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On the Optimal Progressivity of Higher Education Subsidies: the Role of Endogenous Fertility

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Abstract

I develop a simple dynastic model in the style of Barro and Becker, with endogenous fertility and human capital accumulation, to quantify the optimal progressivity of higher education subsidies. I find that the optimal policy is characterised by a higher degree of progressivity than current U.S. education subsidies. Additionally, the relation between progressivity of education policy and welfare/ population growth is hump-/ U-shaped respectively. While an assumption of endogenous fertility is quantitatively important, heterogeneity in fertilities is sufficient to generate these results. This is because welfare gains from more progressive subsidies are driven not only by decreases in fertility rates of low income individuals, but also by the fact that their children transit to states associated with higher incomes and, consequently, relatively low fertilities.

JEL Codes: J13, J24, I22.

Keywords: Higher education subsidies, Endogenous fertility, Heterogeneous agents.

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1 Introduction

In this paper I quantify the optimal degree of progressivity of higher education subsidies in a dynamic dynastic model with market luck shock, intergenerational transmission of human capital in the form of parental investments in college education, and endogenous fertility. The assumption of endogenous fertility is novel for this kind of economic framework, and therefore distinguishes this study from the existing literature. Progressive higher education subsidies provide insurance against negative ability shock and serve as a policy instrument for redistribution of income across ex-ante heterogeneous agents.[1] However, this policy might disincentivise high-ability individuals from investing in higher education for their children, who are also likely to be highly-able. This creates a standard equity-efficiency tradeoff for utilitarian social planner.

This study modifies the social planner's problem by considering endogenous fertility, and finds that a welfare-maximising higher education subsidisation policy is characterised by a higher degree of progressivity than current U.S. policy.[2] The main driver behind this result is the assumption of endogenous fertility. When education subsidies become more progressive, the share of low-productivity individuals decreases, not only because this category of agents invest more in the education of their children but also because they cut their fertility rates and their children transit to the states corresponding to higher productivity, and consequently, relatively lower fertilities. In contrast, the counterpart model with exogenous fertility predicts that the introduction of more progressive higher education subsidies does not lead to any material welfare gains.

This study combines two strands of literature on education subsidisation and endogenous fertility, which are discussed in more detail below. Studies analysing

[1]Progressive subsidies imply that low-income individuals are subsidised at higher rates compared to high-income individuals.

[2]In this paper I employ a Millian social welfare criterion equal to the expected welfare of a new born individual and follow the tradition of the Ramsey taxation problem, implying full information and simple functional forms of taxes and subsidies.

optimal education policies typically do not account for endogenous fertility (see Loury, 1981; Benabou, 2002; Caucatt & Kumar, 2003; Bohacek & Kapicka, 2012; Abbot et al., 2013). Studies which analyse both education policies and endogenous fertility do not consider heterogeneity of agents nor, in the main, the implications of individuals' fertility decisions for the distribution of productivity types (see Doepke & De la Croix, 2004; Moav, 2005; Baudin, 2011). This paper fills this gap and finds that consideration of fertility decisions and their implications for the distribution of productivity types is quantitatively important for welfare.

The remainder of the paper is organised as follows. Section 2 presents a review of related literature. Section 3 introduces the model. The calibration is described in section 4. Section 5 begins with an assessment of the benchmark model fit and then presents the main results and decomposes the effects of endogenous fertility and adjustments in factor prices and distribution of productivities.

2 Related Literature

Analysis of optimal education policy places this paper in relation to a large strand of literature focusing on optimal education subsidisation. The most influential analytical studies in this area are those of Loury (1981), Benabou (2002) and Bovenberg & Jacobs (2005). Loury (1981) finds that public education provision increases mean incomes and decreases variation of incomes in a framework with liquidity constraints and heterogeneous agents. Benabou (2002) shows that education policy is superior to redistribution policy from the perspective of income growth, but inferior from the perspective of insurance against negative income shock. Bovenberg & Jacobs (2005) demonstrate that subsidisation of education is an essential component of optimal fiscal policy, as this policy instrument allows for the mitigation of the distortive impact of progressive labor income taxes on human capital accumulation. The main contribution of this paper is that it investigates optimal education policies from a novel perspective - in a model with endogenous fertility.

Similarly to the analytical papers listed above, the existing quantitative studies on higher education policies abstract from endogenous fertility. I follow Caucutt & Kumar (2003) employing a relatively simple general equilibrium model, although a substantial number of studies in this area rely on rich general equilibrium frameworks which allow for detailed modelling of education policy (see Abbott et al., 2013; Kruger & Ludwig, 2013; Bohacek & Kapicka, 2012; Garriga & Knightly, 2007; Akyol & Athreya, 2005).

The results of these studies are mixed. While some papers find that alternative education subsidisation policies may be welfare-improving compared to the current U.S. policy, others do not find any material welfare gains in deviation from the current U.S. status quo. Specifically, Bohacek & Kapicka (2012) find that increases in higher education subsidies in the U.S. to European levels could lead to 1.5 % growth of welfare if reform is financed by higher tax rates. Akyol & Athreya (2005) find that more generous subsidisation of college education compared to the current U.S. policy may be welfare-improving, since subsidies decrease the risk of college completion failure. Analogously, Kruger & Ludwig (2013) demonstrate that the optimal policy would require more generous subsidisation of college education, since it allows for mitigation of the distortive impact of progressive labor income taxation on human capital accumulation.

In contrast, Caucutt & Kumar (2003) and Abbott et al. (2013) do not find any material welfare gains arising from the deviation from current U.S. policy. Additionally, these two papers analyse the optimal degree of progressivity of education subsidies. Specifically, Caucutt & Kumar (2003) find that college subsidies which are more progressive than the current U.S. policy do not lead to any material welfare gains. Similarly, Abbott et al. (2013), employing a rich life-cycle framework, find that introduction of more progressive education subsidisation would lead to relatively small welfare gains, equivalent to a 0.2% increase in life-time consumption. In contrast, this paper finds an almost 1% welfare gain and shows that the assumption of endogenous fertility is essential for this result.

The second strand of literature related to the current study is devoted to endogenous fertility. The studies in this area typically focus on the explanation of inequality patterns (see Moav, 2005; De la Croix and Doepke, 2003; Kremer & Chen, 2002) or implications of different education financing systems and fertility differentials for economic growth (De la Croix and Doepke, 2004).

To the best of my knowledge, the only study in this area which analyses optimal education subsidies in the economy with endogenous fertility is the paper by Baudin (2011). Employing a representative agent framework with endogenous child quality and quantity, Baudin shows that it is optimal to subsidise education and tax the number of children, due to the presence of human capital externalities. In contrast, when the social welfare function allows for preferences for population growth, education might be either subsidised or taxed, depending on the sign of population growth. The fundamental difference of this paper from Baudin's study is the assumption of heterogeneity of agents across productivity types. This assumption leads to a novel type of discrepancy between individual and social planner preferences that is absent in Baudin's paper. Specifically, while deciding on the number and education of their children, individuals do not take into consideration that their decisions affect the distribution of productivity types across agents and, consequently, social welfare.

Methodologically the current paper builds on a seminal study by Becker & Barro (1989) who formalise the child quality-quantity trade off, and Alvarez (1999), who generalise this basic model by the introduction of stochastic ability shocks and endogenous human capital formation, as well as Knowles (1999), who further extends this setting to a general equilibrium model. The current paper can be seen as one of the first attempts to model parental investments in the college education of their children and analyse optimal higher education subsidisation in this type of economic environment.

3 The Model

I consider an overlapping-generations model populated by a continuum of three-period lived individuals. A period in the model is equal to a 25 year interval, so that the model periods correspond to actual life ages 0-25, 26-50 and 51-75 respectively.

Given that a mother's average age at first birth is 26 (for 2011) and estimated life expectancy is 78 for both sexes (for the period 2005-2010) in the U.S., timing in the model is consistent with a real economy.[3] Agents are referred to as children in the first period of their lives, young adults or parents in the second period, and old adults in the third period.

All decisions are taken by young adults. At the beginning of adulthood individuals are characterised by state vector $x = (z, s)$ where z is market luck shock, and s is investments in college education. Adult individuals choose the number of their children and investments in the higher education of each child. Higher education is subsidised by government. For simplicity I assume that there is no social security system in the economy. Therefore, savings are the only source agents can rely on to finance their consumption after retirement.

