Demand Uncertainty, Forecasting, and Monopolistic Equilibrium*

Jacek A. Cukrowski
&(5*((, 3UDJXH &]HFK 5HSXEOLF

0D\

$EVWUDFW

7KLV SDSHU FRQVLGHUV WKH UROH RI IRUHFDVWLQJ DQG GDWD SURFHVVLQJ LQ D PRQRSROLVWLF ILUP IDFLQJ XQFHUWDLQ GHPDQG 7KH PRQRSRO\ LV FRQVLGHUHG LQ D G\QDPLF PXOWLSHULRG PRGHO ,QWHUWHPSRUDO OLQNV DUH GHWHUPLQHG E\ H[SHQGLWXUHV RQ GHPDQG DQDO\VLV LQ D SUHVHQW SHULRG DQG EHQHILWV IURP WKLV DFWLYLW\ L H VPDOOHU YDULDQFH RI WKH SUHGLFWLRQ HUURU LQ IXWXUH SHULRGV $VVXPLQJ WKDW WKH PRQRSRO\ PD[LPL]HV LWV WRWDO GLVFRXQWHG H[SHFWHG XWLOLW\ IURP WKH SURILW LQ LQGHILQLWH WLPH WKH RXWFRPHV ZH GHULYH FRPH XQGHU WKH IROORZLQJ KHDGLQJV)LUVW ZH VKRZ WKDW WKH RSWLPDO IRUHFDVWLQJ VWUDWHJ\ LV VWDWLRQDU\ 6HFRQG EDVHG RQ WKH DQDO\VLV RI WKH VWHDG\ VWDWH ZH ILQG WKDW RQO\ PRQRSROLHV PDQDJHG E\ ULVN DYHUVH LQGLYLGXDOV DUH FRQFHUQHG DERXW GHPDQG IRUHFDVWV 7KLUG ZH H[PLQH WKH H[LVWHQFH DQG XQLTXHQHVV RI WKH RSWLPDO GHPDQG IRUHFDVWLQJ VWUDWHJ\ LQ ULVN DYHUVH PRQRSROVWLF ILUPV)LQDOO\ ZH LQYHVWLJDWH WKH ZHOIDUH HIIHFWV RI GHPDQG IRUHFDVWLQJ LQ WKH ULVN DYHUVH PRQRSRO\ 7KH UHVXOWV LQGLFDWH WKDW GHPDQG IRUHFDVWLQJ LQ D PRQRSROVWLF ILUP DOZD\V LQFUHDVHV WKH H[SHFWHG YDOXH RI WKH SURGXFHU V VXUSOXV EXW LWV LPSDFW RQ WKH H[SHFWHG GHDGZHLJKW ORVV DQG WKH H[SHFWHG FRQVXPHU VXUSOXV FDQQRW EH XQDPELJXRXVO\ GHWHUPLQHG DQG GHSHQGV RQ WKH SURSHUWLHV RI VWRFKDVWLF GHPDQG

. H\ZRUGV 0RQRSRO\ GHPDQG XQFHUWDLQW\ IRUHFDVWLQJ GDWD SURFHVVLQJ

-(/ &ODVVLILFDWLRQ ' ' /

---

*7KLV ZRUN ZDV VXSSRUWHG E\ WKH (6& *UDQW DW &(5*((, , ZLVK WR WKDQN ' U . UH§LPLU ¤LJL– IRU LQWHUHVWLQJ DQG XVHIXO GLVFXVVLRQV RQ WKH VXEMHFW PDWWHU RI WKLV SDSHU , DP DOVR JUDWHIXO WR 3URI -DQ . PHQD DQG ' U 0LODQ +RUQLD‡HN IRU KHOSIXO VXJJHVWLRQV DQG YDOXDEOH FRPPHQWV

$EVWUDNW

7DWR SUgFH KRGQRWm UROL SURJQo]RYgQm D]SUDFRYgQm GDW Y PRQRSROLVWLFNk ILUPÀ NWHUg ‡H0m QHXU‡LWk SRSWgYFH 0RQRSR0 MH KRGQRFHQ Y G\QDPLFNkP PRGH0X SUR YmFH ‡DVRY‹FK REGREm 0H]L‡DVRYk SURSRMHQm MH GgQR Y‹GDML QD DQD0‹]X SRSWgYN\ Y VRX‡DVQkP REGREm D X'LWN\ MH' S0\QRX] WkWR DQD0‹]\ W]Q PHQ§m YDULDQFH FK\E\ SUHGLNFH Y S¸m§WmFK REGREmFK =D S¸HGSRN0DGX 'H PRQRSR0 PD[LPDOL]XMH FH0NRYRX GLVNRQWRYDQRX R‡HNgYDQRX X'LWH‡QRVW]LVNX Y QHRPH]HQkP ‡DVH PRKRX E‹W QD§H Y‹V0HGN\ VKUQXW\ GR QgV0HGXmFmFK ERGÖ 8Ng]D0L MVPH 'H RSWLPg0Qm VWUDWHJLH SURJQR]RYgQm MH VWDFLRQgUQm = DQD0‹]\ XVWg0HQkKR VWDYX S0\QH 'H SRX]H PRQRSR0\ ¸m]HQk MHGQRW0LYFL V DYHU]m YÖ‡L UL]LNX EHURX Y rYDKX SRSWgYNRYk SURJQo]\ =NRXPD0L MVPH H[LVWHQFL D MHGQR]QD‡QRVW RSWLPg0Qm VWUDWHJLH SURJQo]RYgQm SRSWgYN\ X PRQRSR0LVWLFN‹FK ILUHP V DYHU]m N UL]LNX =NRXPD0L MVPH WDNk GRSDG\ SURJQo]RYgQm SRSWgYN\ PRQRSR0LVWLFN‹FK ILUHP V DYHU]m N UL]LNX QD E0DKRE\W 9‹V0HGN\ XND]Xm 'H SURJQo]RYgQm SRSWgYN\ PRQRSR0LVWLFN‹FK ILUHP YHGH NH]Y‹§HQm R‡HNgYDQk KRGQRW\ S¸HE\WNX Y‹UREFH ’ÖV0HGN\ SURJQo]RYgQm QD R‡HNgYDQRX XPUWYHQRX]WUgWX D QD R‡HNgYDQ‹ VSRW¸HELWH0VN‹ S¸HE\WHN QHPRKRX E‹W MHGQR]QD‡QÀ VWDQRYHQ\ D]g0H'm QD Y0DVWQRVWHFK VWRFKDVWLFNk SRSWgYN\

1. Introduction

,Q HFRQRPLF WKHRU\ WKH ILUP LV XVXDOO\ DQDO\]HG XQGHU WKH SUHVXPSWLRQ WKDW LW RSHUDWHV LQ D QRQVWRFKDVWLF HQYLURQPHQW ,Q SUDFWLFH KRZHYHU WKH ILUP LV QHYHU VXUH DERXW D QXPEHU RI YDULDEOHV VXFK DV IDFWRU SULFHV WKH H[DFW VKDSH RI WKH SURGXFWLRQ IXQFWLRQ RU WKH GHPDQG FXUYH DQG VR RQ (YHQ LI WKH ILUP NQRZV LWV FRVW VWUXFWXUHV ZLWK FHUWDLQW\ LW YHU\ UDUHO\ LI HYHU NQRZV SUHFLVHO\ WKH GHPDQG FRQGLWLRQV LW IDFHV (DUOLHU FRQWULEXWRUV WR WKH WKHRU\ RI WKH ILUP KDYH GHPRQVWUDWHG WKDW WKH XQFHUWDLQW\ RI GHPDQG FKDQJHV PDQ\ RI WKH VWDQGDUG UHVXOWV GHULYHG IRU WKH ILUP XQGHU FHUWDLQW\ H J XQGHU XQFHUWDLQW\ WKH PRQRSRO\ LV QRW LQ JHQHUDO LQYDULDQW EHWZHHQ TXDQWLW\ VHWWLQJ RU SULFH VHWWLQJ EHKDYLRU FKDQJHV LQ IL[HG FRVWV DUH QRW LUUHOHYDQW IRU WKH RSWLPDO RXWSXW SULFH GHFLVLRQV RI WKH ILUP IDFLQJ XQFHUWDLQ GHPDQG HWF [1]

7KH ILUP IDFLQJ XQFHUWDLQ GHPDQG KDV EHHQ VXEMHFW RI D YDULHW\ VWXGLHV VHH H J 6DQGPR /HODQG /LP RU +H\ ,Q PRVW RI WKHVH SDSHUV WKH ILUP V EHOLHIV DERXW GHPDQG DUH VXPPDUL]HG LQ D VXEMHFWLYH SUREDELOLW\ GLVWULEXWLRQ ZKLFK FDQQRW EH FKDQJHG E\ WKH ILUP V DFWLRQV 7KH IDFW WKDW WKH ILUP PD\ EH DEOH WR SUHGLFW FKDQJHV LQ GHPDQG RU DW OHDVW WR GHFUHDVH WKH UDQJH RI SRVVLEOH YDULDWLRQV LV XVXDOO\ QHJOHFWHG LQ WKH VWDQGDUG WUHDWPHQWV RI HFRQRPLF EHKDYLRU XQGHU XQFHUWDLQ\ +RZHYHU WKH DELOLW\ RI WKH ILUP WR SUHGLFW GHPDQG DOWKRXJK QRW DOZD\V SHUIHFW PD\ DIIHFW D QXPEHU RI SDUDPHWHUV RI HFRQRPLF HTXLOLEULXP VHH H J 1HOVRQ IRU DQ DQDO\VLV RI XQFHUWDLQ\ DQG SUHGLFWLRQ LQ FRPSHWLWLYH PDUNHWV

,Q WKH SUHVHQW SDSHU ZH DQD\]H WKH RSWLPDO IRUHFDVWLQJ VWUDWHJLHV RI WKH PRQRSROLVWLF ILUP RSHUDWLQJ LQ WKH PDUNHW ZLWK XQFHUWDLQ GHPDQG L H ZH DVVXPH WKDW WKH UHODWLRQVKLS EHWZHHQ TXDQWLWLHV GHPDQGHG DQG PDUNHW SULFHV UDQGRP\ YDULHV IURP SHULRG WR SHULRG ZKHUH WKH PDUNHW DQD\VLV LV FRVW\ DQG WLPH FRQVXPLQJ ,Q SDUWLFXODU ZH VWXG\ WKH EHKDYLRU RI WKH PRQRSR\ LQ WKH PDUNHW ZKHUH WKH WRWDO GHPDQG FRPHV IURP D 0DUJH EXW ILQLWH QXPEHU RI VRXUFHV 7KH GHPDQG FXUYH LQ HDFK LQGLYLGXDO VRXUFH FKDQJHV UDQGRP\ IURP SHULRG WR SHULRG EXW LQ DQ\ WLPH SHULRG GHPDQG FKDQJHV DUH DVVXPHG WR EH FRUUHODWHG ZLWK WKH FKDQJHV SULRU WR WKLV SHULRG UHIOHFWLQJ D FHUWDLQ LQHUWQHVV LQ FRQVXPHU EHKDYLRU 6LQFH JDWKHULQJ DQG SURFHVVLQJ LQIRUPDWLRQ UHTXLUHV WLPH WKH VXP RI LQGLYLGXDO GHPDQGV L H WKH WRWDO GHPDQG FDQQRW EH LQVWDQWHQRXV\ GHWHUPLQHG LQ SDUWLFXODU ZH DVVXPH WKDW WKH UHVXOWV RI WKH PDUNHW DQD\VLV DUH DYDLODEOH RQ\ DIWHU WKH HQG RI WKH SHULRG &RQVHTXHQW\ WKH ILUP V RXWSXW SULFH GHFLVLRQV KDYH

[1] 6HH /HODQG IRU GHWDLOV

WR EH PDGH EDVHG QRW RQ WKH FXUUHQW GHPDQG IXQFWLRQ EXW RQ LWV SUHGLFWLRQ

,Q HDFK SHULRG WKH SURILW PD[LPL]LQJ PRQRSROLVWLF ILUP VHWV WKH YROXPH RI RXWSXW VLQFH LW KDV D KLJK FRPPLWPHQW YDOXH ZLWKLQ D SHULRG RI WLPH L H WKH RXWSXW GHFLVLRQV DUH LUUHYHUVLEOH ZLWKLQ WKH WLPH XQLW 7KH SULFH LV DVVXPHG WR EH PRUH IOH[LEOH DQG FDQ FKDQJH WR VRPH H[WHQW GXH WR UHDO PDUNHW FRQGLWLRQV +RZHYHU WKH ILUP LV VWLOO DVVXPHG WR EH XQDEOH WR 0HDUQ WKH WUXH GHPDQG IXQFWLRQ GXULQJ WKH XQLW RI WLPH [2] DQG FRQVHTXHQWO\ KDV WR UHO\ RQO\ RQ WKH UHVXOWV RI WKH GHPDQG DQD0\VLV

1RWH WKDW E\ DOORZLQJ IRU D VPDOO SULFH DGMXVWPHQW ZH DYRLG WKH SURE0HP RI LQYHQWRULHV DQG DQ\ SRWHQWLDO 0RVVHV FRQQHFWHG ZLWK WKHP [3] VHH =DEH0 IRU DQ DQD0\VLV RI WKH EHKDYLRU RI WKH PRQRSR0LVWLF ILUP LQ D PX0LSHULRG PRGH0 ZLWK LQYHQWRULHV

,I WKH ILUP ZHUH DE0H WR SUHGLFW GHPDQG SHUIHFW0\ L H LI LW NQHZ GHPDQG IRU LWV SURGXFWLRQ ZLWKRXW HUURU WKHQ LW ZRX0G SURGXFH DQ RSWL0D0 RXWSXW DQG HDUQ WKH KLJKHVW SURILW +RZHYHU VLQFH GHPDQG IRUHFDVWV DUH EDVHG RQ SDVW GDWD WKH SUHGLFWLRQ HUURU DSSHDUV DQG FRQVHTXHQW0\ WKH ILUP V RXWSXW GHFLVLRQV GHYLDWH IURP WKH RSWL0D0 7DNLQJ LQWR DFFRXQW WKDW PRUH SUHFLVH SUHGLFWLRQV UHTXLUH PRUH UHVRXUFHV WR EH GHYRWHG WR GDWD SURFHVVLQJ LQ WKH ILUP EHWWHU SUHGLFWLRQV RQ WKH RQH KDQG PDNH WKH ILUP V RXWSXW GHFLVLRQV F0RVH WR RSWL0D0 EXW RQ WKH RWKHU KDQG WKH\ LQGXFH DGGLWLRQD0 FRVWV FRQVHTXHQW0\ WKH VH0HFWLRQ RI WKH RSWL0D0 GHPDQG IRUHFDVWLQJ VWUDWHJ\ IDFHV WKH WUDGLWLRQD0 WUDGH RII SURE0HP ZKLFK LV DW WKH FRUH RI DQ\ HFRQR0LF DQD0\VLV

