CHARLES UNIVERSITY

FACULTY OF SOCIAL SCIENCES

Center for Economic Research and Graduate Education

Dissertation Thesis

2026 **Artem Razumovskii**

CHARLES UNIVERSITY

FACULTY OF SOCIAL SCIENCES

Center for Economic Research and Graduate Education

Artem Razumovskii

Essays in Economic Theory

Dissertation Thesis

Prague 2026

Dissertation Committee:

Doc. PhDr. Jan Zápal, Ph.D. (CERGE-EI, Chair)

Yiman Sun, Ph.D. (CERGE-EI)

Ole Jann, Ph.D. (CERGE-EI)

Referees:

Ludmila Matysková, Ph.D. (University of Alicante)

Orestis Troumpounis, Ph.D. (Ca' Foscari University of Venice)

Author: **Artem Razumovskii**

Supervisor: **Doc. PhDr. Jan Zápal, Ph.D.**

Year of the defense: 2026

Abstract

The first chapter of the dissertation focuses on optimal and self-imposed interim deadlines for naïve, sophisticated, and partially-sophisticated agents in a single-project context. I find that for each type, there is a unique design for an exogenous interim deadline that maximizes the agent's welfare. However, only the sophisticated agent would self-impose an optimal interim deadline, while the naïve agent would not apply a self-imposed deadline at all. The partially-sophisticated agent sets a nonoptimal self-imposed deadline, which can actually decrease her own welfare. The main result is that the partially-sophisticated agent who is relatively less present-biased would *decrease* her own welfare by setting a self-imposed deadline, and the partially-sophisticated agent who is relatively more present-biased would *increase* her welfare given the same degree of sophistication.

The second chapter studies the symmetry and uniqueness of the optimal pair of interim deadlines in the case of two parallel projects. I find that imposing an interim deadline in one project may shift the optimal timing for the interim deadline in the parallel project to earlier or later. The size of the effect and the uniqueness of the optimal pair of timings for interim deadlines in both projects depend on the agent's present bias. The optimal pair is not unique, and the distance between deadlines in the parallel project is higher for a less present-biased agent.

The third chapter, co-authored with Misha Gipsman[1] and Artyom Jelnov[2], investigates the removal of trade barriers by the toll collector under the threat of conflict with a central authority. We show that the toll collector relaxes the barriers and allows more merchants to reach the market when under the threat of conflict. However, the toll collector allows only a few merchants to cross, maintaining a significant restriction on trade.

Abstrakt

První kapitola se zaměřuje na optimální a osobně uválené průběžné termíny pro naivní, sofistikované a částečně sofistikované agenty v kontextu jednoho projektu. Zjišťuji, že pro každý typ existuje jedinečný návrh exogenního mezitímního termínu, který maximalizuje

[1]Ariel University, Israel. E-mail address: mishag@ariel.ac.il
[2]Ariel University, Israel. E-mail address: artyomj@ariel.ac.il

blahobyt agenta. Pouze sofistikovaný agent by si však sám stanovil optimální mezitímní termín, zatímco naivní agent by osobně uválený termín neuplatnil vůbec. Částečně sofistikovaný agent stanoví neoptimální osobně uválený termín a jejím zavedením může dokonce snížit svůj vlastní blahobyt. Hlavním výsledkem je, že částečně sofistikovaný agent, který je relativně méně orientován na přítomnost, by použitím osobně uváleného termínu snížil svůj vlastní blahobyt a částečně sofistikovaný agent, který je relativně více orientován na přítomnost, by při stejné míře sofistikovanosti svůj blahobyt zvýšil.

Druhá kapitola zkoumá symetrii a jedinečnost optimální dvojice průběžných termínů v případě dvou paralelních projektů. Zjišťuji, že uložení průběžného termínu v jednom projektu může posunout optimální načasování průběžného termínu v paralelním projektu dříve nebo později. Velikost tohoto efektu a jedinečnost optimální dvojice načasování průběžných termínů v obou projektech závisí na současném vychýlení agenta. Optimální dvojice není jedinečná, stejně jako vzdálenost mezi termíny v paralelním projektu je větší pro méně přítomnostně vychýleného agenta.

Třetí kapitola zkoumá odstraňování obchodních překážek výběrčím mýta pod hrozbou konfliktu s ústředním orgánem. Ukazuji, že výběrčí mýtného uvolňuje bariéry a umožňuje většímu počtu obchodníků dostat se na trh pod hrozbou konfliktu. Výběrčí mýtného však umožní přechod pouze několika málo obchodníkům, čímž zachovává významné omezení obchodu.

Keywords

Time-Inconsistent Preferences, Present Bias, Deadlines, Procrastination, Self-Control, Rent-Seeking, Toll Collector, Trade Barriers, Asymmetric Valuation

Klíčová slova

Časově Nekonzistentní Preference, Přítomnostní Vychýlení, Termíny, Prokrastinace, Sebekontrola, Rent Seeking, Výběrčí Mýtného, Obchodní Bariéry, Asymetrické Oceňování

Length of the work: 187,138 characters with spaces

Declaration

1. I hereby declare that I have compiled this thesis using the listed literature and resources only.
2. I hereby declare that my thesis has not been used to gain any other academic title.
3. I fully agree to my work being used for study and scientific purposes.

In Prague on March 3, 2026

Artem Razumovskii

Acknowledgement

I would like to extend my heartfelt gratitude to my supervisor at CERGE-EI, Jan Zápal, for his insightful feedback, invaluable guidance, continuous encouragement, and unwavering support throughout my PhD journey. I am equally grateful to the members of my dissertation committee, Ole Jann and Yiman Sun, for their constructive comments, thoughtful discussions, and honest evaluations, all of which significantly contributed to enhancing the quality of my research.

