AUCTIONS, ACTIONS, AND THE FAILURE OF INFORMATION AGGREGATION

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We study a market in which k identical and indivisible objects are allocated using a uniform-price auction where $z > k$ bidders each demand one object. Before the auction, each bidder receives an informative but imperfect signal about the state of the world. The good that is auctioned is a common-value object for the bidders, and a bidder's valuation for the object is determined *jointly* by the state of the world and an action that he chooses after winning the object but before he observes the state. We show that there are symmetric equilibria in which the auction price is completely uninformative about the state of the world and aggregates no information even in an arbitrarily large auction. In the equilibrium that we construct, because prices do not aggregate information, agents have strict incentives to acquire costly information before they participate in the market. Also, market statistics other than price, such as the amount of rationing and bid distributions contain extra information about the state. Our findings sharply contrast with past work which shows that in large auctions where there is no ex-post action, the auction price aggregates information.

Keywords: Auctions, Large markets, Information Aggregation
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Contents

> "We must look at the price system as a mechanism for communicating information if we want to understand its real function....The most significant fact about this system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action..." Hayek (1945).

1. Introduction

One important reason to trust markets arises from the belief that market prices accurately summarize the vast array of information held by market participants. Whether this belief is justified, that is, whether prices efficiently aggregate information dispersed among agents that are active in an economy is a central economic question addressed by past research. In certain auction markets, prices do in fact effectively aggregate dispersed information. Specifically, consider a market in which a large number of identical common-value objects are sold through a uniform-price auction. In such an auction, if the bidders each have an independent signal about an unknown state of the world and if this unknown state determines the value of the object, then the equilibrium price converges to the true value of the object as the number of objects and the number of bidders grow arbitrarily large. Therefore, the auction price reveals information about the unknown state of the world. Wilson (1977), Milgrom (1979), and Pesendorfer and Swinkels (1997) have shown that this remarkable result holds under quite general assumptions.

In many situations, however, the common value of an object is not determined solely by the unknown parameters of the environment, i.e., the unknown state of the world. Rather, the object's value is also a function of how the object is utilized; in turn, the optimal way to utilize the object can depend on the unknown state of the world. For example, suppose that a large tract of land is to be divided and sold to farmers in smaller parcels through a uniform-price auction. Each farmer who successfully acquires a parcel of land in the auction needs to decide which crop to grow (e.g., wheat or rice). However, there is uncertainty about future crop prices as well as which crop grows best on that land. Alternatively, consider a uniform-price auction in which bandwidth is sold to telecommunication companies. Each winner must decide whether to use conventional technology or adopt an unconventional new one. However, there is uncertainty about future demand drivers (such as customer tastes) which will determine which technology is more profitable. In both of these examples, the winner of an object in the auction (a piece of land in the first and bandwidth in the second) must choose an action which will itself affect the value that the winner derives from the object. Moreover, this action must be taken after the auction is finalized but before some payoff-relevant uncertainty is resolved.

In the examples discussed above, if the auction price provides additional information that reduces uncertainty, i.e., if the auction price aggregates information, then the winners would make better decisions when choosing their action (which crop to grow or which technology to adopt). However, none of the past work on information aggregation in auctions explores how the information revealed by the auction price is used after the auction is completed. In contrast; in this paper we explicitly model how the information about the state of the world is used after a common-value auction is completed; in our model, the auction's winners must decide on an action in order to put the objects acquired into productive use and the optimal choice of action depends on the true state of the world. We show that such large common-value auctions have symmetric equilibria in which the equilibrium price reveals no information about the state of the world. Moreover, in such equilibria the nonrevealing price leads to inefficiency: a substantial number of buyers who acquire an object in the auction inefficiently choose the wrong action. Our result suggests that if information is useful for efficient decision making, then the equilibrium price may not aggregate all the information relevant for the decision and thus inefficiency may persist even in a large market. This finding stands in stark contrast to earlier studies which show that prices aggregate information when there is no immediate use for this information.

More specifically, we study a model in which k identical and indivisible objects are allocated using a uniform-price auction in which $z > k$ bidders each demand one unit of the good. Before the auction, each bidder receives an informative but imperfect signal about the state of the world. In the auction, bidders choose their bids as a function of their signal, the k highest bidders are allocated one unit of the object, and all bidders who win an object pay a uniform price equal to the $k+1$st highest bid. The good that is auctioned is a common-value object for the bidders and a bidder's valuation for the object is determined *jointly* by the state of the world and an action that he chooses after winning the object but before he observes the state. In a large market, if the market clearing price were to aggregate all information, then actions would be chosen efficiently and competition would necessarily drive the price of the object to its *efficient-use value*.

We explore a number of properties of markets as the numbers of bidders and the objects grow proportionately; however, our primary focus is on the informativeness of prices. An outsider who could observe the signals of an arbitrarily large number of bidders would learn the state of the world perfectly. Motivated by such an outsider's perspective, we say that prices fully aggregate information if an outsider can figure out the state of the world almost perfectly just by observing the equilibrium price.

We present two main results. *In our first main result*, we construct a particular sequence of symmetric equilibria in which, as the market grows arbitrarily large, the limit price conveys no information about the true state of the world and remains strictly below the efficient use-value of the object. In the equilibria we construct, a strictly positive fraction of agents choose the wrong action because the price conveys no new information. Therefore, inefficiency persists even in a large market whose outcome would have been efficient if one could observe all of the bidders' signals. Also, because the equilibrium price does not convey new information, agents have strict incentives to acquire costly information both before they participate in the auction and after the objects have been allocated.

A prominent property of the equilibrium which we construct is that equilibrium bids are nondecreasing in the signal that an agent receives, i.e., the equilibrium is monotone. In order to explore the robustness of our first result, we also study arbitrary symmetric equilibria which are monotone. *In our second result*, we characterize equilibrium behavior in any symmetric, monotone equilibrium and we provide a sufficient condition under which such equilibria exist. We then use our characterization to show that no sequence of symmetric, monotone equilibria can fully aggregate information. Moreover, a non-negligible fraction of the auction's winners choose the wrong action in such equilibria.

In a nutshell, our results suggest that the auction price may not aggregate information if the information is useful for productive efficiency. In our model, market statistics other than price, such as the amount of rationing and bid distributions, are informative. Therefore, whether these statistics are observed after an auction is finalized can affect how much information is aggregated by prices. Moreover, dynamic trading in which traders engage in multiple rounds of activities may augment the accumulation of useful information.

In order to obtain our result, we make a number of strong assumptions that jointly restrict the signal distribution and the value of an object as a function of the state and the chosen action. However, the key driver of our result is the assumption that we make on the interaction between the unknown states and the actions that are chosen. In particular, our main assumption requires that the optimal action is different in each state and that choosing the wrong action in a given state is particularly costly. This assumption implies that it is best to own the object when the state is known with certainty, and worst to own the object when there is significant uncertainty about the state.

Relation to the literature. This paper is closely related to earlier work which studies information aggregation in large auctions. Wilson (1977) studied second-price auctions with common value for one object for sale, and Milgrom (1979) extended the analysis to any arbitrary number of objects. Both of these papers show that as the number of bidders gets arbitrarily large, price converges to the true value of the object, but only provided that there are bidders with arbitrarily precise signals about the state of the world. Pesendorfer and Swinkels (1997) further generalize the analysis to the case where there are no arbitrarily precise signals. They show that prices converge to the true value of a common-value object in all symmetric equilibria if and only if both the number of identical objects and the number of bidders who are not allocated an object grow without bound. Pesendorfer and Swinkels (2000) generalize the analysis further to a mixed private, common-value environment. Finally, Kremer (2002) shows that the information aggregation properties of auctions are more general than the particular mechanisms studied before; he does this by providing a unified approach that uses the statistical properties of certain order statistics.[1] Our model is closest to Pesendorfer and Swinkels (1997). The main difference from theirs is that in our model the object's value is jointly determined by the unknown state of the world and the action that the owner of the object later takes.[2]

Our work also relates to the literature on costly information acquisition in rational-expectations models, such as Grossman and Stiglitz (1976, 1980) and Grossman (1981). These papers explain the conceptual difficulties in interpreting prices as both allocation devices and information aggregators. Specifically, they argue that if consumers and producers need to undertake a costly activity in order to acquire information, then equilibrium prices cannot reveal the state of the world perfectly. Their reasoning is as follows: if prices were to reveal the state perfectly, then no agent would have an incentive to pay for information in the first place; but if no agent acquires information, then the prices cannot reveal the state as there is no information to aggregate. However, as was the case for auction markets, these papers do not explicitly consider how the information revealed by the market price could be used by the market participants once they have completed their trade in the market. In our model, since prices do not aggregate information, agents have a strict incentive to acquire information. This finding contrasts with the findings of Grossman and Stiglitz (1980), who argue that agents have no incentive to acquire information precisely because prices are so efficient in aggregating information.

Our model is related to work by Bond and Eraslan (2010) who show that trade between two agents with the same preferences is possible if the value of the object traded is jointly determined by an unknown state of the world and an ex-post action that the eventual owner of the object will undertake. In their model, trade is precluded by a no-trade theorem without

[1]See Hong and Shum (2004) for a calculation of the rate at which price converges to the true value in large common-value auctions. Jackson and Kremer (2007) show that the result of Pesendorfer and Swinkels (1997) does not generalize to an auction with price discrimination. See Kremer and Skrzypacz (2005) for related results concerning the link between information aggregation and the properties of order statistics.

[2]There is extensive work on information aggregation and the role of prices in various other market contexts. For example, see Reny and Perry (2006) and Cripps and Swinkels (2006) for large double auctions; Vives (2011) and Rostek and Weretka (2010) for markets for divisible objects; Rubinstein and Wolinsky (1985, 1990), Osborne and Rubinstein (2010), Lauermann (2013), Lauermann and Wolinsky (2011, 2012), Lauermann and Virág (2012), Golosov et al. (2011), Ostrovsky (2012) for search markets. In the voting context, Feddersen and Pesendorfer (1997) show that information is aggregated in large elections as long as the voting rule is not unanimous, and Bhattacharya (2013) shows that this result hinges critically on the assumption that the preferences voters comove in the same direction with new public information.

any ex-post action. It is the ex-post action and the consequent value of information that lead to the possibility of trade. Our model shares the feature that the eventual owner of an object undertakes an action once trading is complete. However, whereas they consider a bilateral bargaining framework and focus on the possibility of trade, we analyze an auction framework with a large number of strategic bidders and focus on information aggregation.

A prominent feature of our equilibrium construction is pooling by bidders who receive signals that lie in a certain range. In a recent paper, Lauermann and Wolinsky (2012) study an auction where the seller needs to solicit bidders for the auction and this makes the number of participants of the auction dependent on the state of the world. Similar to our paper, their model also features pooling in equilibrium. There is pooling in their model because bidding the pooling bid provides insurance against winning too frequently in the low-payoff state. The reason why pooling is sustained in our model is different: winning at the pooling bid provides the bidder with additional information which he can then use to make a better informed action choice.

Finally, there is a growing literature which investigates the impact of financial markets on the real economy. Past work in this literature explores situations where speculators' private information about the success prospects of an action to be chosen by a manager (or a CEO or a central banker) is partially revealed through its effect on the prices of financial assets or derivatives. Knowing this, managers also factor in financial asset price information when deciding on their course of action. Papers in this literature then argue that such feedback loops may decrease the information content of prices. A recent paper by Bond et al. (Forthcoming) surveys this literature.

2. Model

We consider a sealed-bid, uniform-price auction. There are z bidders with unit demand and k identical objects. We denote the ratio of objects to bidders (i.e., market tightness) by $\kappa := \frac{k}{z} < 1$. The set of states of the world is $\Omega := \{L, R\}$ with a generic element ω. The state of the world is drawn according to a common prior $\pi \in [0,1]$, where π denotes the prior probability that the state is R. Each bidder i observes a private signal s_i that belongs to the set of signals $S = [0,1]$, and submits a bid $b_i \in [0,\infty)$. Each of the k highest bidders receives an object and is called a winner; all other bidders are called losers.[3] Each winner pays a price p which is equal to the $(k+1)^{st}$ highest bid.

The bidders' signals are independently and identically distributed conditional on the state of the world. Each bidder's signal distribution has a cumulative distribution function $F(.|w)$ with a continuous density $f(.|w)$. If a bidder believes that the probability of state R is p,

[3] To rank bids that are tied, nature picks a ranking of bidders at random with each ranking equally likely.

then we say that the bidder's likelihood ratio is $\rho := p/(1-p)$. For a bidder who receives signal $s \in [0,1]$, we denote his likelihood ratio, slightly abusing notation, to be $\rho(s)$, defined as follows:

$$\rho(s) = \frac{\pi}{1-\pi}\frac{f(s|R)}{f(s|L)} = \rho_0 \frac{f(s|R)}{f(s|L)},$$

where $\rho_0 := \pi/(1-\pi)$ denotes the prior likelihood ratio derived from the common prior π. In addition to our assumption that the density function is continuous we make the following assumption on the signal distributions:

Assumption 1 (MLRP) *The likelihood ratio $\rho(s)$ is a strictly increasing function of s.*[4]

In what follows, we refer to the m^{th} highest value among n signals by Y_n^m. Let $s_R^\kappa \in S$ and $s_L^\kappa \in S$ be the unique signals that satisfy $F(s_R^\kappa|R) = 1-\kappa$ and $F(s_L^\kappa|L) = 1-\kappa$. Recall that $\kappa < 1$ is the market tightness, i.e., the ratio of objects to bidders. Intuitively, in a large market there are approximately as many bidders with signals above s_R^κ as there are objects in state R. Therefore, if we were to allocate the objects to the bidders with higher signals first, then, in state R, the bidders who receive an object would be approximately those bidders whose signals exceed s_R^κ.

The payoff of a bidder who does not win an object is equal to zero. We assume that a bidder who wins an object must choose an action from a finite set of actions denoted by A. This action, together with the state of the world, determines the winner's valuation for the good. Although all our of arguments go through with an arbitrary, finite number of actions, to keep exposition simple, *we assume that* $A = \{l, r\}$. A winning bidder's payoff is jointly determined by the auction price p, the action that he chooses $a \in A$, and the state of the world $\omega \in \Omega$. In particular, we assume that a winning bidder's payoff is equal to $v(a,\omega) - p$, where the function $v(a,\omega)$ gives the winner's valuation for the object. In what follows, we assume, without loss of generality, that $v(r,R) \geq v(l,L)$ and we make the following main assumption:

[4] All of the results in this paper go through without alteration if we drop Assumption 1 and our assumption that the density function f is continuous. With two states, we can always ensure that the likelihood ratio function $\rho(s)$ is nondecreasing in s by reordering the signals, i.e., we can ensure that MLRP holds in its weak form. In this paper, we make a stronger assumption to simplify our proofs. For further reference, see our working paper Atakan and Ekmekci (2012) where we prove identical results but without Assumption 1.