: KHUHDV WKH QHFHVVLW\ DQG RUJDQL]DWLRQD0 DVSHFWV RI GDWD SURFHVVLQJ IRU WKH SXUSRVH RI SUHGLFWLQJ GHPDQG LQ WKH ILUP KDV EHHQ UHFHQW0\ SUHVHQWHG E\ 5DQHU DQG 9DQ =DQGW 0LWW0H KDV EHHQ GRQH WR FRQVLGHU WKH H[WHQW RI GHPDQG IRUHFDVWLQJ RQ HFRQR0LF HTXL0LEULXP 7KH SXUSRVH RI WKH SDSHU LV WR GHYH0RS D PRGH0 RI D PRQRSR0LVWLF ILUP ZKLFK IDFHV VWRFKDVWLF GHPDQG DQG ZKLFK

---

[2] 2XU DQD0\VLV LV UH0DWHG WR WKH TXDQWLW\ VHWWLQJ EHKDYLRU DFFRUGLQJ WR ZKLFK WKH ILUP VHWV WKH TXDQWLW\ H[DQWH DQG WKHQ WKH RXWSXW LV VR0G DW D SULFH REWDLQHG LQ WKH PDUNHW +RZHYHU XQ0LNH LQ WKH VWDQGDUG TXDQWLW\ VHWWLQJ PRGH ZH DVVXPH WKDW WKH ILUP FDQQRW 0HDUQ WKH WUXH GHPDQG FRQGLWLRQV LQVSLWH RI VRPH DGMXVWPHQW RI SULFH GXH WR PDUNHW SUHVVXUH 7KH UHDVRQ LV WKDW WKH WUXH GHPDQG FKDQJHV RYHU WLPH VR WKH ILUP LV QHYHU VXUH ZKHWKHU WKH FKDQJH LQ SULFH PHDQV PRYHPHQW WRZDUGV WKH WUXH GHPDQG IXQFWLRQ RU UHI0HFWV VKLIWV RI WKDW IXQFWLRQ 7KXV RXU DVVXPSWLRQ LV PRUH UHVWULFWLYH EXW D0SR PRUH UHD0LVWLF

[3] $0WHUQDWLYH0\ WR FLUFXPYHQW WKH SURE0HP RI LQYHQWRULHV RQH FDQ WKLQN WKDW WKH PRQRSR0LVWLF ILUP XQGHU VWXG\ SURGXFHV VHUYLFHV RU SHULVKDE0H JRRGV

LQ RUGHU WR LPSURYH LWV SULFH RXWSXW GHFLVLRQV LV DEOH WR PDNH IRUHFDVWV

7KH SDSHU SURYLGHV DQ DQDO\VLV RI WKH RSWLPDO GHPDQG IRUHFDVWLQJ VWUDWHJ\ LQ PRQRSROLVWLF ILUPV DQG LPSOLFDWLRQV RI GHPDQG DQDO\VLV RQ RXWSXW SULFH GHFLVLRQV RI WKH PRQRSRO\ DQG RQ WKH GLVWULEXWLRQ RI ZHIDUH ,Q 6HFWLRQ VWRFKDVWLF GHPDQG LV FKDUDFWHUL]HG DQG IRUPDO\ GHILQHG 6HFWLRQ FRQWDLQV D GHVFULSWLRQ RI WKH GDWD SURFHVVLQJ IRU WKH SXUSRVH RI GHPDQG IRUHFDVWLQJ LQ WKH ILUP ,Q 6HFWLRQ WKH REMHFWLYHV RI D PRQRSROLVWLF ILUP DUH SUHVHQWHG DQG DQDO\]HG 6HFWLRQ H[DPLQHV WKH RSWLPDO RXWSXW SULFH GHFLVLRQV RI WKH PRQRSRO\ : HIDUH LPSOLFDWLRQV RI IRUHFDVWLQJ DFWLYLWLHV LQ WKH PRQRSROLVWLF ILUP DUH SUHVHQWHG LQ 6HFWLRQ

2. Stochastic Demand

&RQVLGHU D PDUNHW IRU D VLQJOH FRPPRGLW\ LQ ZKLFK GHPDQG FRPHV IURP D ODUJH QXPEHU RI LGHQWLFDO VRXUFHV 1 2QH FDQ WKLQN RI WKHVH VRXUFHV WR EH ZKROHVDOH ILUPV VKRSV RU HYHQ FRQVXPHUV 7R LQWURGXFH XQFHUWDLQW\ DVVXPH WKDW GHPDQG LQ HDFK LQGLYLGXDO VRXUFH L L 1 DW DQ\ SHULRG RI WLPH W W LV DQ LQWHJHU QXPEHU ∞ W ∞ FDQ EH GHVFULEHG E\ WKH IROORZLQJ LPSOLFLW GHPDQG UHODWLRQVKLS [4]

$$I\ T_{i,t}\ S_t\ \eta_{i,t}$$

ZKHUH

$T_{i,t}$ $T_{i,t}\geq$ LV D TXDQWLW\ GHPDQGHG DW SULFH S_t $S_t\geq$

$\eta_{i,t}$ GHQRWH LGHQWLFDOO\ GLVWULEXWHG UDQGRP YDULDEOHV VSHFLILHG E\ SUREDELOLW\ GHQVLW\ IXQFWLRQV

7KH UHVWULFWLRQV SODFHG RQ DUH WKDW IRU DQ\ SDUWLFXODU YDOXH RI $\eta_{i,t}$ WKH UHODWLRQVKLS EHWZHHQ S_t DQG $T_{i,t}$ LV GRZQZDUG VORSLQJ DQG WKDW WKH PARGHU YDOXHV RI $\eta_{i,t}$ DUH DVVRFLDWHG ZLWK JUHDWHU GHPDQG / HODQG / LP 7KXV GHPDQG LQ HDFK VRXUFH FDQ EH H[SUHVVHG DV HLWKHU

$$T_{i,t}\quad T_{i,t}\ S_t\ \eta_{i,t}\quad \partial T_{i,t}\ S_t\ \eta_{i,t}\ \partial S_t\quad \text{DQG}\quad \partial T_{i,t}\ S_t\ \eta_{i,t}\ \partial \eta_{i,t}\ !$$

RU

[4] 6HH / HODQG RU / LP

$$S_t \quad S_t\, T_{i,t}\, \eta_{i,t} \qquad \partial S_t\, T_{i,t}\, \eta_{i,t} \quad \partial T_{i,t} \qquad \text{DQG} \quad \partial S_t\, T_{i,t}\, \eta_{i,t} \quad \partial \eta_{i,t} \;!$$

' HILQH $\eta_{i,t}^{\circ} \equiv (>\eta_{i,t}@$ ZKHUH (LV DQ H[SHFWDWLRQ RSHUDWRU WKHQ IRU DQ\ YDOXHV RI S_t DQG $T_{i,t}$ DQG VXIILFLHQWO\ FRQFHQWUDWHG GLVWULEXWLRQV RI $\eta_{i,t}$ VHH 6DPXHOVRQ RU /LP ZH FDQ DSSUR[LPDWH $T_{i,t}\, S_t\, \eta_{i,t}$ DQG $S_t\, T_{i,t}\, \eta_{i,t}$ DURXQG $\eta_{i,t}^{\circ}$ DV

$$T_{i,t}\, S_t\, \eta_{i,t} \approx T_{i,t}\, S_t\, \eta_{i,t}^{\circ} \qquad \eta_{i,t} - \eta_{i,t}^{\circ} \quad \partial T_{i,t}\, S_t\, \eta_{i,t}^{\circ} \quad \partial \eta_{i,t}$$

$$S_t\, T_{i,t}\, \eta_{i,t} \approx S_t\, T_{i,t}\, \eta_{i,t}^{\circ} \qquad \eta_{i,t} - \eta_{i,t}^{\circ} \quad \partial S_t\, T_{i,t}\, \eta_{i,t}^{\circ} \quad \partial \eta_{i,t}$$

ZKHUH $\partial T_{i,t}\, S_t\, \eta_{i,t}^{\circ} \quad \partial \eta_{i,t}$ DQG $\partial S_t\, T_{i,t}\, \eta_{i,t}^{\circ} \quad \partial \eta_{i,t}$ GHQRWH SDUWLDO GHULYDWLYHV RI WKH GHPDQG DQG LQYHUVH GHPDQG IXQFWLRQV ZLWK UHVSHFW WR UDQGRP YDULDEOHV $\eta_{i,t}$ HYDOXDWHG DW WKHLU H[SHFWHG YDOXHV $\eta_{i,t}^{\circ}$ L 1

1RWH WKDW QR UHVWULFWLRQV DUH LPSRVHG RQ WKH VLJQ RI FURVV SDUWLDO GHULYDWLYHV RI WKH GHPDQG DQG LQYHUVH GHPDQG IXQFWLRQV $\partial^2 T_{i,t}\, S_t\, \eta_{i,t} \quad \partial S_t \partial \eta_{i,t}$ DQG $\partial^2 S_t\, T_{i,t}\, \eta_{i,t} \quad \partial T_{i,t} \partial \eta_{i,t}$ 7KH\ FRXOG EH QHJDWLYH SRVLWLYH RU HTXDO WR]HUR GHSHQGLQJ RQ WKH SDUWLFXODU IRUP RI WKH LPSOLFLW GHPDQG IXQFWLRQ

5DQGRP YDULDEOHV $\eta_{i,t}$ FRXOG PRYH XS RU GRZQ LQ UHVSRQVH WR FKDQJHV LQ WKH YDULDEOHV RPLWWHG IURP D FRUUHFW GHPDQG VSHFLILFDWLRQ [5] VXFK DV IRU LQVWDQFH LQWHUHVW UDWHV LQIODWLRQ SHUVRQDO LQFRPH SULFHV RI RWKHU JRRGV HWF 0XFK RI WKHVH PRYHPHQWV KRZHYHU PLJKW EH GXH WR IDFWRUV ZKLFK DUH KDUG WR FDSWXUH VXFK DV IRU H[DPSOH FKDQJHV LQ WKH ZHDWKHU RU LQ FRQVXPHUV WDVWHV 7KXV LQ PDQ\ FDVHV LW PD\ EH GLIILFXOW RU LPSRVVLEOH WR H[SODLQ IOXFWXDWLRQV LQ GHPDQG WKURXJK WKH XVH RI D VWUXFWXUDO PRGHO 0RUHRYHU LW PLJKW KDSSHQ WKDW HYHQ LI VWDWLVWLFDO\ VLJQLILFDQW UHJUHVVLRQ HTXDWLRQV FRXOG EH HVWLPDWHG WKH UHVXOW FRXOG QRW EH XVHIXO IRU IRUHFDVWLQJ SXUSRVHV IRU H[DPSOH ZKHQ H[SODQDWRU\ YDULDEOHV ZKLFK DUH QRW 0DJJHG PXVW WKHPVHOYHV EH IRUHFDVWHG ,Q VXFK VLWXDWLRQV DQ DOWHUQDWLYH PHDQV RI REWDLQLQJ SUHGLFWLRQV RI $\eta_{i,t}$ KDYH WR EH XVHG 7KH HDVLHVW ZD\ LV WR SUHGLFW FKDQJHV LQ $\eta_{i,t}$ EDVHG RQ WKH DQDO\VLV RI WKHLU PRYHPHQWV LQ WKH SDVW 6XFK IRUHFDVWV KRZHYHU DUH SRVVLEOH RQO\ LI WKH UDQGRP YDULDEOHV $\eta_{i,t}$ DUH REVHUYDEOH DQG LI WKH\ DUH FRUUHODWHG ZLWK WKHLU SUHYLRXV YDOXHV

$\$OWKRXJK UDQGRP YDULDEOHV \eta_{i,t} \quad L \quad 1$ FDQQRW EH GLUHFWO\ REVHUYHG ZH DVVXPH WKDW WKH ILUP FDQ HDVLO\ JHW LQIRUPDWLRQ DERXW TXDQWLWLHV GHPDQGHG LQ HDFK LQGLYLGXDO VRXUFH IRU D JLYHQ SULFH $S_t \quad T_{i,t}\, S_t\, \eta_{i,t} \quad L \quad 1$ DQG FRQVHTXHQWO\ LW FDQ GHWHUPLQH WKH GHYLDWLRQ IURP WKH H[SHFWHG YDOXH $\eta_{i,t}^{\circ}$ LQ DQ\ SHULRG W DV

[5] $(>\eta_{i,t}@ \neq$ UHIOHFWV WKH FDVH RI RPLWWHG YDULDEOHV L 1

$$\nu_{i,t} \quad \eta_{i,t} - \eta_{i,t}^{\circ} \approx \gg T_{i,t}\, S_t\, \eta_{i,t} \quad - T_{i,t}\, S_t\, \eta_{i,t}^{\circ} \; @ \quad \partial T_{i,t}\, S_t\, \eta_{i,t}^{\circ} \quad \partial \eta_{i,t}$$

1RWH WKDW E\ JHWWLQJ WKH LQIRUPDWLRQ DERXW WKH WUXH TXDQWLW\ GHPDQGHG DW D FHUWDLQ SULFH S_t LQ SHULRG W $T_{i,t}\, S_t\, \eta_{i,t}$ IURP DQ\ JLYHQ VRXUFH L L 1 WKH PRQRSROLVW FDQ GHWHUPLQH WKH GLIIHUHQFHV $\eta_{i,t} - \eta_{i,t}^{\circ}$ VLQFH WKH H[SHFWHG TXDQWLW\ GHPDQGHG $T_{i,t}\, S_t\, \eta_{i,t}^{\circ}$ DQG WKH LPSDFW RI WKH UDQGRP YDULDEOH $\eta_{i,t}$ RQ GHPDQG HYDOXDWHG DW WKH H[SHFWHG YDOXH $\partial T_{i,t}\, S_t\, \eta_{i,t}^{\circ} \; \partial \eta_{i,t}$ DUH ERWK NQRZQ DQG WKHQ XVLQJ DSSUR[LPDWLRQV DQG LW FDQ ZRUN RXW WKH WUXH GHPDQG DQG WKH LQYHUVH GHPDQG FXUYHV IRU HDFK SDUWLFXODU VRXUFH SURYLGHG WKDW WKH SUREDELOLW\ GLVWULEXWLRQ RI WKH UDQGRP YDULDEOH $\eta_{i,t}$ LV FRPSDFW DQG FRQFHQWUDWHG