I deeply appreciate the enlightening conversations, thought-provoking comments, and challenging questions from Avner Shaked, which often provided me with fresh perspectives on my work.

My sincere thanks also go to my colleagues and friends at CERGE-EI and the Academic Skills Center. I am especially thankful to Larbi Alaoui, Alexander Frug, Rastislav Rehák, Evgeniya Dubinina, Maxim Senkov, and Pavel Ilinov for their valuable feedback, suggestions, and support.

Lastly, I would like to acknowledge the Student Office Affairs team — particularly Tereza Kulhankova — for their consistent assistance and support throughout my studies.

In Prague on
March 3, 2026

Artem Razumovskii

Table of Contents

Introduction

The first two chapters of the dissertation investigate the behavior of a time-inconsistent agent under interim deadlines, her welfare, and the optimal design of deadlines. The first chapter focuses on optimal interim deadlines for naïve, sophisticated, and partially-sophisticated agents in a single-project context, while the second chapter studies the symmetry and uniqueness of the optimal pair of interim deadlines in the case of two parallel projects. The third chapter investigates the removal of trade barriers by a toll collector under the threat of conflict with a central authority.

People are partially time inconsistent, and many have difficulties committing to a detailed schedule for a project. In the first chapter, I study optimal interim deadlines and how they affect the behavior and resulting welfare of a present-biased agent. I consider a model in which there are three types of agent in terms of how the agent understands her present bias: naïve, sophisticated, and partially-sophisticated. For each type, there is a unique design for an exogenous interim deadline that maximizes the agent's welfare. However, only the sophisticated agent would self-impose an optimal interim deadline, while the naïve agent would not apply a self-imposed deadline at all. The partially-sophisticated agent sets a nonoptimal self-imposed deadline that can even decrease her own welfare. The main result is that the partially-sophisticated agent who is relatively less present-biased would *decrease* her own welfare by using a self-imposed deadline, and the partially-sophisticated agent who is relatively more present-biased would *increase* her welfare given the same degree of sophistication.

In the second chapter, I study optimal interim deadlines for a procrastinator in the context of multiple parallel projects. While deadlines are well studied in the context of single and repeated projects, I focus on the spillover effects of imposing an interim deadline in one project on optimal timing for the interim deadline in the parallel project. I study how the timing for the interim deadline in one project affects the procrastinator's behavior, her welfare, and the optimal timing for the interim deadline in the parallel project. I find that imposing the interim deadline in one project may shift the optimal timing for the interim deadline in the parallel project to earlier or later. If the interim deadline in one project is imposed earlier than the optimal timing for the interim deadline in the parallel project before the interim deadline in the first project is imposed, this optimal timing (in the parallel project) shifts to later. Similarly, if the interim deadline in the first project is imposed later than the optimal timing, the second shifts to earlier. Further, the size of the effect and the uniqueness of the optimal pair of timings for interim deadlines in both projects depend on the agent's present bias. For the more present-biased agents and for the agents who are close to being time-consistent, there is a unique pair of optimal timings, and it is optimal to impose the interim deadlines in the same period. However, there are less present-biased agents, for whom it is optimal to delay the interim deadline in one of the projects, and there are two symmetric pairs of optimal timings.

In the third chapter, my co-authors Misha Gipsman, Artyom Jelnov, and I explore removing trade barriers. As a historical example, we consider merchants who must pay a landlord (the Lord) to cross the Lord's land to gain access to a market. The Lord controls how many merchants will cross his land by imposing a toll payment. On the other side, a central authority (the King) is interested in social welfare and thus is inclined to eliminate this barrier. We show that the Lord relaxes the barriers and allows more merchants to reach the market under the threat of conflict with the King. However, the Lord allows only a few merchants to cross, maintaining a significant restriction on trade. The probability that the King will win the conflict and remove barriers remains low. The presence of multiple Lords on the same road leads to a monopoly or to an oligopoly. The situation changes rapidly only when several roads are present and the Lords have to compete with each other on tolls: they voluntarily remove the barriers.

1 Interim Deadline for Procrastinators

Published as CERGE-EI Working Paper Series No 769

1.1 Introduction

Procrastination is common and by definition[3] suboptimal. Many of those who have engaged in procrastination would have preferred in hindsight to stick to their original project plan to produce better outcomes. In this paper, I analyze the behavior of a procrastinator under differently designed interim deadlines, describe the optimal deadline that maximizes an agent's welfare, and study how the agent would self-impose an interim deadline. In real life, different situations are considered procrastination. In this paper, I focus on the present-biased agent who works on a project for several periods. In each period, the agent chooses how much effort to invest in the project. The amount of effort invested is associated with immediate costs and increases the reward the agent receives at the end of the project. The setup is simple and ubiquitous, so most life activities can be connected to the framework. For example, any job can be divided into intervals in which we work on a project every working day in a month, and then we receive a salary based on the effort we have invested. Students study every day at school during a semester to receive the final grades at the end, and runners train daily to achieve the best possible results in a competition. In all these situations, people invest effort across several periods and earn an outcome at the end based on their accumulated effort expended.

An agent who is a procrastinator invests less effort than she planned in advance. Persons who experience self-control problems[4] use different tools, such as deadlines, commitment

[3] I follow the definition provided by Ericson & Laibson, 2019 (page 29): "we will define someone as procrastinating if, when completing a costly task, there is a delay that appears suboptimal from their own perspective."

