks are designated with letter B . Each bank's assets are displayed along the according

row and banks liabilities are displayed along the according column. For example, matrix element (2,5) shows how much Bank 1 has lent to Bank 5. The last two columns designate non-liquid assets (NLA) and liquid assets (LA), that is, cash, respectively. The last row are assets from the rest of the world, that is, deposits.

Figure 3 displays a visual outline of the financial system displayed on Table 3. Each bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial system there are four further indicators. First, the red ball gives an indication about the percentage of the financial systems a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

Table 4 displays systemic risk (SR) as well as banks' contribution to it ($B1 - B15$), all in the baseline setting. All values are obtained as outlined in Sub-Section 3.7.

	B 1	B 2	B 3	B 4	B 5	B 6	B 7	B 8	B 9	B 10	B 11	B 12	B 13	B 14	B 15	NLA	LA
B 1	0	0	0	57	46	14	0	47	110	94	14	99	0	39	0	0	50
B 2	0	0	0	58	47	15	0	48	113	97	14	102	0	40	0	0	50
B 3	0	0	0	3	2	1	0	2	6	5	1	5	0	2	0	466	50
B 4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	812	50
B 5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	753	50
B 6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	575	50
B 7	0	0	0	56	45	14	0	46	108	92	13	97	0	38	0	0	50
B 8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	755	50
B 9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1108	50
B 10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1020	50
B 11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	571	50
B 12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1048	50
B 13	0	0	0	57	46	15	0	47	110	95	14	100	0	39	0	0	50
B 14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	711	50
B 15	0	0	0	57	46	15	0	47	110	94	14	100	0	39	0	0	50
Deposits	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	0	0

Table 3: Financial System in Baseline Setting

The table displays the financial system for the baseline setting. Banks are designated with letter *B*. Each bank's assets are displayed along the according row and banks liabilities are displayed along the according column. For example, matrix element (2,5) shows how much Bank 1 has lent to Bank 5. The last two columns designate non-liquid assets (NLA) and liquid assets (LA), that is, cash, respectively. The last row are assets from the rest of the world, that is, deposits.

Financial System in Baseline Setting

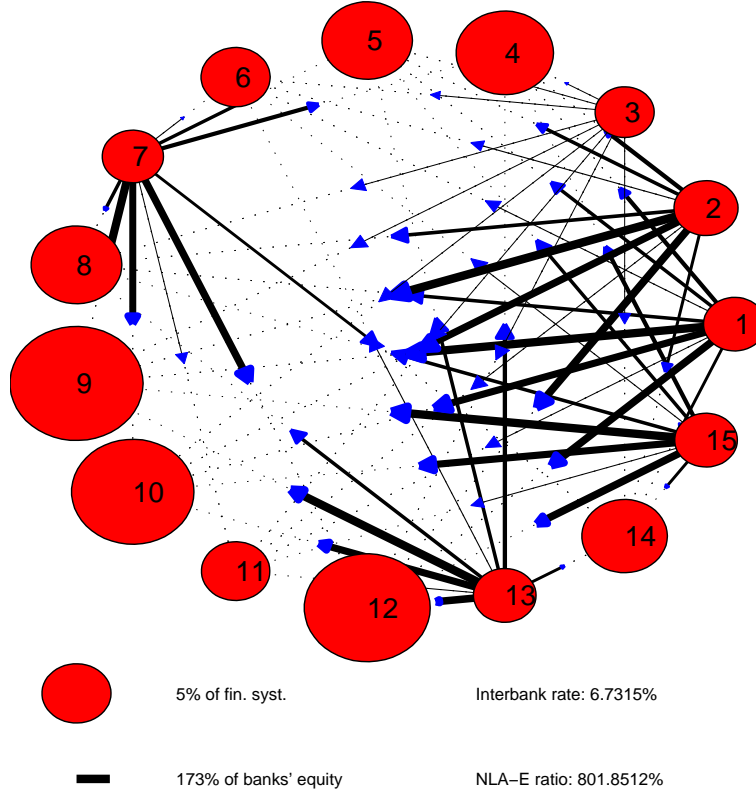


Figure 3: Financial System in Baseline Scenario

The figure displays an outline of the financial system emerging in the baseline setting. Each bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial system there are four further indicators. First, the red ball gives an indication about the percentage of the financial systems a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

B 1	B 2	B 3	B 4	B 5	B 6	B 7	B 8
0.00	0.00	0.03	0.05	0.04	0.03	0.00	0.04
B 9	B 10	B 11	B 12	B 13	B 14	B 15	SR
0.06	0.06	0.03	0.05	0.00	0.04	0.00	0.43

Table 4: Systemic Risk and Banks' Contribution in the Baseline Setting Without Central Bank Intervention

The table displays banks' contribution to systemic risk, as well as overall systemic risk (SR), in the baseline setting. Note that values have been rounded.

Figure 4 displays selected financial system realizations at different liquidity requirements, with all remainder model parameters kept at their baseline value. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity. On the figure, four developments can be identified with increasing values of the liquidity requirement. First, the financial system becomes more concentrated, that is, few banks get very big relative to the remainder banks in the financial system. Second, the number of banks which engage in interbank lending increases. Third, the interest rate on the interbank market increases. Fourth, the relative amount of non-liquid asset investments to banks' equity decreases. The mechanisms which drive these developments are as follows. When banks have to hold a larger fraction of their deposits as liquid assets, can invest less in the interbank market (via supplying funds to the market) and non-liquid investments. The latter banks try to replace the funds from their deposits which they now have to hold in cash via increasing their demand for funds on the interbank market. This pushed up the interest rate which in turn makes it more profitable for banks whose return on non-liquid assets is now below the interest rate on the interbank market to invest their funds into interbank lending instead of non-liquid assets. The number of banks engaging in interbank market lending activities thus increases. At the same time this

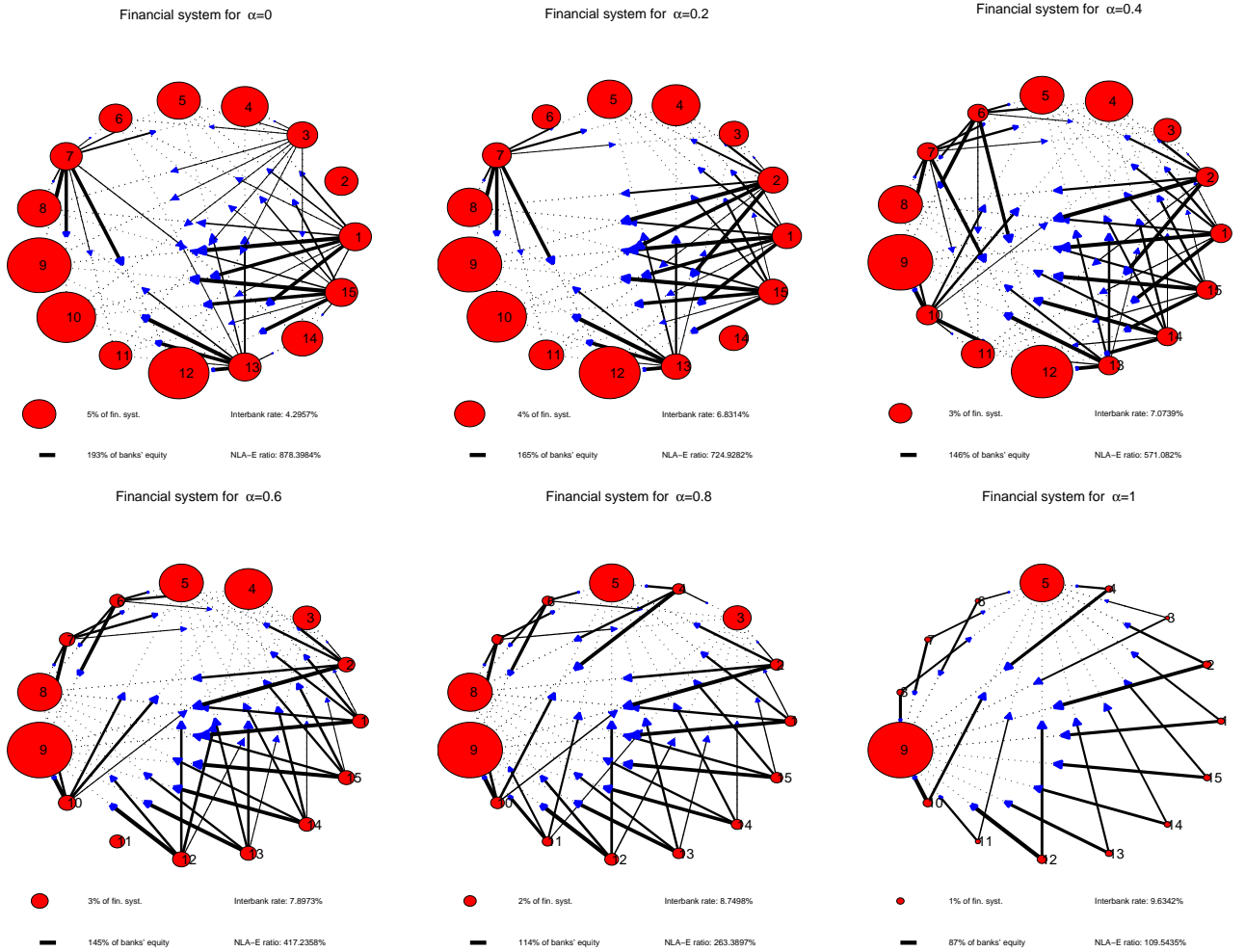


Figure 4: Financial System Structures and Liquidity Requirement

The figure displays selected financial system realizations at different liquidity requirements, with all remainder model parameters kept at their baseline value. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

decreases the amount of overall investment into non-liquid assets in the financial system.

Figure 5 shows a visualization of systemic risk and banks' contribution to it when the liquidity ratio, α , increases. The figure displays systemic risk (y-axis on panel 16, bottom right) and banks' contribution to it (y-axis on panels 1 to 15) as solid lines over different values of the liquidity requirement ratio, with all other model parameters kept as in the baseline setting. The dotted lines are the two standard deviation error bands. On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

On the figure, three developments can be identified when the liquidity requirement increases. First, systemic risk (panel 16) decreases. Second, the amount of non-liquid asset investment relative to banks' equity decreases. Third, interbank lending relative to banks' equity first increases and then decreases again. Note that the contribution of banks to systemic risk is usually higher for banks which have borrowed because (i) they hold many non-liquid assets which can cause firesales spirals in the financial system and, (ii), since they have borrowed, they can directly transmit shocks in the financial system if they default on their debt.

The mechanism for these developments is outlined in the following. Increasing the capital requirement ratio leads banks to hold more of their deposits in cash instead of investing them into non-liquid assets or the interbank market. Banks which want to keep their level of non-liquid asset investment thus increase their demand for funds on the interbank market. The interest rate for interbank funds thus increases. As outlined before, banks whose return on non-liquid asset investments is now below the yield on the interbank market start providing funds to other banks, which decreases the amount of non-liquid asset investment in the financial system. The bell-shaped development of interbank lending relative to banks' equity is driven by two opposite effects. First, the number of banks providing liquidity on

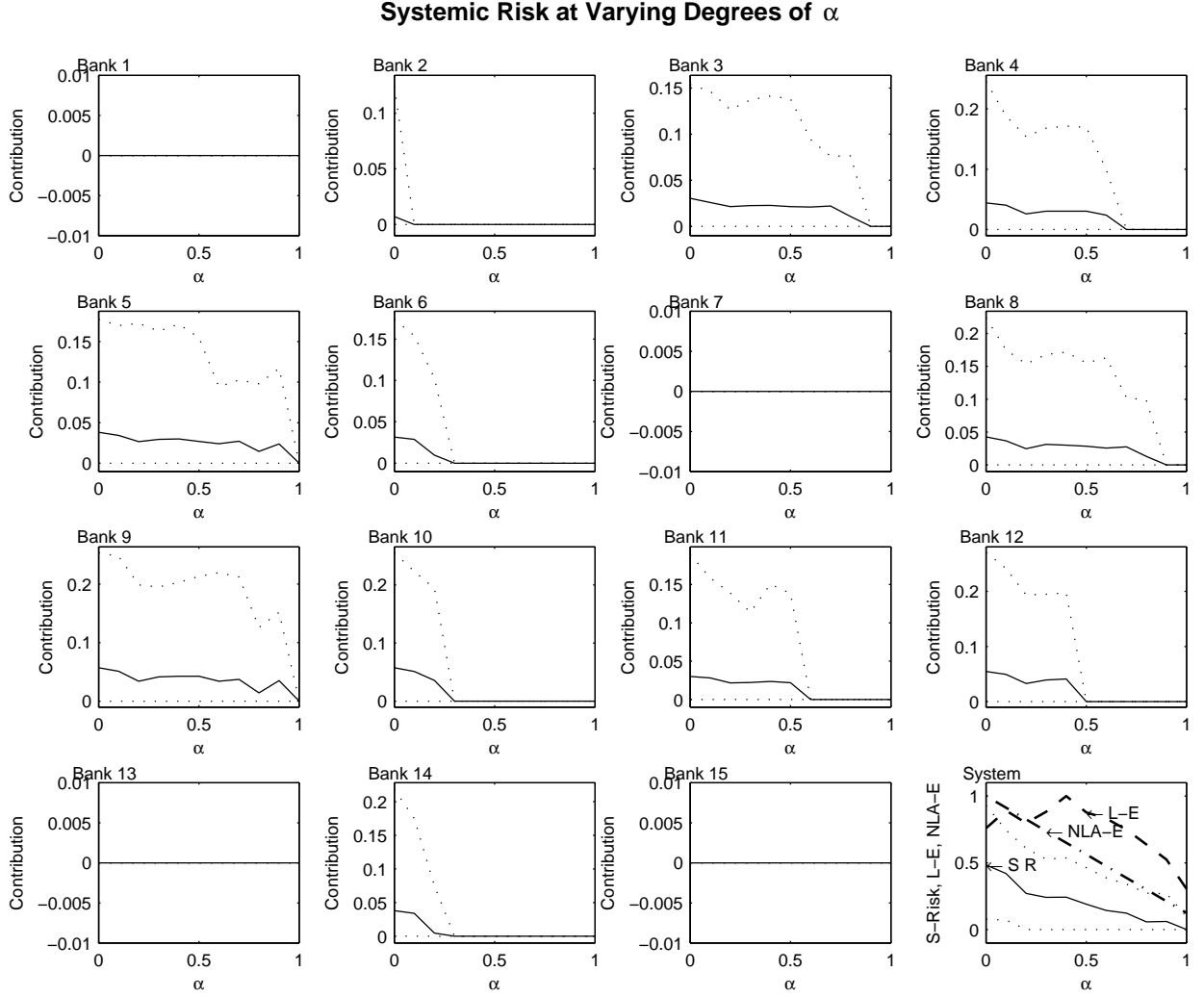


Figure 5: Systemic Risk and Liquidity Requirement

The figure displays systemic risk (y-axis on panel 16, bottom right) and banks' contribution to it (y-axis on panels 1 to 15) as solid lines over different values of the liquidity requirement ratio, with all other model parameters kept as in the baseline setting. The dotted lines are the two standard deviation error bands. On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

the interbank market increases, however, second, the amount each bank provides on the interbank market gets more and more limited by the increasing liquidity requirement. The latter effect ultimately prevails.