Assumption 2 *The valuation function satisfies the following three inequalities:*

(1) $v(l, L) > v(r, L)$,

(2) $v(l, L) > v(l, R)$,

(3) $v(l, R) \geq 0$ *and* $v(r, L) \geq 0$.

Note that if the valuation function does not satisfy inequality (1), then r is a weakly dominant action. Also, if the valuation function does not satisfy inequality (2), then a bidder's valuation for the good is higher in state R than in state L regardless of the action he chooses. The final inequality ensures that owning the good is weakly preferred to not owning the good given any prior over the states. Therefore, if inequality (3) is satisfied, then it is individually rational for all the bidders to participate in the auction. Consider, to fix ideas, a simple case where $v(l, R) = v(r, L) = 0$. In this case, Assumption 2 implies that a bidder's *valuation for the good is positive if his action matches the state* of the world, and his *valuation for the good is zero if his action does not match the state* of the world.

Remark 1 *Assumption 2 is the substantive assumption which allows us to argue that information is not aggregated in our model. The main implication of Assumption 2, which we use in many of our arguments, is the fact that a bidder's expected valuation for the good is a nonmonotonic function of the probability that he assigns to state R. In contrast, if either inequality* (1) *or* (2) *is not satisfied, then a bidder's expected valuation for the good is a monotonic function of the probability that he assigns to state R. In either of these two cases, Pesendorfer and Swinkels (1997)'s findings apply under Assumption 1 and therefore information is aggregated in every symmetric equilibrium of a large market. See section 5.1 for a more detailed discussion of this nonmonotonicity.*

2.1. Strategies, equilibrium, and values. Each bidder submits a bid after observing his signal. A bidding strategy for player i is a measure H_i on $[0, 1] \times [0, \infty)$ with marginal distribution $F(s) = \pi F(s|R) + (1 - \pi)F(s|L)$ on its first coordinate (see Milgrom and Weber (1985)). The set of all bidding strategies is Σ. A strategy is pure if there is a function $b : [0, 1] \to [0, \infty)$ such that $H(\{s, b(s)\}_{s \in [0,1]}) = 1$.[5]

Each winner chooses an action from the set of actions A. Hence, the action strategy is a mapping from a bidder's signal, his bid, and the winning price to an action, $a_i : S \times [0, \infty) \times [0, \infty) \to A$. Since the bidders' actions do not affect other bidders' payoffs, confining attention to pure strategy actions is without loss of generality.

[5]If b represents H, then so does any function that is equal to b at almost every $s \in [0, 1]$.

In the rest of the paper, we will restrict attention to pure symmetric Nash equilibria, which are equilibria where each bidder uses the same pure bidding strategy, i.e., $b_i = b_j$ for every two bidders i and j. The term $\Pr_b$ denotes the joint probability distribution over states of the world, signal and bid distributions, allocations, and prices, where this distribution is induced by the pure and symmetric bidding strategy profile where each bidder uses the bidding strategy b.

Below we define a bidder's value as a function of his beliefs but we work with the likelihood ratio instead of working directly with beliefs for analytic convenience.[6] The value function u is a bidder's expected valuation for an object as a function of the likelihood ratio ρ. In particular, let $u : [0, \infty) \to \mathbb{R}$ be the function defined by

$$u(\rho) = \max_{a \in \{l,r\}} \left\{ \frac{1}{\rho + 1} v(a, L) + \frac{\rho}{\rho + 1} v(a, R) \right\}.$$

Note that $u(0) = v(l, L)$ and $\lim_{\rho \to \infty} u(\rho) = v(r, R)$. Let $\rho^* \in (0, \infty)$ be the unique solution to the equation:

$$\frac{1}{\rho + 1} v(l, L) + \frac{\rho}{\rho + 1} v(l, R) = \frac{1}{\rho + 1} v(r, L) + \frac{\rho}{\rho + 1} v(r, R).$$

This cutoff is the likelihood ratio that makes a bidder indifferent between action l and r. Such a cutoff always exists because of Assumption 2. *In what follows, we extensively use the fact that the value function $u(\cdot)$ is strictly decreasing in the interval $[0, \rho^*]$ and strictly increasing in the interval $[\rho^*, \infty)$.* For a depiction of the value function see figure 1.

3. Large Markets and the Failure of Information Aggregation

In this section we present our main result as Theorem 1. In Theorem 1, we construct a sequence of equilibria for auctions $\{\Gamma_z\}_{z=1}^{\infty}$ where the z^{th} auction Γ_z has z bidders and $\lfloor \kappa z \rfloor$ objects for sale.[7] In the remainder of the paper, we will proceed as if κz is an integer for expositional simplicity. We assume that the other parameters of the auctions, i.e., (v, F, π, κ), are constant along the sequence and satisfy all the assumptions that we have already made. For the sequence of equilibria we construct, equilibrium price reveals no information about the state of the world at the limit where there is an arbitrarily large number of bidders. Although the limit equilibrium price reveals no information, bidders *do* learn some information about the state of the world from the fact of winning because, despite the price is independent of the state, the probability of winning depends on the state. However, the amount of

[6] The whole analysis could be redone working directly with beliefs as there is a one-to-one mapping between likelihood ratios and beliefs.

[7] The term $\lfloor \kappa z \rfloor$ refers to the highest integer not bigger than κz.

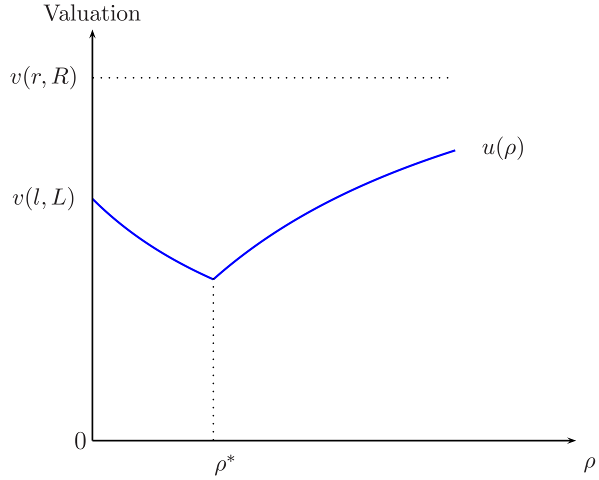

Figure 1: Continuation value as a function of the likelihood ratio of the bidder who wins a unit in the auction, before he makes the action choice. Assumption 2 implies that $u(\rho)$ is a nonnegative, nonmonotonic function which is minimized at ρ^* as depicted here.

information that they learn is limited and therefore incorrect ex-post actions are frequently undertaken.

3.1. Information aggregation. Here we formally define information aggregation and its failure. Our object of study is a sequence of bidding functions $\boldsymbol{b} = \{b_z\}_{z=m}^{\infty}$. We say that the sequence $\boldsymbol{b}$ is an equilibrium sequence if b_z is part of a symmetric Nash equilibrium of Γ_z for each z.

Suppose that the number of bidders z is large. In this case, the law of large numbers implies that observing the signals $(s_1, ..., s_z)$ conveys precise information about the state of the world $\omega \in \{L, R\}$. The bidding function b_z determines a price p^* for the auction Γ_z given any realization of signals $(s_1, ..., s_z)$. We say that information is aggregated in the auction if this price p^* also conveys precise information about the state of the world. More precisely, (i) if the likelihood ratio $\frac{\Pr_{b_z}(p^*|R)}{\Pr_{b_z}(p^*|L)}$ is close to zero (i.e., if it is arbitrarily more probable that we observe such a price when $\omega = L$), then an outsider who observes price p^* learns that the state is L. Alternatively, (ii) if the likelihood ratio $\frac{\Pr_{b_z}(p^*|R)}{\Pr_{b_z}(p^*|L)}$ is arbitrarily large, then an outsider who observes price p^* learns that the state is R. If the probability that we observe a price that satisfies either (i) or (ii) is arbitrarily close to one, then we say that the equilibrium sequence $\boldsymbol{b}$ fully aggregates information. Conversely, if the likelihood ratio $\frac{\Pr_{b_z}(p^*|R)}{\Pr_{b_z}(p^*|L)}$ is close to one, i.e., if we are equally likely to observe price p^* in either of the two states, then an outsider who observes price p^* learns arbitrarily little information about the state of the world. If the probability that we observe such a price is arbitrarily close to one, then we say that the equilibrium sequence $\boldsymbol{b}$ aggregates no information. The precise definitions are as follows:

Definition 1 *An equilibrium sequence* **b** ***aggregates no information*** *if, for any* $\epsilon > 0$,

$$\lim_{z\to\infty} \Pr_{b_z}\left(p_z \in \left\{p \in [0,\infty) : \frac{\Pr_{b_z}(p|R)}{\Pr_{b_z}(p|L)} \in (1-\epsilon, 1+\epsilon)\right\}\right) = 1.$$

An equilibrium sequence **b** ***fully aggregates information*** *if, for any* $\epsilon > 0$,

$$\lim_{z\to\infty} \Pr_{b_z}\left(p_z \in \left\{p \in [0,\infty) : \frac{\Pr_{b_z}(p|R)}{\Pr_{b_z}(p|L)} \in [0,\epsilon) \cup (1/\epsilon, \infty))\right\}\right) = 1.$$

Remark 2 *Our definition of information aggregation takes the perspective of an outside observer who only sees the auction price. Our definition differs from the definition provided by Pesendorfer and Swinkels (1997). In their model, the state of the world is defined as the value of the object and each bidder receives a signal about that value. Therefore, they say that information is aggregated if the equilibrium prices converge to the true value of the object as the market grows large. In their setup, each state represents a distinct value for the object, so when information is aggregated in their model with their definition, then it is also aggregated under our definition. Conversely, if information aggregation fails using our definition, then it will also fail under the definition of Pesendorfer and Swinkels (1997). We should note that in the equilibria we construct, some bidders who participate in the auction obtain additional information compared to an outside observer but these bidders nevertheless remain imperfectly informed about the state of the world (for more, see Remarks 5 and 7).*

3.2. Failure of information aggregation. Our main theorem shows that if, in addition to Assumptions 1 and 2, the two assumptions described below are satisfied, then there exists an equilibrium sequence **b** which aggregates no information. Recall that $s_R^\kappa \in S$ is the signal such that $F(s_R^\kappa|R) = 1-\kappa$. The first assumption we require for the theorem is as follows:

Assumption 3 $\rho(0) > 0$ *and* $u(\rho(0)) < u(\rho(s_R^\kappa))$.

The first part of this assumption requires that even a bidder with signal 0 is not sure that the state is L. The second part of the assumption is a joint restriction on the prior, the informativeness of signals 0 and s_R^κ, and values. This assumption is likely to hold in economic situations where state R is sufficiently more likely a priori, or in environments where higher signals are more informative than lower signals, or in economic situations where choosing the correct action in state R is considerably more profitable than choosing the correct action in state L. A simpler case in which Assumption 3 is satisfied is when all the bidders would choose action r if they acted solely on the information contained in their private signal, i.e., if $\rho(0) = \frac{f(0|R)}{f(0|L)}\rho_0 \geq \rho^*$. In such a case, the agents' valuation for the good is an increasing function of their signals.

Remark 3 *For any f under which signal zero is not perfectly informative, i.e., $\frac{f(0|R)}{f(0|L)} > 0$, any $\kappa \in (0,1)$, and any ρ^*, there exits a cutoff likelihood ratio $\underline{\rho} > 0$ such that if the prior likelihood ratio ρ_0 exceeds $\underline{\rho}$, then Assumption 3 is satisfied. In other words, Assumption 3 is satisfied for sufficiently large ρ_0.*[8]

The second assumption we require for the theorem is as follows:

Assumption 4 $u(\rho(s_R^\kappa)) < v(l, L)$.

Under this assumption, if a bidder who received signal s_R^κ chooses an action based solely on this signal, then this bidder's expected valuation is lower than $v(l, L)$. Therefore, a sufficiently strong additional signal in favor of state L can increase the expected valuation of such a bidder. This assumption is likely to hold in economic situations where there are not too many bidders per available unit.[9] See figure 2 for a depiction of a situation where both Assumption 3 and 4 are satisfied.

Remark 4 *For any f under which signal zero is not perfectly informative, i.e., $\frac{f(0|R)}{f(0|L)} > 0$, any $\kappa \in (0,1)$, and any ρ^*, there exits a cutoff likelihood ratio $\bar{\rho} > 0$ such that if the prior likelihood ratio ρ_0 is less than this cutoff, then Assumption 4 is satisfied, i.e., Assumption 4 is satisfied if ρ_0 is sufficiently small. Moreover $\bar{\rho} > \underline{\rho}$.*[10] *Combining this insight with the similar finding in Remark 3 we conclude that both Assumption 3 and 4 are satisfied if the prior likelihood ratio lies in the open interval $(\underline{\rho}, \bar{\rho})$.*

Our main theorem is below. All proofs are in the Appendix.

Theorem 1 *If Assumptions 1, 2, 3 and 4 are satisfied, then there exists an equilibrium sequence $\boldsymbol{b}$ which reveals no information.*

Remark 5 *A strictly positive fraction of the auction's winners choose the wrong action in the equilibrium sequence that we construct for Theorem 1. This inefficiency occurs because*

[8]To see why this statement is true, note that if $\rho_0 = \frac{f(0|L)}{f(0|R)}\rho^*$, then $\rho(0) = \rho^*$ and therefore $u(\rho(0)) < u(\rho(s_R^\kappa))$ because $u(\cdot)$ is minimized at ρ^*.

[9]If there are few bidders per available unit, i.e., if the market is not particularly tight and κ is close to one, then s_R^κ is small. However, if s_R^κ is either sufficiently close to ρ^* or smaller than ρ^*, then $u(\rho(s)) < v(l, L)$ for all $s \le s_R^\kappa$. Therefore, there is a cutoff $\bar{\kappa}$ such that for all $\kappa \ge \bar{\kappa}$ we have $u(\rho(s_R^\kappa) < v(l, L)$.