[4] To describe this behavior, I assume the agent has time-inconsistent preferences. In other words, the agent's preferences change over time, which can lead to the difference between planned effort for future

contracts, and self-control penalties,[5] to overcome procrastination. I describe and analyze the present-biased agent's behavior under an interim deadline and shed new light on how the partially-sophisticated agent affects her welfare by setting a self-imposed deadline. I investigate the effect of an interim deadline on the agent's behavior and welfare, leaving aside other instruments and incentives to overcome procrastination. I consider an interim deadline to be a unique instrument that can be used to improve the agent's performance and welfare. In my model, the deadline specifies the goal and the period.[6] To meet the deadline, the agent has to achieve a goal by this period. Deadlines are common in modern life: students' homework, preparing for exams, and job contracts all involve deadlines. Often, we restrict our future selves by setting deadlines on purpose to increase our performance or to achieve a certain goal. Thus, the questions of how an imposed deadline will affect us and how to set deadlines optimally are important and relevant in the modern world.

A deadline can be a powerful restriction on future actions and the effect of a deadline on the agent's welfare can be negative. While some experimental papers document a positive effect of deadlines on agents' performance and welfare (e.g., Ariely & Wertenbroch, 2002), others document no effect (e.g., Bisin & Hyndman, 2020) or even a negative effect (e.g., Burger, Charness, & Lynham, 2011). In this paper, first, I investigate how the design of an interim deadline affects the agent's behavior and the resulting welfare, and characterize the optimal interim deadline that maximizes the agent's welfare. Second, I study how the agent would self-impose a interim deadline and how this would affect her resulting welfare.

---

periods and actual expended effort when that period comes. The agent's behavior is suboptimal from the current period perspective (current preferences) but differs from the optimal behavior from the general perspective. Because the agent's preferences change from period to period, optimal agent's behavior also changes across periods and the agent evaluates her overall performance differently in different periods. For models similar to the one in this paper, it is standard to take so-called "long-run" preferences to evaluate the agent's performance (e.g., Herweg & Müller, 2011; O'Donoghue & Rabin, 1999a; O Donoghue, Rabin, et al., 2006). The idea is to consider the situation when the entire project is in the future and the agent evaluates different periods in the project similarly. Typically, the optimal agent's behavior in this situation coincides with the behavior of the time-consistent agent.

[5] See Giné, Karlan, and Zinman (2010); Houser, Schunk, Winter, and Xiao (2018); Trope and Fishbach (2000).

[6] For the formal definition, see Definition 3.

The agent can be one of three types based on how she understands her present bias: naïve, sophisticated, and partially-sophisticated. The naïve agent is not aware of her time inconsistency and does not realize that she will suffer from self-control problems in the future (believes she is time-consistent), while sophisticated and partially-sophisticated agents are aware. The sophisticated agent correctly knows her present bias and fully predicts her future behavior. The partially-sophisticated agent underestimates her present bias, but cannot correctly predict her future. This is the framework in which I study how the agent would self-impose the deadline and how this deadline would affect her welfare. The analysis is divided into two parts.

Firstly, I investigate how the imposed deadline affects the agent's behavior and welfare. I consider the situation when the interim deadline is imposed exogenously for the purpose of maximizing the agent's welfare. Because my aim is to analyze how the imposed deadline affects the agent, I focus on the situation when the agent is restricted to satisfying the deadline condition, leaving aside the question of who the person might be who is interested in maximizing the agent's welfare, and assuming that the agent has relatively significant losses when she does not meet the deadline.[7] In this setup, I find that the agent continually procrastinates from the beginning of the project to the deadline. In other words, the agent postpones effort from earlier periods to later periods that are closer to the deadline. As a result, the later interim deadline can accumulate a larger amount of effort in the last periods and even decrease the agent's welfare. Thus, depending on the design, the interim deadline may increase or decrease the agent's welfare. I find that there exists a unique interim deadline that maximizes the agent's welfare and that the design for this deadline depends only on the degree of the agent's present bias. Under the same interim deadline, the naïve agent would postpone more effort for future periods compared to sophisticated and partially-sophisticated

[7]One might think about specific principal-agent problems or the social planner problem. While I study how the imposed deadline affects the agent and focus on when the agent has to meet the deadline, the results are valid when the agent has a zero outside option. Because all results correspond to the agent's behavior that yields positive welfare, it is sufficient to set the reward to zero when the agent misses the deadline to motivate the agent to meet the deadline.

agents. Thus, the naïve agent's welfare will be the lowest. The sophisticated agent would behave more like the time-consistent agent than naïve and partially-sophisticated agents, and would gain the highest welfare. The partially-sophisticated agent will fall between naïve and sophisticated agents. As a result, the optimal deadline for the naïve agent is in the earlier period, for the sophisticated agent it is in the later period, and for the partially-sophisticated agent it is in between.

Second, I study how the agent would self-impose the interim deadline and how the deadline would affect the agent's welfare compared to when the agent behaves with no deadline. Because the naïve agent believes she is time-consistent, she has no incentive to impose a deadline on herself and she would not set any self-imposed deadline. By contrast, both sophisticated and partially-sophisticated agents are aware of their self-control problems and would use a self-imposed deadline to affect their future selves. While the sophisticated agent always uses the self-imposed deadline that improves her welfare and sets the deadline optimally, I find that the self-imposed deadline may increase or decrease the agent's welfare (compared to no deadline) depending on the combination of the agent's present bias and sophistication level.[8] Fixing the sophistication level, the agent who is relatively less present-biased would be worse off with a self-imposed deadline, while the agent who is relatively more present-biased would be better off.