Next, we turn to investigating the effect of increasing the capital requirement ratio in the financial system, γ . Figure 6 displays the evolution of the financial system structures at increasing values of capital requirement. On the figure, four developments can be identified

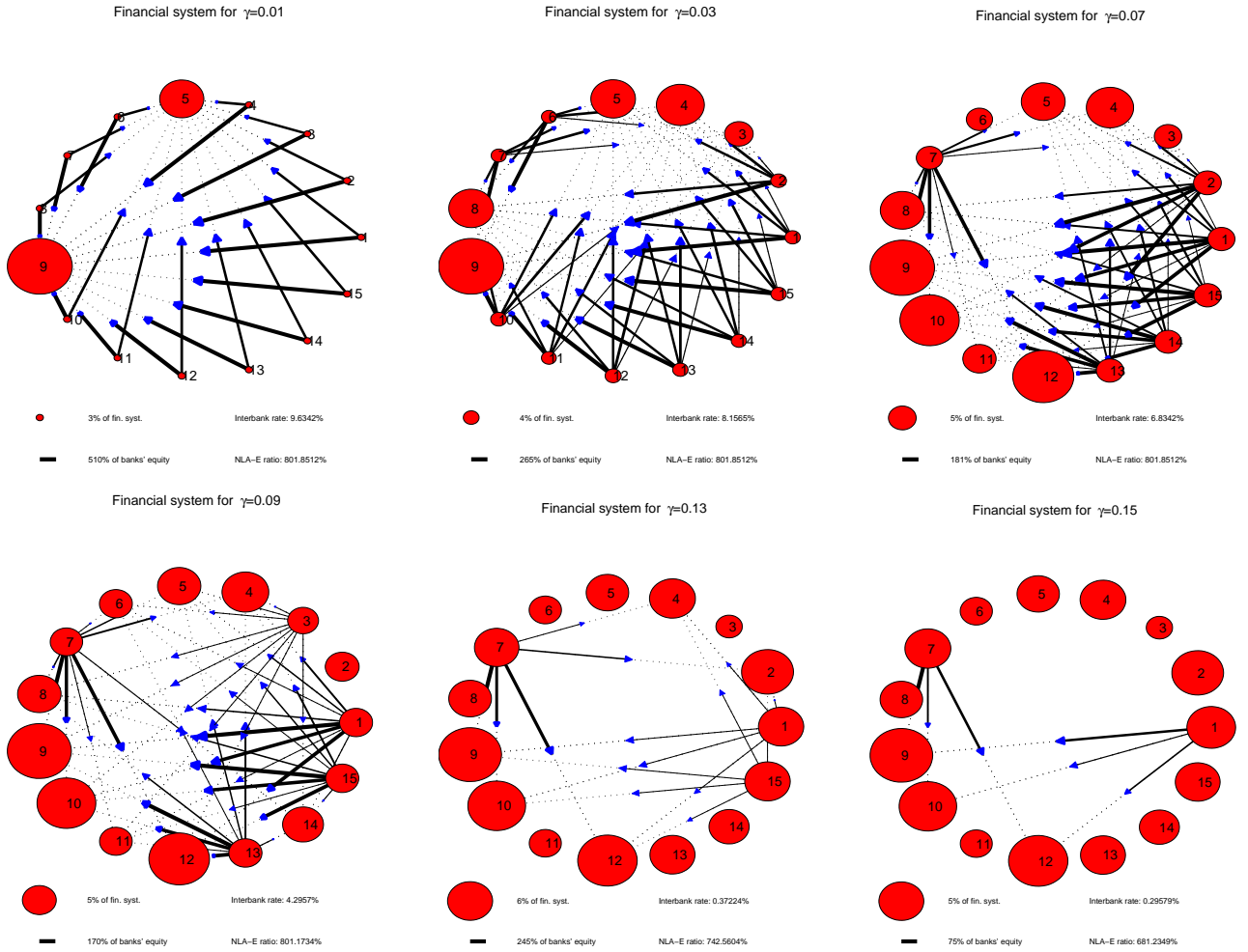


Figure 6: Financial System Structures and Capital Requirement Ratio

The figure displays selected financial system realizations at different liquidity requirements, with all remainder model parameters kept at their baseline value. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

when the capital requirement ratio is increased. First, the system becomes less concentrated, that is, the banks become more similar in size. Second, the interest rate on the interbank market decreases. Third, the amount of investment into non-liquid assets relative to banks' equity is first stable and then decreases. Fourth, less banks engage in lending funds on the interbank market.

The mechanisms underlying these developments are outlined in the following. When the capital requirement ratio is increased, banks can leverage less ('leverage effect') and thus demand less funds on the interbank market. This causes the interbank market interest rate to decrease. As the interest rate decreases, it becomes more profitable for banks to invest into non-liquid assets because they obtain a higher return relative to the interbank market. This switching-in of banks into non-liquid asset investments counterbalances the 'leverage effect' for some time. However, ultimately it prevails and the ratio of non-liquid asset investment to banks' equity decreases. Since banks leverage less, the system also becomes less concentrated.

Figure 7 shows the effect of changes in the capital requirement ratio on systemic risk and banks' contribution to it.

On the figure, three developments can be identified when the capital requirement ratio is increased. First, the ratio of non-liquid assets to banks' equity is first stable but starts to decrease at high levels of capital requirement. Second, bank lending decreases. Third, systemic risk initially increases and then decreases again.

The mechanisms underlying these developments are outlined in the following. When the capital requirement ratio is increased, banks can leverage less which decreases their demand for funds on the interbank market and causes the interest rate for interbank funds to decrease. As outlined before, the ratio of non-liquid assets to banks' equity decreases because ultimately the leverage effect prevails over the relatively higher number of banks engaging in non-liquid asset investments. Finally, systemic risk initially increases when more banks start borrowing money in the interbank market, causing the financial system to become vulnerable to shocks to a higher number of banks. However, since with an increasing capital

Systemic Risk at Varying Degrees of γ

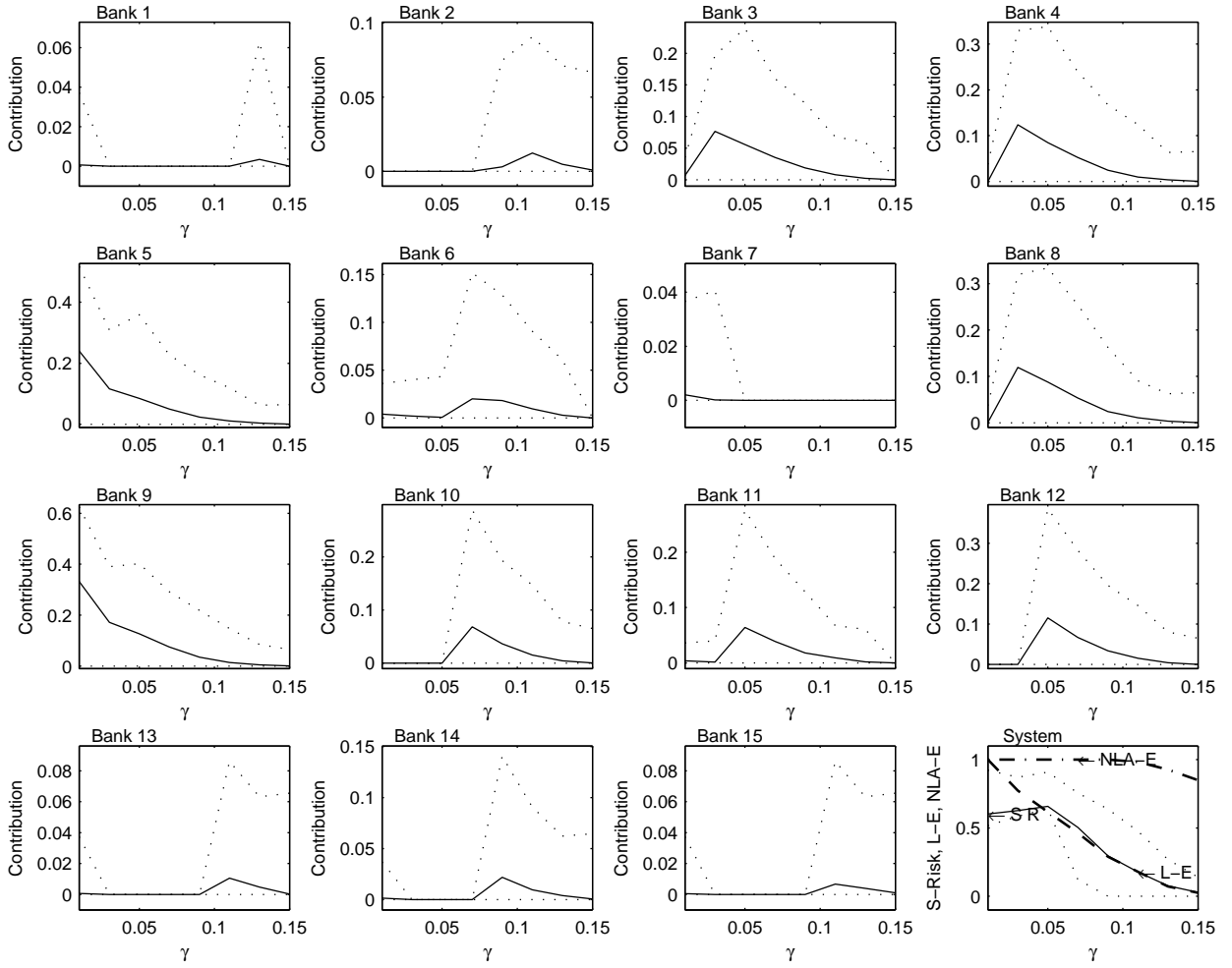


Figure 7: Systemic Risk and Capital Requirement Ratio

The figure displays systemic risk (y-axis on panel 16, bottom right) and banks' contribution to it (y-axis on panels 1 to 15) as solid lines over different values of the liquidity requirement ratio, with all other model parameters kept as in the baseline setting. The dotted lines are the two standard deviation error bands. On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

requirement ratio less and less leverage is possible, the interbank lending channel ultimately dries out, causing systemic risk to decrease again.

In the following we will investigate the effects of increasing the risk weight on non-liquid assets, χ_1 . Figure 8 displays the evolution of the financial systems at increasing values of the risk weight on non-liquid assets. The effects and mechanisms at work when increasing the risk weight on non-liquid assets are similar to the effects and mechanisms which arise when increasing the capital requirement ratio.

Figure 9 shows the effect of increasing the risk weight on non-liquid asset investments on systemic risk and banks' contribution to it. As before, the effects and mechanisms at work when increasing the risk weight on non-liquid assets are similar to the effects and mechanisms which arise when increasing the capital requirement ratio.

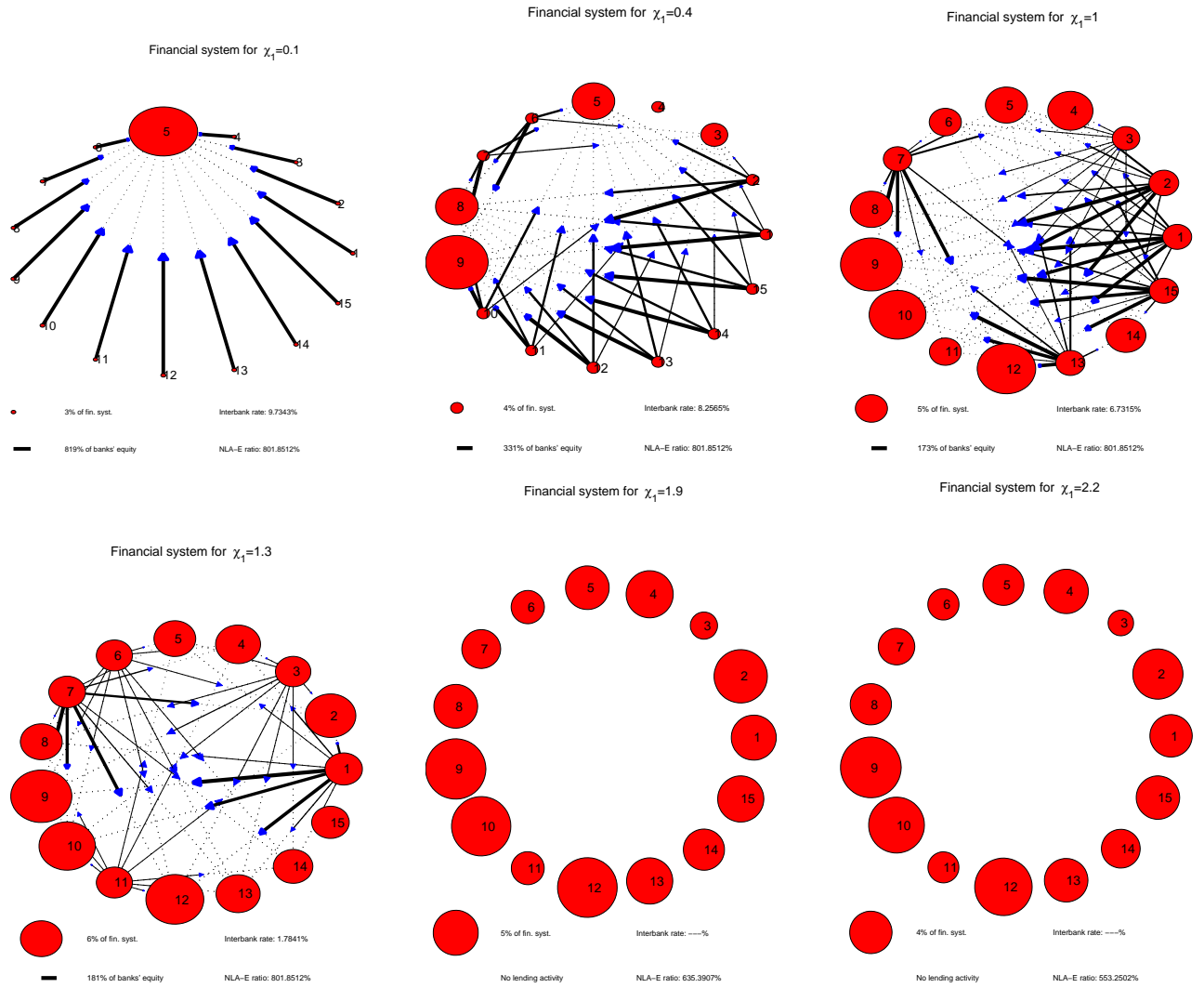


Figure 8: Financial System Structures and Derivative Risk Weights

The figure displays selected financial system realizations at different liquidity requirements, with all remainder model parameters kept at their baseline value. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