[10]Define $\bar{\rho}$ as the unique $\rho \in (0,1)$ which satisfies the equality $u(\bar{\rho}\frac{f(s_R^\kappa|R)}{f(s_R^\kappa|L)}) = v(l, L)$. It is clear that Assumption 4 is satisfied iff $\rho \in (0, \bar{\rho})$. Note also that if $\rho_0 = \bar{\rho}$, then $u(\rho_0\frac{f(0|R)}{f(0|L)}) < v(l, L) = u(\bar{\rho}\frac{f(s_R^\kappa|R)}{f(s_R^\kappa|L)})$, i.e., if $\rho_0 = \bar{\rho}$, then Assumption 3 is satisfied. Therefore, $\bar{\rho} > \underline{\rho}$.

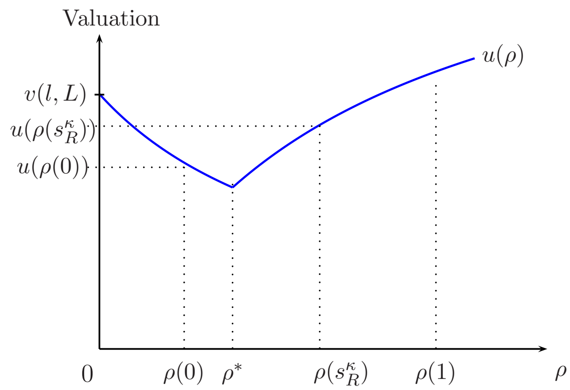

Figure 2: This figure shows the initial range of beliefs, expressed as likelihood ratios, on the belief-value graph. These beliefs and values satisfy Assumptions 3 and 4.

the equilibrium price reveals no information. We provide the details for this inefficiency in section 3.4, item (i).

We prove this theorem by constructing an equilibrium sequence which aggregates no information. In this construction, each bidding function b_z in the equilibrium sequence $\boldsymbol{b}$ is a nondecreasing function of s. Assumption 3 allows us to construct an equilibrium sequence in which each bidding function b_z is nondecreasing in s. Assumption 4, on the other hand, allows us to ensure that the equilibrium sequence that we construct aggregates *no information* about the state of the world.

In the equilibrium that we construct, information is not aggregated by the price because agents with different signals submit the same "pooling" bid (see figure 3) and because the auction price is equal to this pooling bid in both states. We now provide some intuition for Theorem 1 by *i*) arguing that if the bidding function is nondecreasing, then there must be pooling; and *ii*) arguing that pooling can be sustained in equilibrium.

Assume that the bidding function is strictly increasing and therefore that there is no pooling bid. Consider a bidder who receives a signal $\epsilon > 0$ that is arbitrarily close to the lowest signal (i.e., zero). If the auction price is equal to this bidder's bid, then this bidder is almost certain that the state is L in a sufficiently large auction. However, then a bidder who receives signal zero would rather outbid a bidder who receives signal ϵ. This is because the bidder with signal zero is more convinced that the state is L than the bidder with signal ϵ. This, however, contradicts that the bidding function is strictly increasing.[11]

In our construction, a pooling bid can be sustained because agents have an incentive to learn the state in order to use this information while choosing their action. Specifically, when

[11]Note that this argument crucially hinges on the *nonmonotonicity* of the value function (see figure 2 and section 5.1).

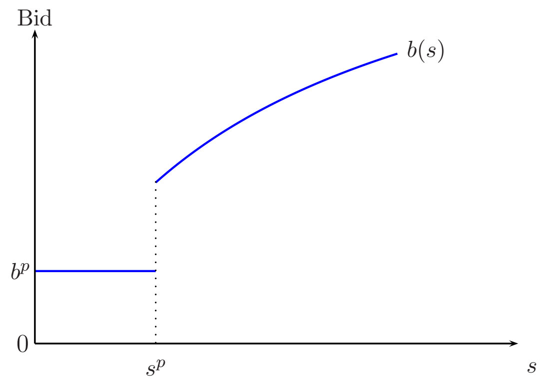

Figure 3: A typical equilibrium bidding function. Buyers with signals below a cutoff s^p bid a pooling bid b^p, and those with signals above s^p bid according to the bidding function in Pesendorfer and Swinkels (1997), i.e., $b(s) = v(r, R)\Pr(\omega = R|s_1 = s, Y_{n-1}^k = s) + v(r, L)\Pr(\omega = L|s_1 = s, Y_{n-1}^k = s)$ for $s > s^p$.

the price is equal to the pooling bid, objects are allocated using *rationing* among the bidders who choose the pooling bid.[12] A bidder who chooses the pooling bid and wins an object through rationing at a price equal to the pooling bid obtains more information about the state of the world, compared to the case in which he instead chooses a higher bid, avoids rationing, and wins an object. This is because winning an object under rationing is more likely in state L than in state R. In other words, rationing is a lottery whose odds depend on the state of the world. Moreover, a sufficiently strong signal in favor of state L is valuable for some agents because the value function is *nonmonotonic*. If a bidder who chooses the pooling bid increases his bid, then he acquires an object more frequently because he avoids rationing when the price is equal to the pooling bid. However, in this case he forgoes the extra piece of information that comes from winning under rationing. Because this extra piece of information is sufficiently valuable for bidders who choose the pooling bid, these bidders refrain from increasing their bids.

3.3. Sketch of the construction. In this section, we sketch the ideas behind constructing the equilibrium sequence $\mathbf{b}$ whose existence is claimed in Theorem 1. Specifically, we construct an equilibrium in which no information is aggregated in a hypothetical market with a continuum of bidders with mass one and a continuum of objects with mass κ. Focusing on a hypothetical market with a continuum of bidders allows us to capture the main properties of the equilibrium sequence $\mathbf{b}$ for a market with a finite but large number of bidders while allowing us to avoid the more technical details involved in describing such equilibria for finite markets. In what follows we repeatedly use the fact that the value function $u(\cdot)$ is strictly

[12] See Stiglitz and Weiss (1981), Bester (1985, 1987), and Lauermann and Wolinsky (2012) for other models where there is rationing in equilibrium.

decreasing in the interval $[0, \rho^*]$ and strictly increasing in the interval $[\rho^*, \infty)$. Note that Assumption 3 implies that $\rho^* < \rho(s_R^\kappa)$. This is because otherwise the fact that $u(\cdot)$ is strictly decreasing on $[0, \rho^*]$ would imply that $u(\rho(s_R^\kappa)) < u(\rho(0))$ which contradicts Assumption 3. See figure 2 for a depiction.

We construct an equilibrium in which the equilibrium bidding function b is constant on the interval $[0, s^p]$ for some cutoff signal $s^p > s_R^\kappa$, which we calculate further below (i.e., $b(s) = b^p$ for all $s \in [0, s^p]$), and the bidding function is equal to the strictly increasing bidding function in Pesendorfer and Swinkels (1997) on the interval $(s^p, 1]$, i.e., $b(s) = v(r, R)Pr(\omega = R|s_1 = s, Y_{n-1}^k = s) + v(r, L)Pr(\omega = L|s_1 = s, Y_{n-1}^k = s)$ for $s \in (s^p, 1]$. We call the bid b^p (i.e., the bid submitted by all bidders with signals in the interval $[0, s^p]$) the *pooling bid* or *pooling price* and we set this bid b^p equal to $u(\rho(s^p))$, i.e., the expected utility of an agent who receives the cutoff signal s^p. See figure 3 for a depiction of the bidding function b. This equilibrium has the following properties:

(i) The auction price is equal to the pooling price in either state of the world, and hence conveys no additional information about the state of the world. The auction price is always equal to the pooling price because s^p exceeds s_R^κ. The fact that s^p exceeds s_R^κ implies that for any price $p' > b^p$, the mass of bidders who submit a bid greater than or equal to p' is strictly smaller than the mass of available objects, i.e., the measure of the set $\{s : b(s) \geq p'\}$ is strictly less than κ in both states.

(ii) Bidders with signals that exceed s^p bid above the pooling price therefore, they are always allocated an object and always choose action r. These bidders choose action r because they obtain no new information from the auction price and because choosing r is optimal based solely on their private signal.

(iii) Bidders with signals smaller than s^p who are allocated an object, i.e., the bidders who bid the pooling price, take action l. Although the price conveys no information about the state, the fact that a bidder wins an object by bidding the pooling price is a strong signal favoring state L which induces that bidder to choose action l. Winning an object by bidding the pooling price is a strong signal favoring state L because the mass of bidders bidding the pooling price exceeds the mass of objects to be allocated to bidders who bid the pooling price. Moreover, a bidder is more likely to be allocated a good in state L than in state R. We discuss this issue in more detail below.

Calculating the cutoff signal s^p. The cutoff signal s^p is the signal which leaves a bidder indifferent between bidding the pooling bid and bidding slightly above the pooling bid. As a first step in calculating s^p, we calculate a bidder's payoff if he bids slightly above the pooling bid, and if he bids the pooling bid and wins an object.

Payoff from bidding slightly above the pooling bid. If a bidder bids above the pooling bid, then he wins an object with certainty. The posterior belief of a bidder who wins an object by bidding above the pooling bid is equal to his initial beliefs. This is because the auction price is always equal to the pooling bid in this equilibrium and conveys no information. Consequently, the expected value of the object to a bidder with signal s if he bids above the pooling bid is $u\left(\rho(s)\right)$.

Payoff from bidding the pooling bid. We now calculate the value of the object for a bidder who receives the cutoff signal s^p if he bids the pooling bid and wins a unit, when $s^p \geq s_R^\kappa$. In such an event, this bidder has an *extra* piece of information, which comes from the fact that he wins a unit while bidding the pooling bid. In particular, a fraction $1 - F(s^p|\omega)$ of bidders bid strictly above the pooling bid and each wins an object with certainty regardless of the state. The fraction of objects that remains to be delivered to bidders who choose the pooling bid is $\kappa - (1 - F(s^p|\omega))$. Since the number of objects remaining to be delivered is less than the number of bidders, there is *rationing* among the bidders at the pooling bid. Consequently, the belief of type s^p (represented as the likelihood ratio) if he bids the pooling bid and wins the object is as follows:

$$\rho^p(s^p) := \rho(s^p)\frac{\kappa - (1 - F(s^p|R))}{F(s^p|R)} \Big/ \frac{\kappa - (1 - F(s^p|L))}{F(s^p|L)} = \rho(s^p)\frac{\kappa - (1 - F(s^p|R))}{\kappa - (1 - F(s^p|L))}\frac{F(s^p|L)}{F(s^p|R)},$$

where the ratio $\Delta(s^p) := \frac{\kappa-(1-F(s^p|R))}{F(s^p|R)} / \frac{\kappa-(1-F(s^p|L))}{F(s^p|L)}$ reflects the extra information that a bidder learns from winning an object at the pooling bid. If a bidder with signal s^p bids the pooling bid and wins the object, then the expected value of the object to him is equal to $u\left(\rho^p(s^p)\right)$.

Remark 6 *It is straightforward to verify that $\Delta(s^p) < 1$, that is, winning an object at the pooling bid is more likely in state L than in state R; winning an object at the pooling bid is therefore an additional signal in favor of state L. In the context of the auction models of Pesendorfer and Swinkels (1997) or Milgrom and Weber (1982), the fact that $\Delta(s^p) < 1$ is commonly referred to as the* ***loser's curse****.[13] Intuitively, the loser's curse holds because if the state is L, then the MLRP assumption implies that fewer bidders choose a bid which exceeds the pooling bid, and therefore more goods are left over to be allocated to the bidders who choose the pooling bid.*

As we stated above, the cutoff signal s^p is the signal which leaves a bidder indifferent between bidding the pooling bid and bidding slightly above the pooling bid. In other words,

[13] The loser's curse is defined in the setting with finitely many bidders; however, the idea extends naturally to the hypothetical setting with a continuum of bidders.

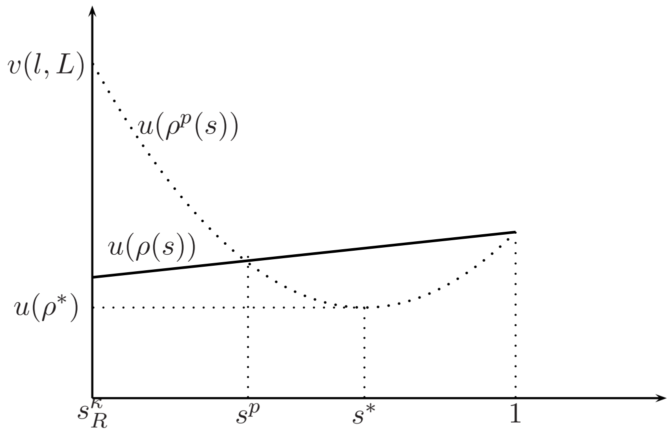

Figure 4: This figure depicts the value of the object to the cutoff type as a function of the choice of the cutoff type.

the cutoff signal is defined implicitly by the following equation:

$$u\left(\rho(s^p)\right) = u\left(\rho^p(s^p)\right).$$

We now argue that this cutoff signal is unique. Specifically, we show that there is a unique signal $s \in [s_R^\kappa, 1)$ such that $u\left(\rho(s)\right) = u\left(\rho^p(s)\right)$, and we denote this signal by s^p. We argue that the functions $u\left(\rho(\cdot)\right)$ and $u\left(\rho^p(\cdot)\right)$ are depicted accurately by figure 4 and thus have a unique point of intersection. That is, the function $u\left(\rho(\cdot)\right)$ is strictly increasing on the interval $[s_R^\kappa, s^*]$ where s^* denotes the unique signal such that $\rho^p(s^*) = \rho^*$ (see figure 5). The function $u\left(\rho^p(\cdot)\right)$ strictly exceeds $u\left(\rho(\cdot)\right)$ at s_R^κ, is strictly lower than $u\left(\rho(\cdot)\right)$ at s^*, and is strictly decreasing in the interval $[s_R^\kappa, s^*]$. Thus the two functions must cross at a unique point $s^p \in (s_R^\kappa, s^*)$. Moreover, $u\left(\rho^p(\cdot)\right)$ remains strictly below $u\left(\rho(\cdot)\right)$ on $[s^*, 1)$ and therefore the two functions do not cross again except at $s = 1$. To see why $u(\rho^p(s_R^\kappa)) > u(\rho(s_R^\kappa))$, note that $\rho^p(s_R^\kappa) = 0$ and that $v(l, L) > u(\rho(s_R^\kappa))$ by *Assumption 4*. Therefore, $u(\rho^p(s_R^\kappa)) = u(0) = v(l, L) > u(\rho(s_R^\kappa))$. The function $u(\rho(s))$ is strictly increasing because $\rho(s)$ is strictly increasing in s, because $\rho(s) \geq \rho(s_R^\kappa) > \rho^*$, and because $u(\rho)$ is strictly increasing in the interval (ρ^*, ∞). Also, the function $u(\rho^p(s))$ is strictly decreasing in s for all $s \in [s_R^\kappa, s^*]$ because $\rho^p(s) < \rho^*$ for all for all $s \in [s_R^\kappa, s^*)$. Finally, to complete the argument, note that $u(\rho(s)) > u(\rho^p(s))$ for all $s \in [s^*, 1)$ because $u(\rho)$ is strictly increasing in the interval (ρ^*, ∞) and because $\rho^* \leq \rho^p(s) = \rho(s)\Delta(s) < \rho(s)$ for all $s \in [s^*, 1)$.