3.1 Human capital production

In the model, an individual's human capital h corresponds to his labor market productivity and depends on both market luck shock z and investment in college education s, according to the following human capital production technology:

$$h = z(\kappa + s)^{\eta}$$

where $0 < \eta < 1$, $\kappa > 0$. Clearly, this functional form satisfies the standard properties including $\frac{\partial h}{\partial s} > 0$ and $\frac{\partial^2 h}{\partial^2 s} < 0$. Additionally, marginal return in the case of $s = 0$ is finite: $\frac{\partial h}{\partial s}|_{s=0} < \infty$. The latter assumption makes the model consistent with the data, suggesting that there are no infinite returns on education, especially at the college stage (see Psacharopoulos & Patrinos, 2004). The functional form of the human capital production function is close to that employed in De la Croix & Doepke (2003) and Erosa & Koreshkova (2007).

I assume that market luck shock is correlated across generations. The assumption of correlation of z across generations allows us to capture the importance of

[3] Sources: Central Intelligence Agency: the World Factbook; World Population Prospects, 2012.

factors including the effect of networks and neighbourhoods. Individuals who are successful on the labor market are more likely to form a network with other top earners and live in richer neighbourhoods than those who are less lucky on the labor market. These factors are very likely to improve the labor market prospects of their children. Individuals' human capital is also affected by the investment in college education chosen by their parents. Since the focus of this paper is higher education subsidisation, I do not model primary and secondary schooling. Instead I assume that all children are endowed with an exogenous positive level of human capital, captured by parameter κ and interpreted as compulsory primary and secondary education. College education is financed by parents and subsidised by government.[4] The assumption of parental choice of their children's education is supported by the findings of Belley & Lochner (2007) who, employing NLSY97 data, demonstrate that family income is an important determinant of college attendance and quality of college. Additionally, individuals are eligible for student loans. For simplicity, the share of college expenditures covered by loans is assumed to be exogenous.[5]

One of the key assumptions of this paper is the modelling of investments in college education of children as a continuous choice problem, as in Herrington (2015). In other words, parents can choose any non-negative investment in college education of their children $s' \geq 0$. In contrast to the modelling of college investments as a binomial choice problem, typically employed in the literature, this assumption allows us to capture heterogeneity in the costs of different colleges and the amount of time individuals spend on acquisition of higher education.

[4]Given that primary and secondary education in the U.S. is compulsory and publicly provided, while higher education is not compulsory and only partially subsidised, this assumption provides a relatively accurate approximation of the U.S. economy.

[5]Exclusion of student loans from the model would make individuals more financially constrained. Therefore, fertility responses to education subsidies and, consequently, the welfare effect might be inflated.

3.2 Decision problem

In the current model all decisions are taken by young adults characterised by state vector $x = (z, s)$ consisting of ability shock z and investments in college education s, determined by their parents. Young individuals choose consumption c, savings b to finance consumption at old age c_o, the number of their children n and investments in higher education of each child s'. For simplicity, the ability of children z' is assumed to be unobservable for parents when they are making decisions. Therefore, one should think about an individual in a model economy as an "average" agent characterised by state $x = (z, s)$. Old adults exogenously supply $\epsilon < 1$ time units of labor, which allows for the incorporation of a retirement period into the model, and consume their savings $b(1 + r)$.

Young individuals are endowed with one unit of time allocated between labor market activities and child rearing. Raising one child takes fraction ϕ of parental time. Children are also costly in terms of goods, including investment in college education and the exogenous cost of children, interpreted as necessities consisting of expenditures on housing, clothing and food. Government compensates fraction θ of costs of higher education per child. I assume that θ depends on relative parental productivity.[6] The remaining part of college expenditures is paid by parents and their children. Specifically, parents contribute $(1 - \theta(y) - \theta_l)s'$ per child, while children repay $\theta_l s'(1+r_l)$ as a student loan together with interest r_l when they enter adulthood. The student loan ratio θ_l is exogenous and identical for all individuals (discussed in more detail in Section 4).

A young adult's objective is to maximise utility from consumption and the expected utility of each of their children, weighted by altruism discount factor χ and by an increasing and concave function of family size n^{ξ}. Individuals' preferences are modelled in the style of Barro and Becker (1989). The individual decision problem is formulated recursively as follows:

[6]Relative parental productivity is defined as human capital h related to average human capital in the economy.

$$V(z,s) = \max_{c,b,n,s'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \left(\frac{c_o^{1-\sigma}}{1-\sigma} + \chi n^{\xi} E_t \left[V(z',s') | z \right] \right) \right\}$$

subject to

$$c + b + \left[(1 - \theta(y) - \theta_l) s' + g(y,h) \right] n \leq wh(1 - \phi n)(1 - \tau) - \theta_l s (1 + r_l)$$

$$c_o = b(1+r) + wh\epsilon$$

$$h = z(\kappa + s)^{\eta}$$

$$\theta_l + \theta(y) \leq 1$$

where β is the time discount factor, $g(y,h)$ is the cost of children in terms of goods, which is exogenous and defined in Section 4, τ is a proportional labor income tax rate to finance education subsidies, w is wage, and r is interest rate.

The dependence of discount factor χn^{ξ} on the number of children, which is endogenous, together with non-convexities of a budget constraint makes the dynamic programming problem above non-standard. However, as Alvarez & Stokey (1998) show, due to the homogeneity of the utility function, this class of dynamic programming problems can be analysed by similar tools to those used in the standard case.

3.3 Production

The production sector in the economy consists of a large number of firms renting physical capital and effective labor from households in a competitive market. The aggregate production is defined by a standard Cobb-Douglas production function:

$$Y = K^{\alpha} L^{1-\alpha}$$

where K is aggregate capital determined by savings made by the previous generation, and L is aggregate effective labor supply.

3.4 Distribution of agents and population dynamics

As mentioned above, agents are heterogeneous with respect to ability shock z and investments in college education s. Therefore, a formal means of description of heterogeneity in the model is needed. I follow the standard approach employed in the literature. Denote the individual state vector as $x = (z, s) \in X$, where $X = \mathcal{Z} \times \mathcal{S}$, $\mathcal{Z} = [\underline{z}, \bar{z}]$, $\mathcal{S} = [\underline{s}, \bar{s}]$. Then the distribution of individual states across agents is described by probability measure ψ defined on subsets of a state space. Denote the probability space as $(X, B(X), \psi)$, where X is the state space, and $B(X)$ is the Borel σ-algebra on X. Therefore, for each set $B \in B(X)$, $\psi(B)$ shows the fraction of agents whose individual states lie in B as a share of all adult agents. Then for all $B \in B(X)$, the law of motion for the probability measure ψ is given by:

$$\psi'(B) = \frac{\int_X P(x, B) n(x) d\psi(x)}{\gamma}$$

where $P(x, B)$ is a transition function equal to the probability that children of a parent with individual state x will transit to the set B next period, and depends on the transition probability $p(z'|z)$ determined by the stochastic process for market luck shock, $n(x)$ is the number of children chosen by a parent with state x, γ is average fertility, corresponding to the size of the current generation of children relative to that of the current young adults:

$$P(x, B) = \begin{cases} p(z'|z), & \text{if } (z', s(x)) \in B, \\ 0, & \text{otherwise;} \end{cases}$$

$$\gamma = \int_X n(x) d\psi(x).$$

3.5 Stationary equilibrium

The analysis focuses on the concept of stationary equilibrium, according to which factor prices, average fertility, capital, and labor per adult population are constant over time.

Definition. A stationary competitive equilibrium is a set of decision rules $c^*(x)$, $c_o^*(x)$, $n^*(x)$, $s'^*(x)$, $b^*(x)$, factor prices w^* and r^*, interest rate on loans r_l^*, average fertility γ^*, per capita K^*, L^*, average productivity y^* and stationary distribution $\psi^*(x)$ such that:

1) decision rules constitute the solution of the individual's problem with the individual taking factor prices and population growth as given;

2) factor prices w^* and r^* are determined by optimal behaviour of the representative firm:

$w^* = (\frac{K^*}{L^*})^\alpha$, $r^* = (\frac{K^*}{L^*})^{\alpha-1} - \delta$.