Systemic Risk at Varying Degrees of χ_1

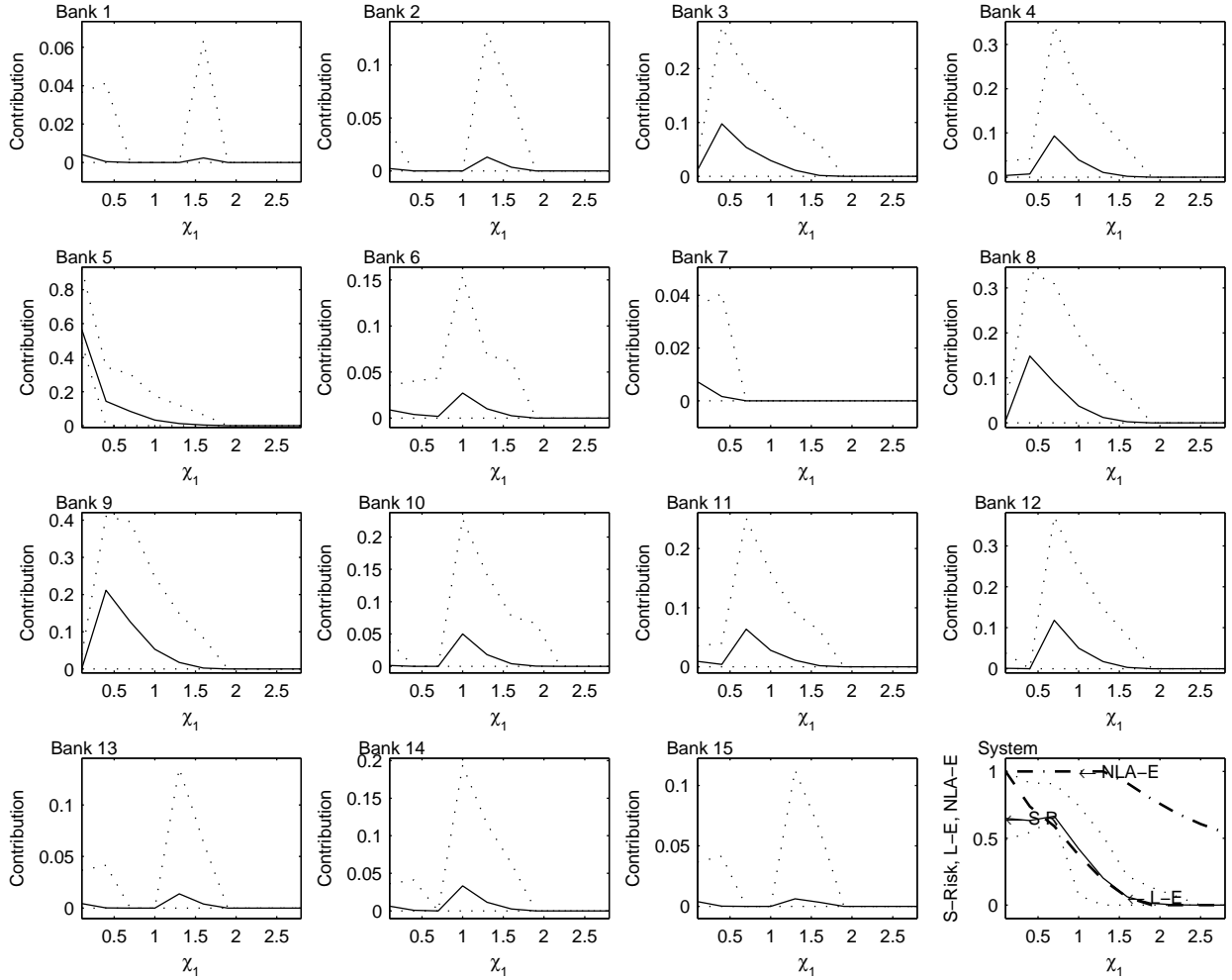


Figure 9: Systemic Risk and Derivative Risk Weights

The figure displays systemic risk (y-axis on panel 16, bottom right) and banks' contribution to it (y-axis on panels 1 to 15) as solid lines over different values of the liquidity requirement ratio, with all other model parameters kept as in the baseline setting. The dotted lines are the two standard deviation error bands. On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

Next we investigate the impact of increasing the risk weight on interbank lending, χ^2 . Figure 10 displays the evolution of the financial system along increasing values of the risk weight on interbank lendings. On the figure, there are three developments which can be identified when the risk weight on interbank lendings is increased. First, the interbank market interest rate is stable and then increases. Second, the ratio of non-liquid assets to banks' equity is first stable and then decreases. Third, in tendency the number of banks engaging in interbank lending increases.

The mechanisms leading to these developments are described in the following. Increasing the risk weight on interbank lending from a certain point onwards reduces supply of lending banks on the interbank market. This causes the interbank interest rate to increase. This in turn increases the number of banks engaging in interbank lending if the interest rate on the interbank market is above their return on non-liquid assets. This causes the non-liquid assets to equity ratio to decline.

Figure 11 shows the effect of increasing the risk weight on interbank lendings on systemic risk and banks' contribution to it. On the figure there are three developments which can be identified when the risk weight on interbank lending is increased. First, systemic risk initially increases and then decreases again. Second, the ratio of non-liquid assets to banks' equity is first stable and then decreases. Third, the ratio of interbank lending to banks' equity is first stable and then decreases.

Increasing the risk weight on interbank lendings reduces supply of funds on the interbank market and thus increases the interbank interest rate to increase. Thus banks which obtain a higher return on interbank lending than on non-liquid asset investments engage as lenders in the interbank market. The higher interconnectedness in the financial system causes systemic risk to increase. However, from some point onwards, the high risk weight on interbank lendings causes the interbank market to dry out which in turn causes systemic risk to decrease again.

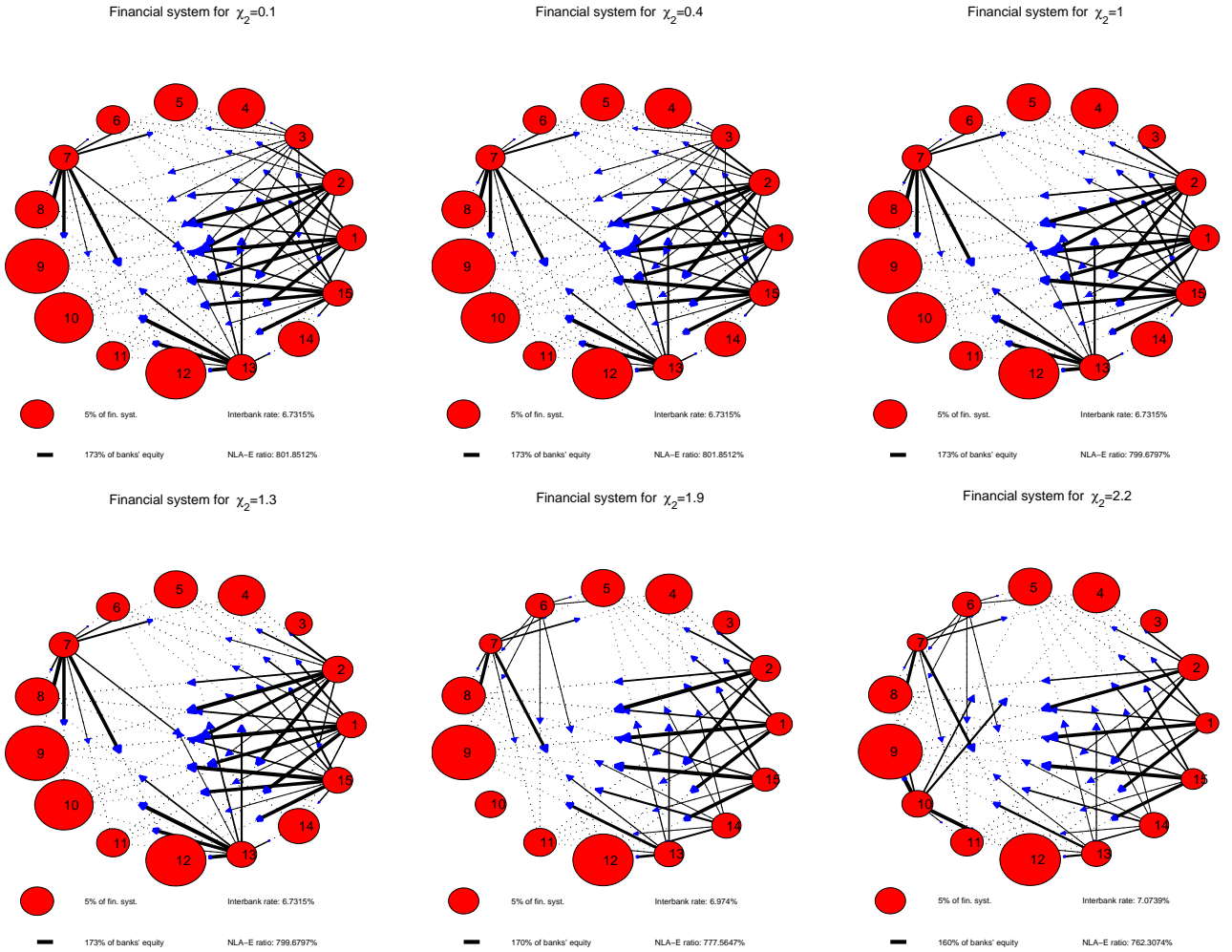


Figure 10: Financial System Structures and Interbank Lending Risk Weights

The figure displays selected financial system realizations at different liquidity requirements, with all remainder model parameters kept at their baseline value. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on average in the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested in non-liquid assets relative to their equity.

Systemic Risk at Varying Degrees of χ_2

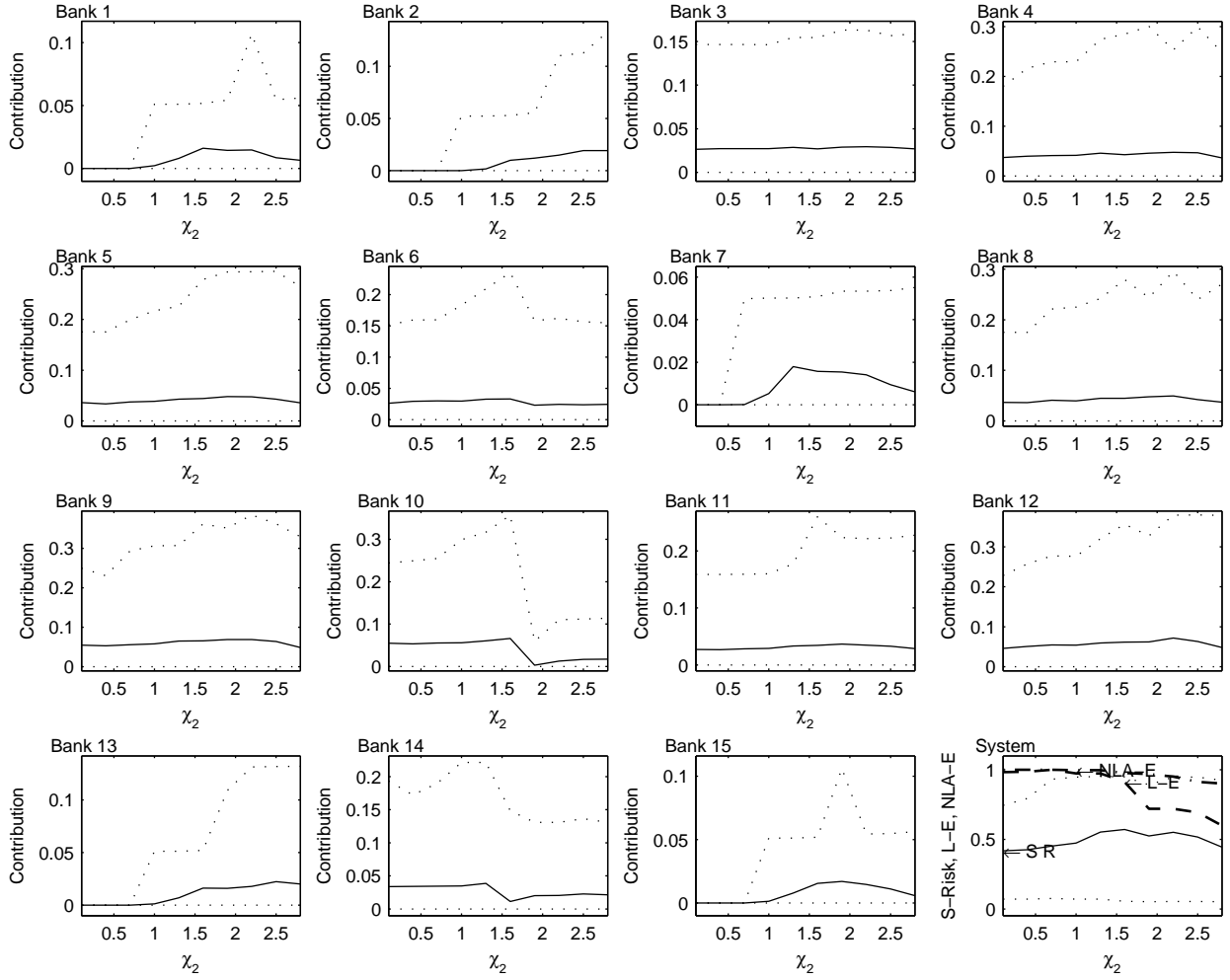


Figure 11: Systemic Risk and Interbank Lending Risk Weights

The figure displays systemic risk (y-axis on panel 16, bottom right) and banks' contribution to it (y-axis on panels 1 to 15) as solid lines over different values of the liquidity requirement ratio, with all other model parameters kept as in the baseline setting. The dotted lines are the two standard deviation error bands. On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

Finally we investigate the impact of introducing risk charges on banks' derivative investments (non-liquid assets) as well as banks' interconnectedness (bank borrowing). To do so, banks will include these levies in their profit optimization function. More specifically, the profit optimization functions given by Equation (1) changes as outlined in Equation (10)

$$\pi_t^i = r_t^{bl} \cdot bl_t^i - (r_t^{bl} + \Delta_t^{bl} + \beta_1) \cdot bb_t^i + (r_t^{nla,i} - \beta_2) \cdot \frac{nla_t^i}{p_t^{nla}} \quad (10)$$

where β_1 is the risk levy for interconnectedness and β_2 is the risk levy for derivative investments.

Figure 12 displays the development of the financial system along increasing values of the systemic risk charges. On the figure, there are three developments which can be identified. First, the number of banks engaging in interbank lending activity decreases. Second, the interest rate on the interbank market decreases. Third, the amount of non-liquid investments decreases slightly.

The penalty parameter on non-liquid assets lowers banks' yield in this asset class and makes them engage in interbank lending activity if their return on non-liquid assets is below the return on the interbank market. Banks' investment in non-liquid assets thus decreases and supply of funds on the interbank market increases. The latter pushes down the interest rate on the interbank market.

The penalty parameter on interbank borrowing puts a wedge between the interest rate banks obtain for lending and the interest rate banks have to pay to borrow. Banks' demand for interbank funding decreases, lowering the interest rate on the interbank market relative to the interest rate which would prevail in absence of the wedge.