We now check that bidders will not want to deviate from the equilibrium we described. We first argue that bidders with signals lower than s^p cannot profitably deviate from the equilibrium by choosing a bid that exceeds the pooling bid, i.e., $b^p = u(\rho(s^p))$. If a bidder with signal $s < s^p$ deviates and bids above the pooling bid, then he wins an object with

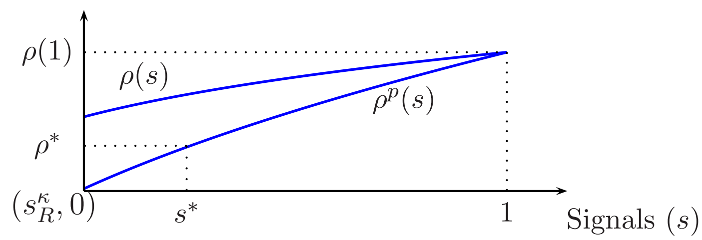

Figure 5: This figure depicts the functions ρ and ρ^p, in the range $[s_R^\kappa, 1]$. Notice that $\rho(s) \geq \rho^p(s)$; $\rho^p(s_R^\kappa) = 0$; and $\rho(1) = \rho^p(1)$.

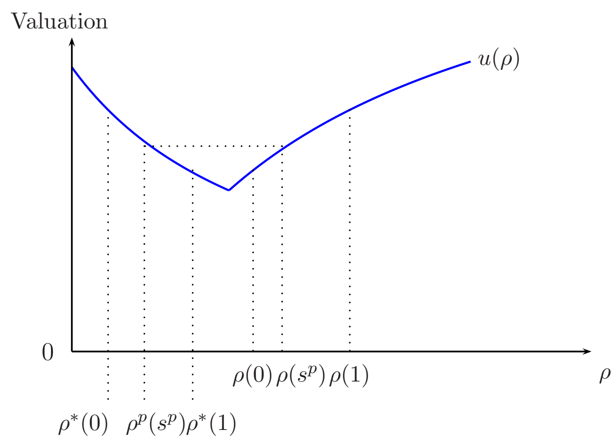

Figure 6: This figure depicts the posterior beliefs of bidders with signals 0, s^p, and 1 under two cases: (i) If they bid the pooling bid and win the object. In this case, their posterior likelihood ratios are $\rho^*(0) = \rho(0)\Delta(s^p)$, $\rho^p(s^p) = \rho(s^p)\Delta(s^p)$, and $\rho^*(1) = \rho(1)\Delta(s^p)$. (ii) if they bid above the pooling bid and win a unit at the pooling price. In this case, the bidders obtain no new information, their posterior and prior likelihood ratios coincide and are equal to $\rho(0)$, $\rho(s^p)$, and $\rho(1)$. A bidder with signal 0 strictly prefers to bid the pooling bid, signal s^p is indifferent between bidding the pooling bid and above it, and signal 1 strictly prefers to bid above the pooling bid.

certainty and pays the pooling bid $b^p = u(\rho(s^p)) > u(\rho(s_R^\kappa))$. In this case, his posterior and prior likelihood ratios coincide and are equal to $\rho(s)$. However, *Assumption 3* implies that $u(\rho(s)) < u(\rho(s^p))$, and therefore we have $u(\rho(s)) < b^p = u(\rho(s^p))$, i.e., the auction price exceeds the expected valuation, conditional on winning, of the bidder with signal s. See figure 6 for a depiction of this argument for the case of $s = 0$.

We now argue that a bidder with signal $s > s^p$ cannot profitably deviate from the equilibrium by choosing the pooling bid. If the bidder sticks to the equilibrium strategy, then he wins an object with certainty and his payoff is equal to $u(\rho(s)) - b^p$, a payoff which is strictly positive. If he deviates instead and chooses the pooling bid, then, conditional on winning, his posterior is equal to $\rho^*(s) := \rho(s)\Delta(s^p)$. To see that this deviation is not profitable, consider two cases. First, suppose that $\rho^*(s) \geq \rho^*$ and recall that $\Delta(s^p) < 1$. However, if $\rho^*(s) \geq \rho^*$, then choosing the pooling bid is not a profitable deviation as $u(\rho^*(s)) - b^p < u(\rho(s)) - b^p$. This is because $\rho^* \leq \rho^*(s) = \rho(s)\Delta(s^p) < \rho(s)$ together with the fact that $u(\cdot)$ is increasing on $[\rho^*, \infty)$ implies that $u(\rho^*(s)) < u(\rho(s))$. Second, suppose that $\rho^*(s) \leq \rho^*$. Note that $\rho^p(s^p) < \rho^*(s)$ because $\rho(s^p) < \rho(s)$ and recall that the pooling bid is equal to $u(\rho^p(s^p))$. However, if $\rho^*(s) \leq \rho^*$, then choosing the pooling bid is again not a profitable deviation as we have $u(\rho^*(s)) - b^p = u(\rho^*(s)) - u(\rho^p(s^p)) < 0$. This is because $\rho^* \geq \rho^*(s) > \rho(s^p)$ together with the fact that $u(\cdot)$ is decreasing on $[0, \rho^*]$ implies that $u(\rho^*(s)) < u(\rho^p(s^p))$. Also, see figure 6.

3.4. Properties of the equilibrium. We have argued that the equilibrium price of the equilibrium that we constructed for Theorem 1 *aggregates no information* even in an arbitrarily large market. There are a number of other novel properties of this equilibrium that we summarize below. Note that none of these properties are present in a standard auction where there are no ex-post actions.

(*i*). A strictly positive fraction of bidders take the *wrong action in equilibrium.* Therefore, inefficiency persists even in a large market in which the outcome would have been efficient if one could use all of the signals observed by the bidders. In particular, *irrespective of the state of the world*, all bidders who win an object at the pooling bid choose action l and all other bidders who win an object choose action r. Therefore, the proportion of bidders choosing the wrong action is equal to $1 - F(s^p|L)$ and $\kappa - (1 - F(s^p|R))$ when the state of the world is L and R, respectively. Both of these numbers are strictly positive because $s^p \in (s_R^\kappa, 1)$.

(*ii*). *The ex-ante expected profit of each bidder is strictly positive in equilibrium.* To see why this is true in the equilibrium that we construct, note that the cutoff bidder who is indifferent between submitting the pooling bid and submitting a higher bid, i.e., the bidder who receives signal s^p, makes zero profit by construction. Moreover, the profit of any bidder who receives signal $s < s^p$ as well as the profit of any bidder who receives signal $s > s^p$ strictly exceeds the cutoff bidder's profit and therefore all bidders except the bidder who receives signal s^p make strictly positive profit. This contrasts with an arbitrarily large auction without ex-post actions as in Pesendorfer and Swinkels (1997) where all bidders make zero profit.

(*iii*). *The value of information, i.e., the value of receiving a signal, is strictly positive for the bidders.* In item (*i*) we argue that a strictly positive fraction of bidders choose the wrong action in equilibrium. Therefore, bidders would be willing to pay a positive sum for a sufficiently informative signal about the state of the world after the auction is completed as such a signal could improve the odds that an agent chooses the correct action. Similarly, bidders would be willing to pay for their initial signals also because the bidders do not expect to fully learn the state from the auction and because almost all bidders make a profit in the auction (item *(ii)*). This stands in contrast to an arbitrarily large auction without ex-post actions which aggregates information (e.g. Pesendorfer and Swinkels (1997)). In such an auction, a bidder would not be willing to pay for a signal as he expects to learn the state perfectly from the auction price and because he expects to make zero profit in the auction.

4. Information Aggregation Failures in Monotone Equilibria

In the previous section, we described equilibria in which no information is aggregated by the price. A prominent property of the equilibrium we described is that equilibrium bids are nondecreasing in the signal that a bidder receives. In this section, in order to demonstrate the robustness of Theorem 1, we characterize all symmetric equilibria in which the bidding function is a *nondecreasing* function of signals (Lemma 1). We then use our characterization to show that information cannot be fully aggregated in equilibria in which the bidding function is nondecreasing (Theorem 2). Moreover, we show that equilibria in which the bidding function is nondecreasing exist under a mildly restrictive condition (Theorem 2). Consequently, our results in this section show that i) the failure of information aggregation is inherent in equilibria in which the bidding function is nondecreasing; and moreover, ii) such equilibria exist for a wide range of parameter values.

Recall that our object of study is a sequence of equilibrium bidding functions $\boldsymbol{b} = \{b_z\}_{z=m}^{\infty}$. We say that a bidding function is nondecreasing (nonincreasing) if $b(s)$ is a nondecreasing (nonincreasing) function of s; and we say that a sequence $\boldsymbol{b}$ is nondecreasing (nonincreasing) if b_z is a nondecreasing (nonincreasing) bidding function for each z. We begin by characterizing nondecreasing equilibrium bidding functions.[14]

Lemma 1 (Characterization) *Suppose that Assumptions 1 and 2 hold. Every equilibrium bidding function b that is nondecreasing satisfies the following conditions:*

(i) There is a cutoff signal $s^p \in [0,1]$ and a pooling bid b^p such that $b(s) = b^p$ for every $s < s^p$, and $b(s) > b^p$ for every $s > s^p$.

(ii) The bidding function $b(s)$ is strictly increasing in the range $(s^p, 1]$.

(iii) Bidders with signals above s^p choose action r and bidders with signals below s^p choose action l when they win an object.

The characterization lemma essentially states that any nondecreasing equilibrium resem-

[14] A straightforward modification of the lemma delivers a characterization of all nonincreasing equilibrium bidding functions as well.

bles the equilibrium that we constructed in the previous section for Theorem 1. More specifically, Lemma 1 shows that in any nondecreasing equilibrium, there is at most one interval, which includes zero, over which the bidding function is constant and equal to the pooling bid; outside of this interval, the bidding function is strictly increasing. Moreover, the bidders who win an object choose l if they have submitted the pooling bid, and choose r if they have submitted a bid above the pooling bid.

As we noted in the previous section, in a large market, if the bidding function is nondecreasing, then there must be pooling. We now provide an intuitive sketch of the remaining arguments for Lemma 1. **Bidders who bid a pooling bid choose action l if they win an object.** On the way to a contradiction, suppose that there is a bidder who bids a pooling bid and chooses action r if he wins an object. Notice that he wins an object only when the auction price is not more than the pooling bid. Moreover, when the price is equal to the pooling bid, there is rationing with strictly positive probability. When the price is equal to the pooling bid, losing a unit is a signal which is favorable for state R because of the *loser's curse*. However, if this bidder deviates from such a strategy by increasing his bid slightly, he ensures that he wins an object whenever the auction price is equal to the pooling price. Such a deviation is profitable, because such a bidder chose action r when he won an object before the deviation (by the hypothesis), and after the deviation, he wins an object in those instances when he had been losing by bidding the pooling bid.

There is only one pooling bid. If the bidders who are bidding a pooling bid choose action l when they win an object at the price equal to the pooling bid, then they also choose action l if the price is lower than the pooling bid, and they make strictly positive profits when the price is lower than their bid. This is because, when the bidding strategy is nondecreasing, lower prices indicate higher likelihood that the state is L. Therefore, the bidder with the lowest signal (i.e., signal zero) would also have chosen action l if he were to bid the highest pooling bid, and won an object. However, the bidder with signal zero then has the highest valuation for the object among all bidders whose bids are less than the highest pooling bid. This implies that the bid chosen by a bidder who receives signal zero must be at least as large as the highest pooling bid. Therefore, the assumption that the bidders use a nondecreasing bidding function implies that there is at most one pooling bid.[15]

Bidders who submit bids above the pooling bid choose action r if they win an object. Suppose s^p is the highest signal for which $b(s^p)$ equals the pooling bid. Assume that a bidder who receives signal $s' > s^p$, where s' is arbitrarily close to s^p, plays l if he wins an object and the auction price is equal to the pooling bid. We now argue that this

[15] This discussion suggests that there may exist equilibria with nonmonotone bidding functions, but we have not managed to characterize such equilibria.

assumption leads to a contradiction. A bidder who receives signal s' prefers submitting a bid that exceeds the pooling bid to submitting the pooling bid because $s' > s^p$. Suppose that the bidder who receives signal s^p deviates and submits a bid that exceeds the pooling bid by an arbitrarily small amount and wins an object at the pooling price. In this event, the posterior of a bidder with signal s^p puts more weight on state l than the posterior of a bidder with signal s'. Therefore, if the bidder who receives signal s' prefers to submit a bid that exceeds the pooling bid, then so does a bidder who receives signal s^p. However, this contradicts the fact that $b(s^p)$ equals the pooling bid.

In the theorem below, we use the characterization given by Lemma 1 to show that monotone equilibria cannot fully aggregate information. Moreover, we establish that a monotone equilibrium sequence exists if Assumption 3 holds, i.e., if $\rho(0) > 0$ and $u(\rho(0)) < u(\rho(s_R^\kappa))$.

THEOREM 2 *Suppose that Assumptions 1 and 2 hold.*

- *(i) If* $\boldsymbol{b}$ *is a nondecreasing equilibrium sequence, then* $\boldsymbol{b}$ *does not fully aggregate information.*
- *(ii) Moreover, if Assumption 3 is satisfied, then a nondecreasing equilibrium sequence* $\boldsymbol{b}$ *exists.*

Item (*ii*) of this theorem establishes existence and item (*i*) of this theorem shows that information is not fully aggregated in a monotone equilibrium. The general idea for item (*i*) is as follows: The price is always equal to the pooling bid in state L. Also, the probability that the price is equal to the pooling bid in state R is bounded strictly away from zero even as the number of bidders grows arbitrarily large. To see why, suppose that the price is equal to the pooling bid with probability close to zero in state R. In such a case, nobody would be willing to choose action r when the price is equal to the pooling bid. But, this contradicts Lemma 1 which shows that all the bidders who submit a bid above the pooling bid play r when they win an object at the price equal to the pooling bid. However, because the price is equal to the pooling bid with strictly positive probability in both states , the *observed price* is the pooling bid with strictly positive probability. Therefore, an outside observer is uncertain about the state when he observes that the auction price is equal to the pooling bid.