The results contribute to the existing literature on behavioral economics, time-inconsistent preferences, and deadlines. To my knowledge, this is the first theoretical paper that considers how an interim deadline affects the agent's effort choice across several periods, and the resulting welfare. This paper characterizes the agent's behavior under the imposed deadline and the optimal design for the interim deadline that maximizes the agent's welfare. The paper also presents a novel finding regarding the impact of a self-imposed deadline on the

[8]The partially-sophisticated agent's underestimation of her present bias can be different. Under the sophistication level, I understand the parameter that describes how close the agent's beliefs about her present bias are to reality. For the formal definition, see Definition 2

welfare of a partially-sophisticated agent. The paper's findings are useful for understanding how deadlines affect behavior and welfare.

The rest of the paper is organized as follows. Section 2 describes related literature and contextualizes this paper into the existing research. Section 3 lays out the model of effort choice with an interim deadline for all agent types. Section 4 analyzes the agents' behavior across periods with and without a deadline. Section 5 provides the results on the optimal exogenous interim deadline for different types. Section 6 describes the results of the self-imposed interim deadline. Section 7 discusses generalizations, limitations, and possible extensions. Section 8 concludes.

1.2 Related Literature

This paper contributes to the literature on time-inconsistent behaviors starting from Strotz (1955),[9] in particular on the quasi-hyperbolic model[10] ((β, δ) - preferences) originally suggested by Phelps and Pollak (1968) and developed by Laibson (1994, 1997) and O'Donoghue and Rabin (1999a, 1999b). I use a finite-horizon model with discrete time and present-biased preferences similar to Herweg and Müller (2011) and Kaur, Kremer, and Mullainathan (2015).[11]

Strotz (1955) and Pollak (1968) distinguish naïve and sophisticated agents, according to how a present-biased agent understands her bias. A naïve agent believes she will behave according

[9]Ericson and Laibson (2019) provides an exhaustive review on the topic of intertemporal choice models.

[10]Researchers aim to describe and explain time-inconsistent behavior by proposing general models and models with specific functional forms. Several researchers have used general models to describe time-inconsistent behavior (e.g., Goldman, 1979, 1980; Peleg & Yaari, 1973; Phelps & Pollak, 1968; Pollak, 1968; Yaari, 1977), while others propose specific functional forms (e.g., Ainslie, 1991, 1992; Ainslie & Haslam, 1992; Ainslie & Herrnstein, 1981; Chung & Herrnstein, 1967; Loewenstein & Prelec, 1992). This paper contributes to the second stream of literature.

[11]The model has undergone many variations and generalizations over past decades (e.g., Cao & Werning, 2016; Harris & Laibson, 2012).

to her current preferences in the future, and is not aware of her self-control problems, while a sophisticated agent is fully aware of her present bias and perfectly predicts her future behavior. Initially, in subsequent work, most researchers assume that the agent has either naïve or sophisticated beliefs,[12] however, recent studies tend to compare the behavior of naïve, sophisticated, and partially-sophisticated agents (e.g., Herweg & Müller, 2011; Hyndman & Bisin, 2022; Kaur et al., 2015). The partially-sophisticated agent is aware of her self-control problem, but underestimates her present bias and cannot correctly foresee her future behavior. In this paper, I analyze and compare the behavior of all three types of agents under exogenous and self-imposed interim deadlines. I characterize the optimal interim deadline for each type and shed new light on how the partially-sophisticated agent would affect her own welfare by using a self-imposed deadline.

In the related literature, deadlines are considered a tool or a commitment device to overcome procrastination which improves the agent's performance. Substantial work has been done on the experimental side, and while the results document a significant demand for the deadlines,[13] the effect on performance is different. Ariely and Wertenbroch (2002) show a positive effect on the agent's performance, and claim that the participants are partially-sophisticated. Burger et al. (2011) document a negative effect, and Bisin and Hyndman (2020) find no effect on the performance. Bisin and Hyndman (2020) studied the effect of deadlines on students who have to complete a single task. The agents must choose the optimal time to exert the immediate costly effort before the deadline. They find that students do not set deadlines optimally and as a result, their deadlines can negatively affect them. This is in line with my results on how partially-sophisticated agents set self-imposed deadlines. Because the partially-sophisticated agent underestimates her present bias, she might set a self-imposed deadline so that it decreases her welfare.

[12] For example, Akerlof (1991) assume naïve beliefs, but Fischer, 2001; Laibson, 1994, 1996, 1997 assume sophisticated beliefs.

[13] This can be interpreted as a support for the fact that the agents are sophisticated or partially-sophisticated.

In addition to studies of deadlines, many researchers have documented demand for other types of commitment devices (e.g., Giné et al., 2010; Houser et al., 2018; Trope & Fishbach, 2000), which supports the assumption that agents are aware of their self-control problems, and emphasizes the importance of studying how commitment devices affect the agent. Kaur et al. (2015) conduct a one-year field experiment to study the workers' behavior under the possibility of taking the dominated contract which elicits future efforts. They found a significant demand for dominated contracts and a positive effect of paydays on effort. They also use the model to predict the agent's behavior. While their model of the agent is similar to mine, they use the model to predict the agent's choice between two contracts. In this paper, I extend the model to describe and analyze the agent's effort choice under the deadline in every period and to analyze the welfare effects.