Figure 13 displays the effect of introducing penalty parameters on banks' derivative investments and banks interbank borrowings on systemic risk and banks' contribution to it. The penalty parameter on derivatives (β_1) lowers banks' return on non-liquid asset investments, thus lowering banks' investments in this asset class. The penalty parameters on interbank borrowing drives a wedge between the interest rate banks obtain for lending

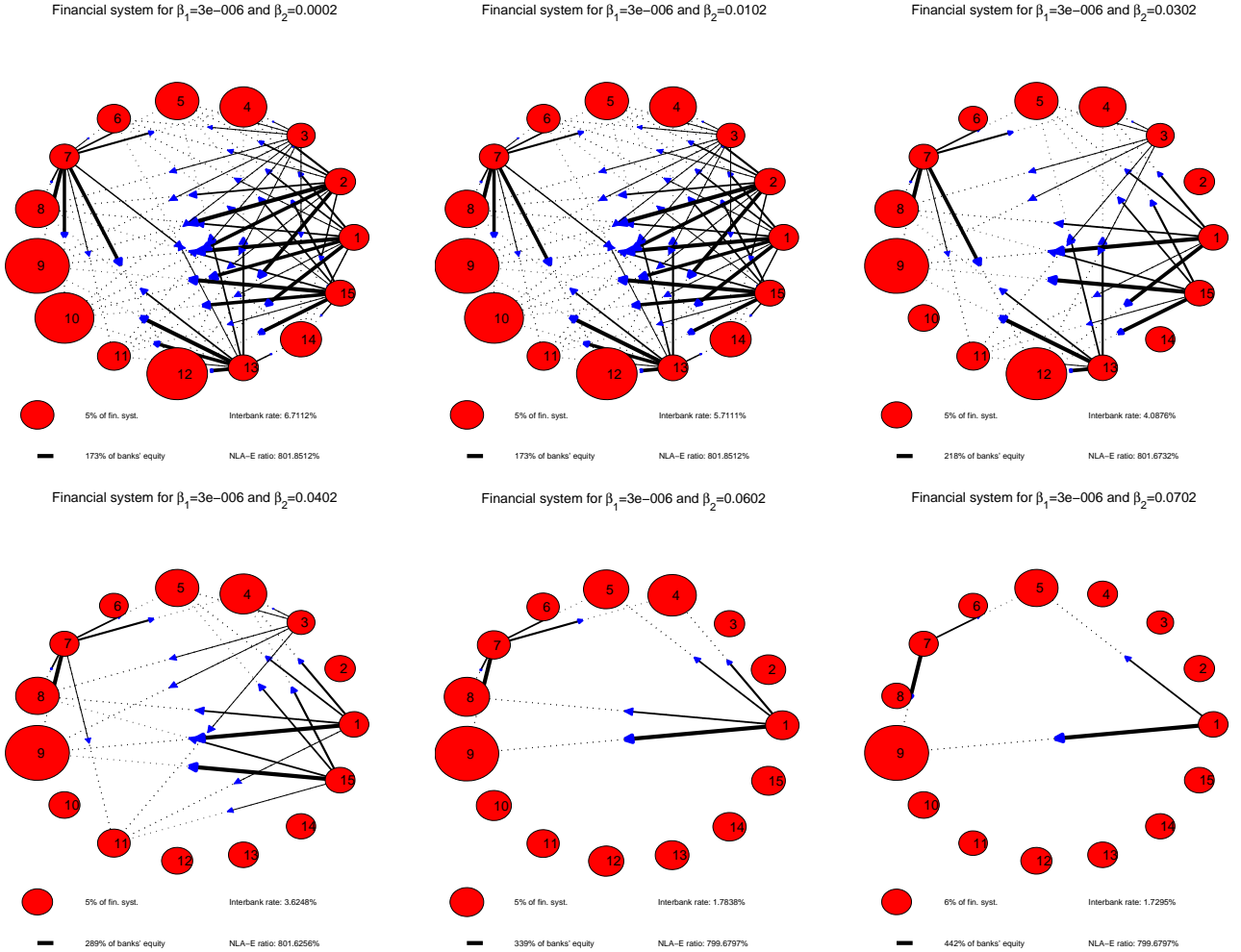


Figure 12: Financial System Structures and Risk Charges

The figure displays selected financial system realizations at different values for systemic risk charges for derivative investments (β_1) and interbank lendings (β_2), with all remainder model parameters kept at their baseline value. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

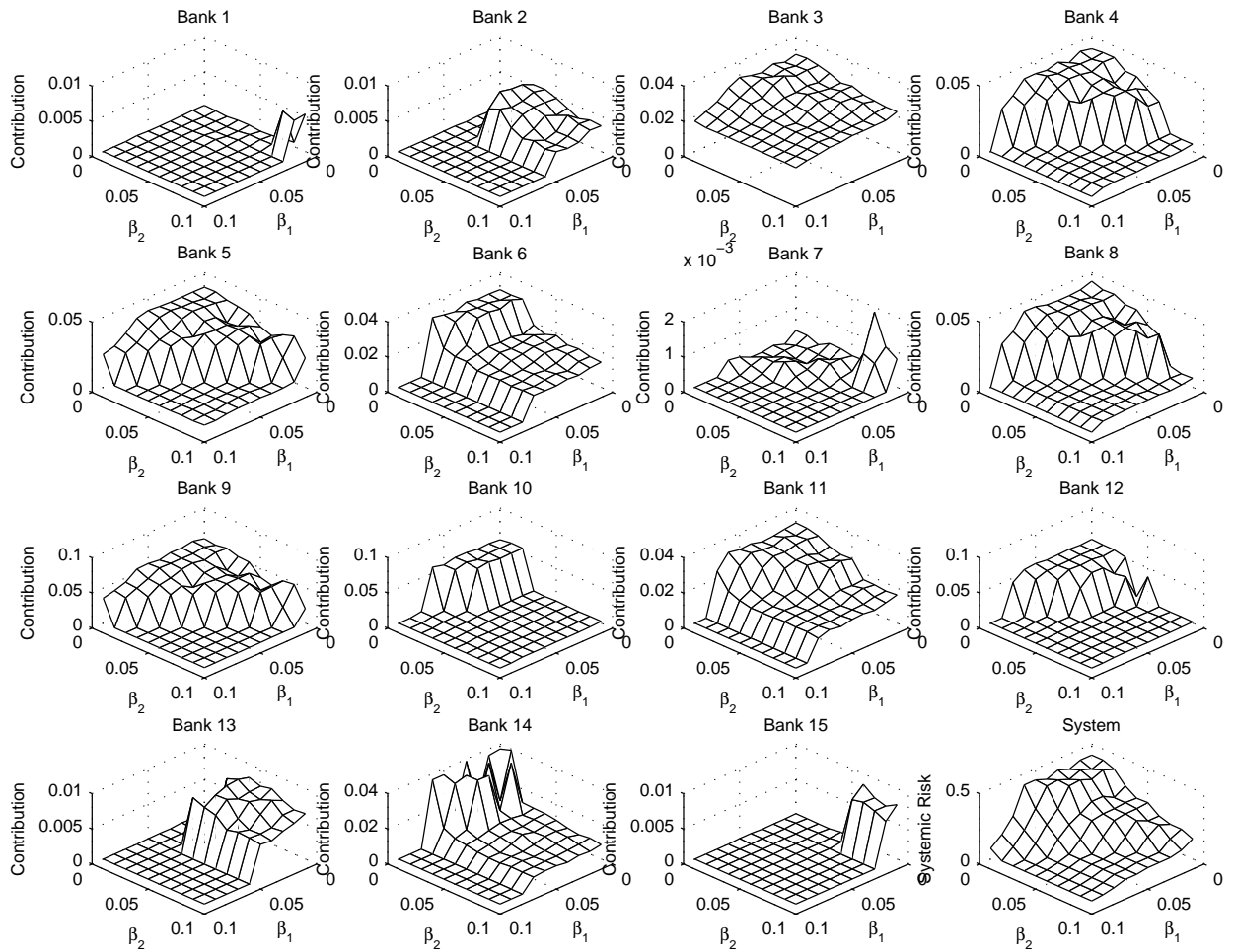


Figure 13: Systemic Risk and Risk Charges

The figure displays systemic risk and banks' contribution to it on the y-axes over a range of increasing values for the penalty parameters for derivatives (β_1) and interbank lendings (β_2) on the z- and x-axes, respectively.

and the rate banks have to pay for borrowing. This causes the interbank market to dry out.

4.2 Analyses with Central Bank Intervention

Central banks intervene in the interbank market both as part of the normal activity of their operational system as well as for unconventional interventions. Both the New York Fed and the ECB achieve the target policy rate by supplying or withdrawing liquidity from the market as part of their normal operational procedures. In times of financial crises and following the disruption of trust in the interbank market as well as the ensuing liquidity hoarding, central banks around the globe have taken unconventional measures also with direct borrowing and lending to individual banks. We therefore want to reconsider the results obtained so far under the assumption that a central bank intervenes in the interbank market.

The central bank is defined as the n^{th} bank. This bank will neither hold cash or non-liquid assets, but will solely supply or demand capital on the interbank market. The main goal is that of achieving its desired target interest rate. We assume that the central bank has unlimited funds and thus cannot default.

Prior to any shock central bank interventions can be characterized as follows. If the target interest rate is below the equilibrium interest rate on the interbank market, namely the equilibrium interest rate achieved in absence of any central bank intervention, the central bank supplies money until the target is achieved. It demands money in the opposite case.

Following endogenous changes in the financial system structure (e.g. through supervisory intervention) the equilibrium interest rate will deviate from the central bank's target: in this case the central bank intervenes via supplying/drawing liquidity to/from the market until the interest rate on the interbank market is within a distance D^i to its desired rate (the default value for D^i is set to 100 basis points).

The parameters in the baseline setting with central bank are the same as displayed on Table 2, with the addition, that the target interest rate of the central bank equals 6.83% which is the equilibrium interest rate in the baseline scenario with $N - 1$ banks in absence of the central bank. Note that the central bank will start intervening, if due to parameter changes the money market rate deviates from its target corridor which is .5 percentages

points around the target rate.

Figure 14 outlines the financial system realization from the baseline scenario. The complete financial system matrix is outlined on Table ??.

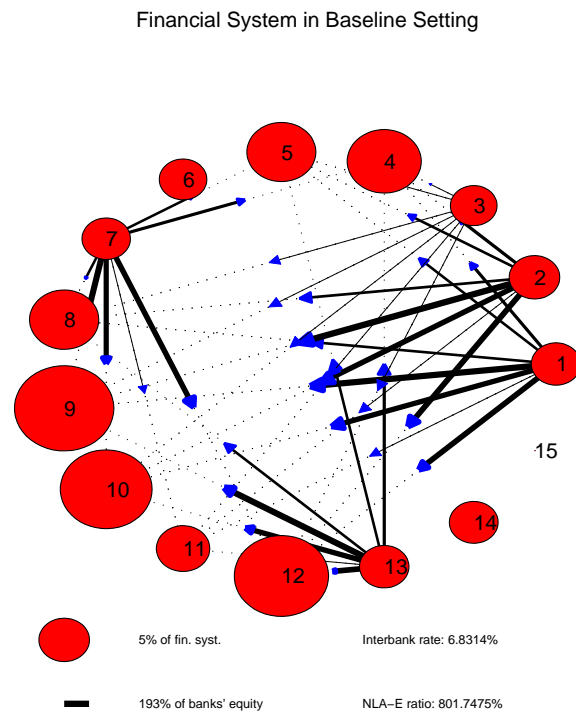


Figure 14: Financial System in Baseline Scenario With Central Bank Intervention

The figure displays an outline of the financial system emerging in the baseline setting with central bank intervention. Each bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank *A* to bank *B* shows that bank *A* has lent money to bank *B*, with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below the stylized financial system there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

Figure 16 displays selected financial systems at different values of the liquidity requirement with central bank intervention. Increasing the liquidity requirement leads to an increasing interest rate until it stabilizes at the upper boundary of the central bank's interest rate corridor. Furthermore, the size of the central bank with respect to the financial system increases and the financial system gets more heterogenous. Figure 16 displays the effect of

	B 1	B 2	B 3	B 4	B 5	B 6	B 7	B 8	B 9	B 10	B 11	B 12	B 13	B 14	B 15	NLA	LA
B 1	0	0	0	63	52	0	0	52	123	105	15	111	0	0	0	0	50
B 2	0	0	0	65	53	0	0	53	126	108	16	113	0	0	0	0	50
B 3	0	0	0	3	3	0	0	3	6	5	1	6	0	0	0	466	50
B 4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	781	50
B 5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	728	50
B 6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	502	50
B 7	0	0	0	62	51	0	0	51	120	103	15	109	0	0	0	0	50
B 8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	730	50
B 9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1048	50
B 10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	969	50
B 11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	563	50
B 12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	995	50
B 13	0	0	0	64	52	0	0	52	123	105	15	111	0	0	0	0	50
B 14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	514	50
B 15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Deposits	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	0	0

Table 5: Financial System in Baseline Setting With Central Bank Intervention

The table displays the financial system for the baseline setting with central bank intervention. Banks are designated with letter *B*. Each bank's assets are displayed along the according row and banks liabilities are displayed along the according column. For example, matrix element (2,5) shows that Bank 1 has lent 57 to Bank 5. The last two columns designate non-liquid assets (NLA) and liquid assets (LA), that is, cash, respectively. The last row are assets from the rest of the world, that is, deposits.

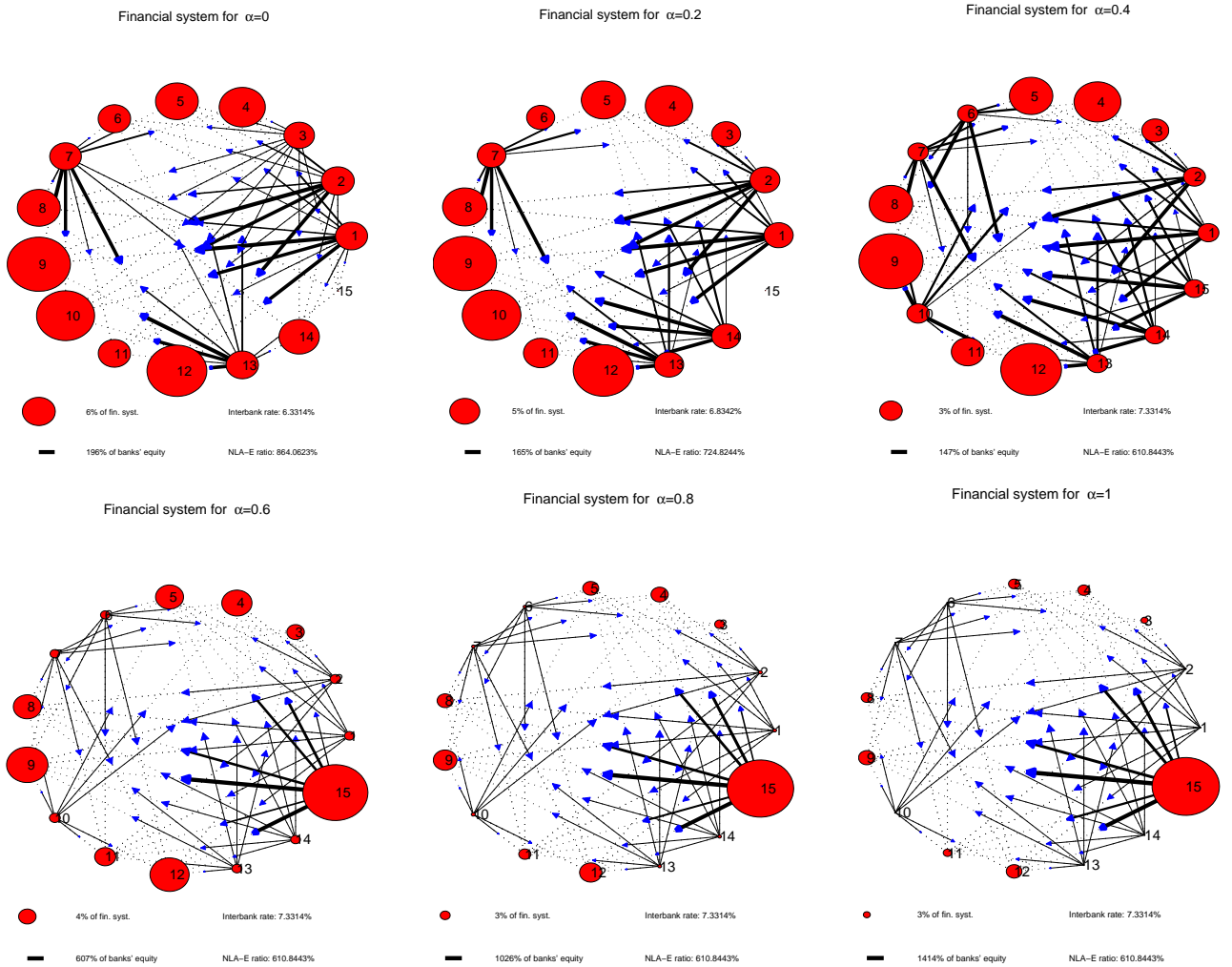


Figure 15: Financial System Structures and Liquidity Requirement With Central Bank Intervention

The figure displays selected financial system realizations at different liquidity requirements, with all remainder model parameters kept at their baseline value. Note that this set up includes central bank intervention which is by default bank 15. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

B 1	B 2	B 3	B 4	B 5	B 6	B 7	B 8
0.00	0.00	0.03	0.04	0.04	0.02	0.00	0.04
B 9	B 10	B 11	B 12	B 13	B 14	B 15	SR
0.05	0.05	0.03	0.04	0.00	0.02	0.00	0.37

Table 6: Systemic Risk and Banks' Contribution in the Baseline Setting

The table displays banks' contribution to systemic risk, as well as overall systemic risk (SR), in the baseline setting with central bank intervention. Note that values have been rounded.

increasing the liquidity requirement on systemic risk and banks' contribution to it. Increasing the liquidity requirement first increases the loan-to-equity ratio and then decreases it again. The non-liquid assets-to-equity ratio first decreases and then stays flat. Similarly, systemic risk at first decreases and then stays flat. As outlined in previous analyses, increasing the liquidity requirement in tendency drives up the interest rate on the interbank market. However, with central bank intervention this effect only applies within the central bank interest corridor, that is, between 6.33% and 7.33%. If the interest rate hits the upper boundary of the corridor, the central bank starts supplying liquidity to prevent the interest rate from increasing further. Thus, borrowing banks obtain funds from the central bank which increases in size relative to the financial system. The non-liquid asset investments only decrease up to the point where the interest rate is capped because of the previously outlined substitution of non-liquid assets from banks to interbank investment if their yield on non-liquid asset investments lies below the interbank market interest rate. The loan-to equity ratio initially increases because of the same substitution effect but then decreases again because with an increasing liquidity requirement banks have less and less fund to lend on the interbank market. Since the non-liquid asset investments and banks' interconnectedness do not decline from some point onwards, systemic risk remains flat.