7R VLPSOLI\ WKH DQDO\VLV DVVXPH WKDW UDQGRP GHYLDWLRQV $\nu_{i,t}$ L 1 IURP WKH H[SHFWHG YDOXHV RI LQGLYLGXDO GHPDQGV DUH LQGHSHQGHQW [6] DQG GHVFULEHG E\ LGHQWLFDO VWDWLRQDU\ VWRFKDVWLF SURFHVVHV ZLWK D PHPRU\ H J E\ DXWRUHJUHVVLYH SURFHVVHV RI DQ\ RUGHU [7] ,Q RWKHU ZRUGV DVVXPH WKDW IRU DQ\ LQGLYLGXDO GHPDQG YDULDQFHV DQG FRYDULDQFHV RI UDQGRP YDULDEOHV $\nu_{i,t}$ DUH LQYDULDQW ZLWK UHVSHFW WR GLVSODFHPHQW LQ WLPH QRWH WKDW E\ GHILQLWLRQ PHDQ YDOXHV RI UDQGRP YDULDEOHV $\nu_{i,t}$ DUH HTXDO WR]HUR L H

$$(\nu_{i,t} \qquad 9DU\ \nu_{i,t} \quad 9DU\ \nu_i \quad \omega^2 ! \quad DQG \; \&RY\ \nu_{i,t}\, \nu_{i,t+s} \neq$$

IRU V L 1 DQG LQWHJHU YDOXHG W ∞ W ∞

8QGHU WKH DVVXPSWLRQV DERYH IRU DQ\ JLYHQ SULFH S_t 3_t $3_t \geq$ WKH YDOXH RI WKH WRWDO GHPDQG IDFHG E\ WKH ILUP LQ SHULRG W D VXP RI LQGLYLGXDO GHPDQGV FRPLQJ IURP DOO VRXUFHV $4_t\, 3_t\, \nu_{1,t}\, \nu_{2,t} \quad \nu_{N,t}$ FDQ EH UHSUHVHQWHG DV

$$Q_t\, P_t\, \nu \;_t\, \nu \;_t \quad \nu_{N\,t} \;\approx\; \sum_{i=}^{N} \gg q_{i\,t}\, P_t\, \eta_{i\,t}^{\circ} \quad + \; \nu_{i\,t}\, \frac{\partial q_{i\,t}\, P_t\, \eta_{i\,t}^{\circ}}{\partial \eta_{i\,t}} \; @ \; =$$

$$= \sum_{i=}^{N} \; q_{i\,t}\, P_t\, \eta_{i\,t}^{\circ} \quad + \; \frac{\partial q_{i\,t}\, P_t\, \eta_{i\,t}^{\circ}}{\partial \eta_{i\,t}} \; \sum_{i=}^{N} \nu_{i\,t} \; =$$

---

[6] ,Q JHQHUDO VSHFLILFDWLRQV RI VWRFKDVWLF SURFHVVHV GHVFULELQJ LQGLYLGXDO GHPDQGV VKRXOG LQFOXGH DOVR D FRPPRQ QRLVH ZKLFK FRXOG UHIOHFW WKH DJJUHJDWH GHPDQG VKRFNV L H ZKLFK FRXOG HTXDOO\ DIIHFW DOO VRXUFHV RI GHPDQG EXW WR VLPSOLI\ WKH H[SRVLWLRQ ZH ZLOO QHJOHFW WKLV FRPPRQ FRPSRQHQW

[7] $ VLPLODU VWUXFWXUH RI GHPDQG ZDV DVVXPHG E\ 5DQQHU DQG 9DQ =DQGW

$$= \sum_{i=}^{N} q_{i\,t}\, P_t\, \eta_{i\,t}^{\circ} + \frac{\partial q_{i\,t}\, P_t\, \eta_{i\,t}^{\circ}}{\partial \eta_{i\,t}}\, \nu_t$$

ZKHUH

$$\nu_t = \sum_{i=}^{N} \nu_{i\,t}$$

7DNLQJ H[SHFWDWLRQ ZH REWDLQ

$$\mathrm{E}\!>\! Q_t\, P_t\, \eta_{\ t}\, \eta_{\ t} \quad \eta_{N\,t}\,@ \approx \sum_{i=}^{N} q_{i\,t}\, P_t\, \eta_{i\,t}^{\circ} = Q_t\, P_t\, \eta_t^{\circ}$$

ZKHUH

$$\eta_t^{\circ} = \sum_{i=}^{N} \eta_{i\,t}^{\circ}$$

IRU DQ\ LQWHJHU YD0XHG W ∞ W ∞

6LQFH UDQGRP YDULDE0HV $\nu_{i,t}$ DUH WLPH LQYDULDQW WKH H[SHFWHG YD0XH $4_t\ 3_t\ \eta_t^{\circ}$ FDQ EH UHSUHVHQWHG DV $4\ 3\ \eta^{\circ}$ ZKHUH $\eta^{\circ}\ \eta_t^{\circ}$ IRU DQ\ LQWHJHU W ∞ W ∞

1RWH WKDW IRU DQ\ L L 1 LQWHJHU W ∞ W ∞ DQG S_t 3 ZH KDYH

$$\frac{\partial q_{i\,t}\, p_t\, \eta_{i\,t}^{\circ}}{\partial \eta_{i\,t}} = Q\ P\, \eta^{\circ} \qquad \frac{\partial p_t\, q_{i\,t}\, \eta^{\circ}{}_{i\,t}}{\partial \eta_{i\,t}} = P\ Q\, \eta^{\circ}$$

DQG

$$\frac{\partial\ q_{i\,t}\, p_t\, \eta_{i\,t}^{\circ}}{\partial p_t \partial \eta_{i\,t}} = Q\ P\, \eta^{\circ} \qquad \frac{\partial\ p_t\, q_{i\,t}\, \eta^{\circ}{}_{i\,t}}{\partial q_{i\,t} \partial \eta_{i\,t}} = P\ Q\, \eta^{\circ}$$

ZKHUH $4_2\ 3\ \eta^{\circ}$ $3_2\ 4\ \eta^{\circ}$ GHQRWH SDUWLD0 GHULYDWLYHV ZLWK UHVSHFW WR WKH VHFRQG DUJXPHQW[8] HYD0XDWHG LQ η° DQG $4_{1,2}\ 3\ \eta^{\circ}$ $3_{1,2}\ 4\ \eta^{\circ}$ DUH FURVV SDUWLD0 GHULYDWLYHV HYD0XDWHG LQ η° 7KXV DW DQ\ SHULRG W IRU D JLYHQ SULFH 3 WKH GHYLDWLRQ IURP WKH H[SHFWHG WRWD0 TXDQWLW\ GHPDQGHG $;_t$ 3 FDQ EH GHWHUPLQHG DV

$$X_t\, P = Q\ P\, \eta^{\circ}\ \nu_t$$

---

[8] +HQFHIRUWK QXPHULFD0 VXEVFULSWV ZLOO GHQRWH SDUWLD0 GHULYDWLYHV XQ0HVV RWKHUZLVH VSHFLILHG

DQG WKH WRWDO GHYLDWLRQ IURP WKH SULFH 3 FRUUHVSRQGLQJ WR WKH H[SHFWHG WRWDO TXDQWLW\ GHPDQGHG 4 $<_t$ 4 LV

$$Y_t Q = P Q \eta^{\circ} \nu_t$$

6LQFH LPPHGLDWH FRPSXWDWLRQV DUH QRW SRVVLEOH DQG WKH ILUP V RXWSXW SULFH GHFLVLRQV KDYH WR EH PDGH SULRU WR WKH NQRZOHGJH RI WKH PDUNHW SULFH WKH UHVXOW FRPSXWHG LQ SHULRG W FDQ EH XVHG RQO\ LQ VXEVHTXHQW SHULRGV L H GHYLDWLRQV ; $_t$ 3 DQG $<_t$ 4 FDQ EH HVWLPDWHG EDVHG RQ WKH UHVXOWV FRPSXWHG LQ WKH SDVW DQG FRQVHTXHQWO\ D0ZD\V ZLWK FHUWDLQ HUURU ,W KDV WR EH VWUHVVHG KRZHYHU WKDW WKH YDULDQFH RI WKH HUURU LQ WKH HVWLPDWLRQ LQFUHDVHV ZLWK WKH WLPH HODSVHG IURP REVHUYDWLRQV RI LQGLYLGXDO GHPDQGV WR WKH PRPHQW ZKHQ GHFLVLRQV DUH PDGH VHH 5DQHU DQG 9DQ =DQGW IRU D GHWDLOHG GLVFXVVLRQ 7KHUHIRUH WKH PRQRSRO\ IDFHV QRW RQO\ WKH UDWKHU VWDQGDUG SURE0HP RI ILQGLQJ DSSURSULDWH HVWLPDWLRQ GHPDQG EXW DOVR WKH SURE0HP RI ILQGLQJ WKH RSWLPDO FRVW RI WKHVH HVWLPDWLRQ VLQFH WKH GDWD SURFHVVLQJ LV LQKHUHQW0\ FRVW\ DQG WKH DFTXLULQJ DQG DQDO\]LQJ RI PRUH SLHFHV RI LQIRUPDWLRQ DQG LQ SDUWLFXODU PRUH UHFHQW LQIRUPDWLRQ KDV WR EH ZHLJKHG DJDLQVW WKH LQFUHDVLQJ FRVW RI VXFK DQ HQGHDYRU

,Q JHQHUDO WKH ILUP PD\ ILQG LW DGYDQWDJHRXV WR FRPSXWH LQ VXEVHTXHQW SHULRGV VD\ W−P P GHYLDWLRQV IURP WKH PHDQ YD0XHV RI UDQGRP YDULDE0HV $\eta_{i,t-m}$ FRPLQJ IURP GLIIHUHQW VXEVHWV RI VRXUFHV VD\ 6_{t-m} P DQG XVH WKHP IRU WKH HVWLPDWLRQ RI WKH WRWDO GHYLDWLRQ IURP WKH H[SHFWHG GHPDQG LQ SHULRG W DV ZH ZLOO VHH 0DWHU UDWLRQDO VWUDWHJ\ UHTXLUHV WKDW VRXUFHV RI GHPDQG VKRX0G EH DQD0\]HG F\FOLFDO0\ RQH DIWHU WKH RWKHU

' HQRWH WKH UHVXOWV FRPSXWHG LQ VXEVHTXHQW SHULRGV DV

$$\nu_{t-m}^{S_{t-m}} \qquad \nu_{t-}^{S_{t-1}}$$

,I VXEVHWV 6_{t-m} 6_{t-1} FRQWDLQ Q_{t-m} Q_{t-1} $Q_{t-m} \geq$ $Q_{t-1} \geq$ VRXUFHV RI LQGLYLGXDO GHPDQGV UHVSHFWLYH0\ WKHQ WKHUH H[LVWV DQ LQWHJHU QXPEHU . $\leq . \leq 1$ VXFK WKDW

$$\sum_{i=}^{k} n_{t-i} \leq N \qquad \sum_{j=}^{K} n_{t-j}$$

7KXV WKH HVWLPDWH $\hat{v}_t$ RI WKH WRWDO GHYLDWLRQ ν_t EDQ EH FRPSXWHG DV

$$\hat{v}_t = \sum_{j=}^{k} \hat{v}_t^{S_{t-j}} + \frac{N - \sum_{j=}^{K} n_{t-j}}{n_{t-K-}} \hat{v}_t^{S_{t-K-1}}$$

ZKHUH $\hat{v}_t^{S_{t-m}}$LV D IRUHFDVW IRU SHULRG W RI WKH VXP RI GHYLDWLRQV IURP PHDQ YD0XHV RI UDQGRP YDULDE0HV $\eta_{i,t}$ FRPLQJ IURP WKH VRXUFHV LQF0XGHG LQ WKH VHW 6_{t-m} P .

6LQFH DOO DYDL0DE0H SUHGLFWLRQV RI SDUWLDO GHYLDWLRQV $\hat{v}_t^{S_{t-m}}$ P . FDQ EH UHSUHVHQWHG DV 0LQHDU FRPELQDWLRQV RI WKH WUXH YD0XHV RI FRUUHVSRQGLQJ SDUWLDO GHYLDWLRQV LQ WKH SDVW WKH H[SHFWHG YD0XH RI WKH HUURU LQ WKH SUHGLFWLRQ RI WKH WRWD0 GHYLDWLRQ $\tilde{v}_t = v_t - \hat{v}_t$HTXD0V]HUR)XUWKHUPRUH LWV YDULDQFH DVVXPLQJ WKDW GHYLDWLRQV IURP WKH H[SHFWHG YD0XHV RI LQGLYLGXD0 GHPDQGV $v_{i,t}$ DUH LQGHSHQGHQW LGHQWLFDO\ GLVWULEXWHG DQG WLPH LQYDULDQW LV

ZKHUH $$\sigma_t = Var\ \tilde{v}_t = \sum_{j=}^{K} n_{t-j}\sigma_{tj} + N - \sum_{j=}^{K} n_{t-j}\ \sigma_{t\,K-}$$

$$\sigma_{t\,m} = Var[v_{i\,t} - \hat{v}_{i\,t}\ m] = E\{[v_{i\,t} - \hat{v}_{i\,t}\ m]^2\}$$

LV WKH YDULDQFH RI WKH HUURU LQ HVWLPDWLRQ ZLWK 0DJ P P . RI WKH GHYLDWLRQ RI WKH UDQGRP YDULDE0H $\eta_{i,t}$ IURP LWV PHDQ YD0XH L 1 DQG $\hat{v}_{i\,t}\ m$ GHQRWHV WKH HVWLPDWLRQ ZLWK 0DJ P P . RI WKH GHYLDWLRQ RI WKH UDQGRP YDULDE0H $\eta_{i,t}$ IURP LWV PHDQ YD0XH L 1

7KH IRUHFDVW RI WKH WRWD0 GHPDQG DQG WKH WRWD0 LQYHUVH GHPDQG DUH JLYHQ DV

$$\hat{Q}_t\ P = Q\ P\ \eta^\circ + Q\ P\ \eta^\circ\ \hat{v}_t$$

DQG

$$\hat{P}_t\ Q = P\ Q\ \eta^\circ + P\ Q\ \eta^\circ\ \hat{v}_t$$

UHVSHFWLYH0\ ([SHFWHG HUURUV LQ WKHVH IRUHFDVWV HTXD0]HUR 6LQFH WRWD0 GHPDQG LQ SHULRG W FDQ EH VSHFLILHG DV 4_t 3 4 3 η° ; $_t$ 3 4 3 η° 4_2 3 η° v_t DQG WRWD0 LQYHUVH GHPDQG LQ SHULRG W FDQ EH UHSUHVHQWHG DV 3_t 4 3 4 η° $<_t$ 4 3 4 η° 3_2 4 η° v_t WKH YDULDQFHV RI WKH SUHGLFWLRQ HUURUV DUH GHWHUPLQHG DV

$$P\ -\hat{Q}_t P\ ^2 = Var[\ Q\ P\ \eta^\circ\ v_t\ -\hat{v}_t\ ] = Q\ P\ \eta^\circ\ V_t$$