From the theoretical literature, the Phelps and Pollak (1968) and Laibson (1994) model includes several generalizations and is used in different studies. Harris and Laibson (2012) introduce a continuous time version, and Hyndman and Bisin (2022) provide an adaptation for studying the optimal stopping time problem. Hsiaw (2013) studies the interaction between goal and self-control and Grenadier and Wang (2007) model investment-timing decisions with time-inconsistent preferences. Cao and Werning (2016) generalize the Harris and Laibson (2012) model to study flexible dynamic savings in continuous time. My model is similar to Herweg and Müller (2011), who study the agent's effort choice in two projects during two periods. Instead of two projects and two periods, I consider an N-period model with an agent pursuing a single project.

Deadlines in theoretical literature are often presented as a time when the agent loses the possibility to complete a task with immediate costs and delayed reward. The problem then is in finding the optimal stopping time when the agent has to stop waiting and complete the task instantly (e.g., Hyndman & Bisin, 2022; O'Donoghue & Rabin, 1999b). Other papers consider the agent who invests a certain level of effort in the task during several periods and then is rewarded according to the accumulated efforts after the deadline. Herweg and Müller (2011) study a two-period model with a time-inconsistent agent and compare the behavior of the agents with naïve and sophisticated beliefs. Instead of imposing deadlines, Kaur et al. (2015) provide a choice between two contracts that specify the pay rates based on whether the agent achieves the goal by the predefined moment. Their contracts can be considered as a deadline that specifies the goal and time when the agent is incentivized to achieve his goal. In this paper, I define the deadline in a similar way: the interim deadline specifies the period and goal the agent is supposed to achieve by that period.

The results of the experimental papers suggest that deadlines can differently affect performance and agents' welfare, which is in line with the results of this paper. Most of the existing theoretical literature considers situations different from the setup in this paper, while a few papers study similar setups but different extensions. This paper is a logical continuation of the existing studies. Its results can be used in future research and its conjectures in different fields, including behavioral economics, management, and contract design.

1.3 Model

1.3.1 Agent

The agent has to perform a project for several (N) periods. In each period $t \in \{1,..., N\}$, she chooses an effort level $e_t \geq 0$ that she expends in the current period. In period t, she also forms the effort plan for the future periods: $e_n \geq 0$, $n \in \{t+1,..., N\}$. However, the agent does not have to follow the formed plan when future periods come. The expended effort level e_t in the current period is associated with the immediate costs $c(e_t)$ and the reward after the end of the project for the agent. The cost function is assumed to be the same for every period and is taken in a quadratic form:

$$c(e_t) = \frac{e_t^2}{2} \tag{1}$$

The agent is rewarded after the end of the project in period $N+1$ according to the reward function $R\left(\sum_{t=1}^{N} e_t\right)$. The reward function is assumed to be in an additive form:

$$R\left(\sum_{t=1}^{N} e_t\right) = \sum_{t=1}^{N} e_t \tag{2}$$

The convex cost function represents the fact that the agent becomes tired when she works more hours on a given day. The quadratic form is the simplest and most popular to model the convex costs. However, the agent is rested and faces the same marginal cost at the beginning of any day. At the end of the project, the agent is paid according to her total expended effort. It is assumed that one unit of expended effort is associated with one utility unit at the end of the project. This motivates the choice of the additive linear reward function.

I study the agent's behavior with time-inconsistent preferences with quasi-hyperbolic discounting or so-called (β, δ) – preferences (Laibson, 1997; O'Donoghue & Rabin, 1999a; O'Donoghue & Rabin, 1999b).[14] Precisely, the agent's intertemporal preferences at the chosen period

[14] (β, δ) – preferences in period t can be presented as the following utility function:

$$U_t = u_t + \beta\delta u_{t+1} + \beta\delta^2 u_{t+2} + \beta\delta^3 u_{t+3} + \beta\delta^4 u_{t+4} + ...$$

Here β is a present bias parameter and δ is a long-run discount factor, U_t is total utility, and u_t is the utility in period t. Under these preferences, the agent chooses the current and future consumption/effort in period t maximizing total utility U_t. Moving to period $t+1$, the agent's preferences change and can be described with:

$$U_{t+1} = u_{t+1} + \beta\delta u_{t+2} + \beta\delta^2 u_{t+3} + \beta\delta^3 u_{t+4} + ...$$

$t \in \{1,..., N\}$ can be represented by intertemporal utility function U_t:

$$U_t = u_t + \beta \left[\sum_{n=t+1}^{N+1} \delta^{n-t} u_n \right] \tag{3}$$

u_t represents the agent's instantaneous utility from period t; $\delta \in (0,1]$ – is a standard discount factor; and $\beta \in (0,1]$ – is a present bias parameter. The present bias parameter here is crucial and presents the time inconsistency in the model. In any fixed period t, the agent weights the current period more than the future when $\beta < 1$.[15] In order to focus on the agent's procrastination problem, I abstract away from the standard exponential discounting and set $\delta = 1$.[16] Because the agent faces the cost functions in each period $t \in \{1,..., N\}$ and the reward function after the project at period $N + 1$, her preferences in period t can be presented by intertemporal utility U_t:

$$U_t = -c(e_t) + \beta \left[- \sum_{n=t+1}^{N} c(e_n) + R\left(\sum_{n=1}^{N} e_n \right) \right] = -\frac{e_t^2}{2} + \beta \left[- \sum_{n=t+1}^{N} \frac{e_n^2}{2} + \sum_{n=t}^{N} e_t \right] \tag{4}$$

The agent is modeled as a sequence of intertemporal selves at each period $t \in \{1,..., N\}$. At any chosen period t, the agent maximizes her intertemporal utility U_t by choosing the effort

Again, the agent chooses current and future consumption/effort in period $t+1$ maximizing the total utility U_{t+1}. In period $t+1$, the intertemporal substitution between periods $t+1$ and any further period has changed compared to period t. Specifically, under assumption $0 < \beta < 1$, the agent prefers to consume more and puts in less effort in period $t+1$ compared to her plan for the period $t+1$ in period t; in other words, the agent procrastinates.