Systemic Risk at Varying Degrees of α

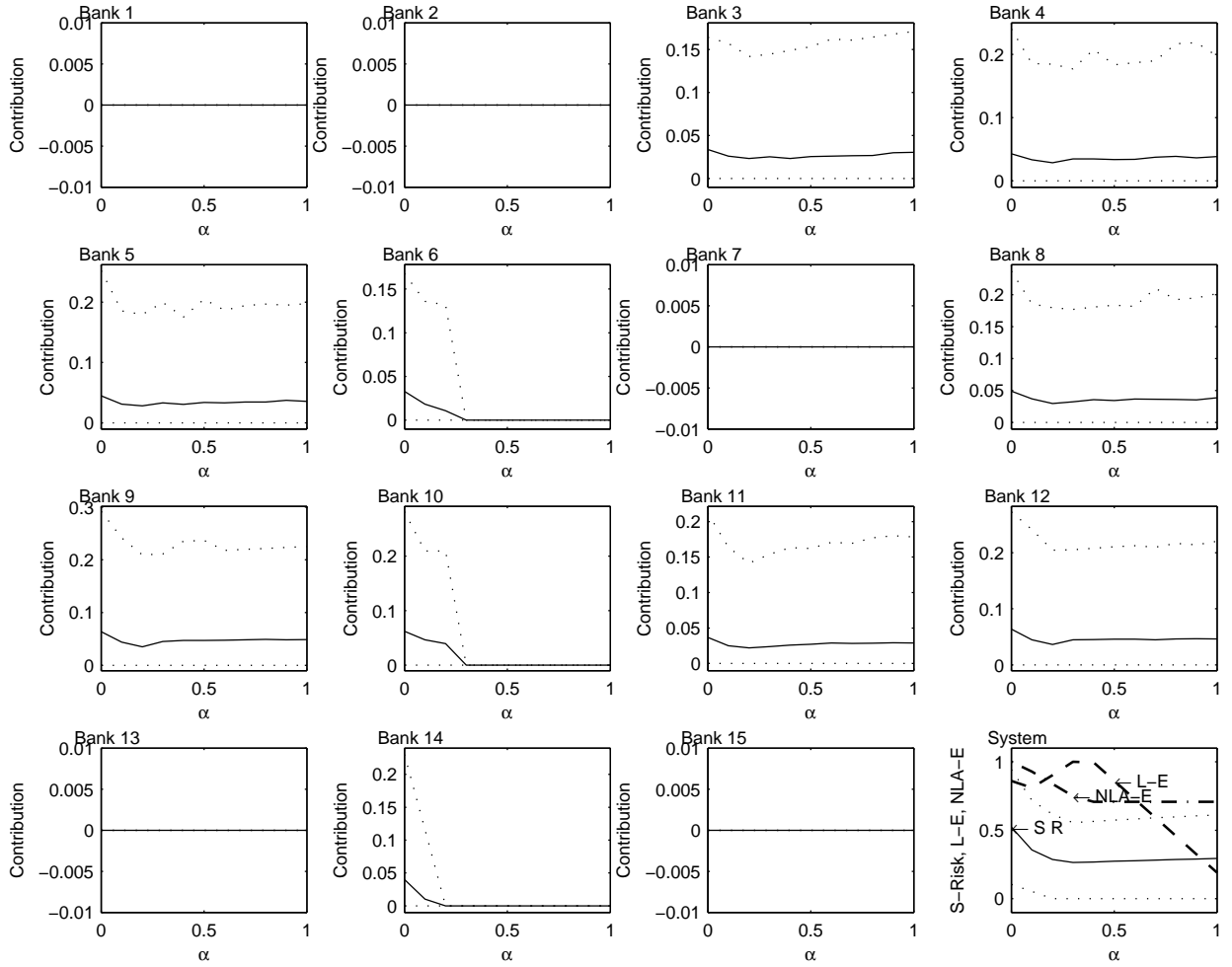


Figure 16: Systemic Risk and Liquidity Requirement With Central Bank Intervention

The figure displays systemic risk (y-axis on panel 16, bottom right) and banks' contribution to it (y-axis on panels 1 to 14) as solid lines over different values of the liquidity requirement ratio, with all other model parameters kept as in the baseline setting. Note that this set up includes central bank intervention which is by default bank 15. The dotted lines are the two standard deviation error bands. On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

Figure 17 displays selected financial systems at various capital requirement ratios with central bank intervention. Increasing the capital requirement ratio, lowers the interest rate on the interbank market within the interest rate corridor of the central bank. At low levels of capital requirement, the central bank is very big and at very high capital requirement ratios the central bank is very small, both with respect to the financial system. Overall, the financial system gets more homogenous with an increasing capital requirement ratio and the number of lending banks increases. Figure 18 displays systemic risk and banks' contribution to it with central bank intervention when the capital requirement ratio is increased. Increasing the capital requirement ratio leads to a decrease in systemic risk as well as to a decreasing non-liquid assets-to-equity ratio and loan-to-equity ratio. At relatively low capital requirement ratios banks demand for interbank liquidity pushed up the interest rate to the upper bound of the interest rate corridor. Increasing the capital requirement ratio then results in less demand of funds on the interbank market, reducing the necessary interest rate stabilizing intervention by the central bank. At some point, when the interbank interest rate is within the central bank corridor, the central bank does not intervene. However, as banks an leverage less and less and the interes rate sinks, les banks provide liquidity on the interbank market because it becomes profitable for them to invest in non-liquid assets. When the lower bound of the central bank's interest rate corridor is hit it starts absorbing liquidity from the market to stabilize the interest rate.

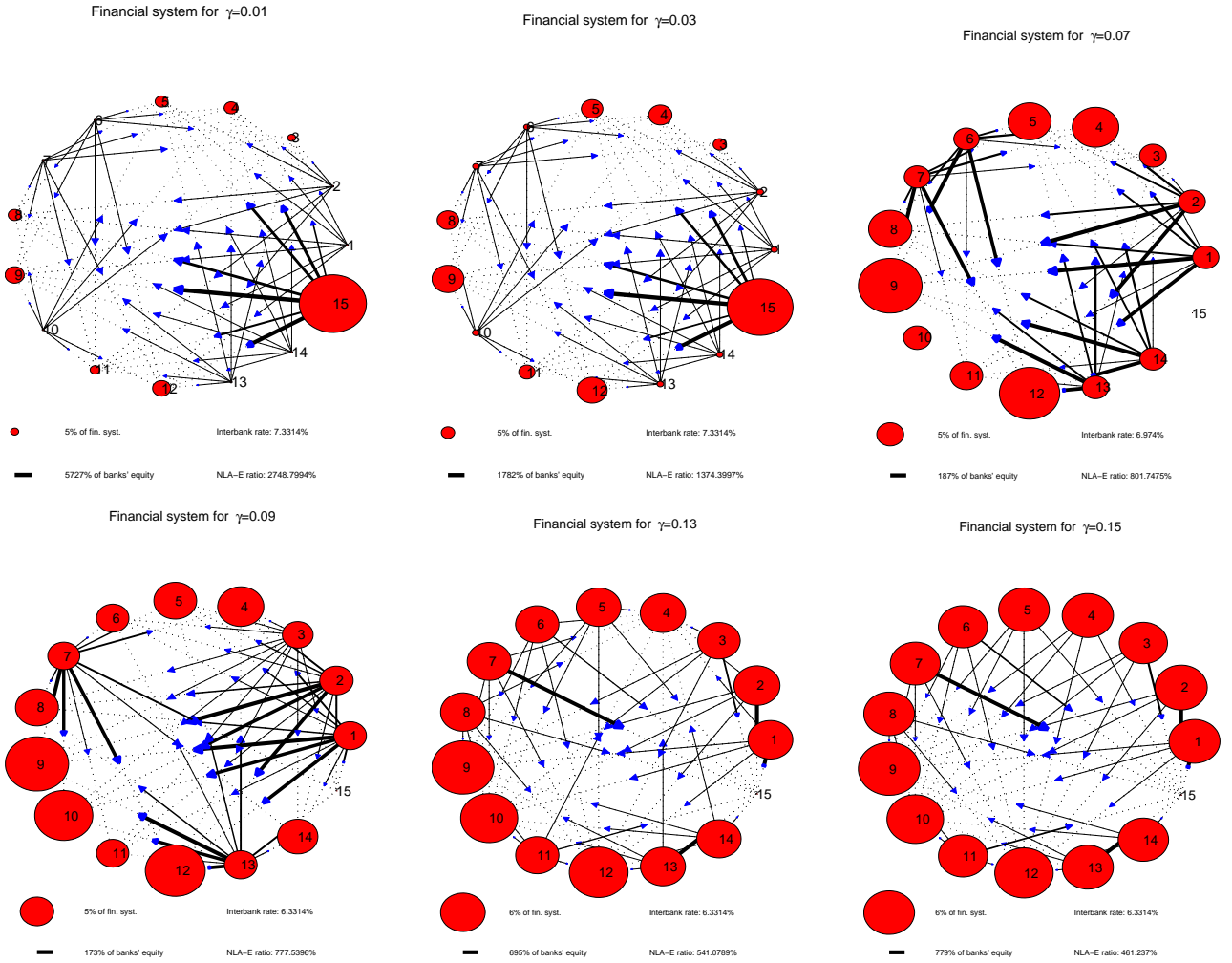


Figure 17: Financial System Structures and Capital Requirement Ratio With Central Bank Intervention

The figure displays selected financial system realizations at different capital requirement ratios, with all remainder model parameters kept at their baseline value. Note that this set up includes central bank intervention which is by default bank 15. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

Systemic Risk at Varying Degrees of γ

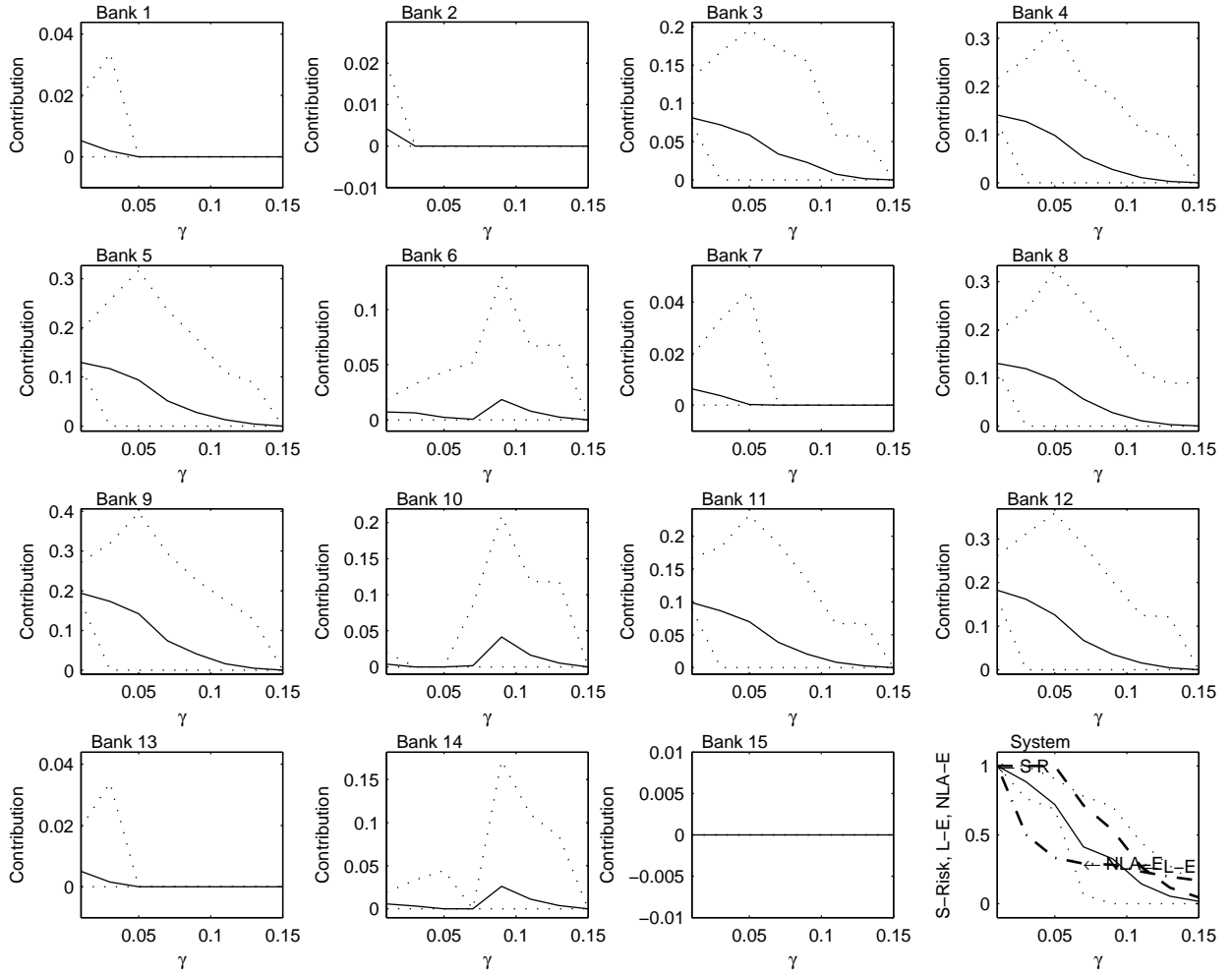


Figure 18: Systemic Risk and Capital Requirement Ratio With Central Bank Intervention

The figure displays systemic risk (y-axis on panel 16, bottom right) and banks' contribution to it (y-axis on panels 1 to 14) as solid lines over different values of the capital requirement ratio, with all other model parameters kept as in the baseline setting. Note that this set up includes central bank intervention which is by default bank 15. The dotted lines are the two standard deviation error bands. On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

Figure 19 displays selected financial systems at various levels of the risk weight for non-liquid asset investments. On the figure, the interest rate tends to decrease from the upper central bank corridor to the lower central bank corridor, the banking system becomes more homogenous with respect to banks' size, and investments into non-liquid assets rapidly go down, all when the risk weight on non-liquid assets is increased. Figure 20 displays systemic risk and banks' contribution to it when the risk weight on non-liquid assets is increased. Increasing the weight on derivatives results in less investments in that asset class, furthermore interbank market lending and systemic risk decrease.