3) markets clear:

a) market of goods: $Y^* + (1-\delta)K^* = \int_X \left[c^*(x) + \frac{1}{\gamma^*}c_o^*(x) + \gamma^* K^* + g(y,x)n^*(x) + \theta(x)s'^*(x)n^*(x)\right] d\psi^*(x)$ where relative parental productivity $y = \frac{h}{\int_X h d\psi^*(x)}$;

b) market of capital: $K^* = \int_X \frac{1}{\gamma^*} b^*(x) d\psi^*(x)$;

c) labor market: $L^* = \int_X \left[h(1-\phi n^*(x)) + \frac{1}{\gamma^*}h\epsilon\right] d\psi^*(x)$;

4) government budget is balanced: $\tau w^* L^* = \int_X \theta(x)s'^*(x)n^*(x)d\psi^*(x)$[7];

5) loans: government finances loans to the current generation of children from the repayments of loans by the current generation of adults so that $\theta_l \int_X s'^*(x)n^*(x)d\psi^*(x) = (1+r_l)\theta_l \int_X s d\psi^*(x)$, since the economy is in a steady state $1 + r_l^* = \gamma^*$;

6) a stationary distribution $\psi^*(x)$ is consistent with households' decision rules and stochastic process for z.

In general it is difficult to guarantee the existence and uniqueness of equilibrium in heterogeneous-agents models. I follow intuition from Knowles (1999), who analyses a heterogeneous-agents model with endogenous fertility. The presence of market luck shock guarantees that the child of the parent with the lowest realisation of market luck shock may draw the highest realisation of market luck shock and vice versa. In other words, both the American dream and the American nightmare are possible. Together with the assumption of minimal human capital of children whose parents

[7] All aggregates are per total number of adults in the economy. Similarly to the representative agent models, the total number of individuals is irrelevant for equilibrium and can be normalised to any positive number.

choose not to invest in their college education (due to $\kappa > 0$) this guarantees the absence of poverty traps. In other words, an individual with no college education may draw high realisation of market luck shock and become rich. Moreover, due to mean-reverting properties of the market luck shock and decreasing marginal return on human capital, extremely high productivity is also not an absorbing state.

4 Calibration

The benchmark model presented above is calibrated to match salient characteristics of the U.S. economy. For all statistics, except for individuals' fertility rates, I use the U.S. Census data and *National Postsecondary Student Aid Study* (NPSAS) 2011-2012 data. In order to estimate average fertilities for different parental income quintiles, I use data from the *National Longitudinal Survey of Youth* 1979.

4.1 Higher education subsidies and loans

The U.S. government subsidises higher education through financing of public colleges and through direct financial aid to students. Additionally, students may receive financial support from higher education institutions. I model the higher education subsidy as total financial aid in the form of grants and scholarships received from both sources: directly from government (federal, state grants and scholarships) and from institutions (institutional grants and scholarships). As in Herrington (2015), I assume that the higher education subsidy is proportional to the total price of college s. The coefficient of proportionality $\theta(y)$ is a linear function of relative parental productivity y:

$$\theta(y) = \max(a_\theta + b_\theta y, 0).$$

Parameters a_θ, b_θ are estimated from NPSAS data on student financial aid and parental income for undergraduates, 2011 - 2012. For estimation I restrict the sample to the individuals whose parental income is less than 200 thousands dollars (95 % of all observations in the sample fall into this category, additionally, nearly 96 % of all households in the U.S. belong to this income group). The NPSAS data can be analysed solely by online tools. Since neither evaluation of $\theta(y)$ for each individual nor non-linear regression analysis is available via these tools, estimation of a_θ and b_θ based on individual data is unfeasible.

To overcome these difficulties I proceed as follows. First, the whole sample is divided into 24 income groups so that the income interval is equal to 5 thousand dollars for the bottom and middle parts of the income distribution and 50 thousand dollars for individuals with high incomes. For each group I estimate the ratio of average financial aid to average price of college within a group. Second, I regress these group by group ratios of financial aid to price of college on group by group relative parental incomes to obtain the estimates of a_θ and b_θ. The results including standard errors and R^2 are presented in table 2 (the regression fit is depicted in figure 1 in Appendix A1).

Parameter	Value
a_θ	0.51
	(0.014)
b_θ	-0.145
	(0.013)
R^2	0.83

Table 1: Estimates of parameters for $\theta(y)$

For simplicity it is assumed that student loans to finance college expenditures are exogenous and the ratio of the loan to the price of college is homogenous across individuals. According to the NPSAS data 2011-2012, the ratio of loan to the price of college does not vary much across parental income quintiles (from 0.141 to 0.173). Consequently, parameter θ_l is set equal to the average ratio of the loan to the price of college for all individuals (0.15).

4.2 Cost of children in terms of goods

The model assumes that along with time cost of children and expenditures on college, there is a cost of children in terms of goods. This type of cost is interpreted as expenditures on necessities including housing, clothing and food and, therefore, assumed to be exogenous in the model.

According to the *Expenditures on children by families* 2013 report published by the U.S. Department of Agriculture, cost per child is larger in absolute value for families with higher pre tax annual income. However, the cost per child related to parental pre-tax income is lower for richer individuals. In order to capture the empirical properties of this type of cost I employ the following approximation. I assume that the annual cost per child in terms of goods is proportional to the parental effective wage wh, while the coefficient of proportionality decreases with the relative parental effective wage, which is essentially equal to relative parental productivity:

$$g(y,h) = \tilde{g}(y)wh, \text{ where } \tilde{g}(y) = a_g exp(-b_g y), a_g > 0, 0 < b_g < 1.$$

Effective wage is chosen as a scale since the data is provided for annual parental income, which corresponds to effective wage in the model. This assumption is in line with Knowles (1999) who assumes the exogenous cost of children to be proportional to parental income. However, the current study generalises the approach of Knowles by the assumption of a more general functional form and disciplining the model based on the data.

The data from the *Expenditures on children by families* 2013 report is available only for 3 income groups: lowest, middle and highest tertiles. I use estimated cost per child related to life-time parental income proxied by annual income times 25 years (length of young adulthood) for the lowest and highest tertiles, to solve for a_g and b_g.

Income group	Average relative parental income	Average costs per child
Lowest tertile	0.4	0.16
Middle tertile	0.85	0.1
Highest tertile	1.91	0.075

Table 2: Cost of children in terms of goods as a share of life-time parental income

The resulting estimates are presented in the table below. As was assumed, $0 < b_g < 1$ and, therefore, costs of children in terms of goods increase with parental income.

Parameter	Value
a_g	0.18
b_g	0.47

Table 3: Parameters for $\tilde{g}(y)$

Plugging values for the middle income group which were not used for estimation, one would obtain $\tilde{g}(0.85) = 0.12$, which is quite close to the data.

4.3 Other parameters

In this subsection I discuss the estimation of the remaining parameters of the model, including the characteristics of ability shock, preferences and the human capital production function. The following parameters take values which are standard for macroeconomic literature: $\alpha = 0.34$, capital depreciation rate $\delta = 1-(1-0.1)^{25} = 0.928$. I set fraction ϕ of parental time spent per child, equal to 0.09. Given that a model period is equal to 25 years, this corresponds to 2.25 years per child, as in De la Croix and Doepke (2003). The parameter determining labor supply in old adulthood, is set equal to 0.5, since retirement starts at the age 65 in the U.S. This roughly corresponds to the middle of the old adulthood period (51-75 years).

The rest of the parameters, including ρ, σ_z, β, σ, ξ, η, χ, κ are jointly calibrated to minimise the quadratic loss function (sum of squared deviations of statistics predicted by the model from statistics in the data) so that the model replicates salient features of the U.S. economy. The target statistics, and estimated values of the parameters in the data and in the model are presented in the table below.

There is no one-to-one link between parameters of the model and corresponding targeted statistics. The parameters are assigned to the stylised facts based on the principle of sensitivity. Specifically, while intergenerational persistence of earnings is affected by all parameters in the model, this statistic is the most sensitive to persistence ρ in the AR(1) process for z. Similarly, standard deviation σ_z is assigned to the variance of log earnings. The value of β positively affects savings. Therefore, this parameter is assigned to annual capital to GDP ratio. Parameter σ plays an important role in the model with endogenous fertility since it affects the quality-quantity tradeoff through the curvature of the utility function (Knowles, 1999).[8] Higher values of σ imply lower curvature of the utility function. Consequently, as Knowles suggests, individuals would choose the lower number of children. Since σ affects fertilties, I assign this parameter to average fertility rates in the economy. Parameter ξ affects utility from the number of children, and is, therefore, assigned to a fertility differential between the bottom and top income quintiles.