REMARK 7 *Lemma 1 and Theorem 2 together imply that a positive fraction of the auction's winners choose the wrong action. Lemma 1 shows that all bidders who bid above the pooling bid choose action r in a nondecreasing equilibrium. Moreover, Theorem 2 implies that the expected fraction of bidders that bid above the pooling bid is always bounded below by a positive number. Thus, a positive fraction of winners choose the wrong action in state L even in an* *arbitrarily large market.*

5. Discussion

5.1. Pooling bid, loser's curse and nonmonotonicity of the value function A key feature of our equilibrium construction that makes price uninformative about the state of the world is the existence of a pooling bid. In other words, in the equilibrium that we construct, there is an atom in the equilibrium bid distribution at b^p. In sharp contrast, the existence of such a pooling bid is not possible in the symmetric equilibria of the auction models of Pesendorfer and Swinkels (1997) or Milgrom and Weber (1982), where there is no ex-post action. The existence of a pooling bid in our model and the impossibility of pooling in auctions without ex-post actions are both consequences of the ***loser's curse***, i.e., the fact that the probability of winning an object at the pooling bid in state L is strictly higher than the probability of winning an object at the pooling bid in state R. Equivalently, a bidder is more convinced that the state is R when he does not win an object than when he does, provided that the price is the pooling bid and he bid the pooling bid.

Intuitively, not winning an object at the pooling bid, when the auction price is equal to the pooling bid, is a strong signal in favor of state R. Therefore, whenever the auction price is equal to the pooling price, a bidder would rather increase his bid slightly and ensure that he wins an object in Pesendorfer and Swinkels (1997)'s model, because any news in favor of state R is good news.

In our framework, however, the value function is nonmonotonic in the belief of the bidder that the state is R which is a consequence of Assumption 2. In particular, the bidders who bid the pooling bid take action l when they win a unit. Hence, for such a bidder a signal in favor of state R is bad news, unless this signal is overwhelmingly strong. Therefore, the loser's curse argument does not preclude pooling. On the contrary, deviation from the pooling bid to a slightly higher bid makes such bidders win a unit more frequently, albeit with a different belief about the state, which makes them worse off.

To see why information could be aggregated if Assumption 2 is not satisfied and if the value function is monotonic, suppose that $v(r, R) > v(l, L) = v(r, L) = 0 \geq v(l, R)$, that is, action l is weakly dominated by action r. In this case, v does not satisfy inequality (1) and our model coincides with Pesendorfer and Swinkels (1997)'s model with two states of the world $\Omega = \{L, R\}$ where the value of the object is equal to zero in state L and equal to $v(r, R)$ in state R. In the unique symmetric equilibrium of the auction with z bidders and k objects, the bidding function $b_z(s) = v(r, R) \Pr(\omega = R | s_1 = s, Y^k_{z-1} = s)$ for every $s \in (0, 1)$. This function is strictly increasing in s because the signal distribution satisfies MLRP. Notice that as z gets larger, bidders who receive signals s^κ_R and s^κ_L determine the equilibrium prices

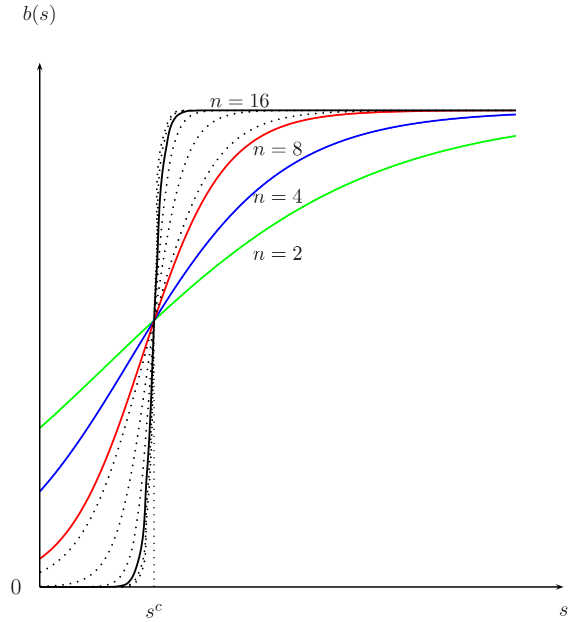

Figure 7: This figure depicts the equilibrium bidding functions in Pesendorfer and Swinkels (1997). These functions converge to a step function that has a jump at a signal s^c that satisfies $s^\kappa_L < s^c < s^\kappa_R$.

in states R and L respectively. A key observation that delivers information aggregation is that $\lim_{z\to\infty} b_z(s^\kappa_R) = v(r, R)$ and $\lim_{z\to\infty} b_z(s^\kappa_L) = 0$, i.e., prices converge to $v(r, R)$ and 0 in states R and L respectively. Intuitively, the bids of s^κ_R and s^κ_L are distinct from each other, and hence prices reveal which of these types won the last unit, and hence set the price (see also Figure 7).

In contrast, in the equilibrium that we construct in Theorem 1 there is a pooling bid that is chosen by bidders who receive signals in $[0, s^p]$. Moreover, $s^p > s^\kappa_R$, and therefore the price is equal to the pooling bid in both states and hence is uninformative. The key is that the bidder with signal s^κ_R and s^κ_L bid the same price, and hence price cannot distinguish the identity of the signal that determined the price.

For completeness, we note that if inequality (1) is satisfied but if inequality (2) is not (e.g., if $v(r, R) > v(l, R) > v(l, L) > v(r, L)$), then the value function is again strictly increasing in ρ and the auction price aggregates information. Although this configuration does not directly fit into Pesendorfer and Swinkels (1997)'s framework, a slight modification of their arguments would show that the symmetric equilibrium bidding function is given by $b_z(s) = u(\rho(s_1 = s, Y^k_{z-1} = s))$ for every $s \in (0, 1)$. It is then straightforward to show that the equilibrium price converges to $v(r, R)$ and $v(l, L)$ in states R and L, respectively.

5.2. Discussion of the assumptions and the limitations of the results. There are three important limitations for the results that we presented in this paper. First, we limited our attention to a model with only two states. Second, we focused only on monotone equilibria and our results do not rule out the possibility that information is aggregated in other nonmonotone equilibria of the auction which we have not characterized. We focused on monotone equilibria because, when taken together with MLRP, monotonicity allows us to use the standard auction theory results related to order statistics. Third, we constructed a monotone equilibrium for the large auction only under a restrictive assumption (Assumption 3).

We focused on two states because this allows us to conveniently summarize a posterior by just one number. In a model with more states, beliefs need to be specified using a measure which is a multidimensional vector. This makes it more difficult to compare beliefs obtained by updating after various events. For example, comparing the value given the posteriors after winning at the pooling bid with the value given the posterior if one deviates and wins with certainty is not straightforward. In a model without actions but with many states (e.g., Pesendorfer and Swinkels (1997)), the posterior beliefs are important only up to the expectation of a bidder's value given the posterior. This is because the agent only cares about the posterior expectation of the object's value. In such a case, MLRP characterizes how the expectation behaves in a tractable manner. In our case with actions, the whole posterior distribution could be potentially relevant because the agent needs to choose an action and the value from the action can depend on the state in a nonlinear fashion.

We constructed a monotone equilibrium for the large auction under Assumption 3 and a monotone equilibrium may fail to exist in our model if Assumption 3 is not satisfied.[16] However, equilibrium nonexistence is a known problem for auctions where the bidders' private information cannot be ordered in a natural way (see Jackson (2009)). In fact, a stronger version of Assumption 3, which requires that all buyers would choose action r if they acted solely on their private signal, is one way to ensure that a bidder's pre-auction value is increasing in his signal.

Too see why we require an appropriate assumption to construct a nondecreasing equilibrium consider the following example: Suppose that signal zero perfectly reveals that the state is L, i.e., $u(\rho(0)) = u(0) = v(l, L)$. This implies that the pooling bid b_z^p must be equal to $v(l, L)$ in a sufficiently large market. This is because the agent with signal 0, who submits the pooling bid, would otherwise increase his bid slightly to avoid rationing. However, $b_z^p = v(l, L)$ cannot be the equilibrium pooling bid because types with any positive signal that is close to zero in any finite market can not be entirely sure that the state is L even

[16] Assumption 3 can possibly be relaxed further. In the equilibrium that we construct, we require that the bidder with the cut-off signal s^p earns zero profit. It is also possible to construct monotone equilibria without this zero-profit requirement and the condition needed for the existence of an equilibrium without the zero-profit requirement would be weaker than Assumption 3.

conditional on winning the object. Hence, their value for the object is strictly less than b_z^p, which contradicts that they submit the pooling bid. Thus, in order to sustain a monotone equilibrium, we need to put a restriction on the magnitude of information conveyed by signal 0 (such as Assumption 3).

6. Conclusion

In this paper, we explored the role of market prices in aggregating information about the correct use of objects. In our set-up, multiple homogeneous goods are allocated among multiple bidders via a Vickrey-type auction. Our main finding is that, when prices contain information about the ex-post actions that the owners of the object will take, then prices may not reveal all the information available in the market. In the extreme case, prices reveal no information about the state of the world, and a non-negligible fraction of the objects are thus used incorrectly. We interpret our results as suggesting that it is too much to expect prices alone to reveal the state of the world perfectly. Also, our results highlight that markets have several statistics other than price, such as the amount of rationing, volume of trade, and bid distributions, that are relevant for aggregating information.

A. Organization of the Appendix

We start by proving Theorem 2, item (*ii*) instead of Theorem 1, because the construction we use for the former is used for the latter theorem. Later we prove Theorem 1, Lemma 1 and then we prove Theorem 2 item (*i*), using Lemma 1. We present the proofs of some technical lemmata that we use in the proofs of our theorems and Lemma 1 in the online Appendix.

B. Proof of Theorem 2, item (*ii*)

B.1. Method used for the construction The construction has two main steps. In the first step, we show that in a large market with size z, there exists a cutoff signal, s_z^p, such that in a monotonic bidding profile b_z where all types below s_z^p bid a pooling bid, the following two properties are satisfied: *i*) The value of the object to bidders with signals $s < s_z^p$, who win a unit by bidding the pooling bid, is not less than the value of the object to such bidders if they were to win a unit by bidding above the pooling bid, and when the price is equal to the pooling bid, *ii*) the value of the object to bidders with signals $s > s_z^p$ when they bid above the pooling bid and the price is equal to the pooling bid is not less than if such bidders were to bid the pooling bid and win a unit. In this step, we also determine the value of the pooling bid.

The second step shows that under Assumption 3, when z is sufficiently large, no bidder has a profitable deviation from the bidding profiles constructed in step 1 and thus the bidding profile constitutes an equilibrium of the auction game.

B.2. Step 1: Cutoff type For any $s \in (0,1)$, $s' \in S$ and $z \in \mathbb{Z}$ define the following:

$$\rho_z^-(s', s) := \frac{\Pr\left(Y_{z-1}^{\kappa z} \le s, s_1 = s', 1 \text{ wins the lottery}|R\right)}{\Pr\left(Y_{z-1}^{\kappa z} \le s, s_1 = s', 1 \text{ wins the lottery}|L\right)},$$
$$\rho_z^+(s', s) := \frac{\Pr\left(Y_{z-1}^{\kappa z} \le s, s_1 = s'|R\right)}{\Pr\left(Y_{z-1}^{\kappa z} \le s, s_1 = s'|L\right)}.$$

The event that "1 wins the lottery" corresponds to the event that bidder 1 wins a prize (or equivalently one unit of the object) in the following auxiliary lottery whose odds depend on the signal distribution across the bidders. The lottery has q prizes allocated equally likely to o people, where the number of prizes $q = max\{0, \kappa z - |j \in \{2, ..., z\} : s_j > s|\}$ and the number of people is $o = 1 + |j \in \{2, ..., z\} : s_j \le s|$.[17]

Intuitively, $\rho_z^-(s', s)$ is the posterior likelihood ratio for type s', when he bids the pooling bid and wins a unit, where the bidders who bid the pooling bid are those with signals less than s. The second function, $\rho_z^+(s', s)$ is the posterior likelihood ratio for type s', when he bids above the pooling bid and wins a unit at a price equal to the pooling bid, where the bidders who bid the pooling bid are those with signals less than s.

REMARK 8 *Observe that both $\rho_z^-(s, s')$ and $\rho_z^+(s, s')$ are continuous in both arguments. This is because the cdf $F(s|\omega)$ admits a continuous density function $f(s|\omega)$ for each ω.*

Abusing notation,[18] we make the following definitions:

$$\rho_z^-(s) := \rho_z^-(s, s) \text{ and } \rho_z^+(s) := \rho_z^+(s, s).$$

Intuitively, $\rho_z^-(s)$ is the posterior likelihood ratio of a cutoff type s when he bids a pooling bid that only types in the range $[0, s]$ bid, and when he wins an object with such a bid.

REMARK 9 *From Pesendorfer and Swinkels (1997, Lemma 7, page 1272) and $f(0|L) \neq f(0|R)$ we know that i) $\rho_z^-(s) < \rho_z^+(s)$ for any $s \in (0,1)$, and ii) $\rho_z^-(s', s) < \rho_z^+(s', s)$ for any $s, s' \in (0,1)$. This is called the loser's curse. Moreover, as we later show in Lemma O.2 in the online appendix, $\rho_z^-(s, s')$ and $\rho_z^+(s, s')$ are both strictly increasing in s and s' because of MLRP.*

Since $\rho_z^-(s)$ and $\rho_z^+(s)$ are both increasing functions, $u(\rho_z^-(s))$ and $u(\rho_z^+(s))$ are both at most single-troughed functions. Now we make two observations about values of the functions

[17] The index sets exclude the number 1 since it is reserved for the bidder who is doing these calculations for his best response.

[18] We use the same notation for both functions although they take different number of arguments.

$\rho_z^-(s)$ and $\rho_z^+(s)$ when s is close to zero and when s is close to one.