DQG

$$Q - \hat{P}_t Q @ = Var>P \quad Q\eta° \quad \nu_t - \hat{\nu}_t @ = P \quad Q\eta° \quad Va$$

UHVSHFWLYHO\

$VVXPLQJ WKDW WKH H[SHFWHG GHPDQG FXUYH DQG WKH VKDSH RI WKH SUREDELOLW\ GLVWULEXWLRQ RI WKH IRUHFDVW HUURU DUH NQRZQ XQFHUWDLQ GHPDQG ZKLFK WKH ILUP IDFHV LQ WKH SHULRG W LV FKDUDFWHUL]HG E\ WKH YDULDQFH RU WKH VWDQGDUG GHYLDWLRQ RI WKH SUHGLFWLRQ HUURU L H XQFHUWDLQ GHPDQG DQG LQYHUVH GHPDQG FXUYHV LQ WKH SHULRG W DUH GHVFULEHG E\ $t 3 σt η° DQG 3t 4 σt η° UHVSHFWLYHO\

7DNLQJ LQWR DFFRXQW WKDW YDULDELOLW\ RI GHPDQG GHFUHDVHV WKH TXDOLW\ RI RXWSXW SULFH GHFLVLRQV L H SULFH RXWSXW GHFLVLRQV GHYLDWH IURP WKH RSWLPDO WKDW ZRXOG EH PDGH LI WKH YDULDQFH ZHUH HTXDO WR]HUR DQG WKDW UHVXOWV RI GHPDQG DQD\VLV FDQ EH XVHG RQO\ DIWHU WKH HQG RI WKH SHULRG LQ ZKLFK WKH\ ZHUH FRPSXWHG WKH VPDOOHVW YDULDQFH RI WKH SUHGLFWLRQ HUURU FRUUHVSRQGV WR WKH FDVH ZKHQ DOO VRXUFHV RI GHPDQG DUH DQD\]HG LQ WKH SUHFHGLQJ SHULRG 7KH DQD\VLV RI WKH WRWDO GHPDQG LQ HDFK SHULRG KROZHYHU UHTXLUHV WKDW D QXPEHU RI HFRQRPLF UHVRXUFHV WR GHYRWHG WR GDWD SURFHVVLQJ LQ WKH ILUP L H LW LQGXFHV VLJQLILFDQW FRVWV WKDW FDQQRW DZD\V EH RIIVHW E\ WKH H[SHFWHG EHQHILW IURP WKH RXWSXW SULFH GHFLVLRQ ZLWK D 0RZHU ULVN RI HUURU 7KXV LQVWHDG RI H[DPLQLQJ GHPDQG FRPLQJ IURP DOO VRXUFHV LQ HDFK SHULRG WKH ILUP FDQ VHTXHQWLDOO\ DQD\]H GHPDQGV FRPLQJ IURP FHUWDLQ VXEVHWV RI VRXUFHV RI GHPDQG ,Q WKLV FDVH KROZHYHU WKH ILUP KDV WR GHWHUPLQH WKH RSWLPDO QXPEHUV RI VRXUFHV RI GHPDQG WKDW VKRXOG EH DQD\]HG LQ VXFFHHGLQJ SHULRGV

3. Data-Processing in Forecasting Problems

,W IROORZV IURP WKH SUHYLRXV VHFWLRQ WKDW LQ WKH PRGHO FRQVLGHUHG WKH SXUSRVH RI WKH PDUNHW DQD\VLV VHFWRU RI WKH ILUP LV WR VXPPDUL]H WKH DFWXDO GHYLDWLRQV IURP WKH PHDQ YDOXHV RI LQGLYLGXDO GHPDQGV FRPLQJ IURP D FHUWDLQ QXPEHU RI VRXUFHV Q Q≤1 ZKHUH 1 LV WKH WRWDO QXPEHU RI VRXUFHV RI GHPDQG 7DNLQJ LQWR DFFRXQW WKDW DGGLWLRQ LV WKH DVVRFLDWLYH RSHUDWLRQ FRPSXWDWLRQDO SURFHVVHV LQ WKH IRUHFDVWLQJ SUREOHPV FDQ EH GHVFULEHG E\ WKH G\QDPLF SDUDOOHO SURFHVVLQJ PRGHO RI DVVRFLDWLYH FRPSXWDWLRQ XVHG IRU WKH PRGHOOLQJ LQIRUPDWLRQ SURFHVVLQJ LQ HQWHUSULVHV VHH 5DGQHU DQG 5DGQHU DQG 9DQ =DQGW DQG RU /LSPDQ

,Q WKLV PRGHO FRPSXWDWLRQDO SURFHVVHV LQ WKH ILUP DUH UHSUHVHQWHG DV LQ DQ LGHDOL]HG SDUDOOHO FRPSXWHU L H LW LV DVVXPHG WKDW HDFK FRPSXWDWLRQDO HOHPHQW LQ WKH ILUP LV PRGHOOHG DV D SURFHVVRU ZKLFK FRQWDLQV

DQ LQILQLWH PHPRU\ ZKHUH GDWD DUH VWRUHG FDOOHG D EXIIHU DQG D UHJLVWHU ZKHUH VXPPDWLRQV DUH PDGH (DFK SURFHVVRU FDQ UHDG D VLQJOH LWHP RI GDWD IURP LWV PHPRU\ PXOLS\ WKH YDOXH E\ RQH RU DQ\ RWKHU FRQVWDQW DQG DGG WKH UHVXOW WR WKH UHJLVWHU UHVHWWLQJ LW HTXDO WR WKH UHVXOWLQJ VXP /RDGLQJ DQG DGGLQJ D VLQJOH GDWXP WR WKH FRQWHQWV RI WKH UHJLVWHU LV FDOOHG DQ RSHUDWLRQ 7KH WLPH LV DVVXPHG WR EH WKH VDPH ZKDWHYHU WKH YDOXHV RI GDWD DGGHG DUH RU ZKHQ D GDWXP LV DGGHG WR WKH FOHDUHG UHJLVWHU L H WR]HUR 0RUHRYHU D SURFHVVRU FDQ VHQG WKH FRQWHQWV RI LWV UHJLVWHU WR DQ RXWSXW RU WR WKH EXIIHU RI DQ\ RWKHU SURFHVVRU WKURXJK D FRPPXQLFDWLRQ FKDQQHO LQ]HUR WLPH VHH 5DGQHU DQG 9DQ =DQGW IRU GHWDLOV

(DFK SURFHVVRU KDV D OLPLWHG FDSDFLW\ LQ WKDW WKHUH LV D PD[LPXP QXPEHU RI RSHUDWLRQV LW FDQ FRPSXWH SHU XQLW RI WLPH +RZHYHU WKH VSHHG RI FRPSXWDWLRQ LQ HDFK LQGLYLGXDO SURFHVVRU GHSHQGV RQ WKH UHVRXUFHV DOORFDWHG WR LW 7KH UHODWLRQVKLS EHWZHHQ WKH UHVRXUFHV DOORFDWHG WR D VLQJOH SURFHVVRU DQG WKH QXPEHU RI RSHUDWLRQV LW FDQ FRPSXWH LQ D XQLW RI WLPH LV GHWHUPLQHG E\ WKH WHFKQRORJ\ RI GDWD SURFHVVLQJ DQG JLYHQ LQ IXQFWLRQDO IRUP DV WKH LQIRUPDWLRQ SURFHVVLQJ IXQFWLRQ) N 5₊→5₊ 7KLV IXQFWLRQ LV XQGHUVWRRG DV D SURGXFWLRQ IXQFWLRQ LQ LQIRUPDWLRQ SURFHVVLQJ DQG VSHFLILHV WKH QXPEHU RI RSHUDWLRQV SHU XQLW RI WLPH WKDW FDQ EH PDGH LQ D VLQJOH SURFHVVRU WR ZKLFK UHVRXUFHV N DUH DOORFDWHG WR VLPSOLI\ WKH DQDO\VLV ZH DVVXPH WKDW WKHUH LV RQ\ RQH IDFWRU RI SURGXFWLRQ $Q LPSRUWDQW DVVXPSWLRQ JHQHUDO\ PDGH DERXW SURGXFWLRQ IXQFWLRQ LV GLPLQLVKLQJ PDUJLQDO SURGXFW RI LQSXW ,Q WKH DQDO\VLV EHORZ ZH ZLOO DVVXPH WKDW WKLV LV DOVR D SURSHUW\ RI WKH LQIRUPDWLRQ SURFHVVLQJ IXQFWLRQ 7KHUHIRUH ZH ZLOO DVVXPH WKDW WKH LQIRUPDWLRQ SURFHVVLQJ IXQFWLRQ LV VWULFW\ LQFUHDVLQJ DQG FRQFDYH LQ N L H G) N G N ! G²) N GN²

7KH GXUDWLRQ RI D VLQJOH RSHUDWLRQ G FDQ EH GHWHUPLQHG DV) N DQG FRQVHTXHQW\ LW LV DOVR D IXQFWLRQ RI WKH UHVRXUFHV N HPSOR\HG LQ WKH SURFHVVRU FRQVLGHUHG G N) N ,Q JHQHUDO GLIIHUHQW DPRXQWV RI WKH UHVRXUFHV FRXOG EH DOORFDWHG WR HDFK LQGLYLGXDO SURFHVVRU VHH &XNURZVNL KRZHYHU LQ RUGHU WR VLPSOLI\ WKH DQDO\VLV DQG IRFXV RQ WKH HIIHFWV RI GHPDQG IRUHFDVWLQJ RQ WKH PRQRSROLVWLF HTXLOLEULXP ZH ZLOO DVVXPH WKDW DOO SURFHVVRUV DUH LGHQWLFDO DQG WKHLU QXPEHU LV H[RJHQRXV\ JLYHQ

(DFK SURFHVVRU DGGV GDWD LWHPV LQ D VHULDO IDVKLRQ 7KXV WR VSHHG XS WKH FRPSXWDWLRQDO SURFHVV GDWD SURFHVVLQJ FDQ EH GRQH LQ SDUDOOHO XVLQJ PRUH WKDQ RQH SURFHVVRU L H LQ D GHFHQWUDOL]HG FRPSXWDWLRQDO VWUXFWXUH *LEERQV DQG 5\WWHU VKRZ VHH DOVR 5DGQHU WKDW WKH OHQJWK RI WKH VKRUWHVW VHTXHQFH RI RSHUDWLRQV QHHGHG WR DGG Q GDWD LWHPV LQ GHFHQWUDOL]HG VWUXFWXUH ZLWK 5 LGHQWLFDO SURFHVVRUV LV GHWHUPLQHG E\ WKH IROORZLQJ H[SUHVVLRN

$$C_R(n) \quad \lfloor \ \rfloor \ \lceil 0\mathrm{RJ}_2\, R \quad n\, \mathrm{PRG}\, R \rceil$$

ZKHUH EUDFNHWV ⌊ ⌋ DQG ⌈ ⌉ GHQRWH URXQGLQJ GRZQ DQG XS WR WKH QHDUHVW LQWHJHU UHVSHFWLYH0\

5DQGQHU VKRZV WKDW LI Q 5 DQG Q 5 DUH DOO 0DUJH WKHQ WKH 0HQJK RI WKH VKRUWHVW VHTXHQFH RI RSHUDWLRQV QHHGHG WR VXPPDUL]H Q GDWD LWHPV LQ WKH VWUXFWXUH ZLWK 5 SURFHVVRUV FDQ EH DSSUR[LPDWHG DV

$$\hat{C}_R\, n \ = \frac{n}{R} + 0\mathrm{RJ}\, R$$

7DNLQJ LQWR DFFRXQW WKDW HDFK RSHUDWLRQ WDNHV G N XQLWV RI WLPH WKH GH0D\ LQ WKH FRPSXWDWLRQDO SURFHVV LQ WKH VWUXFWXUH ZLWK LGHQWLFDO SURFHVVRUV FDQ EH DSSUR[LPDWHG DV

$$\hat{D}_P\, n \ = \frac{\hat{C}_P\, n}{F\, k}$$

,W IROORZV IURP DQG WKDW WKH QXPEHU RI GDWD LWHPV ZKLFK FDQ EH VXPPDUL]HG LQ RQH SHULRG FDQ EH VSHFLILHG DV

$$Q \quad 5) \ N \ - \ 5\ 0\mathrm{RJ}_2 5$$

7KH FRVW RI UHVRXUFHV QHHGHG WR VXPPDUL]H Q GDWD LWHPV LQ WKH VWUXFWXUH ZLWK 5 SURFHVVRUV HTXDO0V

$$5\rho N \quad 5\rho)^{-1}\ Q\ 5 \quad 0\mathrm{RJ}_2 5$$

ZKHUH ρ LV WKH XQLW FRVW RI WKH UHVRXUFHV DOORFDWHG WR FRPSXWDWLRQDO SURFHVV 7KH WRWDO FRVW RI GDWD SURFHVVLQJ LQ D VLQJOH SHULRG LV

$$9_R\ Q \quad Q\gamma \quad 5\rho)^{-1}\ Q\ 5 \quad 0\mathrm{RJ}_2 5$$

ZKHUH Qγ LV WKH FRVW RI JDWKHULQJ GDWD XVHG LQ FRPSXWDWLRQ γ LV WKH XQLW FRVW RI GDWD LWHPV

,Q WKH DQD0\VLV EHORZ ZH ZLOO WKLQN RI Q DV D FRQWLQXRXV YDULDE0H UDWKHU WKDQ LQWHJHU YD0XHG DQG FRQVHTXHQW0\ ZH ZLOO FRQVLGHU WKH GDWD SURFHVVLQJ FRVW IXQFWLRQ 9_R Q DV D FRQWLQXRXV IXQFWLRQ RI Q 7KH VKDSH RI WKLV IXQFWLRQ LV GHWHUPLQHG E\ WKH IRUP RI WKH LQIRUPDWLRQ SURFHVVLQJ IXQFWLRQ) N ,Q SDUWLFX0DU LI WKH LQIRUPDWLRQ SURFHVVLQJ IXQFWLRQ) N LV FRQFDYH LQ N WKHQ WKH IXQFWLRQ

9_R Q LV VWULFWO\ LQFUHDVLQJ DQG FRQYH[LQ Q L H $G9_R$ Q GQ! $G^2 9_R$ Q GQ²! 7KLV IRUP RI WKH GDWD SURFHVVLQJ FRVW IXQFWLRQ ZLOO EH DVVXPHG LQ WKH DQDO\VLV EHORZ