The mechanisms leading to the effects described above are similar to the effects taking place when the capital requirement ratio is increased.

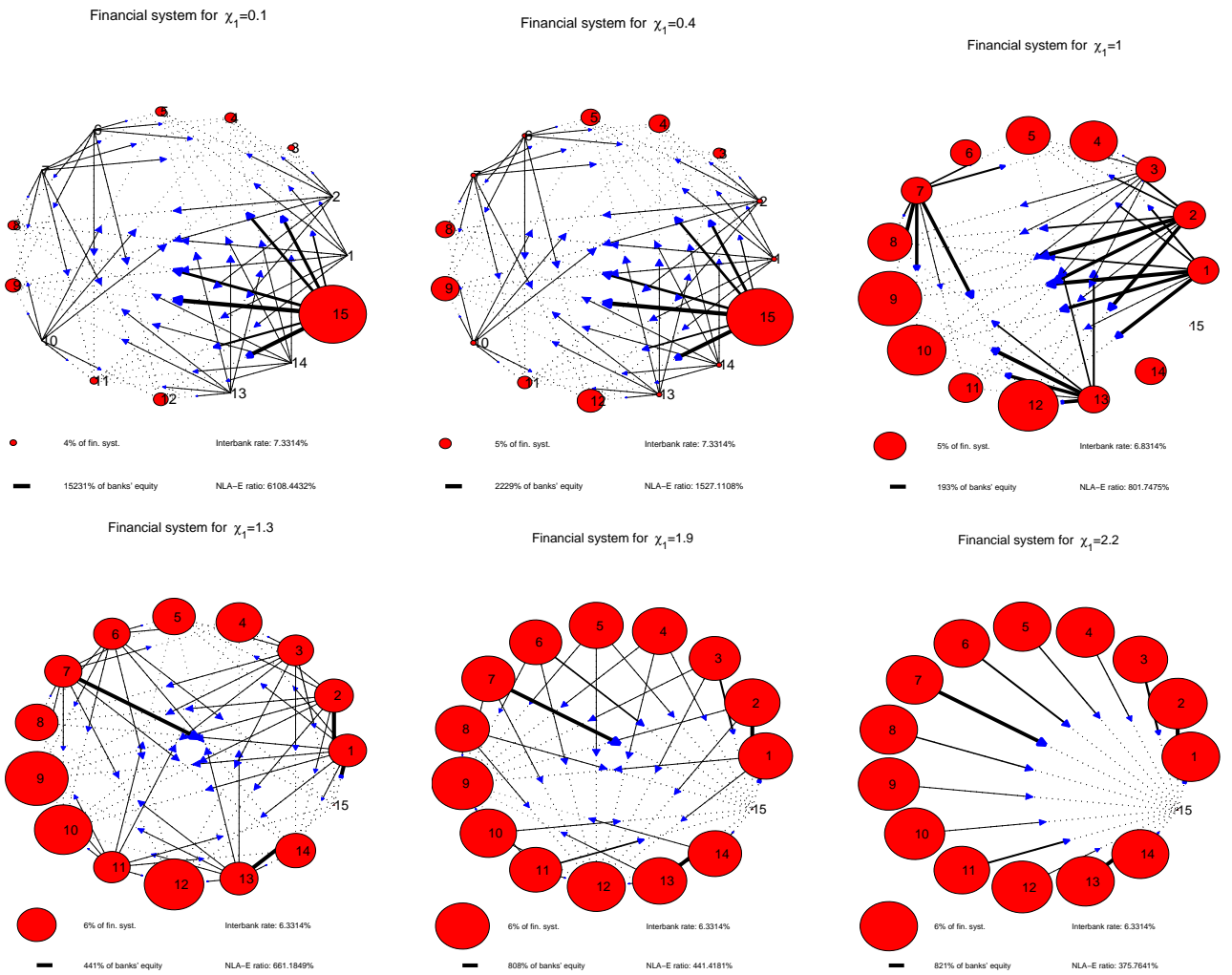


Figure 19: Financial System Structures and Derivative Risk Weights With Central Bank Intervention

The figure displays selected financial system realizations at different risk weights for derivative investments, with all remainder model parameters kept at their baseline value. Note that this set up includes central bank intervention which is by default bank 15. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

Systemic Risk at Varying Degrees of χ_1

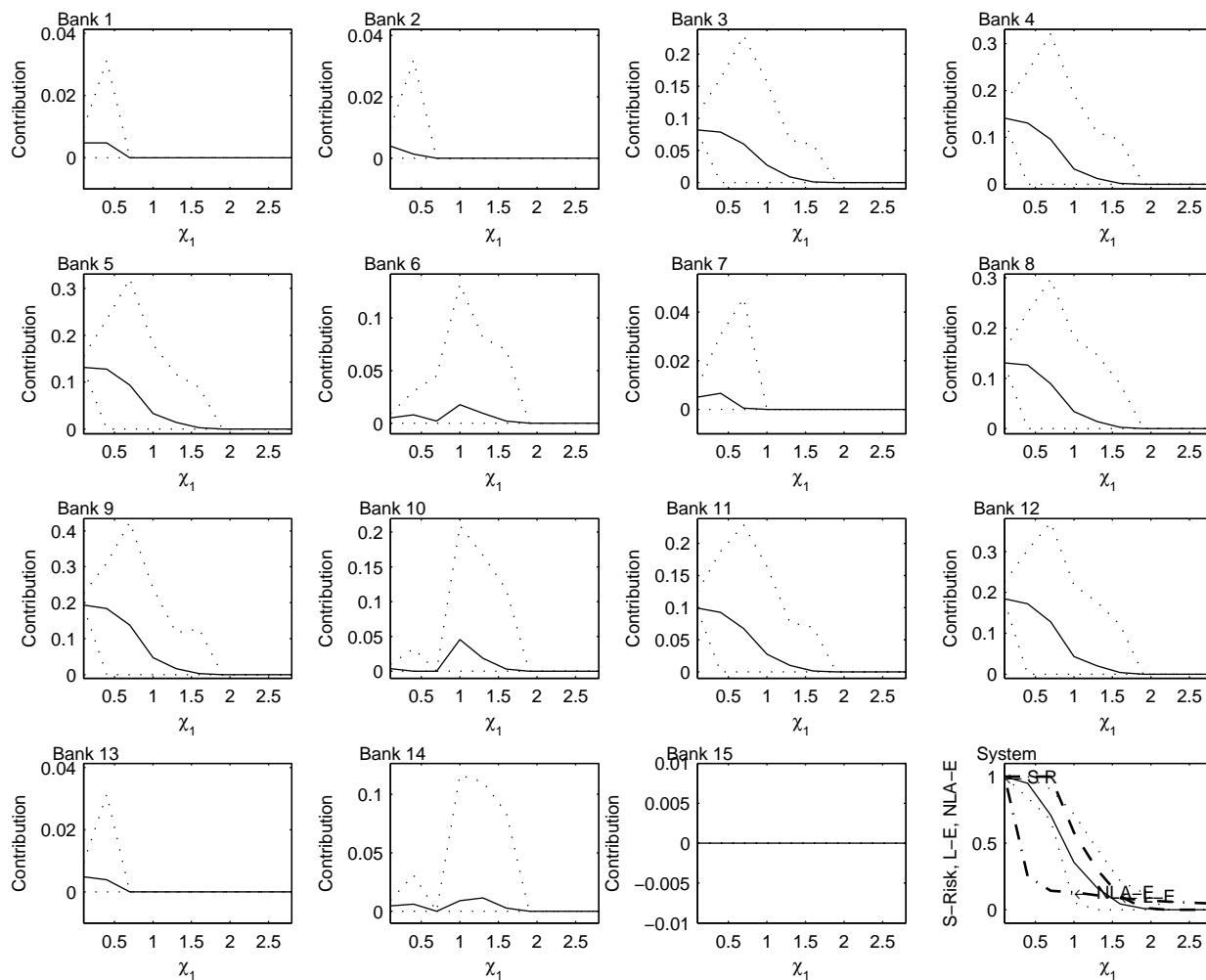


Figure 20: Systemic Risk and Derivative Risk Weights With Central Bank Intervention

The figure displays systemic risk (y-axis on panel 16, bottom right) and banks' contribution to it (y-axis on panels 1 to 14) as solid lines over different values of the risk weight for derivatives, with all other model parameters kept as in the baseline setting. Note that this set up includes central bank intervention which is by default bank 15. The dotted lines are the two standard deviation error bands. On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

Figure 21 displays selected financial systems at various risk weights for banks' interbank lending. Increasing the risk weight for interbank lending leaves derivative investments almost unaffected, because the interest rate can only increase little between the interest rate corridor boundaries. Furthermore, when the interest rate on the interbank market increases, more banks start lending on the interbank market.

Figure 22 displays systemic risk and banks contribution to it when the risk weight on interbank lending is increased. When increasing the risk weight on interbank lending, systemic risk increases slightly, the loan-to-equity ratio decreases and the non-liquid assets-to-equity ratio decreases slightly.

Increasing the risk weight on interbank lending leads to less supply of funds to the interbank market. The interest rate thus tends to decrease which in turn increases the number of banks engaging in interbank lending because they obtain a higher yield than from derivative investments. When the interbank interest rate hits the upper bound of the central bank's interest rate corridor, the central bank starts supplying funds to stabilize it. In the model systemic risk increases slightly because the banking system becomes more interconnected and non-liquid asset investments only decrease slightly up to the point when the central bank starts supplying funds to the market.

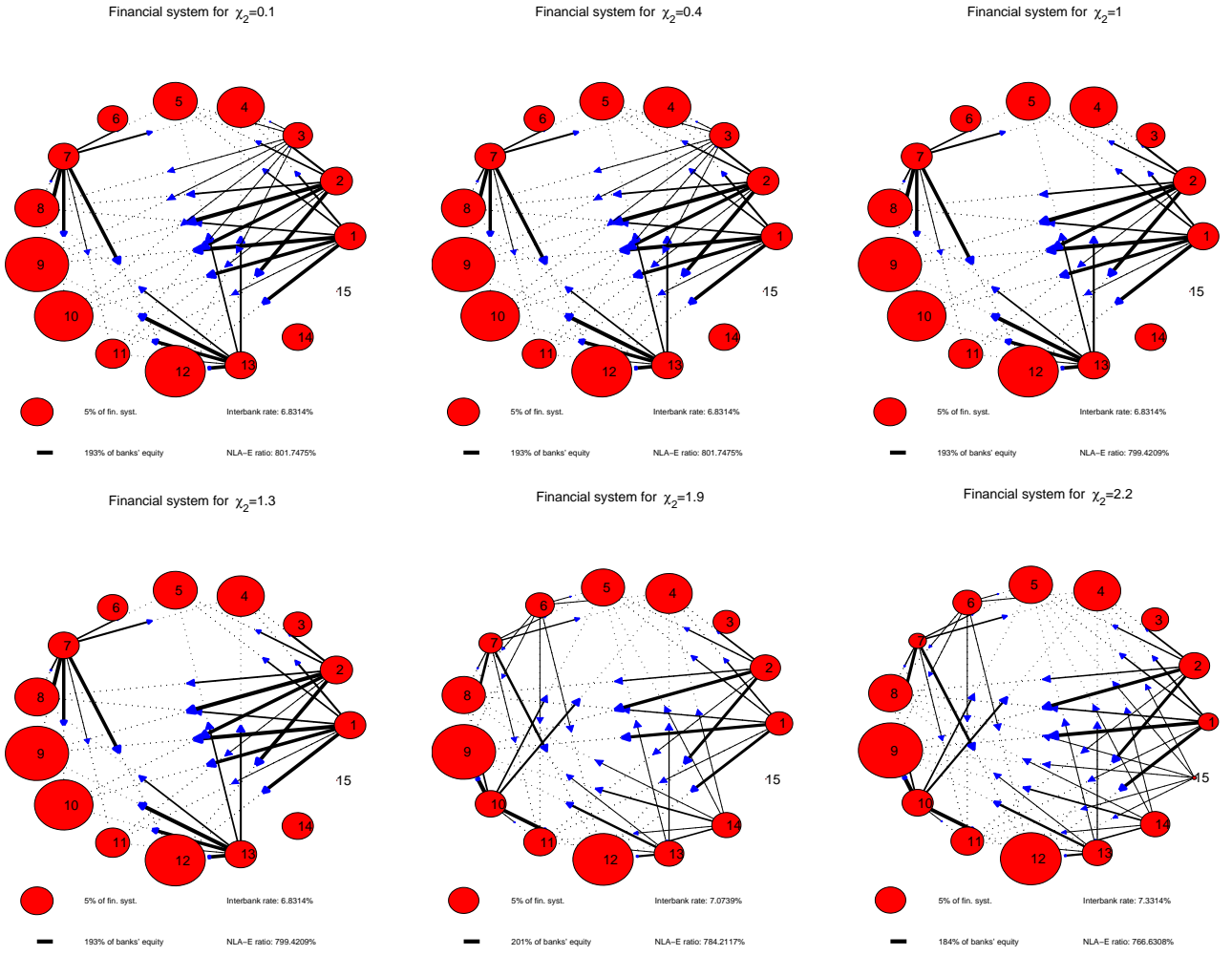


Figure 21: Financial System Structures and Interbank Lending Risk Weights With Central Bank Intervention

The figure displays selected financial system realizations at different risk weights for interbank lendings, with all remainder model parameters kept at their baseline value. Note that this set up includes central bank intervention which is by default bank 15. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