REMARK 10 *Note that the inequality* $u(\rho(0)) < u(\rho(s_R^\kappa))$*, which we assume in Assumption 3, implies that* $\rho(s_R^\kappa) > \rho^*$*. We will use the fact that* $\rho(s_R^\kappa) > \rho^*$ *in many of our arguments.*

LEMMA 2

1. *$\exists\varepsilon > 0$ and a Z_1 such that $\rho_z^-(s) < \rho^*$, $\rho_z^+(s) < \rho^*$, and $u(\rho_z^-(s)) > u(\rho_z^+(s))$ for every $s \leq \varepsilon$ and every $z > Z_1$.*
2. *$\exists\varepsilon > 0$ and a Z_2 such that $\rho_z^-(s) > \rho^*$, $\rho_z^+(s) > \rho^*$, and $u(\rho_z^-(s)) < u(\rho_z^+(s))$ for every $s \geq 1-\varepsilon$ and every $z > Z_2$.*

PROOF:

1. $\exists\varepsilon > 0$ such that $\lim_{z\to\infty} \rho_z^+(\varepsilon) = 0$, because of MLRP. Since $\rho_z^-(s) < \rho_z^+(s)$ for $s \in (0,1)$, $\lim_{z\to\infty} \rho_z^-(\varepsilon) = 0$, and since $u(\rho)$ is strictly decreasing in the range $[0,\rho^*]$, we have that $u(\rho_z^-(s)) > u(\rho_z^+(s))$ for $s \leq \varepsilon$ when $z > Z_1$ for some integer Z_1.

2. For any $s > s_R^\kappa$, $\lim_{z\to\infty} \rho_z^-(1-\varepsilon) = \rho_0 \frac{\kappa-(1-F(s|R))}{\kappa-(1-F(s|L))}$ as we show in Lemma O.1 in the online appendix. This, together with Assumption 3 imply that $\exists\varepsilon > 0$ such that $\lim_{z\to\infty} \rho_z^-(1-\varepsilon) > \rho^*$. Since $\rho_z^-(s) < \rho_z^+(s)$, and $\rho_z^-(s)$ and $\rho_z^+(s)$ are strictly increasing functions, we have that $\rho_z^-(s) > \rho^*$ and $\rho_z^+(s) > \rho^*$ for every $s \geq 1-\varepsilon$ when z is sufficiently large. Since $u(\rho)$ is strictly increasing in the range $[\rho^*, 1]$, we have that $u(\rho_z^-(s)) < u(\rho_z^+(s))$ for $s \geq 1-\varepsilon$. □

LEMMA 3

1. *For every $z > \max\{Z_1, Z_2\}$, there is a unique $s_z^p \in (\varepsilon, 1-\varepsilon)$ that satisfies the equality $u(\rho_z^-(s)) = u(\rho_z^+(s))$.*
2. *When such an s_z^p exists, $\rho_z^+(s_z^p) > \rho^*$, and $\rho_z^-(s_z^p) < \rho^*$.*

PROOF: 1. Let s_z^1 and s_z^2 be the unique signals that solve the equalities $\rho_z^-(s) = \rho^*$ and $\rho_z^+(s) = \rho^*$, respectively. Note that $s_z^1, s_z^2 \in (0,1)$ and are well-defined when $z > \max\{Z_1, Z_2\}$ from Lemma 2, and because ρ_z^- and ρ_z^+ are both continuous and strictly increasing. Moreover, because $\rho_z^-(s) < \rho_z^+(s)$, $s_z^1 > s_z^2$. In the range $[0, s_z^2]$, $u(\rho_z^-(s)) > u(\rho_z^+(s))$, and in the range $[s_z^1, 1)$, $u(\rho_z^+(s)) > u(\rho_z^-(s))$.

In the range $[s_z^2, s_z^1]$, $u(\rho_z^-(s))$ is strictly decreasing and $u(\rho_z^+(s))$ is strictly increasing. Therefore, $u(\rho_z^-(s)) - u(\rho_z^+(s))$ is strictly negative in $[0, s_z^2]$, strictly increasing in $[s_z^2, s_z^1]$, and strictly positive in $[s_z^1, 1)$. Therefore, by the intermediate value theorem, there is a unique signal s_z^p, in the range (s_z^2, s_z^1) that satisfies the equality $u(\rho_z^-(s)) = u(\rho_z^+(s))$.

2. As we argued above, such a signal is in the range (s_z^2, s_z^1), and hence $\rho_z^+(s_z^p) > \rho^*$ and $\rho_z^-(s_z^p) < \rho^*$ from the definition of s_z^1, s_z^2, and from the monotonicity of ρ_z^- and ρ_z^+. □

B.3. Setting the pooling bid and its properties We now determine the bidding function, b_z, for $z > \max\{Z_1, Z_2\}$. The bidding function $b_z(s)$ is constant and equal to $b_z^p := u(\rho_z^-(s_z^p)) = u(\rho_z^+(s_z^p))$ for bidders with signals in the interval $[0, s_z^p]$ and is strictly increasing and equal to $u(\rho(s_1 = s, Y_{z-1}^{\kappa z} = s))$ in the region $(s_z^p, 1]$. Notice that, the part of the bidding function that is strictly increasing coincides with the Pesendorfer and Swinkels (1997) equilibrium bidding function, in the case where bidders are taking action r. Moreover, $u(\rho(s_1 = s, Y_{z-1}^{\kappa z} = s)) > u(\rho(s_1 = s_z^p, Y_{z-1}^{\kappa z} \leq s_z^p)) = u(\rho_z^+(s_z^p)) = b_z^p$ for every $s \geq s_z^p$. This follows from MLRP together with $\rho_z^+(s_z^p) > \rho^*$ (Lemma 3). The following corollary follows from Lemma 3, and we use it frequently while checking that bidders have no profitable deviations.

Corollary 1 *The posterior likelihood ratio of types lower than s_z^p, conditional on winning at price b_z^p is less than ρ^*, and types higher than s_z^p, conditional on the price being b_z^p has a posterior likelihood ratio that is more than ρ^*. In particular,*

$$\rho(s_1 = s, Y_{z-1}^{\kappa z} \leq s_z^p, 1 \text{ wins with } b_z^p) < \rho^* \text{ for } s \leq s_z^p \text{ and } \rho(s_1 = s, Y_{z-1}^{\kappa z} \leq s_z^p) > \rho^* \text{ for } s \geq s_z^p.$$

B.4. Step 2: Checking deviations In this step, we will show that the bidding function we constructed in step 1 is an equilibrium when z is large (i.e., when $z > Z'$ for some integer Z') by showing that no type has a profitable deviation from the proposed bidding strategy profile. In the following we assume that z is large enough that s_z^p exists, i.e., $z > \max\{Z_1, Z_2\}$.

B.4.1. *Bidders with signals above s_z^p.* Pick a type $s > s_z^p$. We will first show that for any type $s' \in (s_z^p, s)$ the following inequality holds:

$$(4) \qquad u(\rho(s_1 = s, Y_{z-1}^{\kappa z} = s')) > b_z(s') = u(\rho(s_1 = s', Y_{z-1}^{\kappa z} = s'))$$

This inequality follows because $\rho(s_1 = s, Y_{z-1}^{\kappa z} = s') > \rho(s_1 = s', Y_{z-1}^{\kappa z} = s')$ by MLRP and because $\rho(s_1 = s', Y_{z-1}^{\kappa z} = s') > \rho(s_1 = s', Y_{z-1}^{\kappa z} \leq s') = \rho_z^+(s') \geq \rho^*$, again by MLRP and Lemma 3. Therefore, type s has no profitable deviation to bid in the interval $(b_z^p, b(s))$. A similar calculation shows that such a type has no profitable deviation to bid above $b(s)$. Next, we will argue that such a type does not find it profitable to bid b_z^p. To show this, we will prove two inequalities:

$$(5) \qquad u(\rho_z^+(s, s_z^p)) > b_z^p$$

$$(6) \qquad u(\rho_z^+(s, s_z^p)) \geq u(\rho_z^-(s, s_z^p)).$$

To see that these inequalities suffice to prove that bidding b_z^p is not a profitable deviation, notice that, first, such a type makes a strictly positive profit when the price is above b_z^p and below $b_z(s)$. The first inequality above says that, when bidding above b_z^p, type s has a positive payoff when the price is equal to b_z^p. The second inequality says that, the payoff to a bidder when he bids above b_z^p and the price is b_z^p is not less than when he bids b_z^p and wins a unit. Moreover, the probability of winning a unit by bidding above b_z^p is strictly larger than winning by bidding b_z^p. Hence, bidding b_z^p cannot not be a profitable deviation. Below we prove these two inequalities:

The first inequality follows because $\rho_z^+(s, s_z^p) > \rho_z^+(s_z^p) \geq \rho^*$ by MLRP and Lemma 3.

Now we prove that the second inequality holds. There are two cases to consider. Either $\rho_z^-(s, s_z^p) \geq \rho^*$ or $\rho_z^-(s, s_z^p) < \rho^*$. In the former case, $\rho_z^+(s, s_z^p) \geq \rho_z^-(s, s_z^p)$ together with the facts that both are at least ρ^* and u is increasing when $\rho \geq \rho^*$ deliver the desired inequality. In the latter case, $\rho_z^-(s, s_z^p) > \rho_z^-(s_z^p)$. Since both are less than ρ^*, and since $u(\rho)$ is decreasing in that range, $u(\rho_z^-(s, s_z^p)) < u(\rho_z^-(s_z^p)) = b_z^p < u(\rho_z^+(s, s_z^p))$.

B.4.2. *Bidders with signals below s_z^p.* In this part of the proof, we will take z sufficiently large and use Assumption 3. The next lemma shows that bidders with signals below s_z^p have a nonpositive payoff if they bid above b_z^p and if the price is equal to b_z^p.

LEMMA 4 *$\exists Z_3 \in \mathbb{Z}$ such that $u(\rho_z^+(s, s_z^p)) \leq b_z^p$ for every $s < s_z^p$, every $z > Z_3$.*

The proof of Lemma 4 is given further below. Now we argue that bidders with signals less than s_z^p do not have profitable deviations to bid strictly above b_z^p if the hypothesis of Lemma 4 is satisfied. As shown in Lemma 4, such types lose money if they bid above b_z^p and if the price is equal to b_z^p. If a type $s < s_z^p$ bids strictly above $b(s')$ for some $s' > s_z^p$, we will now argue that he loses money when the price is equal to $b(s')$. If $\rho(s_1 = s, Y_{z-1}^{\kappa z} = s') \geq \rho^*$, then $u(\rho(s_1 = s, Y_{z-1}^{\kappa z} = s')) < b(s') = \rho(s_1 = s', Y_{z-1}^{\kappa z} = s')$. If $\rho(s_1 = s, Y_{z-1}^{\kappa z} = s') < \rho^*$, then $b_z(s') > b_z^p \geq u(\rho_z^+(s, s_z^p)) = u(\rho(s_1 = s, Y_{z-1}^{\kappa z} \leq s_z^p)) > u(\rho(s_1 = s, Y_{z-1}^{\kappa z} = s'))$, where the second inequality follows from Lemma 4, and the third inequality follows from MLRP.

PROOF OF LEMMA 4: If $\rho_z^+(s, s_z^p) \geq \rho^*$, then $u(\rho_z^+(s_z^p)) = b_z^p > u(\rho_z^+(s, s_z^p))$ because of MLRP and because $u(\rho)$ is increasing when $\rho \geq \rho^*$. The rest of the proof shows either directly that $u(\rho_z^+(s, s_z^p)) \leq b_z^p$ or indirectly by showing that $\rho_z^+(s, s_z^p) \geq \rho^*$ for every $s < s_z^p$ when z is sufficiently large. In the latter case proving that $\rho_z^+(0, s_z^p) \geq \rho^*$ suffices because, $\rho_z^+(0, s_z^p) \leq \rho_z^+(s, s_z^p)$.

At this point, we would like to remind the reader that s_z^p is the unique solution to the equality $u(\rho_z^-(s_z^p)) = u(\rho_z^+(s_z^p))$, and $b_z^p = u(\rho_z^-(s_z^p))$. Also, Recall that s_R^κ is the signal such that $F(s_R^\kappa|R) = 1 - \kappa$. We'll make our argument in the following steps under the assumption that the limit of the sequence $\{s_z^p, b_z^p\}_{z=1}^\infty = (s^p, b^p)$ exists and then we will verify this in Lemma O.1 in the online appendix. The next three claims are the steps of the proof of Lemma 4.

Claim 1 $s^p \geq s^\kappa_R$.

Proof: On the way to a contradiction, suppose that $s^p < s^\kappa_R$. Then, $\lim_{z\to\infty} \rho^+_z(s^p_z) = 0$, because $\lim_z P\left(Y^{\kappa z}_{z-1} \leq s^p_z | L\right) = 1$ and $\lim_z P\left(Y^{\kappa z}_{z-1} \leq s^p_z | R\right) = 0$ if $s^p < s^\kappa_R$. This contradicts the assertion in Corollary 1 that $\rho(s_1 = s, Y^{\kappa z}_{z-1} \leq s^p_z) > \rho^*$ for $s > s^p_z$. □

Claim 2 *If $s^p > s^\kappa_R$, then $\exists Z_4 \in \mathbb{Z}$ such that $u(\rho^+_z(s, s^p_z)) < b^p_z$, for every $z > Z_4$.*

Proof: Since $F(s^p|\omega) > 1 - \kappa$ for $\omega \in \Omega$, $\lim_{z\to\infty} Pr(Y^{\kappa z}_{z-1} \leq s^p_z) = 1$ for $\omega \in \Omega$. Hence, $\lim_{z\to\infty} \rho(s_1 = 0, Y^{\kappa z}_{z-1} \leq s^p_z) = \rho(s_1 = 0)$. Moreover, the same argument delivers that $\lim_{z\to\infty} \rho^+_z(s^p_z, s^p_z) = u(\rho(s^p))$. By Assumption 3, $u(\rho(0)) < u(\rho(s^\kappa_R))$, and if $s^p > s^\kappa_R$, then $\rho(s^p) > \rho^*$ and hence $u(\rho(0)) < u(\rho(s^p)) = b^p$. It follows then that $\exists Z_4 \in \mathbb{Z}$ such that $u(\rho^+_z(s, s^p_z)) < b^p_z$, for every $s < s^p_z$, for every $z > Z_4$. □

Claim 3 *If $s^p = s^\kappa_R$, then $\exists Z_5 \in \mathbb{Z}$ such that $u(\rho^+_z(s, s^p_z)) < b^p_z$, for every $z > Z_5$.*

Proof: This is the case when prices may indeed reveal some information. We'll start by arguing that the pooling bids, b^p_z, converge to $u(0)$.