The remaining parameters η and κ characterise human capital production technology, and parameter χ corresponds to a parental altruism factor. These parameters affect parental incentives to invest in the education of their children. Parameter η determines marginal return on investments in education. Therefore, the targeted statistic for η is the college wage premium. Parameter κ also affects marginal returns on investments in education and corresponds to the share of individuals without higher education. The remaining parameter χ influences the contribution of the expected value of children to overall parental utility and, consequently, affects the importance of child quality for parents. Since education subsidies are proportional to total investments in college education chosen by parents, public expenditures on college are influenced by χ as well. Therefore, this parameter is assigned to the share of public expenditures on higher education in GDP.

[8] Additionally, in a Barro & Becker type of model $\sigma \in (0,1)$. This guarantees that utility increases with the number of children.

Description	Parameter	Value	Target	Data	Model
Persistence in AR(1) process for z	ρ	0.31	Intergenerational persistence of log earnings	0.47	0.473
Std dev of noise in AR(1) process for z	σ_z	0.52	Variance of log earnings	0.36	0.363
Time discount factor	β	0.61	Annual capital to GDP ratio	3	2.99
Risk aversion parameter	σ	0.6	Average fertility rate	1.04	1.036
Curvature of altruism discount factor	ξ	0.21	Fertility differential	1.41	1.42
Elasticity of HC output w.r.t. input	η	0.36	Wage premium of higher education	1.61	1.62
Altruism discount factor	χ	0.55	Public expenditures on higher education, share of GDP	0.95%	0.946%
Minimal human capital level parameter	κ	0.0065	Share of individuals, no higher education	42%	40%

Table 4: Estimates of jointly calibrated parameters

Notes. The fertility differential is defined as a ratio of average fertility rates of the bottom quintile of income distribution to the top quintile. The estimated values of intergenerational persistence and variance of log earnings are taken from Corak (2012) and Mulligan (1997) respectively. The remaining stylised facts are author's calculations. Average fertility rates for different parental income quintiles are estimated based on NLSY79. Wage premium is evaluated as a ratio of average monthly earnings of full-time workers with at least some college to those of workers with no college based on Survey of Income and Program Participation, 2008, US Census Bureau. The share of public expenditures on higher education in GDP (for the year 2011) is taken from Education at a Glance 2014.

5 Results

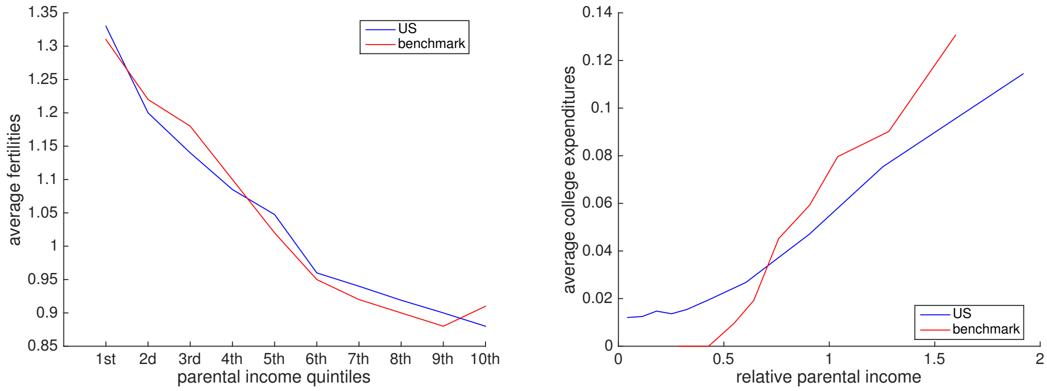
5.1 Benchmark model fit

As table 4 demonstrates, the benchmark model predicts targeted statistics quite well: nearly all stylised facts are almost perfectly matched. Small discrepancies in the cases of the share of individuals with no higher education, wage premium and average fertilities could be explained by the fact that the model is highly non-linear. Consequently, there is no guarantee of the existence of parameters delivering a precise match.

Additionally, the model also replicates certain important properties of the U.S. economy which were not targeted in the calibration exercise. Specifically, the number of children decreases with parental income, while investments in college education per child increase with parental income in the U.S. (see figure 1 below). These observations are consistent with the results of empirical studies including Jones & Tertilt (2008), Hanushek (1992) and the empirical portai of De la Croix & Doepke's 2009 study. The model replicates these properties of the data quite closely.

Interestingly, as Alvarez (1999) demonstrates, the Barro-Becker model does not guarantee the replication of these stylised facts in general. Using a dynastic formulation of the current model, I show that investment in education per child s' increases with parental education s (see Appendix A1). Additionally, as figure 1 demonstrates, investments in the education of children predicted by the model lie in a reasonable range. In contrast, the negative relation between fertilities and parental income cannot be guaranteed in this model. However, the assumption of time cost of children, which implies that opportunity costs are higher for high income parents, and investment in education of children increases with parental income, deliver a quite accurate match of the fertility-income profile.

The results presented above suggest that the model fits the relevant characteristics of the U.S. economy quite well, including those that were not directly targeted by calibration. Therefore, the model may serve as a proper laboratory for quantitative analysis of higher education subsidisation policies.



1a: Fertilities 1b: Investments in education

Figure 1: Incomes, fertilities and investments in education

Notes. The fertility-income profile is estimated based on NLSY 1979 data. The data on total investment in higher education (accounting for number of years of schooling) by parental income is not available. To overcome these restrictions, the following approximation is employed. For each parental income group, average years of schooling is estimated based on the data on the highest level of education earned by total family income, provided by the NPSAS. Then average years of schooling is multiplied by average student budget from NPSAS data. Finally, resulting expenditures on education are normalised by average life-time income.

5.2 Policy experiments

In this subsection I solve for welfare-maximising progressivity of higher education subsidies. The analysis focuses on a so-called small reform in the style of Piketty & Saez (2012, 2013) implying that total spending on higher education is fixed at the absolute current U.S. level. I focus on the class of linear policies:

$$\theta(y) = \begin{cases} 1 - \theta_l, & \text{if } a_l + b_l y \geq 1 - \theta_l; \\ a_l + b_l y, & \text{if } 0 < a_l + b_l y < 1 - \theta_l; \\ 0, & \text{otherwise.} \end{cases}$$

Parameter a_l is a free parameter, which can take either positive or negative values and is interpreted as progressivity of education subsidies. Given a_l, parameter b_l is chosen so that the absolute level of public expenditures on higher education is the same as in the U.S. economy. Higher values of a_l implies that b_l is negative and the function $\theta(y)$ decreases with y. Therefore, subsidies are progressive. In contrast, if a_l becomes sufficiently low or negative, the corresponding values of b_l are likely to be positive. This implies that the education subsidy $\theta(y)$ becomes regressive.

The current study follows the Millian social welfare criterion, which ranks allocations of different population size by comparing the expected utility of a newborn individual:

$$W(a_l) = \int_X V(x, a_l) d\psi(x, a_l).$$

On the one hand, more progressive subsidies could allow for better insurance against negative ability shock. On the other hand, since market luck shock is correlated across generations, more progressive subsidisation may result in efficiency lost. This is a standard equity-efficiency trade off. However, the assumption of endogenous fertility modifies this classical social planner's problem, due to its non-trivial effect on the distribution of productivity types across agents. Consequently, in the model with endogenous fertility more progressive subsidisation of higher education may be desirable from the welfare maximisation perspective, because this policy stimulates low income individuals to not only invest more in the education of their children, but also cut their fertility rates. Therefore, the share of low productivity individuals declines and welfare increases.

5.2.1 Welfare and population growth

Figures 1 and 2 below depict average per capita welfare and fertility rates in a steady state equilibrium as functions of a_l (given that parameter b_l is adjusted to match education budget as in the U.S. economy). Figure 3 depicts per capita welfare as a function of average fertility rates for different education policies. The number markers on the figure 3 correspond to the values of progressivity a_l.