4. Monopoly Under Uncertainty

: H QRZ PRYH RQ WR RXU PDLQ IRFXV RI LQWHUHVW WKH EHKDYLRU RI D PRQRSROLVWLF ILUP RSHUDWLQJ LQ XQLPLWHG WLPH LQ WKH ZRUOG ZLWK XQFHUWDLQ GHPDQG DV GHVFULEHG LQ 6HFWLRQ ZKHQ WKH JDWKHULQJ DQG SURFHVVLQJ RI LQIRUPDWLRQ DUH FRVWO\ 7R FRQFHQWUDWH RQ WKH QRUPDWLYH LVVXH RI WKH SULFH RXWSXW GHFLVLRQV RI WKH ILUP ZH DVVXPH WKDW WKHUH LV QR VSLOO EHWZHHQ VKDUHKROGHUV DQG PDQDJHPHQW XWLOLW\ IXQFWLRQV L H PDQDJHUV RI WKH ILUP PD[LPL]H WKH H[SHFWHG XWLOLW\ RI WKHLU VKDUHKROGHUV DV D UHVXOW WKH SULQFLSDO DJHQW SUREOHP LV DYRLGHG DQG WKH GHFLVLRQV LQ WKH ILUP DUH PDGH E\ D JURXS RI GHFLVLRQ PDNHUV ZLWK VXIILFLHQWO\ VLPLODU SUHIHUHQFHV WR JXDUDQWHH WKH H[LVWHQFH RI D JURXS SUHIHUHQFH IXQFWLRQ [9] ,W HQVXUHV WKDW WKH EHKDYLRU RI WKH ILUP XQGHU GHPDQG UDQGRPQHVV REH\V WKH D[LRPV RI WKH 1HXPDQ 0RUJHQVWHUQ XWLOLW\ WKHRU\

7DNLQJ LQWR DFFRXQW WKDW WKH OLIH RI WKH ILUP LV XQOLPLWHG WKH ILUP V RSWLPL]DWLRQ SUREOHP FDQ EH UHSUHVHQWHG DV WKH IROORZLQJ LQILQLWH KRUL]RQ GLVFRXQWHG G\QDPLF SURJUDPPLQJ SUREOHP

$$\max_{Q_t \; n_t} \; \sum_{t=}^{\infty} \beta^t \; \mathrm{E}\{ U> \Pi_t \; Q_t \; \sigma_t \; \eta^\circ \; n_t \; @ \}$$

ZKHUH σ_{t+1} J σ_t Q_t ZLWK σ_0 1$\omega^{2\ 1/2}$

E LV DQ H[SHFWDWLRQ RSHUDWRU
8 · GHQRWHV WKH XWLOLW\ IXQFWLRQ RI WKH UHSUHVHQWDWLYH VKDUHKROGHU
Π_t · LV WKH SURILW RI WKH ILUP LQ WKH SHULRG W W
4_t LV D TXDQWLW\ SURGXFHG LQ WKH SHULRG W W
Q_t GHQRWHV WKH QXPEHU RI LQGLYLGXDO GHPDQGV DQD\]HG LQ WKH SHULRG W W
σ_t LV WKH VWDQGDUG GHYLDWLRQ RI WKH HUURU LQ WKH SUHGLFWLRQ RI WKH WRWDO GHYLDWLRQ RI WKH UDQGRP YDULDEOHV $\eta_{i,t}$ L 1 IURP WKHLU PHDQV LQ WKH SHULRG W W
η° LV WKH VXP RI WKH PHDQ YD0XHV RI UDQGRP YDULDEOHV $\eta_{i,t}$ L 1
1 LV WKH WRWDO QXPEHU RI VRXUFHV RI GHPDQG
ω^2 LV WKH YDULDQFH RI WKH VWRFKDVWLF SURFHVV XQGHU\LQJ HDFK LQGLYLGXDO

[9] See, for example, Sandmo (1971) for a detail discussion.

GHPDQG DURXQG LWV PHDQ
β LV WKH GLVFRXQW IDFWRU β∈

7KH FRVW RI JDWKHULQJ DQG SURFHVVLQJ LQIRUPDWLRQ LQ D JLYHQ SHULRG DQG WKH EHQHILWV IURP WKLV DFWLYLW\ LQ WKH IXWXUH SHULRGV L H VPDOOHU YDULDQFH RI WKH SUHGLFWLRQ HUURU VSHFLI\ WKH 0LQN ZKLFK FRQQHFWV WKH SUHVHQW ZLWK WKH IXWXUH ,Q RWKHU ZRUGV LQ WKH PRGHO FRQVLGHUHG WKHUH LV DQ LQWHUWHPSRUDO WUDGH RII EHWZHHQ KLJKHU FRVWV RI GDWD SURFHVVLQJ WRGD\ DQG WKH IXWXUH EHQHILWV LQ WKH IRUP RI D KLJKHU H[SHFWHG XWLOLW\ 7KXV DORQJ WKH RSWLPDO SDWK WKH FRVW RI DQDO\]LQJ RQH DGGLWLRQDO VRXUFH RI GHPDQG LQ D SHULRG M M KDV WR EH HTXDO]HG ZLWK WKH VXP RI WKH GLVFRXQWHG PDUJLQDO EHQHILWV LQ DOO IXWXUH SHULRGV L H

$$\frac{dV_R\ n_j}{dn_j} = \sum_{t=j+}^{\infty} \beta^t \frac{\partial E\{U>\Pi_t\ Q_t\ \sigma_t\ \eta^\circ\ n_t\ @\}}{\partial \sigma_t} \frac{\partial \sigma_t}{\partial n_j}$$

$VVXPLQJ WKDW DOO WKH SDUDPHWHUV RI WKH PRGHO DUH VWDWLRQDU\ RYHU WLPH WKH RSWLPDO VROXWLRQ WR DQ LQILQLWH KRUL]RQ GLVFRXQWHG G\QDPLF SURJUDPPLQJ SUREOHP LV WLPH LQYDULDQW VHH H J 6DUJHQW 7KXV LQ WKH SUREOHP FRQVLGHUHG WKH RSWLPDO RXWSXW DQG GHPDQG SUHGLFWLQJ VWUDWHJ\ DUH VWDWLRQDU\ L H $_0^* $_1^* $_2^* $^* DQG Q_0^* Q_1^* Q_2^* Q^* ,W LPSOLHV WKDW WKH RSWLPDO YDOXH RI WKH VWDQGDUG GHYLDWLRQ σ^* RI WKH HUURU LQ WKH SUHGLFWLRQ RI WKH WRWDO GHYLDWLRQ RI WKH UDQGRP YDULDEOHV η_{i,t} L 1 IURP WKHLU PHDQV LV VWDWLRQDU\ DQG GHSHQGV RQO\ RQ WKH QXPEHU RI LQGLYLGXDO GHPDQGV DQD\]HG LQ HYHU\ SHULRG σ^* σ Q^* ,W IROORZV WKDW WR HDFK YDOXH RI WKH VWDQGDUG GHYLDWLRQ σ Q^* WKH XQLTXH RQH SHULRG FRVW RI GDWD SURFHVVLQJ FDQ EH UHGXFHG L H WKH FRVW RI GDWD SURFHVVLQJ LQ HDFK SHULRG FDQ EH UHSUHVHQWHG DV D IXQFWLRQ RI WKH VWDQGDUG GHYLDWLRQ LQ WKH VWHDG\ VWDWH

$$*>\sigma\ Q^*\ @ \equiv 9_R\ Q^*$$

6LQFH WKH VWDWLRQDU\ VWDQGDUG GHYLDWLRQ σ Q^* GHFUHDVHV LI WKH QXPEHU RI LQGLYLGXDO GHPDQGV DQD\]HG LQ HDFK SHULRG LQFUHDVHV WKH FRVW RI GDWD SURFHVVLQJ LV D GHFUHDVLQJ IXQFWLRQ RI WKH VWHDG\ VWDWH VWDQGDUG GHYLDWLRQ GLIIHUHQWLDWLQJ ZLWK UHVSHFW WR Q^* JLYHV G9_R Q^* GQ^* ∂* ∂σ^* Gσ GQ^* ! VLQFH G9_R Q^* GQ^*! DQG Gσ GQ^* LW IROORZV WKDW ∂* ∂σ 0 RUHRYHU WKH VKDSH RI WKH IXQFWLRQ 9_R Q^* G9_R Q^* GQ^*! G²9_R Q^* GQ^{*2}! VHH 6HFWLRQ LPSOLHV WKDW * LV D FRQYH[IXQFWLRQ RI σ L H ∂²*>σ Q^* @ ∂σ²!

7KH FRQVLGHUDWLRQ DERYH VKRZV WKDW WKH RSWLPL]DWLRQ SUREOHP XQGHU VWXG\ FDQ EH VROYHG LQ WZR VWHSV)LUVW WKH RSWLPDO RXWSXW $^* DQG WKH RSWLPDO YDOXH RI VWDQGDUG GHYLDWLRQ σ^* FDQ EH GHWHUPLQHG DQG VHFRQG NQRZLQJ σ^* WKH RSWLPDO VL]H RI FRKRUWV RI GDWD VXPPDUL]HG LQ HDFK SHULRG FDQ EH IRXQG

7KXV LQ WKH ILUVW VWDJH WKH ILUP FKRRVHV WKH VWHDG\ VWDWH TXDQWLW\ RI RXWSXW $ DQG WKH YDOXH RI WKH VWDQGDUG GHYLDWLRQ σ ZKLFK PD[LPL]H WKH IROORZLQJ REMHFWLYH IXQFWLRQ

$$\Psi\ Q\ \sigma\ \eta^\circ = \mathrm{E}\{U>\Pi\ Q\ \sigma\ \eta^\circ\ @\}$$

ZKHUH

Π 4 σ η° LV WKH VWHDG\ VWDWH SURILW RI WKH ILUP

Π 4 σ η° 4 3 4 σ η° − & 4 − = − * σ

3 4 σ η° GHQRWHV DQ XQFHUWDLQ LQYHUVH GHPDQG FXUYH
& 4 LV D YDULDEOH FRVW
= LV D IL[HG FRVW
* σ LV WKH FRVW RI GDWD SURFHVVLQJ ZKLFK FRUUHVSRQGV WR WKH VWHDG\ VWDWH VWDQGDUG GHYLDWLRQ σ G* σ Gσ DQG G² * σ Gσ² !

7R VLPSOLI\ WKH DQDO\VLV DVVXPH WKDW WKH VWHDG\ VWDWH HUURU LQ SUHGLFWLRQ RI WKH WRWDO GHYLDWLRQ IURP WKH PHDQ YDOXH η° $\tilde{\nu} = \tilde{\nu}_t = \nu_t - \hat{\nu}_t$ RU DQ\ W LV D QRUPDOO\ GLVWULEXWHG UDQGRP YDULDEOH ZLWK]HUR PHDQ DQG YDULDQFH σ^2 LW FRUUHVSRQGV WR WKH FDVH ZKHQ UDQGRP GHYLDWLRQV IROORZ VWRFKDVWLF SURFHVVHV ZLWK QRUPDOO\ GLVWULEXWHG UDQGRP WHUPV VXFK DV IRU H[DPSOH WKH DXWRUHJUHVVLYH SURFHVV RI DQ\ RUGHU [10]

8QGHU WKLV DVVXPSWLRQ WKH H[SHFWHG LQYHUVH GHPDQG FXUYH ZKHQ WKH DFWXDO GHPDQG LV XQGHUHVWLPDWHG $\tilde{\nu}$ LV VSHFLILHG DV

$$\underline{P}\,Q\,\sigma\,\eta^\circ = P\,Q\,\eta^\circ + \int_{-\infty} \frac{\tilde{\nu}}{\sqrt{\pi\sigma}\;P\;Q\,\eta^\circ} e^{-\frac{\tilde{\nu}^2}{P_2\,Q\,\eta^\circ\,^2\sigma^2}}\, d\tilde{\nu} =$$

$$= P\text{-}Q\,\eta^\circ - P\;Q\,\eta^\circ \frac{\sigma}{\sqrt{\pi}}$$

6LPLODUO\ WKH H[SHFWHG LQYHUVH GHPDQG FXUYH ZKHQ WKH DFWXDO GHPDQG LV RYHUHVWLPDWHG $\tilde{\nu} \geq$ HTXDOV

$$\overline{P}\,Q\,\sigma\,\eta^\circ = P\,Q\,\eta^\circ + \int^{\infty} \frac{\tilde{\nu}}{\sqrt{\pi\sigma}\;P\;Q\,\eta^\circ} e^{-\frac{\tilde{\nu}^2}{P_2\,Q\,\eta^\circ\,^2\sigma^2}}\, d\tilde{\nu} =$$

$$= P\,Q\,\eta^\circ + P\;Q\,\eta^\circ \frac{\sigma}{\sqrt{\pi}}$$

---

[10] It should be stressed that, although the assumption of normal distribution of the random deviations from the expected demand corresponds to the wide class of stochastic processes that would govern stochastic demand, it is chosen solely for simplicity and clarity, and no attempt is made at generality. We believe, however, that many of the qualitative results would hold also in more general, and consequently, more complicated models.