The crucial observation in this case is that $\lim_{z\to\infty} \Pr(\omega = L | p_z = b^p_z, 1 \text{ wins with } b^p_z) = 1$. Intuitively, this is because in state R there are few goods left to be delivered among the bidders who bid the pooling bid whereas in state L there is a significant number of goods left to be delivered among such bidders. Formally, fix an $\epsilon > 0$. Then, $\lim_{z\to\infty} \Pr(Y^{(\kappa-\epsilon)z}_{z-1} > s^p_z | \omega = R) = 0$. Therefore, $\lim_{z\to\infty} \Pr(p_z = b^p_z, 1 \text{ wins with } b^p_z | \omega = R) \leq \frac{\epsilon}{1-\kappa+\epsilon}$. Since this is true for every $\epsilon > 0$, it has to be that

$$(7) \qquad \lim_{z\to\infty} \Pr(p_z = b^p_z, 1 \text{ wins with } b^p_z | \omega = R) = 0.$$

However, $\lim_{z\to\infty} \Pr(p_z = b^p_z | \omega = L) = 1$ because $1 - F(s^\kappa_R | \omega = L) < \kappa$. Moreover, there is an $\epsilon > 0$ such that $\Pr(|\{\text{signals above } s^\kappa_R\}| \leq (\kappa - \epsilon)z | \omega = L) = 1$. Therefore,

$$(8) \qquad \lim_{z\to\infty} \Pr(p_z = b^p_z, 1 \text{ wins with } b^p_z | \omega = L) > 0.$$

Combining equation (7) and inequality (8) delivers that

$$(9) \qquad \lim_{z\to\infty} \Pr(\omega = L | p_z = b^p_z, 1 \text{ wins with } b^p_z) = 1.$$

Since by the assumption of the claim $s^p_z \to s^\kappa_R$, and since $\rho(s^\kappa_R) < \infty$ by strict MLRP, we have $\lim_{z\to\infty} b^p_z = \lim_{z\to\infty} u\left(\rho^-_z\left(s^p_z, s^p_z\right)\right) = u(0)$, which follows from equality (9).

Note that there is an $\epsilon \in (0, \rho^*)$ such that $\lim_{z\to\infty} \rho_z^+(0, s_z^p) > \epsilon$ and $u(\rho^* + \epsilon) < u(\epsilon)$. This inequality follows because, $\rho_z^+(s, s_z^p) \geq \rho^*$ for every $s > s_z^p$ (by Lemma 3 and because of the part of Assumption 3 that $\rho(0) > 0$).

The sequence $\rho_z^+(0, s_z^p)$ has a limit. If this limit is strictly less than $\rho^* + \epsilon$ then we have the following: $\lim_{z\to\infty} u(\rho_z^+(0, s_z^p)) < u(\epsilon) < u(0)$. Since we have shown that $\lim_z b_z^p = u(0)$, $\lim_{z\to\infty}(u(\rho_z^+(0, s_z^p)) - b_z^p) < 0$. If the limit of the sequence $\rho_z^+(0, s_z^p)$ is greater than ρ^*, then $\lim_{z\to\infty}(u(\rho_z^+(0, s_z^p)) - b_z^p) < 0$ by strict MLRP and because $\lim_{z\to\infty} \rho_z^+(0, s_z^p)) < \lim_{z\to\infty} \rho_z^+(s_z^p, s_z^p))$. Hence, there is an integer Z_5 such that $u(\rho_z^+(0, s_z^p)) - b_z^p < 0$ for every $z > Z_5$. To complete the proof of the claim, suppose that $z > Z_5$, and pick any $s < s_z^p$. If $\rho_z^+(s, s_z^p) \geq \rho^*$, then $u(\rho_z^+(s, s_z^p)) - b_z^p < 0$. If $\rho_z^+(s, s_z^p) < \rho^*$, then $u(\rho_z^+(s, s_z^p)) < u(\rho_z^+(0, s_z^p))$ by MLRP, and therefore $u(\rho_z^+(s, s_z^p)) - b_z^p < 0$. □

Let $Z_3 := \max\{Z_4, Z_5\}$, then Claims 1, 2, and 3 together establish Lemma 4. □

C. Proof of Theorem 1

Proof: We will prove this theorem by using the same construction that we used to prove Theorem 2. Since Assumption 3 is assumed, the hypothesis of Theorem 2 is satisfied, and hence the constructed bidding strategies constitute an equilibrium when z is sufficiently large.

We will now show that if Assumption 4 is satisfied, i.e, if $u(0) > u(\rho(s_R^\kappa))$, then the limit of the cutoff types as z goes to infinity, which we denote by s^p is strictly larger than s_R^κ. The implication of this inequality is that equilibrium prices become the pooling bid in both states of the world with probabilities approaching one, and hence prices reveal no information as the market gets arbitrarily large.

In Lemma 4, Claim 1, we showed that any limit point of the cutoffs is at least s_R^κ. Thus it remains to show that s_R^κ is not a limit point of the cutoff types constructed in the sequence of bidding functions. On the way to a contradiction, suppose our claim is not true, i.e., Assumption 4 holds ($u(0) > u(\rho(s_R^\kappa))$) and s_R^κ is the limit point of the cutoff types. Then, as we argued in Lemma 4, Claim 3, the pooling bid converges to $u(0)$, i.e., $\lim_{z\to\infty} b_z^p = u(0)$. Also, we have $\rho_z^+(s_z^p) \geq \rho^*$ from Lemma 3 and we know that $\rho_z^+(s_z^p) < \rho(s_z^p)$.[19] Hence, $b_z^p = u(\rho_z^+(s_z^p)) < u(\rho(s_z^p))$. However, $u(0) = \lim_{z\to\infty} b_z^p \leq \lim_{z\to\infty} u(\rho(s_z^p)) = u(\rho(s_R^\kappa))$ which contradicts to Assumption 4 that $u(0) > u(\rho(s_R^\kappa))$. □

[19] To see why $\rho_z^+(s_z^p) < \rho(s_z^p)$ note that the probability that the price is equal to the pooling bid in state L is at least as great as the probability of the same event in state R. This is because, $\rho(s_1 = s, Y_{z-1}^{\kappa z} \leq a) < \rho(s_1 = s, Y_{z-1}^{\kappa z} \leq 1)$ for any $a < 1$, due to MLRP.

D. PROOF OF LEMMA 1

PROOF: There are two cases to consider, either b is strictly increasing or there is an atom in the bid distribution.

Case 1: If b is strictly increasing, then the first part of the lemma is true by picking $s^p = 0$. The second part of the lemma for this case claims that b is a la Pesendorfer and Swinkels (1997). This follows from a slight modification of the argument in Pesendorfer and Swinkels (1997, Second part of the proof of Proposition 1, page 1272).

Case 2: Suppose that the bidding function has an atom at some bid b^p. Then the monotonicity of the bidding function implies that $b(s) = b^p$ for an interval of signals, $S(b^p) = (s', s^p)$ with $s' < s^p$ and $b(s) = b^p$ for every $s \in S(b^p)$ and $b(s) > b^p$ for every $s > s^p$. Below we will show that there can be at most one atom in the bid distribution and that $s' = 0$.

Step 1: The first step is to show that $\rho(s_1 = s^p, p = b^p, 1 \text{ wins with } b^p) < \rho^*$ when the bidding function b is increasing. On the way to a contradiction, suppose that it's not true. Then due to winner's and loser's curse (see Pesendorfer and Swinkels (1997, page 1272)), types in $S(b^p)$ would deviate and bid slightly above b^p.[20]

Step 2: We show that $\rho(s_1 = s, p, 1 \text{ wins with } b(s)) < \rho^*$ for every $s < s^p$ and $p \le b(s)$.

We first claim that the following is true for every $p' < b^p$ which is in the range of b:

$$\rho(s_1 = s, p') < \rho(s_1 = s, p = b^p, 1 \text{ wins with } b^p).$$

This is a non-trivial claim and the proof is in Lemma O.3 in the online appendix. Moreover, $\rho(s_1 = s, p, 1 \text{ wins with } p) \le \rho(s_1 = s, p)$. This inequality follows from Pesendorfer and Swinkels (1997, Lemma 7, page 1272). Combining the two inequalities in this step with the result in step 1 delivers the claim.

Step 3: We will now argue that all types below s^p bid b^p, i.e., there is at most one atom.

On the way to a contradiction, assume that a positive measure of types bid strictly below b^p and let $s'' < s'$ be such a type. By Lemma O.3 in the online appendix, the probability that type s'' puts on state L were he to bid b^p and the price is any price between his bid and b^p is weakly higher than that of types who are bidding b^p. Formally, for any $p' \le b^p$ that is

[20] Obviously it could be the case that every neighborhood of b^p higher than b^p may be bid by a positive measure of types. If this is the case, the measure of types whose bids are in $(b^p, b^p + \epsilon)$ can be made arbitrarily small by picking ϵ arbitrarily small. Hence, a deviation to a bid less than $b^p + \epsilon$ which does not have an atom can cause a payoff loss when the price is in $(b^p, b^p + \epsilon)$. However this potential loss becomes arbitrarily small as ϵ disappears. On the other side, the payoff gain of this deviation when the price is equal to b^p is strictly positive, due to the loser's curse. Moreover, since the bid distribution has an atom at b^p, the probability that the price is equal to b^p is strictly positive.

in the range of b, the following holds:

$$\Pr(\omega = L | s_1 = s'', p', 1 \text{ wins by bidding } b^p) \geq \Pr(\omega = L | s_1 = s', p', 1 \text{ wins by bidding } b^p).$$

Since bidding slightly below b^p is a feasible strategy, we have that,

$$u(\rho(s_1 = s', b^p, 1 \text{ wins by bidding } b^p)) \geq b^p.$$

Therefore, b^p is weakly less than the value of the object to types who bid b^p conditional on the price being b^p and they winning the object. Since this value is strictly less than the value when the price is strictly lower than b^p, these types make strictly positive profits when the price is strictly less than b^p.

We will now argue that the bid of s'' cannot be an atom. Intuitively, this is because, the value of the object conditional on losing when the price is his bid is strictly larger than the value if the price was strictly above his bid but not higher than b^p, which contradicts his bid being an atom (by Lemma O.3 in the online appendix). More precisely, if $b(s'')$ is an atom, then for any $p' \in (b(s''), b^p]$ such that $b(s) = p'$ for some $s \in (s'', s^p)$,

$$\Pr(\omega = L | s_1 = s'', p = b(s''), 1 \text{ loses by bidding } b(s'')) \geq \Pr(\omega = L | s_1 = s, p', 1 \text{ wins by bidding } p')$$

Therefore,

$$u(\rho(s_1 = s'', p = b(s''), 1 \text{ loses by bidding } b(s''))) \geq u(\rho(s_1 = s, p', 1 \text{ wins by bidding } p')) \geq p' > b(s'').$$

Therefore, type s'' would have an incentive to bid strictly above $b(s'')$, yielding a contradiction to $b(s'')$ being an atom.

Since $b(s'')$ is not an atom, and since s'' can make strictly positive profits when the price is in $(b(s''), b^p]$ by bidding b^p, s'' has a strict incentive to bid b^p, yielding the contradiction that $b(s'') < b^p$.

Step 4: Now we consider bids above b^p and will show that $\rho(s_1 = s^p, p = b^p) > \rho^*$

Since we have shown that there can be at most one atom, b does not have a constant part above s^p. Now we will argue that $\rho(s_1 = s^p, p = b^p) > \rho^*$. This is because, otherwise type 0 would have a profitable deviation to bid above b^p, since if a type above s^p is playing l when the price is b^p, and preferring not to bid b^p, then type 0 would strictly prefer to bid above b^p. Finally, $\rho(s_1 = s, p = b(s')) > \rho^*$ for $s, s' > s^p$ from MLRP.

We now conclude that b has to be a la Pesendorfer and Swinkels (1997) for types above s^p, i.e., for $s > s^p$, $b(s) = u(\rho(s_1 = s, Y^k_{z-1} = s))$. This follows from Pesendorfer and Swinkels

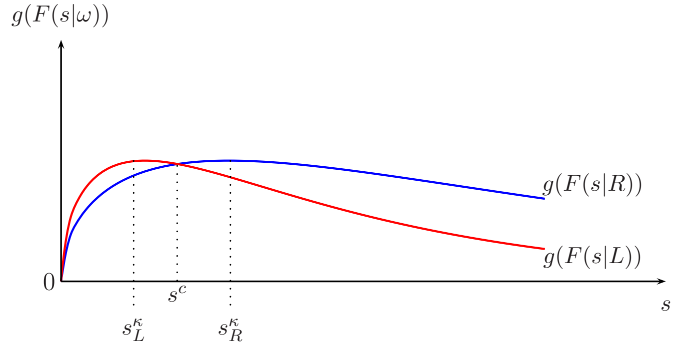

Figure 8: In this figure, $g(t) := t^{(1-\kappa)}(1-t)^\kappa$ and s^c is the unique signal s' such that $g(F(s'|R)) = g(F(s'|L))$. The function $g(t)$ is concave and is maximized at $t^* = 1-\kappa$. Concavity of the function g implies that $s_R^\kappa > s^c > s_L^\kappa$.

(1997), because the value of the object is strictly increasing in the probability that the bidder assigns to state R for types above s^p. □

E. Proof of Theorem 2, item (i)

Proof: If the market size z is sufficiently large, then an equilibrium bidding function cannot be strictly increasing. Because otherwise types that are arbitrarily close to zero would have a profitable deviation to submit bids of higher types. Therefore, due to Lemma 1, such bidding functions have exactly one pooling bid. Let b_z^p be the pooling bid, and let s_z^p be the highest bidder type that bids the pooling bid in the monotonic bidding function b_z. Based on Lemma 1, our first observation is that $\rho(Y_{z-1}^{z\kappa} \leq s_z^p, s_1 = s_z^p) \geq \rho^*$.