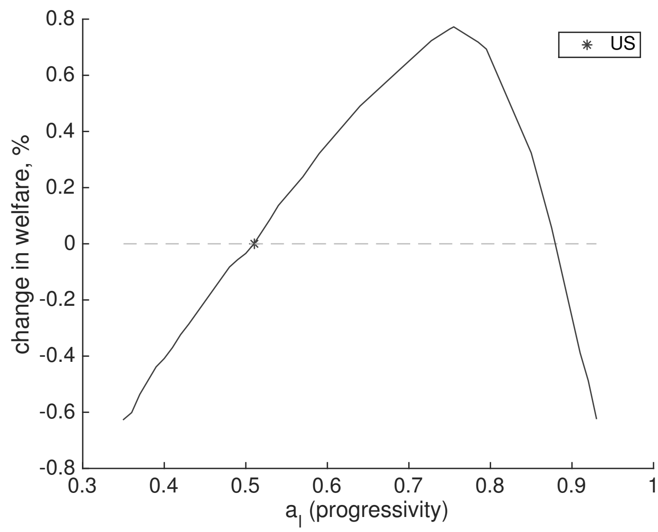


Figure 1: Welfare

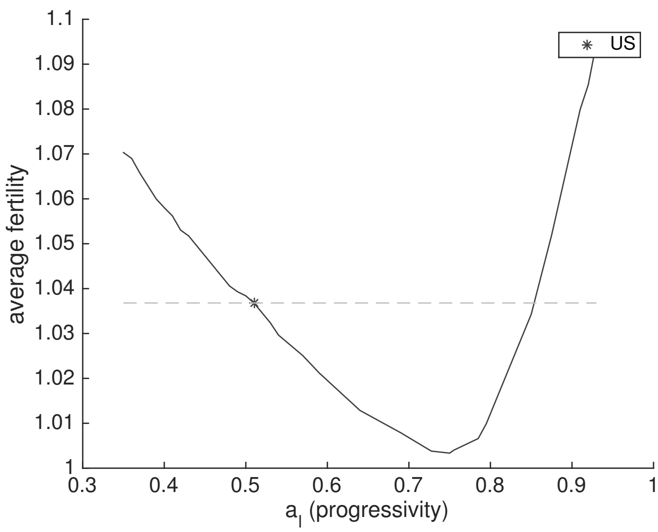


Figure 2: Average fertilities

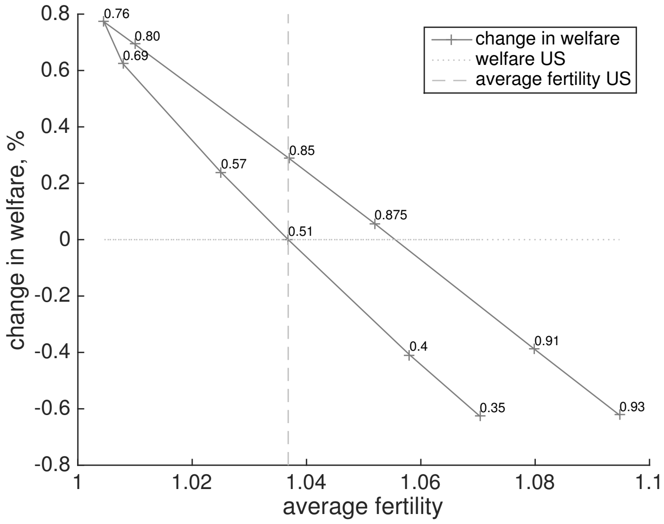


Figure 3: Welfare and average fertilities

Table 6 below presents a comparison of aggregate characteristics in the benchmark case (U.S.) and in the case of welfare-maximising policy. All aggregate variables are normalised to the amount of the working population. Variables including C, Y, K are translated to annual terms and normalised to the GDP in the benchmark economy.

Variable	Description	U.S.	Optimum
S	Investments in higher education, share of U.S. GDP	3.34%	3.28%
C	Total consumption, share of U.S. GDP	80%	81%
Y	Output	100%	100.1%
K	Capital	2.99	3.03
w	Wage	0.2	0.222
r	Interest rate, annual	4.37%	4.3%
τ	Tax	1.14%	1.14%
γ	Average fertility	1.036	1.004
n_d	Fertility differential	1.42	1.17
$\underline{\psi}$	Share of agents, no higher education	0.40	0.04

Table 6: Key economic variables. Benchmark and optimum

As mentioned above, parameter a_l characterises the degree of progressivity of education subsidies. Higher values of a_l correspond to lower (negative) values of slope parameter b_l. As the results demonstrate, the dependance between a_l and welfare is hump-shaped, while dependance between a_l and average fertilities is U-shaped (see figures 1 and 2 above). In order to provide the intuition for this result, I use examples of policies corresponding to $a_l = 0.35, a_l = 0.76$ and $a_l = 0.85$. I label these policies as slightly progressive, optimum and highly progressive respectively. The first policy is slightly progressive since the corresponding slope parameter $b_l = -0.03$, so that subsides are almost flat. Both the second and the third policies are more progressive compared to the benchmark economy, since the corresponding slopes b$_l$ are equal to -0.42, -0.57 respectively versus $b_l = -0.14$ in the benchmark. While the second policy induces the highest per capita welfare and lowest average fertility rates among all linear policies, the third policy leads to lower welfare, whereas average fertility rates are the same as in the U.S. status quo.

First, it is worth mentioning that since education subsidies are proportional, increases in subsidisation rates unambiguously lead to higher total expenditures on children's college education. However, since the share of college expenditures contributed by parents $1 - \theta(y)$ declines, the adjustment in college spending paid by parents has an ambiguous sign. The results suggest the effect of total expenditures on college education dominates and, consequently, college spending paid by parents is positively related to education subsidisation rates.

Now let us return to the example. Assume a less regressive higher education policy corresponding to $a_l = 0.35$ is introduced. Since individuals with low incomes are offered lower education subsidies than in the status quo, this category of agents invest less in the education of their children and consequently, since the cost of children declines, increase their fertility rates (see figures 1a and 1b in Appendix B). Therefore, the share of low income individuals increases. Due to decline in education subsidies agents with low and middle incomes are worse off. High income agents are worse off as well, since increases in fertility rates lead to a decline in the capital-to-labor ratio and, consequently, to a decline in equilibrium wages (see fig. 1c in

Appendix B).[9]

In contrast, when the policy becomes more progressive (see the case of $a_l = 0.76$) than a status quo policy, poor individuals increase investments in education of their children and decrease their fertility rates. Therefore, average productivity increases. Low income individuals are better off. High income individuals are better off as well, since the decline in population growth leads to increases in the capital-to-labor ratio and, consequently, to higher equilibrium wages. Consequently, welfare is higher than in the case of a status quo policy. However, if education policy becomes even more progressive (see the case $a_l = 0.85$), agents with middle and high incomes substantially decrease investments in the education of their children, and increase their fertility rates. Increases in higher education subsidies make low income individuals better off. However, individuals with middle and high incomes are worse off, due to a decrease in education subsidies and lower equilibrium wages. Consequently, welfare declines.

The relationship between progressivity of subsidies and average fertility is U-shaped. As discussed above, increases in progressivity of education subsidies lead to declines in fertilities of low income parents. This effect quantitatively dominates increases in fertilities of high income individuals. Therefore, average fertility declines with progressivity. However, when policy becomes substantially progressive, the fertility rates of a sizeable part of the population, including middle and high income individuals, increase. This effect starts to dominate the decline in fertilities of low income parents. Consequently, when the progressivity of education subsidies increases substantially, average fertility starts to increase.

As can be seen in figure 3, education subsidisation policies associated with higher welfare induce lower average fertility rates than a status quo policy. Interestingly, comparison of the policies that deliver the same average fertility rates demonstrates that more progressive higher education subsidies are superior in terms of welfare

[9]The capital-to-labor ratio is affected by fertility rates since savings are made by the previous generation. Therefore, an increase in population growth leads to a decline in capital per working population and, consequently, contributes to a decline in the capital-to-labor ratio.

compared to their less progressive counterparts (see figure 3). This result suggests that an increase in welfare driven by decline in fertility differentials quantitatively dominates a decline in welfare driven by lower productivity of individuals with high market luck realisations.

5.2.2 Role of distributional effects and heterogeneous fertilities

As discussed above, in an economy with endogenous fertility it might be relatively "easy" to achieve higher welfare by the introduction of policies leading to lower population growth than in the status quo. However, lower fertility rates are not the main factor leading to welfare improvement. Positive welfare gains associated with more progressive education subsidies are mostly driven by distributional effects rather than by a decline in average fertility.

In order to see this, let us evaluate per capita welfare assuming that distribution is fixed, as in the benchmark case. Specifically, first, equilibrium utilities are evaluated in new equilibria corresponding to education subsidies with different degrees of progressivity. Then welfare is calculated by integrating equilibrium utilities over distribution corresponding to the status quo. As figure 2a in Appendix B demonstrates, more progressive subsidisation policies do not lead to any material welfare gains. Therefore, adjustments in distribution are essential.