7KH H[SHFWHG YD0XHV RI WKH ILUP V SURILWV ZKHQ WKH DFWXD0 GHPDQG LV XQGHUHVWLPDWHG 0RZ SURILW DQG RYHUHVWLPDWHG KLJK SURILW HTXD0

$$\underline{\Pi}\ Q\ \sigma\ \eta^{\circ} \;=\; Q\underline{P}\ Q\ \sigma\ \eta^{\circ} \;-\; \mathcal{C}\ Q \;-\; Z \;-\; G\ \sigma$$

DQG

$$\overline{\Pi}\ Q\ \sigma\ \eta^{\circ} \;=\; Q\overline{P}\ Q\ \sigma\ \eta^{\circ} \;-\; C\ Q \;-\; Z \;-\; G\ \sigma$$

UHVSHFWLYH0\

7KH SUREDELOLW\ RI KLJK SURILW HTXD0V VLPLODU\ WKH SUREDELOLW\ RI 0RZ SURILW HTXD0V DQG WKHUHIRUH WKH H[SHFWHG FDQ EH HVWLPDWHG DV

$$\Psi\ Q\ \sigma\ \eta^{\circ} \;=\; -U\!>\ \underline{\Pi}\ Q\ \sigma\ \eta^{\circ}\ @ \;+\; -U\!>\ \overline{\Pi}\ Q\ \sigma\ \eta^{\circ}\ @$$

)LQD00\ WKH ILUP KDV WR ILQG ERWK WKH TXDQWLW\ RI RXWSXW 4* DQG WKH VWDQGDUG GHYLDWLRQ σ* WKDW PD[LPL]H WKH IR0RZLQJ REMHFWLYH IXQFWLRQ

$$\Psi\ Q\ \sigma\ \eta^{\circ} \;=\; -U\ \{\ -\Pi\ Q\ \eta^{\circ} \;-\; QP\ Q\ \eta^{\circ}\ \frac{\sigma}{\sqrt{\pi}} \;-\; G\ \sigma\ \}\ +$$

$$+\ -U\ \{\ \Pi\ Q\ \eta^{\circ} \;+\; QP\ \ Q\ \eta^{\circ}\ \frac{\sigma}{\sqrt{\pi}} \;-\; G\ \sigma\ \}$$

ZKHUH

$$\Pi\ 4\ \eta^{\circ} \quad 4\ 3\ 4\ \eta^{\circ} \;-\; \&\ 4 \;-\; =$$

LV WKH SURILW IXQFWLRQ RI WKH ILUP LI WKH LQYHUVH GHPDQG FXUYH LV 3 4 η° L H ZKHQ WKHUH LV QR XQFHUWDLQW\ RI GHPDQG IRU DQ\ IL[HG YD0XH RI η° WKLV IXQFWLRQ LV DVVXPHG WR EH VWULFW0\ FRQFDYH LQ 4 Π_1 4 η° ! DQG $\Pi_{1,1}$ 4 η°

: KHQ WKH RSWLPD0 YD0XHV 4* σ* DUH FRPSXWHG WKH RSWLPD0 VL]H RI WKH FRKRUWV SURFHVVHG LQ HDFK SHULRG Q* FDQ EH GHWHUPLQHG DV Q* 9_R^{-1}>*-1 σ* @

5. Optimal Steady-State Behavior

7R GHWHUPLQH WKH RSWLPD0 VWHDG\ VWDWH YD0XHV RI WKH TXDQWLW\ SURGXFHG 4* DQG WKH RSWLPD0 YD0XH RI WKH VWDQGDUG GHYLDWLRQ σ* GLIIHUHQWLDWH WKH REMHFWLYH IXQFWLRQ

ZLWK UHVSHFW WR 4 DQG σ DQG UHSUHVHQW WKH ILUVW RUGHU FRQGLWLRQV RI WKH ILUP V RSWLPL]DWLRQ SUREOHP DV

$$\frac{\partial \Psi\ Q\ \sigma\ \eta^{\circ}}{\partial Q} = -U'\ \Pi\ \{\ MR - MC - \frac{\partial > QP\ \ Q\ \eta^{\circ}\ @}{\partial Q}\frac{\sigma}{\sqrt{\pi}}\ \} +$$

$$+ -U'\ \overline{\Pi}\ \{\ MR - MC + \frac{\partial > QP\ \ Q\ \eta^{\circ}\ @}{\partial Q}\frac{\sigma}{\sqrt{\pi}}\ \} =$$

ZKHUH

$$05 - 0\&\ \ 3\ 4\ \eta^{\circ}\ \ 43_{1}\ 4\ \eta^{\circ}\ - G\&\ 4\ \ G4$$

LV WKH GLIIHUHQFH EHWZHHQ WKH YD0XH RI PDUJLQDO UHYHQXH DQG PDUJLQDO FRVW LI WKH GHPDQG LV NQRZQ ZLWK FHUWDLQW\ DQG

$$\frac{\partial \Psi\ Q\ \sigma\ \eta^{\circ}}{\partial \sigma} = -U'\ \Pi\ > -\frac{QP\ \ Q\ \eta^{\circ}}{\sqrt{\pi}} - \frac{dG\ \sigma}{d\sigma}\ @ +$$

$$+ -U'\ \overline{\Pi}\ > \frac{QP\ \ Q\ \eta^{\circ}}{\sqrt{\pi}} - \frac{dG\ \sigma}{d\sigma}\ @ =$$

7KH VHFRQG RUGHU FRQGLWLRQV IRU WKLV PD[LPL]DWLRQ UHTXLUHV WKH +HVVLDQ RI WKH REMHFWLYH IXQFWLRQ

$$\begin{pmatrix} \Psi_{1,1}\ 4\ \sigma\ \eta^{\circ} & \Psi_{1,2}\ 4\ \sigma\ \eta^{\circ} \\ \Psi_{1,2}\ 4\ \sigma\ \eta^{\circ} & \Psi_{2,2}\ 4\ \sigma\ \eta^{\circ} \end{pmatrix}$$

WR EH QHJDWLYH GHILQLWH LW JXDUDQWHHV WKDW WKH REMHFWLYH IXQFWLRQ LV VWULFWO\ FRQFDYH 2QH FDQ VKRZ WKDW LI WKH SURILW IXQFWLRQ ZLWKRXW GHPDQG XQFHUWDLQW\ LV VWULFWO\ FRQFDYH DQG WKH GDWD SURFHVVLQJ FRVW IXQFWLRQ LV FRQYH[G^{2}* σ Gσ^{2}≥ WKHQ WKH VHFRQG RUGHU FRQGLWLRQ KR0GV IRU WKH 0LQHDU DQG FRQFDYH XWLOLW\ IXQFWLRQV VHH WKH $SSHQGL[IRU D GHWDLO DQD0\VLV &RQVHTXHQW\ WKH RSWLPD0 VR0XWLRQ WR WKH RSWLPL]DWLRQ SUREOHP XQGHU FRQVLGHUDWLRQ D0ZD\V H[LVWV DQG LV XQLTXH

)URP WKH ILUVW RUGHU FRQGLWLRQV ZH FDQ LPPHGLDWHO\ GHULYH D VHULHV RI UHVXOWV 5HSUHVHQW MRQG MC D∀ $\frac{\sigma}{\sqrt{\pi}}$ $\frac{\partial > QP\ \ Q\ \eta^{\circ}\ @}{\partial Q}$ B $\frac{U'\ \overline{\Pi}\ \ U'\ \Pi}{+\ U'\ \overline{\Pi}\ \ U'\ \Pi}$

–

DQG

$$\frac{dG\ \sigma}{d\sigma} = \frac{QP\ \ Q\ \eta^{\circ}}{\sqrt{\pi}}\ \frac{U'\ \overline{\Pi}\ \ U'^{B}\underline{\Pi}\ \ -}{U'\ \overline{\Pi}\ \ U'\ \underline{\Pi}\ \ +}$$

UHVSHFWLYHO\ 5HFDOO WKDW WKH FRVW RI GDWD SURFHVVLQJ GHFUHDVHV ZLWK WKH VWDQGDUG GHYLDWLRQ RI WKH SUHGLFWLRQ HUURU L H G* σ Gσ ,W IROORZV WKDW FRQGLWLRQ FDQ EH VDWLVILHG RQO\ LI WKH ULJKW KDQG VLGH RI WKH H[SUHVVLRQ DERYH LV QHJDWLYH 7DNLQJ LQWR DFFRXQW WKDW $ Q σ η° $ Q σ η°IRU DQ\ σ! WKH FRQGLWLRQ LV VDWLVILHG RQO\ LI WKH XWLOLW\ IXQFWLRQ 8 Π LV VWULFWO\ FRQFDYH L H G8 Π GΠ! DQG G²8 Π GΠ² RWKHUZLVH WKH REMHFWLYH IXQFWLRQ Ψ 4 σ η° DOZD\V LQFUHDVHV ZLWK σ L H ∂Ψ 4 σ η° ∂σ!

7KH FXUYDWXUH RI 8 Π UHIOHFWV WKH VKDUHKROGHUV DWWLWXGH WRZDUGV ULVN 6WULFWO\ FRQFDYH XWLOLW\ IXQFWLRQ G8 Π GΠ! DQG G²8 Π GΠ² FRUUHVSRQGV WR ULVN DYHUVH EHKDYLRU 0LQHDU XWLOLW\ G8 Π GΠ! DQG G²8 Π GΠ² GHVFULEHV ULVN QHXWUDOLW\ DQG WKH VWULFWO\ FRQYH[XWLOLW\ IXQFWLRQ G8 Π GΠ! DQG G²8 Π GΠ²! FRUUHVSRQGV WR ULVN 0RYLQJ HFRQRPLF DJHQWV 1RWH WKDW LI WKH XWLOLW\ IXQFWLRQ LV 0LQHDU RU VWULFWO\ FRQYH[WKHQ LW LV VXUH\ QRW LQ WKH ILUP V LQWHUHVW WR IRUHFDVW GHPDQG &RQVHTXHQW\ RQ\ D ULVN DYHUVH ILUP ZLOO EH ZLOOLQJ WR DOORFDWH UHVRXUFHV WR LQIRUPDWLRQ SURFHVVLQJ LQ RUGHU WR SUHGLFW GHPDQG *LYHQ WKDW ILUPV DUH PDQDJHG DFFRUGLQJ WR WKH ZLVKHV RI WKHLU RZQHUV ZKR DUH W\SLFDO\ DVVHW KROGHUV ZH PD\ DVVXPH WKDW D W\SLFDO ILUP LV ULVN DYHUVH VHH H J 6DQGPR RU /HDQG IRU GHWDLO GLVFXVVLRQV

$Q RSWLPDO RXWSXW RI WKH ULVN DYHUVH PRQRSROLVWLF ILUP HQJDJHG LQ GDWD SURFHVVLQJ IRU WKH SXUSRVH RI GHPDQG IRUHFDVWLQJ LV WKH VDPH DV WKH PRQRSROLVWLF RXWSXW ZLWKRXW XQFHUWDLQW\ LI 05–0& VPDOOHU LI 05–0&! DQG JUHDWHU RWKHUZLVH ,W LPSOLHV WKDW WKH ULVN DYHUVH PRQRSRO\ SURGXFHV 0HVV WKDQ LW ZRXOG XQGHU FHUWDLQW\ LI WKH ULJKW KDQG VLGH RI WKH FRQGLWLRQ LV SRVLWLYH L H LI WKH SDUWLDO GHULYDWLYH RI WKH PDUJLQDO UHYHQXH ZLWK UHVSHFW WR η HYDOXDWHG DW η° D PDUJLQDO ULVN SUHPLXP

$$MR\ \ Q\ \eta^{\circ} = \frac{\partial\left[QP\ \ Q\ \eta^{\circ}\right]}{\partial Q}$$

LV JUHDWHU WKDQ]HUR WKH VDPH DV WKH PRQRSROLVWLF RXWSXW ZLWKRXW XQFHUWDLQW\ LI WKLV SDUWLDO GHULYDWLYH LV HTXDO WR]HUR DQG PRUH RWKHUZLVH

,Q JHQHUDO DOO RXWFRPHV DUH SRVVLEOH DQG WKH VLJQ RI WKH PDUJLQDO ULVN SUHPLXP VKRXOG EH DQDO\]HG IRU HDFK SDUWLFXODU IRUP RI VWRFKDVWLF GHPDQG 1RWH KRZHYHU

WKDW LI WKH VWRFKDVWLF GHPDQG IXQFWLRQ VDWLVILHV WKH *principle of increasing uncertainty* [11] / HODQG WKHQ WKH RXWSXW RI WKH SUHGLFWLQJ GHPDQG PRQRSRO\ LV DOZD\V VPDOOHU WKDQ LW ZRXOG EH ZLWKRXW XQFHUWDLQW\ D QHFHVVDU\ DQG VXIILFLHQW FRQGLWLRQ IRU WKLV SULQFLSOH WR KROG LV WKH VDPH VLJQ RI WKH PDUJLQDO UHYHQXH DQG WKH PDUJLQDO ULVN SUHPLXP [12] DQG FRQVHTXHQWO\ WKH UHVXOWV DERYH DUH WR VRPH H[WHQG FRQVLVWHQW ZLWK UHVXOWV GHULYHG E\ / HODQG 2Q WKH RWKHU KDQG LI WKH VWRFKDVWLF GHPDQG IXQFWLRQ GRHV QRW VDWLVI\ WKH SULQFLSOH RI LQFUHDVLQJ XQFHUWDLQW\ IRU H[DPSOH LI WKH LQYHUVH GHPDQG IXQFWLRQ LV VSHFLILHG DV 3 η 4^{-1} – D ZKHUH D! 4 D^{-1} 3! WKHQ WKH SDUWLDO GHULYDWLYH RI WKH PDUJLQDO UHYHQXH ZLWK UHVSHFW WR η HYDOXDWHG DW η° LV HTXDO WR]HUR RU LV QHJDWLYH DQG WKH ULVN DYHUVH PRQRSRO\ SURGXFHV WKH VDPH DV UHVSHFWLYHO\ PRUH WKDQ LW ZRXOG SURGXFH ZLWKRXW GHPDQG XQFHUWDLQW\

1RWH WKDW WKH DEVROXWH YDOXH RI WKH ULJKW KDQG VLGH RI WKH H[SUHVVLRQ GHFUHDVHV LI WKH VWDQGDUG GHYLDWLRQ RI WKH SUHGLFWLRQ HUURU σ GHFUHDVHV LQ WKH 0LPLW LI WKH VWDQGDUG GHYLDWLRQ σ JRHV WR]HUR WKH RSWLPDO RXWSXW RI WKH ULVN DYHUVH PRQRSRO\ ZLWK GHPDQG XQFHUWDLQW\ FRQYHUJHV WR WKH RSWLPDO RXWSXW ZLWKRXW XQFHUWDLQW\ ,W FRQILUPV WKDW LQ DQ\ FDVH EHWWHU IRUHFDVWV PDNH WKH RSWLPDO SULFH RXWSXW GHFLVLRQV RI WKH ULVN DYHUVH PRQRSRO\ FORVHU WR WKRVH WKDW ZRXOG EH PDGH XQGHU FHUWDLQW\ DQG LW LPSOLHV WKDW GHPDQG DQDO\VLV LQ D PRQRSRO\ IUP ZKLFK DIIHFWV WKH YDOXH RI VWDQGDUG GHYLDWLRQ RI WKH SUHGLFWLRQ HUURU σ LQIOXHQFHV WKH GLVWULEXWLRQ RI WKH ZHOIDUH LQ WKH HFRQRP\

6. Implications on Social Welfare

,W LV ZHOO NQRZQ WKDW ZKHQ DQ LQGXVWU\ LV RUJDQL]HG DV D PRQRSRO\ LQVWHDG RI DV D PDQ\ FRPSHWLWLYH IUPV VRPH RI WKH FRPSHWLWLYH FRQVXPHU V DQG SURGXFHU V VXUSOXV LV ORVW DW WKH PRQRSRO\ RXWSXW FKRLFH 6RPH RI WKDW VXUSOXV JHWV WUDQVIHUUHG WR WKH PRQRSROLVW LQ WKH IRUP RI DGGLWLRQDO SURILWV EXW VRPH LV ORVW DOWRJHWKHU

/HW 4 EH WKH RXWSXW SURGXFHG (&6 EH WKH H[SHFWHG FRQVXPHU VXUSOXV (36 EH WKH H[SHFWHG SURGXFHU VXUSOXV (' : / EH WKH H[SHFWHG GHDGZHLJKW ORVV DQG 0HW VXSHUVFULSWV ^{c} ^{m} ° DQG * FRUUHVSRQG WR WKH FRPSHWLWLYH IUP ZLWKRXW XQFHUWDLQW\ WKH PRQRSROLVWLF IUP ZLWKRXW XQFHUWDLQW\ RI GHPDQG WKH ULVN DYHUVH PRQRSROLVWLF

[11] The principle of increasing uncertainty states that the riskiness (or dispersion) of total revenue increases if total expected revenue increases (see Leland, 1972, for a detail discussion).