Let $g(t) := t^{(1-\kappa)}(1-t)^\kappa$ and let $s^c \in (0,1)$ be the unique signal such that $g(F(s^c|L)) = g(F(s^c|R))$. See Figure 8 for a depiction of s^c. Note that

$$\rho(Y_{z-1}^{z\kappa} \leq s_z^p, s_1 = s_z^p) < \rho(Y_{z-1}^{z\kappa} = s_z^p, s_1 = s_z^p) = \rho_0 \left(\frac{g(F(s_z^p|R))}{g(F(s_z^p|L))}\right)^z \left(\frac{f(s_z^p|R)}{f(s_z^p|L)}\right)^2 \left(\frac{F(s_z^p|L)(1-F(s_z^p|L))}{F(s_z^p|R)(1-F(s_z^p|R))}\right).$$

We will now argue that any limit point of the sequence $\{s_z^p\}_{z\geq 1}$ is at least s^c. On the way to a contradiction, suppose not. Then, there is an integer Z' such that $s_z^p < s^c$ for every $z > Z'$. Note that if $s_z^p < s^c$, then $g(F(s'|R)) < g(F(s'|L))$ (as depicted in Figure 8). Therefore, $\lim_{z\to\infty} \rho(Y_{z-1}^{z\kappa} \leq s_z^p, s_1 = s_z^p) = 0$. But this contradicts to the property that $\rho(Y_{z-1}^{z\kappa} \leq s_z^p, s_1 = s_z^p) \geq \rho^*$.

Since $F(s^c|L) > 1-\kappa$, by the Weak Law of Large Numbers it follows that $\lim_{z\to\infty} \Pr(Y_{z-1}^{z\kappa} \leq s_z^p|L) = 1$. Combining this with our initial observation that $\rho(Y_{z-1}^{z\kappa} \leq s_z^p, s_1 = s_z^p) \geq \rho^*$, we obtain that $\liminf_{z\to\infty} \Pr(Y_{z-1}^{z\kappa} \leq s_z^p|R) > 0$. This is because, otherwise we would have that

$\liminf_{z\to\infty} \rho(Y_{z-1}^{z\kappa} \leq s_z^p, s_1 = s_z^p) = 0$, which would be a contradiction to the property that $\rho(Y_{z-1}^{z\kappa} \leq s_z^p, s_1 = s_z^p) \geq \rho^*$.

Hence, the price in state L is equal to the pooling price with a probability that approaches one, and the price in state R is equal to the pooling price with a probability that stays bounded away from zero. Therefore, the pooling price doesn't reveal the state of the world with strictly positive probability in the limit as $z \to \infty$. □

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O. Online Appendix

Lemma O.1 *For any $s > s_R^\kappa$, $\lim_z \rho_z^-(s) = \rho(s)\frac{\kappa-(1-F(s|R)))}{F(s|R)}\frac{F(s|L)}{\kappa-(1-F(s|L)))}$. There is a unique signal s^p which is a limit point of the cutoffs $\{s_z^p\}_{z\geq 1}$. Moreover either $s^p = s_R^\kappa$ or s^p is the unique signal with the property that satisfies for $\bar{\rho}(s) := \rho(s)\frac{\kappa-(1-F(s|R)))}{F(s|R)}\frac{F(s|L)}{\kappa-(1-F(s|L)))}$, $u(\bar{\rho}(s)) = u(\rho(s))$.*

Proof: Pick any convergent sequence of signals $\{s_z^p\}_{z\geq 1}$ with a limit $s^p > s_R^\kappa$. Let *objects taken* denote the random variable which is equal to the minimum of κz and the number of bidders with signals higher than s_z^p. We first note that, $\rho_z^-(s_z^p)$ can be more conveniently expressed by the following equality:

$$\rho_z^-(s_z^p) = \rho(s_z^p)\frac{E\left[\frac{\kappa z-(\text{objects taken})}{z-(\text{objects taken})}|R\right]}{E\left[\frac{\kappa z-(\text{objects taken})}{z-(\text{objects taken})}|L\right]}.$$

Our first observation is the following: Conditional on state ω, $\frac{\text{objects taken}}{z} \to 1 - F(s^p|\omega)$ in probability as $z \to \infty$. Therefore, as $z \to \infty$,

$$E\left[\frac{\kappa z-(\text{objects taken})}{z-(\text{objects taken})}|\omega\right] \to \frac{\kappa-(1-F(s^p|\omega))}{F(s^p|\omega)},$$

and hence,

$$\frac{E\left[\frac{\kappa z-(\text{objects taken})}{z-(\text{objects taken})}|R\right]}{E\left[\frac{\kappa z-(\text{objects taken})}{z-(\text{objects taken})}|L\right]} \to \frac{\frac{\kappa-(1-F(s^p|R))}{F(s^p|R)}}{\frac{\kappa-(1-F(s^p|L))}{F(s^p|L)}}.$$

Therefore as $z \to \infty$, $\rho_z^-(s_z^p) \to \rho(s^p)\frac{\frac{\kappa-(1-F(s^p|R))}{F(s^p|R)}}{\frac{\kappa-(1-F(s^p|L))}{F(s^p|L)}}$. This proves the first claim of the lemma, by taking $\{s_z^p\}_{z\geq 1}$ to be the constant sequence whose elements are equal to $s > s_R^\kappa$. Now let the sequence $\{s_z^p\}_{z\geq 1}$ be the sequence of cutoffs, and let s^p be a limit point of the sequence, and renumber the new sequence so that its limit is s^p. We have already shown that $s^p \geq s_R^\kappa$ in claim 1 in the proof of Lemma 4. So now assume that $s^p > s_R^\kappa$. Since $s^p > s_R^\kappa$, as $z \to \infty$, $\rho_z^+(s_z^p) \to \rho(s^p)$.

Since each s_z^p has the feature that $u(\rho_z^-(s_z^p)) = u(\rho_z^+(s_z^p))$, and since for each $s > s_R^\kappa$, $\rho_z^-(s) \to \rho(s)\frac{\frac{\kappa-(1-F(s|R))}{F(s|R)}}{\frac{\kappa-(1-F(s|L))}{F(s|L)}}$, we have that for $s \in (s_R^\kappa, 1)$,

$$\Delta(s) := u\left(\rho(s)\left(\frac{\kappa-(1-F(s|R)))}{F(s|R)}\frac{F(s|L)}{\kappa-(1-F(s|L)))}\right)\right) - u\left(\rho(s)\right) \overset{(\geq)}{\leq} 0 \text{ for } s \overset{(\leq)}{\geq} s^p.$$

The term $\rho(s)\left(\frac{\kappa-(1-F(s|R)))}{F(s|R)}\frac{F(s|L)}{\kappa-(1-F(s|L)))}\right)$ is strictly increasing and is always strictly less than $\rho(s)$ in the interval $[s_R^\kappa, 1)$, and is strictly negative when s is close to 1, therefore, there should be at most one signal s^p that can be a limit point in the range $(s_R^\kappa, 1)$.

Now suppose that s_R^κ is a limit point. We will show that no signal $s > s_R^\kappa$ can be a limit point. If s_R^κ is a limit point of the sequence, then it should be that $\lim_z u(\rho_z^-(s_z^p)) = u(0)$, and $\limsup_z u(\rho_z^+(s_z^p)) \leq u(\rho(s_R^\kappa))$. Since $u(\rho_z^-(s_z^p)) = u(\rho_z^+(s_z^p))$, it has to be that $u(0) \leq u(\rho(s_R^\kappa))$. But then, for every $s > s_R^\kappa$, $\Delta(s) < 0$.

Hence we have shown that if s_R^κ is a limit point, then it is the unique limit point, and if it is not, and if an $s > s_R^\kappa$ is a limit point, then it is unique. This completes the argument that the sequence has a unique limit point. □

LEMMA O.2 *If $f(0|L) \neq f(0|R)$, then $\rho_z^-(s) > \rho_z^+(s)$ for any $s \in (0,1)$. Moreover both of these functions are strictly increasing in s.*

PROOF: The first claim in this lemma is identical to the argument in Pesendorfer and Swinkels (1997, Lemma 7, page 1272), and is called loser's curse. The claim that $\rho_z^+(s)$ and $\rho_z^-(s)$ are strictly increasing is standard and follows from the MLRP assumption. The proof can be found in the technical appendix of Milgrom and Weber (1982). □

LEMMA O.3 *In an increasing equilibrium bidding function b, if there is an atom at bid b^p, then $\Pr(\omega = L|s_1 = s, p) > \Pr(\omega = L|s_1 = s, p = b^p, 1$ wins with $b^p)$ for any $p < b^p$. Also $\Pr(\omega = L|s_1 = s, p) < \Pr(\omega = L|s_1 = s, p = b^p, 1$ loses with $b^p)$ for any $p > b^p$.*

PROOF: The two claims are proven in a very similar way, so we will only prove the first one, i.e., $\Pr(\omega = L|s_1 = s, p) > \Pr(\omega = L|s_1 = s, p = b^p, 1 \text{ wins with } b^p)$ for any $p < b^p$. This inequality is very intuitive, but we could not find it in any source, so we are proving it directly using the standard combinatorial techniques.

Let the interval of types who are bidding at the atom bid be (s', s''). Then $\Pr(\omega = L|s_1 = s, p) > f(\omega = L|s_1 = s)\Pr(\omega = L|Y_{z-1}^k = s')$. The term $\Pr(\omega = L|s_1 = s, p = b^p, 1$ wins with

b^p) is calculated using the following steps:

$$\begin{aligned}
1 - F_t(s', s''|\omega) &:= \frac{F(s''|\omega) - F(s'|\omega)}{F(s''|\omega)} \\
C_j^{n-1-i}(\omega) &:= \binom{z-1-i}{j} (1 - F_t(s', s''|\omega))^j \left(F_t(s', s''|\omega)\right)^{z-1-i-j} \\
D^i(\omega) &:= \binom{z-1}{i} (1 - F(s''|\omega))^i (F(s''|\omega))^{z-1-i} \sum_{z-1-i \geq j \geq k-i} C_j^{z-1-i}(\omega) \frac{k-i}{j+1} \\
\Pr(s_1 = s, p = b^p, 1 \text{ wins with } b^p|\omega) &= f(s|\omega) \sum_{0 \leq i \leq k-1} D^i(\omega) \\
\Pr(\omega = L|s_1 = s, p = b^p, 1 \text{ wins with } b^p) &= \frac{f(s|L) \sum_{0 \leq i \leq k-1} D^i(L)}{f(s|R) \sum_{0 \leq i \leq k-1} D^i(R)}
\end{aligned}$$

Explanation: The probability that 1 wins with b^p, the price is b^p conditional on ω can be calculated as the sum of the probabilities of winning in each of the following events, $w^{i,j}$ where $i \leq k-1$ bidders bid above s'', and $k - i \leq j \leq z - 1 - i$ bidders bid the pooling bid. The probability of winning conditional on event $w^{i,j}$ is $\frac{k-i}{j+1}$, since there are $k - i$ objects remaining for the $j + 1$ bidders bidding the pooling bid. The above expressions calculate the probability of each event $w^{i,j}$ in each state and calculate the total winning probability in each state. Similarly the term $f(\omega = L|s_1 = s) \Pr(\omega = L|Y_{z-1}^k = s')$ is calculated using the following steps:

$$\Pr(Y_{z-1}^k = s'|\omega) = \tbinom{z-1}{1} f(s'|\omega) \textstyle\sum_{0 \leq i \leq k-1} \tbinom{z-2}{i} (1 - F(s''|\omega))^i (F(s''|\omega))^{z-2-i} C_{k-i-1}^{z-2-i}(\omega)$$
$$f(\omega = L|s_1 = s) \Pr(\omega = L|Y_{z-1}^k = s') = \tfrac{f(s|L) \Pr(Y_{z-1}^k = s'|L)}{f(s|R) \Pr(Y_{z-1}^k = s'|R)}.$$

We will now show the following:

$$\frac{f(s|L) \Pr(Y_{z-1}^k = s'|L)}{f(s|R) \Pr(Y_{z-1}^k = s'|R)} > \frac{f(s|L) \sum_{0 \leq i \leq k-1} D^i(L)}{f(s|R) \sum_{0 \leq i \leq k-1} D^i(R)},$$

or equivalently the following,

$$\frac{\binom{z-1}{1} f(s'|L) \sum_{0 \leq i \leq k-1} \binom{z-2}{i} (1 - F(s''|L))^i (F(s''|L))^{z-2-i} C_{k-i-1}^{z-2-i}(L)}{\binom{z-1}{1} f(s'|R) \sum_{0 \leq i \leq k-1} \binom{z-2}{i} (1 - F(s''|R))^i (F(s''|R))^{z-2-i} C_{k-i-1}^{z-2-i}(R)} > \frac{\sum_{0 \leq i \leq k-1} D^i(L)}{\sum_{0 \leq i \leq k-1} D^i(R)}.$$

Let, $E^i(\omega) := \binom{z-1}{1} f(s'|\omega) \binom{z-2}{i} (1 - F(s''|\omega))^i (F(s''|\omega))^{z-2-i} C_{k-i-1}^{z-2-i}(\omega)$. We first obtain the following identity by direct algebra:

$$\frac{D^i(\omega)}{E^i(\omega)} = \frac{(1 - F_t(s', s''|\omega)) F(s''|\omega)}{f(s'|\omega)} \sum_{k-i \leq j \leq z-i-1} \frac{(k-i)!(z-k-1)!}{(j+1)!(z-j-i-1)!} \left(\frac{1 - F_t(s', s''|\omega)}{F_t(s', s''|\omega)} \right)^{j+i-k}.$$

A simplification of the above identity via a change of variables by letting $u := j - k + i$ delivers the following:

$$D^i(\omega) = E^i(\omega)\frac{(1-F_t(s', s''|\omega))F(s''|\omega)}{f(s'|\omega)} \sum_{0 \le u \le z-k-1} \frac{(k-i)!(z-k-1)!}{(k-i+u+1)!(z-k-u-1)!} \left(\frac{1-F_t(s', s''|\omega)}{F_t(s', s''|\omega)}\right)^u.$$

The following are consequences of MLRP: $\frac{(1-F_t(s',s''|L))F(s''|L)}{f(s'|L)} < \frac{(1-F_t(s',s''|R))F(s''|R)}{f(s'|R)}$, and, for any positive integer u, $\left(\frac{1-F_t(s',s''|L)}{F_t(s',s''|L)}\right)^u < \left(\frac{1-F_t(s',s''|R)}{F_t(s',s''|R)}\right)^u$. Also, for any fixed $u \in \{0, ..., z-k-1\}$, the term $\frac{(k-i)!(z-k-1)!}{(k-i+u+1)!(z-k-u-1)!}$ is strictly increasing in i.

Our final observation is that $\frac{E^i(L)}{E^i(R)}$ is strictly decreasing in i. This observation also follows from the MLRP assumption by doing some algebraic manipulations. Putting these observations together yields the desired result. $\square$