Another important result is that the assumption of heterogenous (exogenous) fertility rates is sufficient to generate the main results of this paper. Assumption of heterogenous fertilities implies that fertility rates are assigned to individuals exogenously, based on the benchmark decision rule. That is, fertility rates negatively depend on parental productivity. In order to demonstrate the role of heterogenous fertility, I run the same policy experiments as in subsection 5.2.1, but assume that the number of children n exogenously depends on parental state variables z and s in the same way as in the benchmark economy.

The results show that similarly to the endogenous fertility (or full adjustment) case analysed above, the dependence between the degree of progressivity of education subsidies and per capita welfare/population growth is hump-shaped/U-shaped (see fig. 2b, 2c in Appendix B). Additionally, the policy maximising per capita welfare is also very similar to the endogenous fertility case ($a_l = 0.75$, $b_l = -0.39$). The individuals' ability to adjust their fertility rates in response to education subsidies amplifies absolute changes in welfare and population growth; but the direction of these changes is the same. The intuition is as follows. When education subsidies become more progressive, low income individuals increase investments in the education of their children. Consequently, their children transit to the states associated with higher human capital and lower fertility rates. On the other hand, individuals with middle and high incomes would invest less in the education of their children. Therefore, their children transit to the states characterised by a lower level of human capital and higher fertility rates then in the benchmark. This leads to higher per capita welfare and lower fertility rates.

5.2.3 Role of endogenous fertility

The key assumption of this paper is endogenous fertility. In this subsection I check the sensitivity of the main results to this assumption.

Assume uniform (exogenous) fertility implying that the number of children is identical across individuals and equal to average fertility in the benchmark economy (1.036). The replication of the same exercise as in subsection 5.2.1, assuming that model parameters are the same as in the benchmark economy, shows that an increase in progressivity of education subsidies does not lead to any material welfare improvement compared to the status quo (see fig. 3a in Appendix B).

However, the model with assumption of exogenous fertility and benchmark parameters generates an unrealistically low share of individuals with no higher education: 18 % versus 40 % in the data. The prediction of other statistics is also imprecise. To address this caveat, I calibrate the model with exogenous fertility to match target statistics in the U.S. data. I assume that parameter ξ is the same as in the benchmark model, since the fertility differential is always equal to 1 in this model. Additionally, parameter σ is also set at the same level as in the benchmark economy, since average fertility is exogenous. The estimates of other parameters and model fit are provided in table 1 in Appendix B. Similar to the case of benchmark model parameters, this model also predicts the absence of any material gains of more progressive policies (see fig. 3b in Appendix B). This result demonstrates that assumption of endogenous fertility is crucial for the main findings of the current paper.

6 Conclusion

In this paper I have developed a dynastic general equilibrium model for analysis of higher education subsidies. The key assumption of the model which distinguishes it from the existing literature is endogenous fertility. Employing this model calibrated to the U.S. economy, I quantify the optimal degree of progressivity of education subsidies. The analysis focuses on linear policies. I find that the optimal policy is characterised by a higher degree of progressivity than current U.S. policy. Additionally, the relation between progressivity of subsidies and welfare/average fertility is hump- and U-shaped respectively.

While the assumption of endogenous fertility is quantitatively important, heterogeneity in fertilities across individuals is sufficient to generate these results. This is because welfare gains of more progressive subsidies are driven not only by declines in fertility rates of low income individuals, but also by the fact that their children transit to the states associated with relatively low fertilities.

7 References

Abbott, B., Gallipoli, G., Meghir, C., & Violante, G. L. (2013). Education policy and intergenerational transfers in equilibrium (No. w18782). National Bureau of Economic Research.

Akyol, A., & Athreya, K. (2005). Risky higher education and subsidies. Journal of Economic Dynamics and Control, 29(6), 979-1023.

Alvarez, F. (1999). Social mobility: the Barro-Becker children meet the Laitner-Loury dynasties. Review of Economic Dynamics, 2(1), 65-103.

Alvarez, F., & Stokey, N. L. (1998). Dynamic programming with homogeneous functions. Journal of economic theory, 82(1), 167-189.

Baudin, T. (2011). Family policies: what does the standard endogenous fertility model tell us? Journal of Public Economic Theory, 13(4), 555-593.

Barro, R. J., & Becker, G. S. (1989). Fertility choice in a model of economic growth. Econometrica: journal of the Econometric Society, 481-501.

Belley, P., & Lochner, L. (2007). The changing role of family income and ability in determining educational achievement (No. w13527). National Bureau of Economic Research.

Benabou, R. (2002). Tax and Education Policy in a Heterogeneous? Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency? Econometrica, 70(2), 481-517.

Bohacek, R., & Kapicka, M. (2012). A quantitative analysis of educational reforms in a dynastic framework. Manuscript, University of California Santa Barbara.

Bovenberg, A. L., & Jacobs, B. (2005). Redistribution and education subsidies are Siamese twins. Journal of Public Economics, 89(11).

Caucutt, E. M., & Kumar, K. B. (2003). Higher education subsidies and heterogeneity: A dynamic analysis. Journal of Economic Dynamics and Control,27(8), 1459-1502.

Corak, M. (2012). Inequality from generation to generation: the United States in comparison.

De La Croix, D., & Doepke, M. (2003). Inequality and growth: why differential fertility matters. The American Economic Review, 93(4), 1091-1113.

De la Croix, D., & Doepke, M. (2004). Public versus private education when differential fertility matters. Journal of Development Economics, 73(2), 607-629.

De la Croix, D., & Doepke, M. (2009). To segregate or to integrate: education politics and democracy. The Review of Economic Studies, 76(2), 597-628.

Erosa, A., & Koreshkova, T. (2007). Progressive taxation in a dynastic model of human capital. Journal of Monetary Economics, 54(3), 667-685.

Hanushek, E. A. (1992). The trade-off between child quantity and quality. Journal of political economy, 84-117.

Hazan, M., & Zoabi, H. (2015). Do highly educated women choose smaller families? The Economic Journal, 125(587), 1191-1226.

Herrington, C. M. (2015). Public education financing, earnings inequality, and intergenerational mobility. Review of Economic Dynamics, 18(4), 822-842.

Jacobs, B., & Bovenberg, A. L. (2010). Human capital and optimal positive taxation of capital income. International Tax and Public Finance, 17(5), 451-478.

Jones, L. E., & Tertilt, M. (2008). Chapter 5 An Economic History of Fertility in the United States: 1826 - 1960. In Frontiers of family economics (pp. 165-230). Emerald Group Publishing Limited.

Knowles, J. (1999). Social Policy, Equilibrium Poverty, and Investment in Children. Manuscript. Philadelphia, Pennsylvania: University of Pennsylvania, Department of Economics.

Kremer, M., & Chen, D. L. (2002). Income distribution dynamics with endogenous fertility. Journal of Economic growth, 7(3), 227-258.

Loury, G. C. (1981). Intergenerational transfers and the distribution of earnings. Econometrica: Journal of the Econometric Society, 843-867.

Moav, O. (2005). Cheap Children and the Persistence of Poverty. The Economic

Journal, 115(500), 88-110.

Mulligan, C. B. (1997). Parental Priorities and Economic Inequality (Chicago: University of Chicago Press).

Piketty, T., & Saez, E. (2012). Optimal labor income taxation (No. w18521). National Bureau of Economic Research.

Piketty, T., & Saez, E. (2013). A theory of optimal inheritance taxation. Econometrica, 81(5), 1851-1886.

Psacharopoulos, G., & Patrinos, H. A. (2004). Returns to investment in education: a further update. Education economics, 12(2), 111-134.

Appendix

Figure 1 below plots the ratio of financial aid to the price of college (proxy for $\theta(y)$) against relative parental income together with regression fit:

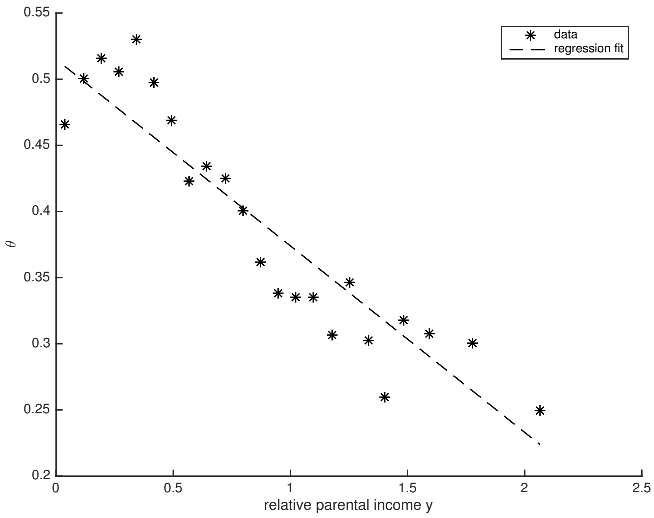


Figure 1: Regression fit for $\theta(y)$

The remaining part of the Appendix provides analytical proof of certain properties of the model and depicts the results.