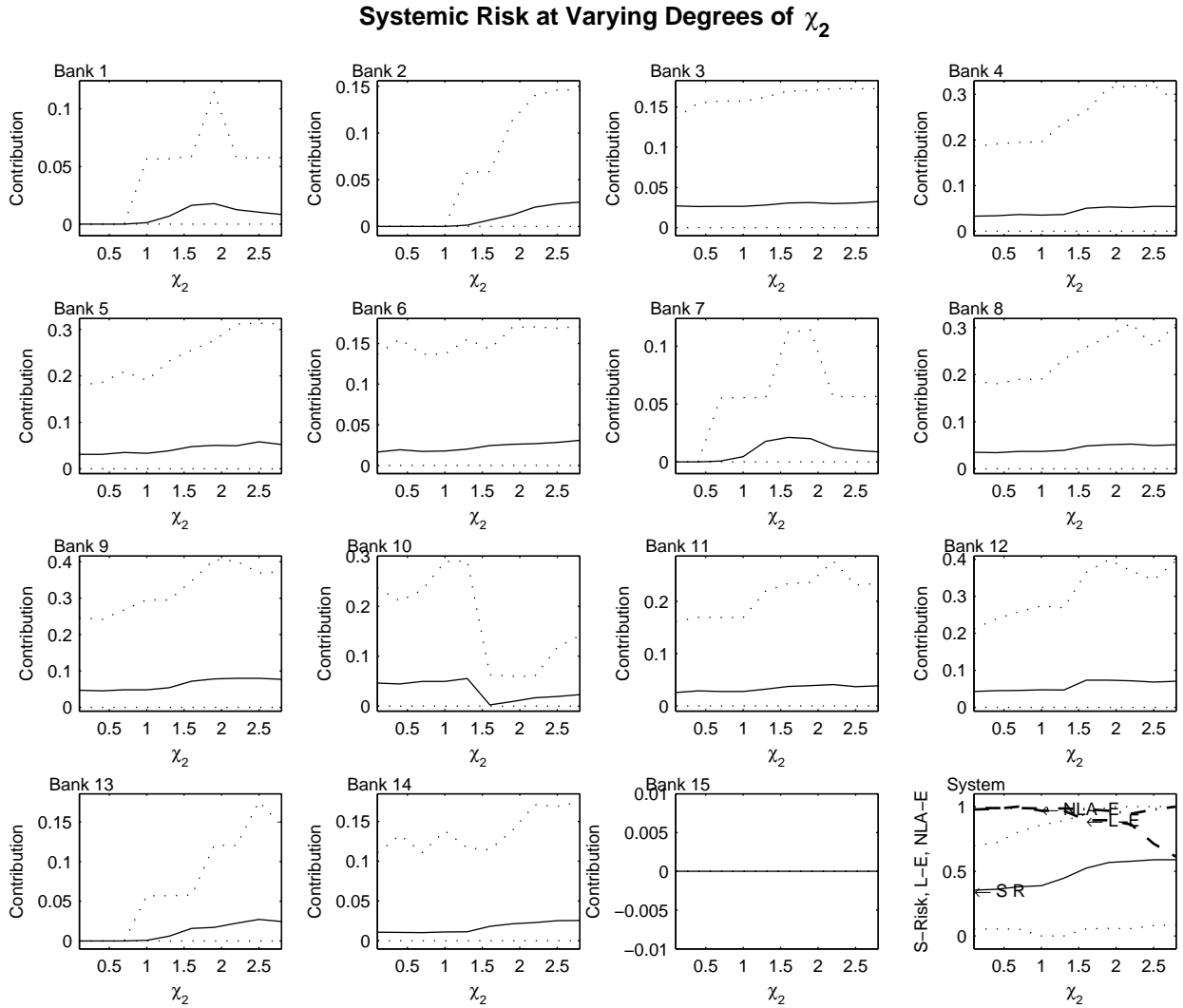


Figure 22: Systemic Risk and Interbank Lending Risk Weights With Central Bank Intervention

The figure displays systemic risk (y-axis on panel 16, bottom right) and banks' contribution to it (y-axis on panels 1 to 14) as solid lines over different values of the risk weight for interbank lending, with all other model parameters kept as in the baseline setting. Note that this set up includes central bank intervention which is by default bank 15. The dotted lines are the two standard deviation error bands. On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

Figure 23 displays selected financial system realizations when increasing the central bank's target rate in a comparative static fashion. Increasing the target rate for the central bank results in decreasing non-liquid asset investments. When the target interest rate is below the equilibrium rate corridor on the interbank market in absence of central bank intervention the central bank is mopping funds to push up the interest rate and when the target rate is above the equilibrium rate corridor, the central bank supplies funds to push down the interest rate. Furthermore, when the target rate increases, banks invest more on the interbank market and less in non-liquid assets.

Figure 24 displays systemic risk and banks' contribution to it when the central bank increases its target interest rate. Increasing the target interest rate results in the loan-to-equity ratio first increasing and then decreasing again, investments in non-liquid assets decrease, and systemic risk goes down. When the central bank increases its target interest rate, it implements the target via supplying less funds to the market (or, if the supply gets negative, via draining funds). When the interest rate then increases it becomes more profitable to engage in interbank lending if their return on non-liquid assets is below the interbank market rate. This causes the lending-to-equity ratio to increase at the expense of non-liquid asset investments. However, this decrease in banks who invest in non-liquid assets ultimately decreases demand for funds on the interbank market (because less banks demand funds there for leveraging up), thus the liquidity exchanged on the interbank market decreases again at increasing levels of the central bank's target rate.

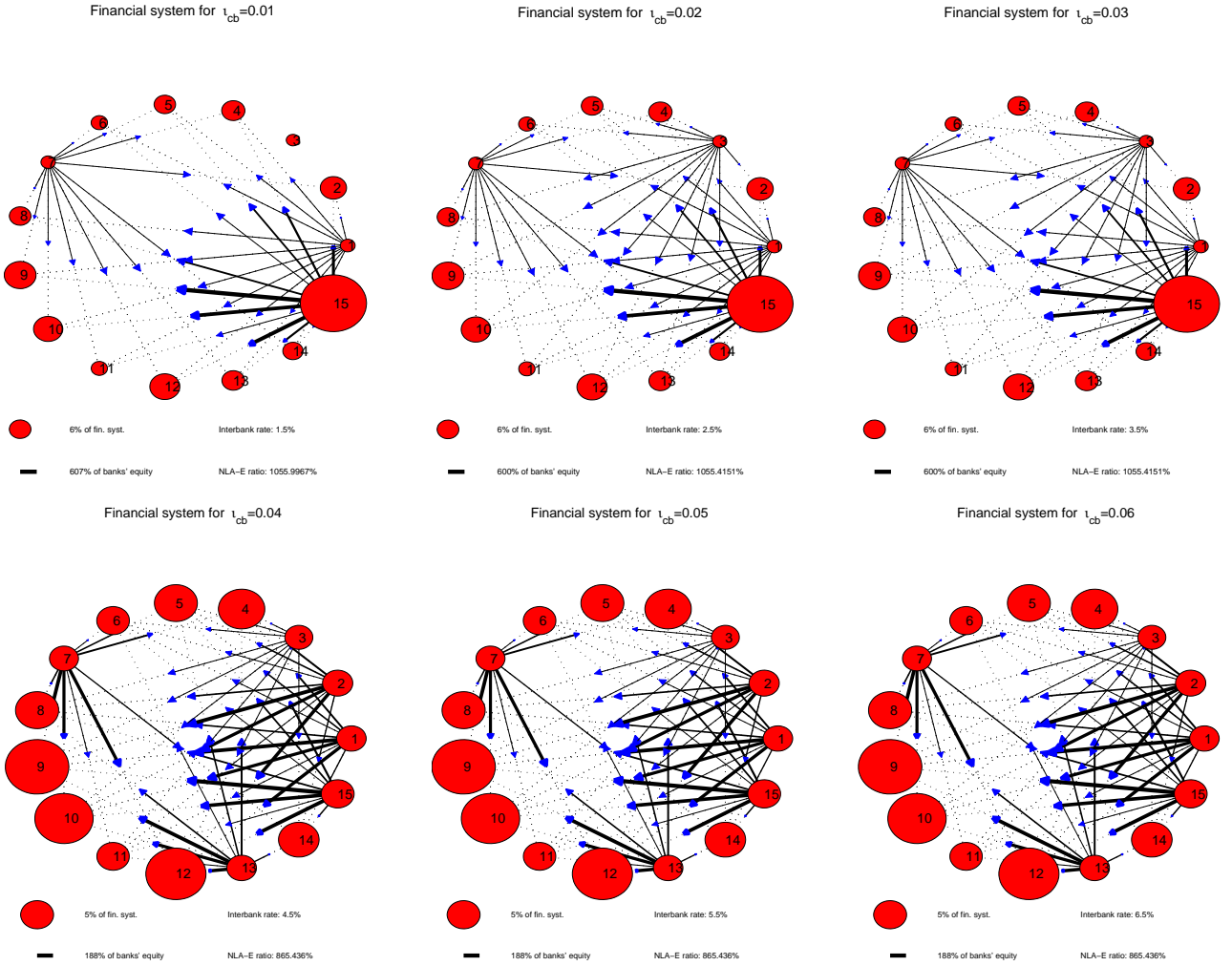


Figure 23: Financial System Structures and Central Bank Target Rate

The figure displays selected financial system realizations at interest rate targets for the central bank, with all remainder model parameters kept at their baseline value. The central bank is by default bank 15. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

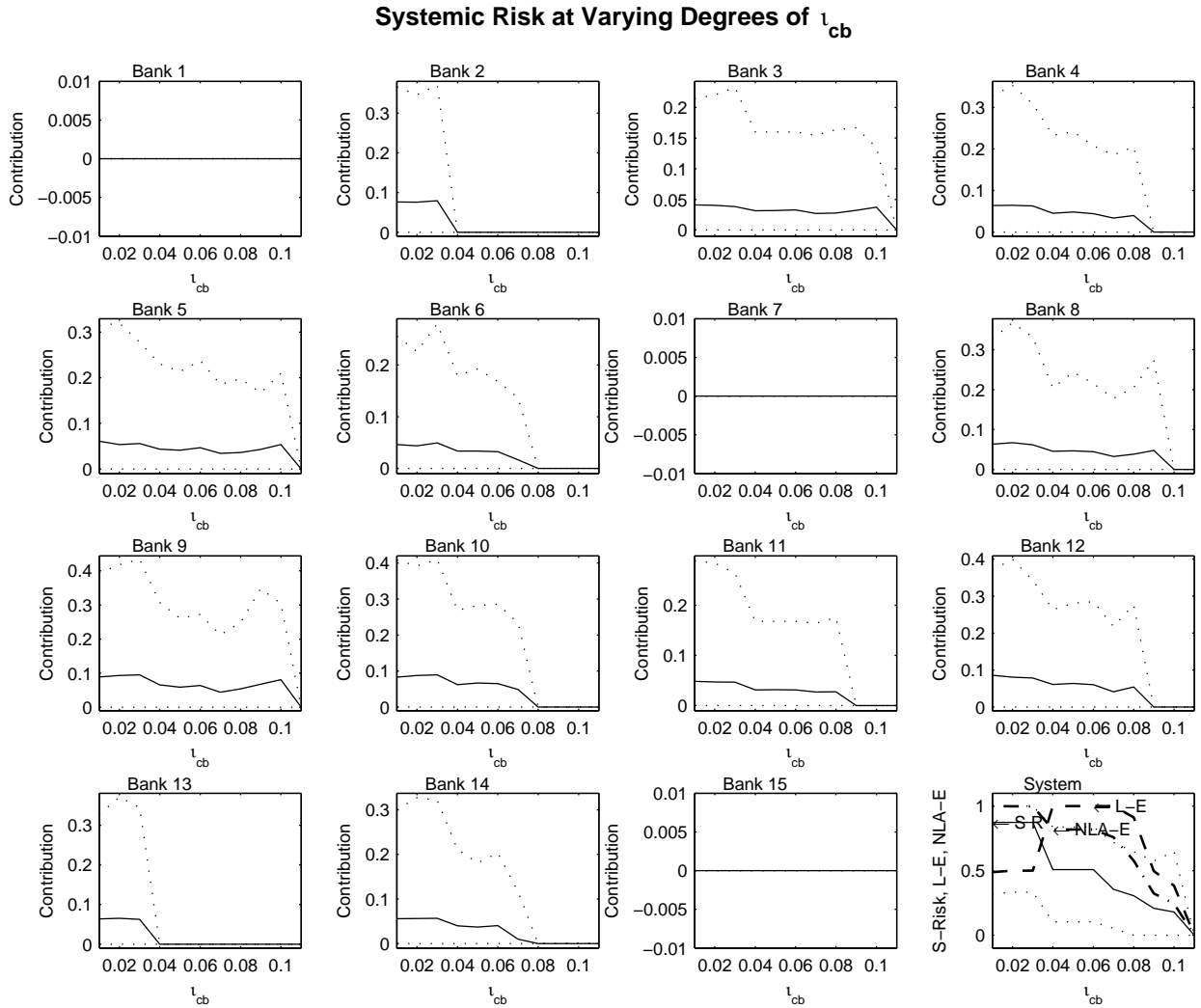


Figure 24: Systemic Risk and Central Bank Target Rate

The figure displays systemic risk (y-axis on panel 16, bottom right) and banks' contribution to it (y-axis on panels 1 to 14) as solid lines over different values of the central bank's target rate, with all other model parameters kept as in the baseline setting. Note that this set up includes central bank intervention which is by default bank 15. The dotted lines are the two standard deviation error bands. On panel 16, the dashed line is the loan-to-equity (L-E) ratio, that is, the sum of all interbank lendings relative to the sum of all banks' equity, and the dash-dotted line is the non-liquid-assets-to-equity (NLA-E) ratio, that is, the sum of all non-liquid assets held by banks relative to the sum of all banks' equity.

Figure 25 displays selected financial systems at different risk charges on derivative investments and interbank lending. Increasing the risk charges on derivative investments and interbank lending results in decreasing non-liquid asset investments, less bank activity and a decreasing interest rate, both on the interbank market. Furthermore the financial system becomes more homogenous with respect to banks' size. Figure 26 displays systemic risk and banks' contribution to it when the risk charges on derivative investments and interbank lending are increased. When the risk charges increase systemic risk and banks' contribution to it in tendency decreases, though the decline is not monotoneously for all banks. As outlined before, the penalty parameter on non-liquid assets lowers banks' yield in this asset class and makes them engage in interbank lending activity if their return on non-liquid assets is below the return on the interbank market. Banks' investment in non-liquid assets thus decreases and supply of funds on the interbank market increases. The latter tends to push down the interest rate on the interbank market. The penalty parameter on interbank borrowing puts a wedge between the interest rate banks obtain for lending and the interest rate banks have to pay to borrow. Banks' demand for interbank funding decreases, lowering the interest rate on the interbank market relative to the interest rate which would prevail in absence of the wedge. In presence of central bank intervention, however, the results stemming from the interest decreases are limited to the lower bound of the central bank's target rate. This results in the central bank mopping up existing liquidity when the lower bound is hit, liquidity which would otherwise be invested in non-liquid investments. Investments in non-liquid assets thus decrease stronger than in the case without central bank intervention. Furthermore, since existing direct links between banks at high levels of risk charges are only between banks and the central banks, the direct shock transmission channel is also more dampened than in the case without central bank intervention. Overall, the latter two effects cause systemic risk to decrease stronger in reaction to systemic risk charges with central bank intervention than in the case without central bank intervention.