[15] β is a present bias parameter that describes how differently the agent weights the future compared to the current moment (today). $\beta = 1$ describes the time-consistent agent, while $0 < \beta < 1$ corresponds to the agent who weighs today relatively more than the future. That agent would prefer to work less and enjoy leisure more today and work more and enjoy leisure less tomorrow.

[16] In the Discussion section, I discuss how the results change when $\delta < 1$.

level for the current period e_t and forming the effort plan for future periods, $\left\{e_n^{(t)}\right\}_{n=t+1}^{N}$, given the effort history and her beliefs about the behavior of her future selves in the next periods. The superscript denotes the period t for the intertemporal agent's self who maximizes the intertemporal utility U_t. In other words, the agent behaves according to the optimal *intertemporal* strategy $\left\{\hat{e}_\tau^{(t)}\right\}_{\tau=t}^{N}$ in period t.

Definition 1: *The agent's* ***optimal intertemporal strategy*** *at period t is the profile of the optimal actions for current and future periods from the perspective of the agent's intertemporal self at period t.*

The optimal *intertemporal* strategy consists of the actions for the current and future periods which maximize the agent's intertemporal utility U_t, given the agent's beliefs about the behavior of her future selves. Thus, the agent behaves according to the current optimal *intertemporal* strategy and invests $\hat{e}_t^{(t)}$ into the project in every period $t \in \{1,..., N\}$. Therefore, the resulted agent's action profile during the project consists of the current actions from the optimal *intertemporal* strategies: $\left\{\hat{e}_t^{(t)}\right\}_{t=1}^{N}$.

1.3.2 Agent Types

The agent can be one of three types, based on how she understands her present bias (β): naïve, sophisticated, or partially-sophisticated. The naïve agent is not aware of her time inconsistency and believes she is time-consistent. The sophisticated agent correctly knows her present bias and fully predicts her future behavior. While the partially-sophisticated agent is aware of her self-control problems, she underestimates her present bias.

In the current period t, the agent invests only the effort level $\hat{e}_t^{(t)}$ into the project. Because the naïve, the sophisticated, and the partially-sophisticated agents differ in terms of how they understand their present bias, they have different beliefs about their future behavior. The naïve agent believes she will behave according to the current optimal *intertemporal* strategy $\left\{\hat{e}_{\tau}^{(t)}\right\}_{\tau=t+1}^{N}$ in the future or, in other words, that β will be equal to 1 in all future next period. Thus, the optimal *intertemporal* strategy for the naïve agent (without a deadline) is the solution to the following utility maximization problem (UMP):

$$\left\{\hat{e}_{\tau}^{(t)}\right\}_{\tau=t}^{N} \in \arg\max_{\{e_\tau\}_{\tau=t}^{N}} \left\{-\frac{e_t^2}{2} + \beta\left[-\sum_{n=t+1}^{N}\frac{e_n^2}{2} + \sum_{n=t}^{N} e_t\right]\right\} \tag{5}$$

$$s.t.: \quad \{e_n\}_{n=1}^{t-1} \;\; are \;\; given$$

On the contrary, the sophisticated agent fully understands her present bias and correctly predicts the behavior of her future selves. In period t, she knows that she will face the same problem (5) in every future period. The partially-sophisticated agent anticipates her future behavior, however, she underestimates her present bias in period t and believes she will face a similar UMP as (5) from period $t+1$ onwards, but with the present bias parameter equal to $\bar{\beta}$ instead of β, where $\beta < \bar{\beta} < 1$:

$$\left\{\hat{e}_{\tau}^{(t+1)}\right\}_{\tau=t+1}^{N} \in \arg\max_{\{e_\tau\}_{\tau=t+1}^{N}} \left\{-c(e_{t+1}) + \bar{\beta}\left[-\sum_{n=t+2}^{N} c(e_n) + R\left(\sum_{n=1}^{N} e_n\right)\right]\right\} \tag{6}$$

$$s.t.: \quad \{e_n\}_{n=1}^{t-1} \;\; are \;\; given$$

Thus, the partially-sophisticated agent forms the effort plan for future periods depending on her sophistication level γ.

Definition 2: *A **sophistication level** is the parameter γ which characterizes how incorrectly the agent estimates her present bias parameter β and is defined by:*

$$\gamma = \frac{1-\bar{\beta}}{1-\beta} \tag{7}$$

1.3.3 Welfare Criteria

Because the agent's preferences change over the periods, the agent evaluates her overall performance differently in different periods. The same agent's action profile gives different utility levels for different intertemporal selves during the project. Therefore, to analyze the agent's welfare, it is standard to take the so-called "long-run" preferences to evaluate the agent's performance:

$$U_0 = \left[-\sum_{t=1}^{N} c(e_t) + R\left(\sum_{t=1}^{N} e_t\right)\right] \tag{8}$$

This approach is in line with O'Donoghue and Rabin (1999a), O Donoghue et al. (2006) and Herweg and Müller (2011). Additionally, these preferences represent the agent who considers the entire project in advance (how she evaluates the performance in a 0-period). The difference then is in multiplication by the constant β.