It can be analytically verified that the model predicts positive dependence between investments in education per child s' and parental education s. Additionally, keeping s, s' positively depends on education subsidy θ.

In order to prove this result, first, write down the dynastic formulation of the individual's problem without higher education subsidies. Using optimality conditions, express c_o as a function of c:

$$c_o = c\left(\beta\left(1+r\right)\right)^{\frac{1}{\sigma}}.$$

Therefore, the problem can be rewritten as:

$$V(z,\bar{h}) = \max_{c,n,\bar{h}'}\{\omega_u \frac{c^{1-\sigma}}{1-\sigma} + \beta\chi n^{\xi} E_t\left[V(z',\bar{h}')|z\right]\}$$

$$\omega c + \left(s' + g(\bar{h},z)\right)n \leq wz\bar{h}(1-\phi n)(1-\tau) + \frac{wz\bar{h}\epsilon(1-\tau)}{1+r}$$

$$\bar{h} = (\kappa + s)^{\eta}$$

$$s' \geq 0$$

$$\omega_u = 1 + \frac{\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}}{(1-\sigma)^{1-\sigma}}$$

$$\omega = 1 + \frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}$$

Now using the human capital production function express s' as a function of child human capital $\bar{h}'$: $s' = \max(\bar{h}'^{\frac{1}{\eta}} - \kappa, 0)$. Denote $\bar{h}'^{\frac{1}{\eta}} = e(\bar{h}')$. Following Alvarez (1999), premultiply the utility function by N^{ξ}, where N is a total size of dynasty with current level of productivity equal to $z\bar{h}$. Then rewrite the problem in a so-called dynastic form:

$$V(\bar{H}, N, z) = \max_{C, \bar{H}', N'} U(C, N) + \beta\chi E_t\left[V(\bar{H}', N', z')|z\right]$$

$$\omega_\lambda C + \left(e\left(\frac{\bar{H}'}{N'}\right) - \kappa + g\left(\frac{\bar{H}}{N}, z\right)\right) N' \leq wz\bar{H}(1-\tau) - wz\frac{\bar{H}}{N}(1-\tau)\phi N' + \frac{wz\bar{H}\epsilon(1-\tau)}{1+r}$$

$$\frac{\bar{H}'}{N'} \geq \kappa^{\eta}$$

where N is a total number of individuals in a dynasty, $C = cN$, $\bar{H} = hN$, $\bar{H}' = h'N'$, $U(C, N) = w_u \frac{(\frac{C}{N})^{1-\sigma}}{1-\sigma} N^{\xi}$, $V(\bar{H}, N, z) = N^{\xi} v(\frac{\bar{H}}{N}, z)$.

The utility function is homogenous of degree ξ with respect to C and N by construction. Additionally, the budget set is a cone. According to Alvarez & Stokey (1998), dynamic programming problems where the objective function is homogenous and the budget set is a cone can be analysed by similar tools as those for standard bounded dynamic programming problems.

Now fix consumption C at some level $\bar{C}$ and write first order conditions with respect to $\bar{H}'$ and N' for interior solution:

$$E_t\left[V_1'(\bar{H}', N', z')|z\right] + \lambda\left(-e_1^{'}\left(\frac{\bar{H}'}{N'}\right)\right) = 0$$

$$E_t\left[V_2'(\bar{H}', N', z')|z\right] + \lambda\left(-e + \kappa + e_1^{'}\left(\frac{\bar{H}'}{N'}\right)\left(\frac{\bar{H}'}{N'^2}\right)N' - g\left(\frac{\bar{H}}{N}, z\right) - (1-\tau)\phi w\frac{\bar{H}}{N}\right) = 0$$

Dividing the first equation above by the second one we get:

$$\frac{E_t\left[V_1'(\bar{H}', N', z')|z\right]}{E_t\left[V_2'(\bar{H}', N', z')|z\right]} = \frac{e_1^{'}\left(\frac{\bar{H}'}{N'}\right)}{e - \kappa - e_1^{'}\left(\frac{\bar{H}'}{N'}\right)\frac{\bar{H}'}{N'} + g\left(\frac{\bar{H}}{N}, z\right) + (1-\tau)\phi wz\frac{\bar{H}}{N}}.$$

Denote $\frac{E_t\left[V_1'(\bar{H}',N',z')|z\right]}{E_t\left[V_2'(\bar{H}',N',z')|z\right]} = \nu(\bar{H}',N',z)$. Since value function $V(\bar{H},N,z) = N^{\xi}v(\frac{\bar{H}}{N},z)$ is homogenous by degree $\xi < 1$, the ratio of its derivatives is homogenous of degree zero with respect to $\bar{H}'$ and N'. This implies that ν can be rewritten as a function of $\frac{\bar{H}'}{N'} = \bar{h}'$ and z. Since the value function is concave, ν is decreasing: $\frac{\partial \nu}{\partial \bar{h}'} < 0$. Plugging the functional form for e, we obtain:

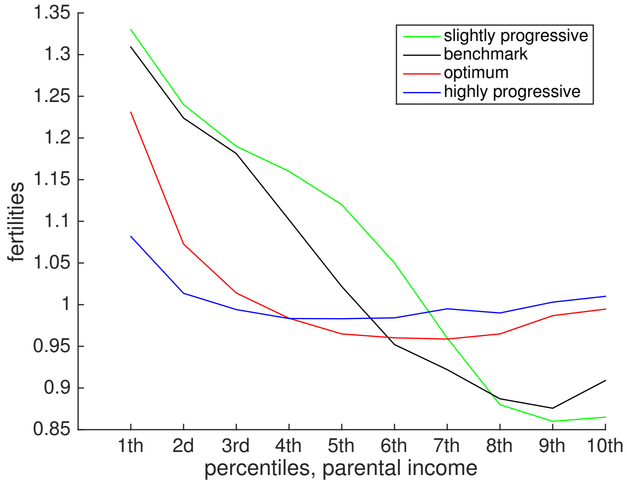
$$\nu(\bar{h}',z) = \frac{e_1'\left(\bar{h}'\right)}{\left(1-\frac{1}{\eta}\right)e\left(\bar{h}'\right) - \kappa + g\left(\bar{h},z\right) + (1-\tau)\phi wz\bar{h}}.$$

Denote the right hand side of the equation above as $\mu(\bar{h},\bar{h}')$. It is straightforward to show that $\frac{\partial \mu}{\partial \bar{h}'} > 0$. Since $\mu(\bar{h},0) = 0$, $\nu(\bar{h}',z) > 0$ and $\frac{\partial \nu}{\partial \bar{h}'} < 0$, this implies that solution $\bar{h}'(\bar{h})$ exists for any $\bar{h} > 0$. Denote the interception point of $\nu(\bar{h}',z)$ and $\mu(\bar{h},\bar{h}')$ as $\bar{h}^*$. Therefore, if $\bar{h}^* < \kappa^{\eta}$, then $\bar{h}'(\bar{h}) = \kappa^{\eta}$, otherwise $\bar{h}'(\bar{h}) = \bar{h}^*$. Since $g_1'(\bar{h},z) > 0$, $\frac{\partial \mu}{\partial \bar{h}} < 0$, therefore, $\bar{h}'$ is a strictly increasing function of h when the solution is interior.