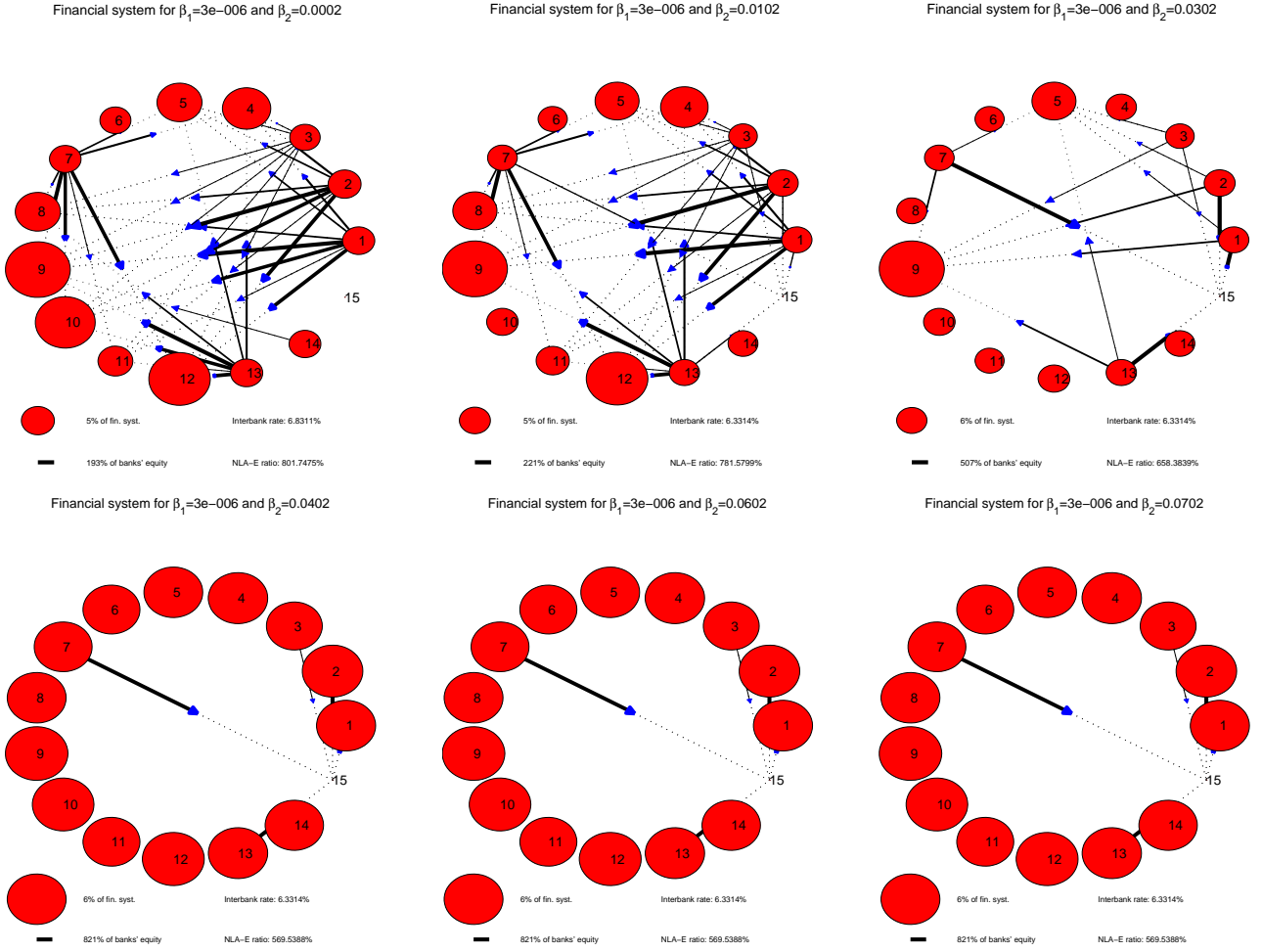


Figure 25: Financial System Structures and Risk Charges With Central Bank Intervention

The figure displays selected financial system realizations at different values for systemic risk charges for derivative investments (β_1) and interbank lendings (β_2), with all remainder model parameters kept at their baseline value. The central bank is by default bank 15. In each of those realizations a bank is represented by a red ball, with the banks' identifiers in the middle of the ball. The diameter of a ball indicates the bank's size, measured by the sum of its risk weighted assets relative to the sum of all risk weighted assets of all banks in the financial system. An arrow pointing from bank A to bank B shows that bank A has lent money to bank B , with the thickness of the arrow indicating the amount of funds lent relative to banks' equity. Below each of the stylized financial systems there are four further indicators. First, the red ball gives an indication about the percentage of the financial system a specific ball designates. Second, the thickness of the black line below gives an indication about how much lending a representative arrow designates relative to banks' equity. Third, the interbank rate is the equilibrium interest rate realizing on the interbank market. Fourth, the non-liquid-assets-to-equity (NLA-E) ratio gives an indication about how much banks have invested on average in non-liquid assets relative to their equity.

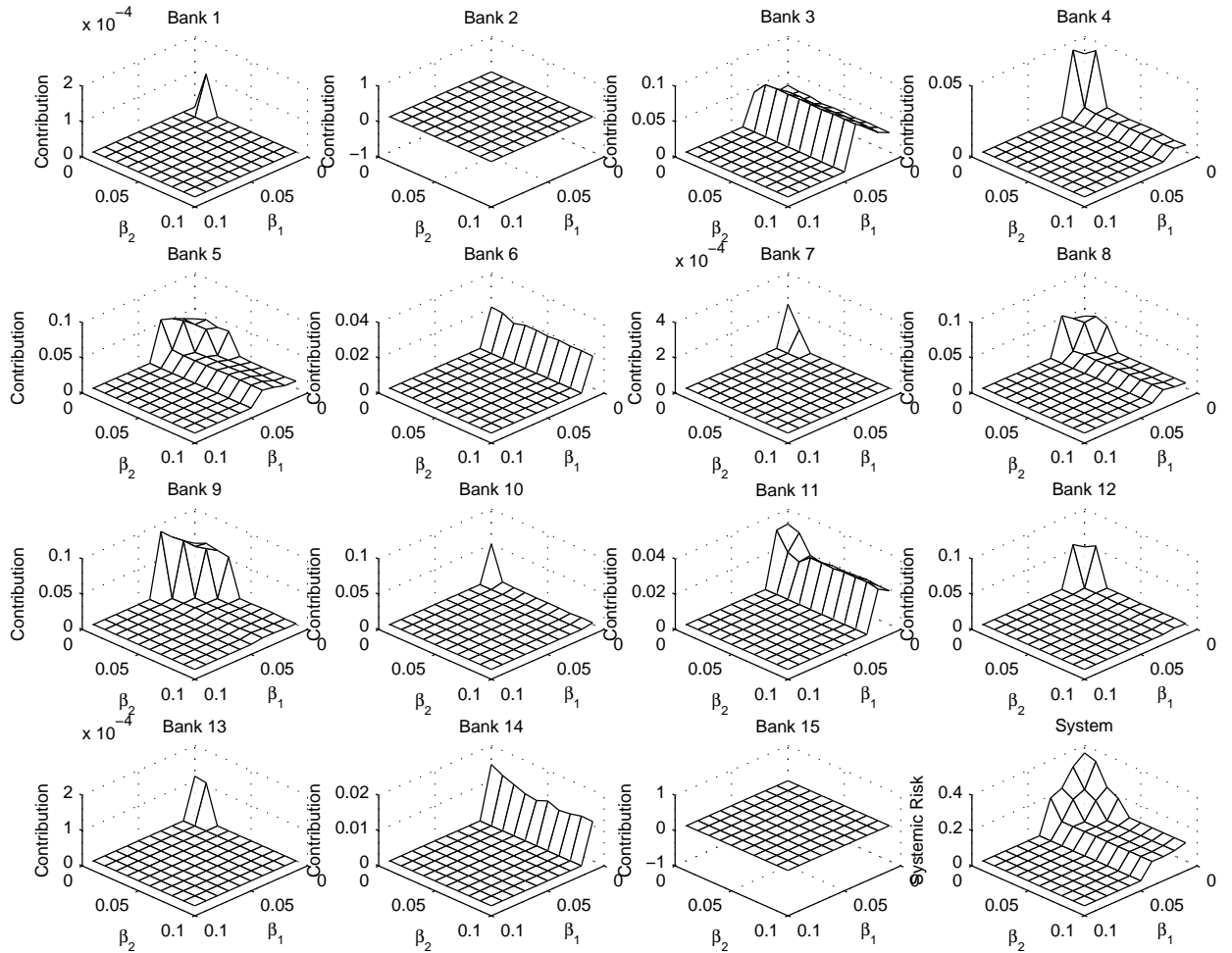


Figure 26: Systemic Risk and Risk Charges With Central Bank Intervention

The figure displays systemic risk and banks' contribution to it on the y-axes over a range of increasing values for the penalty parameters for derivatives (β_1) and interbank lendings (β_2) on the z- and x-axes, respectively. Note that there is central bank intervention which is by default bank 15.

5 Conclusions

One of the major legacies of the recent financial crisis is the quest for measuring, assessing and monitoring systemic risk. So far, this task was made difficult by the mounting complexity of the modern financial systems, all characterized by extensive degrees of interconnections, and the lack of models apt to perform such tasks. We laid down a dynamic network model of banks, in which heterogeneity, network externalities and fire-sale effects contribute to propagate financial shocks through cascades. We have shown that policy and regulatory measures

can reduce systemic risks, directly through higher shock buffers available in the financial system, for example in case of higher capital requirement ratios, or indirectly through banks' optimal reaction to systemic risk charges which lead to the endogenous emergence of a more resilient financial system structure. Note that in most cases there seems to be a trade-off between stability and banks' investments in non-liquid assets –which can be taken as our models' proxy for banks' links with the (exogenous) real economy. Results thus indicate that higher stability might come at the cost of a lower provision of financial products and services to the real economy. Whether this has welfare effects would be interesting to analyze but is beyond the scope of our current model.

Several extensions are possible of our model, ranging from the introduction of maturity mis-match to the analysis of the optimal financial regulator problem. All this is left for future

research.

6 Appendix A. Banks Optimization Problem: Dual Problem

Banks' optimization problem can be reformulated for numerical convenience in the following minimization problem:

$$\min_{bl_t^i, bb_t^i, nla_t^i, c_t^i} -\pi_i = -r_t^{bl} \cdot bl_t^i + (r_t^{bl} + \Delta_t^{bl}) \cdot bb_t^i - r_t^{nla,i} \cdot \frac{nla_t^i}{p_t^{nla}}$$

s.t.

$$-c_t^i \leq -\alpha \cdot d_t^i$$

$$-c_t^i - nla_t^i(p_t^{nla} - (\gamma + \tau^i)\chi_1 p_t^{nla}) - bl_t^i(1 - (\gamma + \tau^i)\chi_2) + bb_t^i \leq -d_t^i$$

$$nla_t^i \geq 0$$

$$c_t^i \geq 0$$

$$bb_t^i \geq 0$$

$$bl_t^i \geq 0$$

$$c_t^i + nla_t^i p_t^{nla} + bl_t^i - bb_t^i = d_t^i + er_t^i$$

where er_t^i is a bank's equity ratio. Note that the objective function is linear in its parameters and subject to a set of linear constraints. The minimization problem can thus be solved via linear programming techniques.

7 Appendix B. Banks Optimization under Shock Transmission

The optimization in the **shock transmission process** can be formulated as follows:¹⁵

$$\max_{nla_t^i, c_t^i} \pi_i = r_t^{bl} \cdot bl_t^i - (r_t^{bl} + \Delta_t^{bl}) \cdot bb_t^i + r_t^{nla, i} \cdot \frac{nla_t^i}{p_t^{nla}}$$

subject to:

$$c_t^i \geq \alpha \cdot d_t^i$$

$$er_t^i = \frac{c_t^i + p_t^{nla} \cdot nla_t^i + bl_t^i - d_t^i - bb_t^i}{\chi_1 \cdot p_t^{nla} \cdot nla_t^i + \chi_2 bl_t^i} \geq \gamma + \tau^i$$

$$c_t^i + nla_t^i p_t^{nla} = d_t^i + er_t^i - bl_t^i + bb_t^i$$

Note that the first two terms in the objective function are constant and thus can be dropped from the optimization problem.

8 Appendix C. The Algorithm

As outlined in Subsection 2.3, a shock to the financial system consists of a random percentage loss of all banks' non liquid assets (Step A) on Figure 2). In Step B), banks re-optimize their holdings of cash and non-liquid assets subject to the constraints outlined in Equations (3) to (5). Note that in this step interbank lending are given and not considered as choice variables. In Step C), bankrupt banks are identified (that is, those that violate one of the constraints in the optimization routine) and a shock to interbank lending is set up to those banks of which the creditor banks have a negative net value (with the net value being the difference between a bank's assets and liabilities). Banks with a negative net value subtract the difference

¹⁵Note that the direct interlinkage structure between banks on the interbank market is taken as given once the system is subject to a shock. Banks thus only engage in optimizing their portfolio with respect to their non-liquid and liquid asset holdings.

between their assets and liabilities, first proportionally from their interbank lending, and if there are no interbank lending left, from their deposits (Step D)). After this shock has been assigned, banks again re-optimize their portfolio (Step B). If there are no interbank shocks to assign and banks do not desire to change their holdings of non liquid assets on their balance sheet, the shock has been transmitted. Systemic risk given the shock is then calculated as the proportion of banks that default in the financial system. Expected systemic risk is obtained via computing the average systemic risk resulting from a large number of random shocks to the financial system, drawn from a multivariate normal distribution which is centered at a loss of 1% and features a variance of 24, for each bank, respectively. **This shock distribution features fat tails and ensures that large shocks to individual banks are possible.**

References

References

- [1] Adrian, Tobias and Markus Brunnermeier , (2009). “CoVaR”. Mimeo, Princeton University.
- [2] Allen, F. and A. Babus (2008). “Networks in Finance.” Working paper, Wharton Financial Institutions Center, University of Pennsylvania.
- [3] Allen, F. and D. Gale, (2000). “Financial Contagion.” *Journal of Political Economy*, 108, p.1-33.
- [4] Allen, F. and D. Gale (2008). “Systemic Risk and Regulation”.
- [5] Anand, K., Gai, P. and M. Marsili, (2011). “Rollover risk, network structure and systemic financial crises”. Forthcoming *Journal of Economic Dynamics and Control*.

- [6] Billio, M. Getmansky, S., Lo, A. and L. Pelizon, 2009. "Measuring Systemic Risk in the Finance and Insurance Sectors". MIT.
- [7] Borio, C., N. Tarashev and K. Tsatsaronis, 2009. "The Systemic Importance of Financial Institutions". BIS Quarterly Review.
- [8] Brunner, A. und J. Krahnen, 2008. "Multiple lenders and corporate distress: Evidence on debt restructuring", *Review of Economic Studies* 75, 2008, S. 415-442.
- [9] Caballero, R. and A. Simsek, 2009. "Fire Sales in a Model of Complexity". Working paper, MIT.
- [10] Cifuentes, R., G. and H. S. Shin (2005). "Liquidity Risk and Contagion." *Journal of European Economic Association*, 3, p.556-566.
- [11] Eisenberg L. and T. Noe (2001). "Systemic Risk in Financial Systems." *Management Science*, 47, p.236-249.
- [12] Financial Stability Oversight Council 2010, U.S. Government.
- [13] Freixas, X., B. Parigi, and J. C. Rochet (2000). "Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank." *Journal of Money, Credit and Banking*, 32, p.611-638.
- [14] Prasanna Gai, Andrew Haldane and Sujit Kapadia, (2011). "Complexity, Concentration and Contagion". Forthcoming *Journal of Monetary Economics*.
- [15] Geanakoplos, J. 2009. "The Leverage Cycle." In D. Acemoglu, K. Rogoff and M. Woodford, eds., NBER Macroeconomic Annual 2009, vol. 24: 1-65.
- [16] Jackson, Matthew, (2009). *Social Economic Networks*. Princeton University Press.
- [17] Krahnen, J. and M. Bluhm, 2010. "Default Risk in an Interconnected Banking System with Endogenous Asset Markets". Frankfurt University.

- [18] Korinek, A. (2011). “Systemic Risk-Taking: Amplification Effects, Externalities, and Regulatory Responses.” Maryland University.
- [19] Lagunoff, R., and L. Schreft (2001). “A Model of Financial Fragility.” *Journal of Economic Theory*, 99, p.220-264.
- [20] Leitner, Y. (2005). “Financial Networks: Contagion, Commitment, and Private Sector Bailouts.” *Journal of Finance*, 60(6), p.2925-2953.
- [21] Mas-Colell, A. (1986). Notes on price and quantity tatonnement. In *Models of Economic Dynamics*, edited by H. Sonnenschein. Lecture Notes in Mathematical Systems No. 264. Berlin: Springer- Verlag.
- [22] Mas-Colell, A. and M. Whinston. *Microeconomic Theory*. Oxford University Press.
- [23] Rochet, J. C. and J. Tirole (1996). “Interbank Lending and Systemic Risk.” *Journal of Money, Credit, and Banking*, 28, p.733-762.
- [24] Shapley, L. (1953): A Value for N-Person Games, Vol. 2: Princeton University Press.
- [25] Stanojevic, R., N. Laoutaris and P. Rodriguez (2010). “On economic heavy hitters: shapley value analysis of 95th-percentile pricing.” *Proceedings of the 10th annual conference on Internet measurement*.
- [26] Uhlig, H. 2010. “A Model of Systemic Bank Run”. *Journal of Monetary Economics*.