1.3.4 Interim Deadline

In this paper, I study the agent's behavior under an exogenously imposed deadline to investigate the effect of the deadline on the agent's behavior and welfare. Then I study how the agent sets the self-imposed deadline and how this deadline affects her welfare.[17] However, in both cases, the interim deadline is an instrument used to maximize the agent's welfare; the agent's "long-run" utility (8).

Definition 3: *An **interim deadline** (ID) is the constraint on the agent's behavior defined by two parameters: **timing** $k \in \{1,..., N\}$ and **goal** $A \geq 0$. The agent is restricted to investing the total level of effort greater or equal to A by the end of period k.*

In other words, if the agent faces an interim deadline $ID = (A, k)$, then her intertemporal selves face the following constraint in every period 1,...,k:

$$\sum_{t=1}^{k} e_t \geq A \tag{9}$$

In the case of the exogenous deadline, the agent is informed about the interim deadline before the project starts and has to meet it. Similarly, in the case of the self-imposed interim deadline, the agent has the possibility to choose the interim deadline $ID = (A, k)$ before the project starts. After the interim deadline is chosen, the agent has no possibility to adjust or

[17] Additionally, in the case of the exogenous deadline, the results of this paper can potentially be useful in different setups. For example, one might think about the situation when there is a principal who designs a deadline contract before a project starts. The agent then has to sign it if she is interested in the project. An incentive for the agent to strictly meet the deadline could be very significant losses (or penalties) if the deadline is not met. For example, a student's cost of one more hour of study needed to pass an exam is incomparable to the loss of failing a course and potentially losing their place on a study program.

cancel it, and must satisfy the deadline as in the endogenous case. Because the agent chooses the interim deadline before the project starts, she chooses the $ID = (A, k)$ to maximize her "long-run" utility (8). Thus, the deadlines are used to maximize the same objective in both cases.

Under the interim deadline, the agent can affect her future selves through her choice of the current effort, and this choice depends on her effort history. Because the different types of agent have different beliefs about their future selves' behavior, the behavior of naïve, sophisticated, and partially-sophisticated agents will be different. After the interim deadline is met at period k, the agent behaves according to the optimal *intertemporal* strategies when no deadline is imposed (5).

1.4 Analysis

The analysis is made in the following order. Firstly, I identify the optimal behavior for the agent with "long-run" preferences, or in other words, I analyze how the time-consistent agent would behave. This would be the first-best outcome, which is not achievable for a time-inconsistent agent. Second, I analyze the behavior of the time-inconsistent agent under no deadline and compare the results with those of a time-consistent agent to measure the present bias effect. Further, I study how the agent would change her behavior under the exogenous interim deadline (9). Based on the agent's response to the imposed deadline, I characterize the parameters (A,k) which maximize the agent's welfare. Finally, I study how the partially-sophisticated agent would set the self-imposed deadline and analyze how the deadline would affect the resulting behavior and welfare.

1.4.1 Time-Consistent Agent

The time-consistent agent (with $\beta = 1$) would behave according to her "long-run" preferences because the optimal strategy would not change across different periods. Thus, the agent behaves according to the solution for the UMP with objective (8), cost function (1), and reward functions (2):

$$\max_{\{e_t\}_{t=1}^{N}} \left\{ -\sum_{t=1}^{N} \frac{e_t^2}{2} + \sum_{t=1}^{N} e_t \right\} \tag{10}$$

The first-order conditions are:

$$-e_t + 1 = 0 \quad , \forall\, t \in \{1,..., N\} \tag{11}$$

As a result, the time-consistent agent expends 1 effort at every period during the entire project, $\{\hat{e}_t\}_{t=1}^{N} = \{1,...,1\}$. The optimal intertemporal strategy for the agent with the "long-run" preferences (agent's self at period 0) then is $\left\{\hat{e}_t^{(0)}\right\}_{t=1}^{N} = \{1,...,1\}$. This behavior maximizes the agent's "long-run" preferences and the corresponding utility level $U_0\left(\left\{\hat{e}_t^{(0)}\right\}_{t=1}^{N}\right) = \frac{N}{2}$ is the upper bound and the unachievable target for the problem with a time-inconsistent agent and an interim deadline.

1.4.2 Time-Inconsistent Agent Under No Deadline

When no deadline is imposed, sophisticated and partially-sophisticated agents have no commitment devices to affect their future selves as the preferences are additive separable.[18] Thus, the resulting agents' action profiles coincide for different agent types.

In period t, the naïve agent solves the UMP (5) and her optimal *intertemporal* strategy is to expend β effort in the current period t and 1 effort in every future period: $\left\{\hat{e}_{\tau}^{(t)}(ND)\right\}_{\tau=t}^{N} = \{\beta, 1, ..., 1\}$. ND denotes here that No Deadline is imposed. The sophisticated agent correctly predicts her future behavior and is aware that she will face the same UMP (5) in every future period. Thus, her optimal *intertemporal* strategy consists of β effort to expend in the current period and foreseen β effort in every future period: $\left\{\hat{e}_{\tau}^{(t)}(ND)\right\}_{\tau=t}^{N} = \{\beta, \beta, ..., \beta\}$. The partially-sophisticated agent predicts her future behavior incorrectly and believes that she will face the UMP (6) in every future period. Thus, her optimal *intertemporal* strategy in period t is to expend β effort in the current period and plan to expend $\bar{\beta}$ effort in every future period: $\left\{\hat{e}_{\tau}^{(t)}(ND)\right\}_{\tau=t}^{N} = \{\beta, \bar{\beta}, ..., \bar{\beta}\}$.