[12] See Leland (1972), Appendix.

ILUP IDFLQJ XQFHUWDLQ GHPDQG ZLWKRXW IRUHFDVWLQJ DFWLYLWLHV DQG WKH ULVN DYHUVH PRQRSRO\ SUHGLFWLQJ XQFHUWDLQ GHPDQG UHVSHFWLYH\ )RU D QHJDWLYH VORSLQJ LQYHUVH GHPDQG FXUYH ZH KDYH WKH IROORZLQJ UHDWLRQVKLSV

G(&6 G4! L H WKH H[SHFWHG YDOXH RI WKH FRQVXPHU V VXUSOXV LQFUHDVHV ZLWK WKH RXWSXW SURGXFHG

G(36 G|4−4m| L H WKH H[SHFWHG YDOXH RI WKH SURGXFHU V VXUSOXV LQFUHDVHV LI WKH GHYLDWLRQ IURP WKH RSWLPDO PRQRSROLVWLF RXWSXW ZLWKRXW XQFHUWDLQW\ GHFUHDVHV

G(' : / G|4−4c| ! L H WKH H[SHFWHG YDOXH RI WKH GHDGZHLJKW ORVV GHFUHDVHV LI WKH GHYLDWLRQ IURP WKH FRPSHWLWLYH RXWSXW GHFUHDVHV

,W IROORZV IURP WKH FRQVLGHUDWLRQ SUHVHQWHG LQ WKH SUHFHGLQJ VHFWLRQ WKDW LI WKH SULQFLSOH RI LQFUHDVLQJ XQFHUWDLQW\ KROGV WKHQ WKH IROORZLQJ FRQGLWLRQ

$$4^{\circ} \quad 4^{*} \quad 4^{m} \quad 4^{c}$$

LV DOZD\V VDWLVILHG VHH)LJXUH 0RUHRYHU GHPDQG IRUHFDVWLQJ LQFUHDVHV WKH RXWSXW RI WKH PRQRSROLVWLF IURP IDFLQJ XQFHUWDLQ GHPDQG 7KDW LV LW GHFUHDVHV WKH GLIIHUHQFHV EHWZHHQ WKH RSWLPDO RXWSXW RI WKH ULVN DYHUVH PRQRSROLVWLF IURP IRUHFDVWLQJ GHPDQG DQG WKH RSWLPDO PRQRSROLVWLF DQG FRPSHWLWLYH RXWSXWV ZLWKRXW XQFHUWDLQ\ $4^{m}-4^{*}$ DQG $4^{c}-4^{*}$ UHVSHFWLYH\ 7KHUHIRUH ZH FDQ FRQFOXGH WKDW LI WKH SULQFLSOH RI LQFUHDVLQJ XQFHUWDLQ\ LV VDWLVILHG WKHQ WKH GHPDQG IRUHFDVWLQJ LQ WKH ULVN DYHUVH PRQRSRO\ LQFUHDVHV WKH H[SHFWHG YDOXH RI WKH FRQVXPHU V VXUSOXV LQFUHDVHV WKH H[SHFWHG YDOXH RI WKH SURGXFHU V VXUSOXV DQG GHFUHDVHV WKH H[SHFWHG YDOXH RI WKH GHDGZHLJKW ORVV

,I WKH SULQFLSOH RI LQFUHDVLQJ XQFHUWDLQ\ GRHVQ W KROG WKHQ WKH RSWLPDO YROXPH RI RXWSXW RI WKH ULVN DYHUVH PRQRSRO\ IRUHFDVWLQJ GHPDQG 4^{*} LV JUHDWHU WKDQ WKH RSWLPDO PRQRSROLVWLF RXWSXW 4^{m} +RZHYHU LW LV DOZD\V VPDOOHU WKDQ LW ZRXOG EH LI GHPDQG LV QRW SUHGLFWHG L H 4^{m} 4^{*} 4° 7KXV LQ WKLV FDVH GHPDQG IRUHFDVWLQJ DFWLYLWLHV GHFUHDVH WKH RXWSXW RI WKH PRQRSROLVWLF IURP IDFLQJ XQFHUWDLQ GHPDQG L H GHFUHDVH 4^{*} DQG $|4^{*}-4^{m}|$ DQG FRQVHTXHQWO\ GHFUHDVH WKH H[SHFWHG FRQVXPHU VXUSOXV DQG LQFUHDVH WKH H[SHFWHG SURGXFHU VXUSOXV 7KH HIIHFWV RI GHPDQG IRUHFDVWLQJ RQ WKH H[SHFWHG GHDGZHLJKW ORVV RI WKH HFRQRP\ FDQQRW EH RQFH IRU DOO XQDPELJXRXVO\ GHWHUPLQHG 2QH FRXOG QRWH KRZHYHU WKDW LI WKH SULQFLSOH RI LQFUHDVLQJ XQFHUWDLQ\ GRHVQ W KROG WKHQ LQ WKH FDVH ZKHQ GHPDQG IOXFWXDWLRQV DUH UHODWLYHO\ VPDOO L H LI 4^{m} 4^{*} 4° 4^{c} GHPDQG IRUHFDVWLQJ LQFUHDVHV WKH GHYLDWLRQ RI WKH RXWSXW SURGXFHG IURP WKH H[SHFWHG FRPSHWLWLYH RXWSXW DQG FRQVHTXHQWO\ LQFUHDVHV H[SHFWHG GHDGZHLJKW ORVV 2Q WKH RWKHU KDQG LI WKH IOXFWXDWLRQV RI GHPDQG DUH ELJJHU L H LI 4^{m} 4^{c} 4° WKHQ D VPDOO VKLIW

LQ WKH PRQRSROLVWLF FKRLFH UHVXOWLQJ IURP GHPDQG IRUHFDVWLQJ DOZD\V GHFUHDVHV WKH H[SHFWHG YDOXH RI WKH GHDGZHLJKW 0RVV VLQFH WKH GHYLDWLRQ IURP WKH RSWLPDO FRPSHWLWLYH RXWSXW GHFUHDVHV +RZHYHU LI WKH UHVXOWLQJ VKLIW LV VR 0DUJH WKDW WKH RSWLPDO PRQRSROLVWLF RXWSXW ZLWK GHPDQG IRUHFDVWLQJ 4* DFKLHYHV WKH FRPSHWLWLYH RXWSXW 4c WKHQ IXUWKHU H[SDQVLRQV RI IRUHFDVWLQJ DFWLYLWLHV ZKLFK 0HDG WR WKH RXWSXW F0RVHU WR WKH RSWLPDO PRQRSROLVWLF RXWSXW FDQ RQ0\ LQFUHDVH WKH H[SHFWHG YD0XH RI WKH GHDGZHLJKW 0RVV

)LJXUH

(IIHFWV RI GHPDQG GHPDQG IRUHFDVWLQJ RQ ZHOIDUH IRU OLQHDU GHPDQG ZLWK DGGLWLYH UDQGRP WHUP $ 05$ 4 η° ! L H WKH SULQFLSOH RI LQFUHDVLQJ XQFHUWDLQW\ KROGV

WKH H[SHFWHG YDOXH RI WKH FRQVXPHU V VXUSOXV

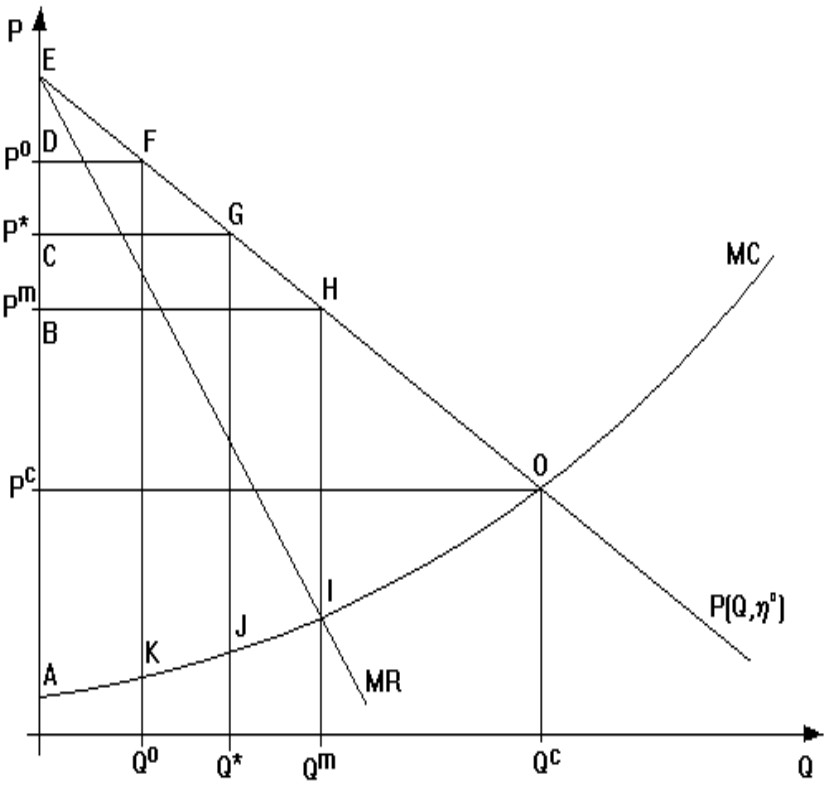
(&6$m DUHD %(+ ! (&6$* DUHD &(* ! (&6$° DUHD '())

WKH H[SHFWHG YDOXH RI WKH SURGXFHU V VXUSOXV

(36$m DUHD $,+% ! (36$* DUHD $-*& ! (36$° DUHD $.)')

WKH H[SHFWHG YDOXH RI WKH GHDGZHLJKW ORVV

('$: /m DUHD ,2+ ('$: /* DUHD -2* ('$: /° DUHD .2))



7KXV WKH RYHUDOO ZHOIDUH HIIHFWV RI GHPDQG IRUHFDVWLQJ LQ WKH ULVN DYHUVH PRQRSROLVWLF ILUP FDQ EH VXPPDUL]HG LQ WKH IROORZLQJ WKUHH VWDWHPHQWV)LUVW GHPDQG IRUHFDVWLQJ LQ D PRQRSROLVWLF ILUP DOZD\V LQFUHDVHV H[SHFWHG SURGXFHU VXUSOXV 6HFRQG LW LQFUHDVHV H[SHFWHG FRQVXPHU VXUSOXV LI WKH SULQFLSOH RI LQFUHDVLQJ XQFHUWDLQW\ LV VDWLVILHG DQG GHFUHDVHV RWKHUZLVH 7KLUG WKH HIIHFW RI GHPDQG IRUHFDVWLQJ RQ WKH H[SHFWHG YDOXH RI WKH GHDGZHLJKW ORVV GHSHQGV QRW RQO\ RQ WKH IRUP RI WKH VWRFKDVWLF GHPDQG FXUYH EXW DOVR RQ WKH PDJQLWXGH RI GHPDQG IOXFWXDWLRQV ,I WKH SULQFLSOH RI LQFUHDVLQJ XQFHUWDLQW\ LV VDWLVILHG WKHQ GHPDQG IRUHFDVWLQJ DFWLYLWLHV DOZD\V GHFUHDVH WKH H[SHFWHG YDOXH RI WKH GHDGZHLJKW ORVV 2WKHUZLVH GDWD SURFHVVLQJ IRU WKH SXUSRVH RI GHPDQG IRUHFDVWLQJ PD\ LQFUHDVH RU GHFUHDVH WKH H[SHFWHG YDOXH RI WKH GHDGZHLJKW ORVV GHSHQGLQJ RQ WKH PDJQLWXGH RI WKH GHPDQG IOXFWXDWLRQV

Conclusions

%\ XVLQJ D VLPSOH PRGHO ZH KDYH WULHG WR H[SODLQ WKH HFRQRPLF DVSHFWV RI GHPDQG IRUHFDVWLQJ LQ D PRQRSROLVWLF ILUP IDFLQJ GHPDQG XQFHUWDLQW\ $VVXPLQJ WKDW WKH ILUP PD[LPL]HV LWV WRWDO GLVFRXQWHG H[SHFWHG XWLOLW\ IURP WKH SURILW LQ LQGHILQLWH WLPH DQG XVHV WLPH VHULHV DQDO\VLV WR SUHGLFW GHPDQG IOXFWXDWLRQV ZH KDYH VKRZQ WKDW WKH RSWLPDO IRUHFDVWLQJ SROLF\ LV VWDWLRQDU\ L H WKDW WKH ILUP KDV WR DQDO\]H WKH VDPH QXPEHU RI VRXUFHV RI GHPDQG LQ HDFK SHULRG)XUWKHUPRUH EDVHG RQ WKH DQDO\VLV RI WKH RSWLPDO EHKDYLRU RI WKH PRQRSROLVWLF ILUPV ZLWK YDULRXV DWWLWXGHV WRZDUGV ULVN ZH KDYH VKRZQ WKDW RQO\ ULVN DYHUVH PRQRSRO\ LV FRQFHUQHG DERXW SUHGLFWLQJ XQFHUWDLQ GHPDQG 0RUHRYHU WKH UHVXOWV LQGLFDWH WKDW WKH RSWLPDO RXWSXW RI WKH PRQRSROLVWLF ILUP ZKLFK XVHV GHPDQG IRUHFDVWV WR LPSURYH LWV SULFH RXWSXW GHFLVLRQV LV DOZD\V FORVHU WR WKH RSWLPDO YDOXH RI WKH RXWSXW ZLWKRXW XQFHUWDLQW\ WKDQ LW ZRXOG EH LI GHPDQG LV QRW DQDO\]HG 7KLV UHVXOWV LQ D UHGLVWULEXWLRQ RI VRFLDO ZHOIDUH ,Q SDUWLFXODU ZH KDYH VKRZQ WKDW GDWD SURFHVVLQJ IRU WKH SXUSRVH RI GHPDQG IRUHFDVWLQJ LQ WKH ULVN DYHUVH PRQRSROLVWLF ILUP DOZD\V LQFUHDVHV WKH H[SHFWHG SURGXFHU VXUSOXV LQFUHDVHV WKH H[SHFWHG FRQVXPHU VXUSOXV LI WKH VWRFKDVWLF GHPDQG IXQFWLRQ VDWLVILHV WKH SULQFLSOH RI LQFUHDVLQJ XQFHUWDLQW\ DQG GHFUHDVHV RWKHUZLVH 7KH HIIHFW RI GHPDQG IRUHFDVWLQJ RQ HDFK PDUNHW ZLWK XQFHUWDLQ GHPDQG VHHPV WR EH ZRUWK\ RI GHWDLOHG LQYHVWLJDWLRQV ,W IROORZV IURP WKH DQDO\VLV DERYH WKDW GDWD SURFHVVLQJ IRU WKH SXUSRVH RI SUHGLFWLQJ GHPDQG KDV