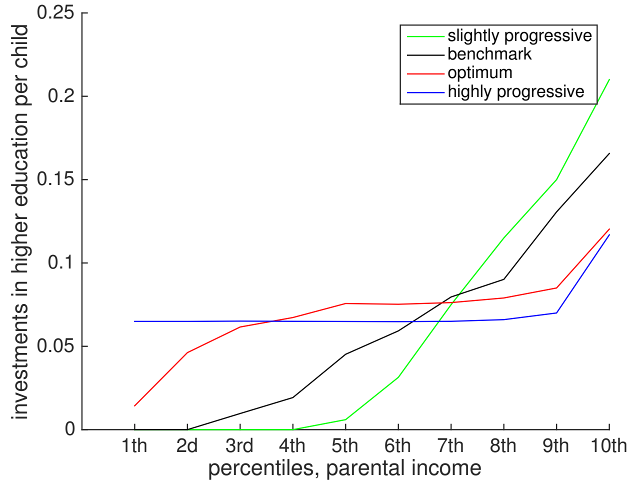
Education subsidies. If one introduces education subsidies into the model, then the equation determining $\bar{h}'(\bar{h})$ modifies as follows:

$$\nu(\bar{h}',z) = \frac{e_1'(\bar{h}')}{\left(1-\frac{1}{\eta}\right)e\left(\bar{h}'\right) - \kappa + \frac{g\left(\bar{h},z\right)+(1-\tau)\phi wz\bar{h}}{1-\theta\left(\bar{h},z\right)}}.$$

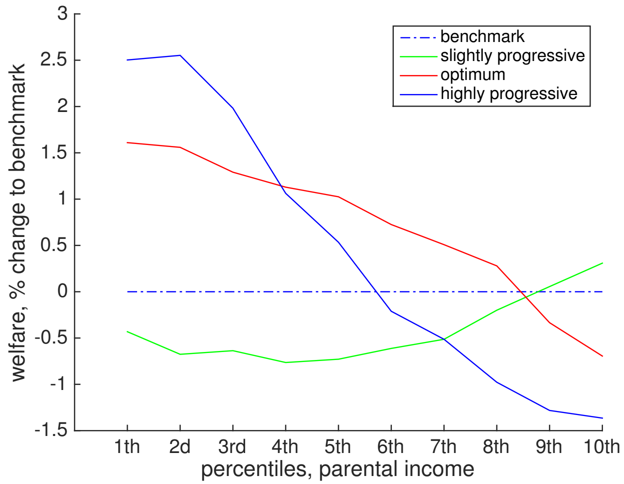
Given that $\theta(\bar{h})$ is a function of $\bar{h}$, dependence between $\bar{h}'$ and $\bar{h}$ can turn to negative (if education subsidies are highly progressive). However, an increase in the level of $\theta(\bar{h},z)$ for given $\bar{h}$ leads to the decrease of the right hand side of the equation above. Consequently, higher subsidies positively affect child human capital $\bar{h}'$.



1a: Fertility

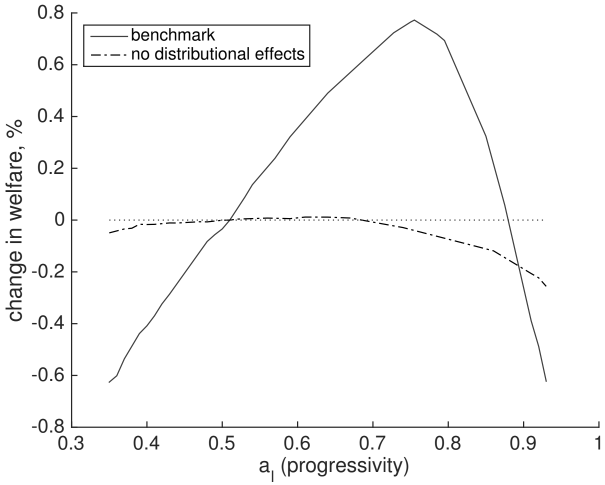


1b: Investments in education per child

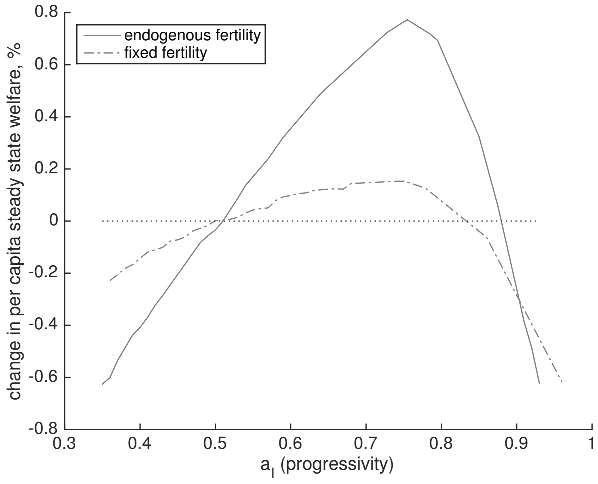


1c: Welfare change.

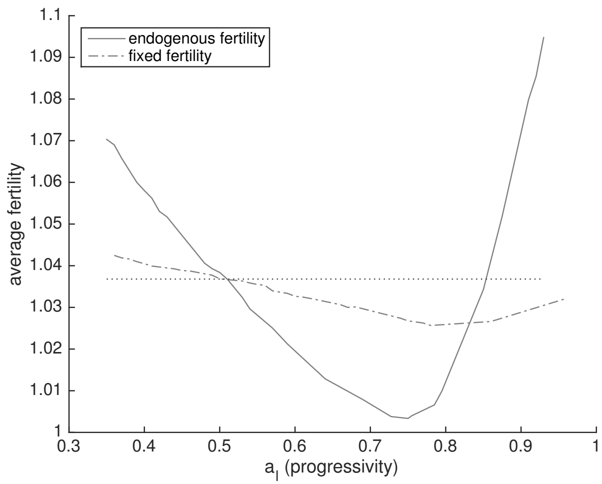
Figure 1: Fertilities, investments in education of children and welfare



2b: Welfare. Elimination of distributional effects.



2c: Welfare. Heterogenous (exogenous) fertility

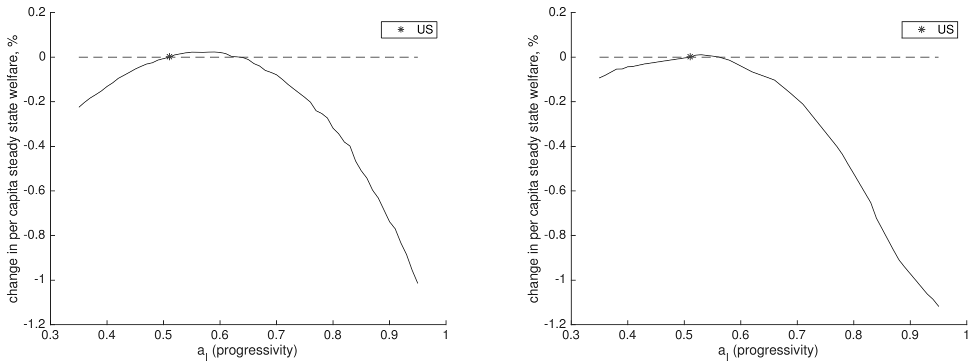


2d: Fertility. Heterogenous (exogenous) fertility

Figure 2: Decomposition of results I

Target	Parameter	Value	Data	Model
Intergenerational persistence of log earnings	ρ	0.32	0.47	0.468
Variance of log earnings	σ_z	0.52	0.36	0.35
Annual capital to GDP ratio	β	0.63	3	3.05
Wage premium of higher education	η	0.55	1.61	1.61
Share of public expenditures on higher education in GDP	χ	0.65	0.95%	0.957%
Share of individuals, no higher education	κ	0.005	42%	40%

Table 1: Estimates of jointly calibrated parameters. Uniform (exogenous) fertility.



3a: Welfare. Uniform (exogenous) fertility. Benchmark parameters

3b: Welfare. Uniform (exogenous) fertility. Calibrated parameters

Figure 3: Decomposition of results II

Abstrakt

V tomto článku studuji jednoduchý dynastický model s endogenní plodností a akumulací lidského kapitálu, podobný Barro a Becker modelům, za účelem kvantifikace optimální progresivity dotací vyššího vzdělání. Zjišťuji, že optimální dotace jsou progresivnější než současném dotace v USA. Zároveň ukazuji, že růst blahobytu s růstem progresivity dotací nejdříve roste a pak klesá, zatímco růst populace s růstem progresivity dotací nejdříve klesá a pak roste. Přestože předpoklad endogenní plodnosti je kvantitativně důležitý, různorodost plodnosti je dostatečným předpokladem k dosažení těchto výsledků. Je to dáno skutečností, že růst blahobytu v důsledku progresivnějších dotací je způsoben nejen poklesem plodnosti nízkopříjmových jednotlivců, ale také sociální mobilitou směrem k vyšším příjmům a tedy k nižší plodnost.

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