7KH UHVHDUFK SUHVHQWHG LQ WKH SDSHU FDQ EH H[WHQGHG LQ VHYHUDO GLUHFWLRQV)RU H[DPSOH WKH HIIHFW RI GHPDQG IRUHFDVWLQJ RQ HQWU\ GHWHUUHQFH LQ WKH PDUNHW ZLWK XQFHUWDLQ GHPDQG VHHPV WR EH ZRUWK\ RI GHWDLOHG LQYHVWLJDWLRQV ,W IROORZV IURP WKH DQDO\VLV DERYH WKDW GDWD SURFHVVLQJ IRU WKH SXUSRVH RI SUHGLFWLQJ GHPDQG KDV

D FKDUDFWHU RI IL[HG FRVW WKHUHIRUH LI GHPDQG IRUHFDVWLQJ LV UHTXLUHG LW KDV WKH HIIHFW RI FUHDWLQJ HFRQRPLHV RI VFDOH ,Q DGGLWLRQ VLQFH SDVW UHVXOWV RI GHPDQG DQDO\VLV FDQ EH XVHG LQ SUHVHQW IRUHFDVWV WKH LQFXPEHQW ILUP KDV D FRVW DGYDQWDJH RYHU WKH SRWHQWLDO HQWUDQW WR EH LQ WKH VDPH SRVLWLRQ DQ HQWUDQW ZRXOG KDYH WR DQDO\]H GHPDQG SULRU WR WKH SHULRG LQ ZKLFK LW HQWHUV WKH LQGXVWU\ ,W VHHPV QDWXUDO WKDW LQ VXFK D VLWXDWLRQ WKH HVWDEOLVKHG PRQRSR\ IDFLQJ WKH WKUHDW RI HQWU\ PLJKW ILQG LW SRVVLEOH DQG RSWLPDO WR GHWHU HQWU\ E\ VHWWLQJ LWV VHOOLQJ SULFHV DERYH WKH PLQLPXP DYHUDJH FRVW 0HYHO ZLWKRXW LQGXFLQJ SRWHQWLDO HQWUDQWV WR HQWHU WKH LQGXVWU\ EXW WKLV SUREOHP ZLOO EH LQYHVWLJDWHG LQ D IRUWKFRPLQJ SDSHU

APPENDIX

Second-order conditions for the firm's maximization problem

&RQVLGHU WKH REMHFWLYH IXQFWLRQ JLYHQ E\ WKH H[SUHVVLRQ 1RWH WKDW LI WKH REMHFWLYH IXQFWLRQ XQGHU FRQVLGHUDWLRQ LV VWULFWO\ FRQFDYH IRU D OLQHDU XWLOLW\ IXQFWLRQ ULVN QHXWUDO ILUP 8 Π ! 8 Π WKHQ LW LV DOVR VWULFWO\ FRQFDYH IRU WKH FRQFDYH XWLOLW\ IXQFWLRQ D ULVN DYHUVH ILUP 8 Π ! 8 Π 7KHUHIRUH WR VKRZ WKDW WKH VHFRQG RUGHU FRQGLWLRQ KROGV IRU D ULVN DYHUVH ILUP LW LV HQRXJK WR SURYH WKDW LW KROGV IRU D ULVN QHXWUDO ILUP

$VVXPH ULVN QHXWUDOLW\ DQG FRQVLGHU WKH +HVVLDQ RI WKH REMHFWLYH IXQFWLRQ JLYHQ E\ WKH H[SUHVVLRQ

7KLV +HVVLDQ LV QHJDWLYH GHILQLWH WKH REMHFWLYH IXQFWLRQ LV VWULFWO\ FRQFDYH LII

$$\Psi_{1,1}\left(\sigma,\eta^{\circ}\right) \quad \text{and} \quad \Psi_{1,1}\left(\sigma,\eta^{\circ}\right)\Psi_{2,2}\left(\sigma,\eta^{\circ}\right) - \left[\Psi_{1,2}\left(\sigma,\eta^{\circ}\right)\right]^{2} > 0$$

RU HTXLYDOHQWO\ LII

$$\Psi_{2,2}\left(\sigma,\eta^{\circ}\right) \quad \text{and} \quad \Psi_{1,1}\left(\sigma,\eta^{\circ}\right)\Psi_{2,2}\left(\sigma,\eta^{\circ}\right) - \left[\Psi_{1,2}\left(\sigma,\eta^{\circ}\right)\right]^{2} > 0$$

7KH VHFRQG SDUWLDO GHULYDWLYH RI WKH REMHFWLYH IXQFWLRQ ZLWK UHVSHFW WR WKH VWDQGDUG GHYLDWLRQ RI WKH SUHGLFWLRQ HUURU σ FDQ EH UHSUHVHQWHG DV

$$\Psi_{\sigma\sigma}\left(Q,\sigma,\eta^{\circ}\right) = \left[U'\left(\Pi - \underline{\Pi}\right) - Q\left(\sigma,\eta^{\circ}\right) + U''\left(\Pi - \underline{\Pi}\right)\left(\Pi - \underline{\Pi}\right)Q\left(\sigma,\eta^{\circ}\right)\right] +$$

$$\left[U'\left(\Pi - \overline{\Pi}\right) - Q\left(\sigma,\eta^{\circ}\right) + U''\left(\Pi - \overline{\Pi}\right)\left(\Pi - \overline{\Pi}\right)Q\left(\sigma,\eta^{\circ}\right)\right] \qquad (A1)$$

$$\left(\Pi - \underline{\Pi}\right)Q\left(\sigma,\eta^{\circ}\right) = \left(\Pi - \overline{\Pi}\right)Q\left(\sigma,\eta^{\circ}\right) = -\frac{d\,G(\sigma)}{d\sigma} \qquad (A2)$$

LV QHJDWLYH L H LI WKH GDWD SURFHVVLQJ FRVW IXQFWLRQ LV FRQYH[LQ σ VHH 6HFWLRQ

7KH VHFRQG SDUWLDO GHULYDWLYH RI WKH REMHFWLYH IXQFWLRQ $\Psi(\sigma)$ ZLWK UHVSHFW WR RXWSXW (Q) FDQ EH UHSUHVHQWHG DV

$$\Psi\ \ Q\sigma\eta^{\circ}\ =\ -\!>U'\,\underline{\Pi}\ \ \underline{\Pi}\ \ Q\sigma\eta^{\circ}\ +\ U''\,\underline{\Pi}\ \ \underline{\Pi}\ \ Q\sigma\eta^{\circ}\ @\ +$$

$$-\!>U'\,\overline{\Pi}\ \ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ +\ U''\,\overline{\Pi}\ \ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ @\qquad \$$$

7KH VHFRQG FURVV SDUWLDO GHULYDWLYH RI WKH REMHFWLYH IXQFWLRQ ZLWK UHVSHFW WR 4 DQG σ LV

$$\Psi\ \ Q\sigma\eta^{\circ}\ =\ -\!>U'\,\underline{\Pi}\ \ \underline{\Pi}\ \ Q\sigma\eta^{\circ}\ +\ U''\,\underline{\Pi}\ \ \underline{\Pi}\ \ Q\sigma\eta^{\circ}\ \ \underline{\Pi}\ \ Q\sigma\eta^{\circ}\ @\ +$$

$$>U'\,\overline{\Pi}\ \ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ +\ U''\,\overline{\Pi}\ \ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ \ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ (\$$$

7KXV IRU WKH OLQHDU XWLOLW\ IXQFWLRQ $8\Pi\ !$ DQG 8Π ZH KDYH

$$\Psi\ \ Q\sigma\eta^{\circ}\,\Psi\ \ Q\sigma\eta^{\circ}\ -\ \Psi\ \ Q\sigma\eta^{\circ}\ =$$

$$U'\,\underline{\Pi}\ \ >\ \underline{\Pi}\ \ Q\sigma\eta^{\circ}\ \ \underline{\Pi}\ \ Q\sigma\eta^{\circ}\ -\ \underline{\Pi}\ \ Q\sigma\eta^{\circ}\ @\ +$$

$$U'\,\overline{\Pi}\ \ >\ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ \ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ -\ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ @\ +$$

$$U'\,\overline{\Pi}\,U'\,\overline{\Pi}\,>\underline{\Pi}\ \ Q\sigma\eta^{\circ}\,\overline{\Pi}\ \ Q\sigma\eta^{\circ}\ +\ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ \ \underline{\Pi}\ \ Q\sigma\eta^{\circ}\ -$$

$$-\ \ \underline{\Pi}\ \ Q\sigma\eta^{\circ}\ \ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ @\qquad \$$$

7DNLQJ LQWR DFFRXQW WKDW $8\Pi_1\ \ 8\Pi_2$ FRQVW ! IRU WKH ULVN QHXWUDO ILUP WKH H[SHFWHG DERYH LV SRVLWLYH LII

$$\underline{\Pi}\ \ Q\sigma\eta^{\circ}\,\underline{\Pi}\ \ Q\sigma\eta^{\circ}\ +\ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ \ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ +$$

$$\underline{\Pi}\ \ Q\sigma\eta^{\circ}\,\overline{\Pi}\ \ Q\sigma\eta^{\circ}\ +\ \overline{\Pi}\ \ Q\sigma\eta^{\circ}\ \ \underline{\Pi}\ \ Q\sigma\eta^{\circ}\ -$$

$$- \;>\underline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\;+\;\overline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\,@\quad !\qquad\qquad \$$$

1RWH WKDW

$$\underline{I}\quad Q\,\sigma\,\eta^{\circ}\;=\;-\overline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\;=\;-\;\frac{}{\sqrt{\pi}}\;\frac{\partial>QP\quad Q\,\eta^{\circ}\,@}{\partial Q}\qquad \$$$

7KHUHIRUH WKH 0DVW WHUP LV HTXDO WR]HUR DQG WKH H[SUHVVLRQ DERYH FDQ EH UHSUHVHQWHG DV

$$\underline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\;>\underline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\;+\;\overline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\,@\;+$$

$$\overline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\;>\overline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\;+\;\underline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\,@\;!\quad \$$$

7DNLQJ LQWR DFFRXQW $ WKH FRQGLWLRQ DERYH LV VDWLVILHG LI

$$\underline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\;+\;\overline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\qquad\qquad \$$$

6LQFH

$$\underline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\;=\;\frac{d\;MR^{B}-\;MC}{dQ}\;-\;\frac{\partial\;>QP\quad Q\,\eta^{\circ}\,@}{\partial Q}\;\frac{\sigma}{\sqrt{\pi}}\qquad \$$$

$$\overline{\Pi}\quad Q\,\sigma\,\eta^{\circ}\;=\;\frac{d\;MR^{B}-\;MC}{dQ}\;+\;\frac{\partial\;>QP\quad Q\,\eta^{\circ}\,@}{\partial Q}\;\frac{\sigma}{\sqrt{\pi}}\qquad \$$$

DQG G 05 – 0& G4 IRU VWULFW FRQFDYH IXQFWLRQ Π 4 43 4 η° –& 4 –= WKH FRQGLWLRQ DERYH LV D0ZD\V VDWLVILHG

,W LPSOLHV WKDW VHFRQG RUGHU FRQGLWLRQ D0ZD\V KR0GV IRU WKH 0LQHDU XWLOLW\ IXQFWLRQ WKH ULVN QHXWUDO ILUP DQG FRQVHTXHQW\ D0VR IRU WKH FRQFDYH XWLOLW\ WKH ULVN DYHUVH ILUP

REFERENCES

&XNURZVNL - $ $Q (FRQRPLF $QDO\VLV RI 1XPHULFDO ' DWD 3URFHVVLQJ LQ WKH)LUP *CERGE-EI Working Paper*

+H\ - ' $ 8QLILHG 7KHRU\ RI WKH %HKDYLRU RI 3URILW 0D[LPL]LQJ /DERXU 0DQDJHG DQG -RLQW 6WRFN)LUPV 2SHUDWLQJ XQGHU 8QFHUWDLQW\ *The Economic Journal*

*LEERQV $ DQG : 5\WWHU *Efficient Parallel Algorithms* &DPEULGJH &DPEULGJH 8QLYHUVLW\ 3UHVV

/HDQG + (7KHRU\ RI WKH)LUP)DFLQJ 8QFHUWDLQ ' HPDQG *American Economic Review*

/LP & 5DQNLQJ %HKDYLRUDO 0RGHV RI WKH)LUP)DFLQJ 8QFHUWDLQ ' HPDQG *American Economic Review*

/LSPDQ / % ,QIRUPDWLRQ 3URFHVVLQJ DQG %RXQGHG 5DWLRQDOLW\ $ 6XUYH\ *Canadian Journal of Economics*

1HOVRQ 5 5 8QFHUWDLQW\ 3UHGLFWLRQ DQG &RPSHWLWLYH (TXLOLEULXP *Quarterly Journal of Economics*

5DGQHU 5 +LHUDUFK\ 7KH (FRQRPLFV RI 0DQDJLQJ *Journal of Economic Literature*

7KH 2UJDQL]DWLRQ RI ' HFHQWUDOL]HG ,QIRUPDWLRQ 3URFHVVLQJ *Econometrica*

5DGQHU 5 DQG 7 9DQ =DQGW ,QIRUPDWLRQ 3URFHVVLQJ LQ)LUPV DQG 5HWXUQ WR 6FDOH *Annales d'Economie et de Statistique*

,QIRUPDWLRQ 3URFHVVLQJ DQG 5HWXUQV WR 6FDOH LQ 6WDWLVWLFDO ' HFLVLRQ 3UREOHPV 0XUUD\ +LOO 1- $7□7 %HOO /DERUDWRULHV

6DPXHOVRQ 3 $ 7KH)XQGDPHQWDO $SSUR[LPDWLRQ 7KHRUHP RI 3RUWIROLR $QDO\VLV LQ 7HUPV RI 0HDQ 9DULDQFH DQG +LJKHU 0RPHQWV *Review of Economic Studies*

6DQGPR $ 2Q WKH 7KHRU\ RI WKH &RPSHWLWLYH)LUP XQGHU 3ULFH 8QFHUWDLQW\ *American Economic Review*

6DUJHQW 7 - *Dynamic Macroeconomic Theory* &DPEULGJH 0$ +DUYDUG 8QLYHUVLW\ 3UHVV

=DEH (0RQRSR0\ DQG 8QFHUWDLQW\ *Review of Economic Studies*