All agent types expend β effort in the current period t. Because there is no instrument to commit to future behavior, the resulting agent's action profile is $\{\beta, \beta, ..., \beta\}$ regardless of the agent type. Because $\beta < 1$, she expends less effort at every period. This behavior is not optimal from the perspective of the agent with "long-run" preferences. The agent expends less effort in every period than the time-consistent agent would expend. This behavior represents procrastination. The resulting agent's welfare then is $U_0\left(\left\{\hat{e}_{t}^{(t)}(ND)\right\}_{t=1}^{N}\right) = \beta(2-\beta)\frac{N}{2}$, which is lower than the time-consistent agent's welfare $\left(\frac{N}{2}\right)$ for any $\beta < 1$.

[18] Generally, the agent can affect the behavior of her future self through the reward function by changing the marginal benefit in future periods when choosing effort in the current period. Herweg and Müller (2011) show that the agent would decrease her effort in the current period to incentivize her future self to invest more effort in the next period under their assumptions on cost and reward functions. However, in this paper, I focus on the effects of deadlines and choose the additive structure of the reward function, so the agent cannot affect the marginal benefit in future periods by choosing the current effort.

1.4.3 Time-Inconsistent Agent Under an Exogenous Interim Deadline

When the exogenous interim deadline $ID = (A, k)$ is imposed, the agent faces the constraint (9) at every period 1,..., k. Therefore, the agent's UMP in period $t \in \{1,..., k\}$ can be written as:

$$\max_{\{e_\tau\}_{\tau=t}^{N}} \left\{ -\frac{e_t^2}{2} + \beta \left[-\sum_{n=t+1}^{N} \frac{e_n^2}{2} + \sum_{n=1}^{N} e_n \right] \right\} \tag{12}$$

$$s.t.: \quad \sum_{\tau=t}^{k} e_\tau \geq A - \sum_{n=1}^{t-1} e_n$$

$$\{e_n\}_{n=1}^{t-1} \ \ are \ \ given$$

I assume that the *goal* A is chosen such that the deadline constraint binds from the first period and consider the situation when the deadline does not bind in the first period further in this section. This means that the agent's optimal *intertemporal* strategy in period 1 with No Deadline, $\left\{\hat{e}_t^{(1)}(ND)\right\}_{t=1}^{N}$, does not satisfy the deadline constraint:

$$\sum_{t=1}^{k} \hat{e}_t^{(1)}(ND) < A \tag{13}$$

In Appendix I, I show that if the deadline constraint binds at some period t, then it binds at any future period $t + 1,..., k$.

Under the interim deadline, the agent's behavior depends on her type, because now the agent can affect her future selves' behavior by investing more or less effort in the current period.

However, the naïve agent would not do that, because she always believes that she will stick to the current effort plan in future periods.

Naïve Agent The naïve agent believes that her preferences are time-consistent and that she will behave according to the current effort plan. Then, she behaves in the current period $t \leq k$ according to the optimal *intertemporal* strategy as a function of imposed ID (9) and her effort history:

$$\left\{\hat{e}_{\tau}^{(t)}\left(ID, \left\{\hat{e}_{n}^{(n)}\right\}_{n=1}^{t-1}\right)\right\}_{\tau=t}^{N} \in \arg\max_{\{e_{\tau}\}_{\tau=t}^{N}} \left\{-\frac{e_t^2}{2} + \beta\left[-\sum_{n=t+1}^{N}\frac{e_n^2}{2} + \sum_{n=1}^{N} e_n\right]\right\} \tag{14}$$

$$s.t.: \quad \sum_{\tau=t}^{k} e_{\tau} \geq A - \sum_{n=1}^{t-1} \hat{e}_{n}^{(n)}$$

$$\left\{\hat{e}_{n}^{(n)}\right\}_{n=1}^{t-1} \quad are \;\; given$$

ID here denotes that the agent solves for the optimal intertemporal strategies under the interim deadline. I omit the dependence on the effort history further in the notations so as not to create complications in the equations. I write $\hat{e}_{\tau}^{(t)}(ID)$ instead of $\hat{e}_{\tau}^{(t)}\left(ID, \left\{\hat{e}_{n}^{(n)}\right\}_{n=1}^{t-1}\right)$ keeping in mind that the agent's choice of effort always depends on her effort history under the interim deadline.

Because the naïve agent is not aware of her time inconsistency and does not anticipate her future behavior, the agent behaves according to the first-order conditions for UMP (14) and keeps the effort ratio the same for the optimal *intertemporal* strategy in every period t:

$$e_t = \beta e_n \quad , \forall\, n \in \{t+1,...,k\} \tag{15}$$

However, in different period t, the agent has to redistribute different total amounts of effort $\left(A - \sum_{n=1}^{t-1} e_n\right)$ across periods. Note that the naïve agent has the same effort ratio (15) for her optimal *intertemporal* strategy when she behaves under no deadline.

Proposition 1: *When the naïve agent faces the interim deadline $ID = (A, k)$, she behaves in periods $\{1,...,k\}$ as follows:*

- *the agent postpones a certain amount of effort to later periods;*
- *the agent expends greater effort in every subsequent period;*
- *the expended effort satisfies equation (16);*

$$\hat{e}_t^{(t)}(ID) = \begin{cases} \beta\alpha\Pi_{l=0}^{t-1}\left(\frac{k-l}{k-l-(1-\beta)}\right) & , \quad t \in \{1,...,k-1\} \\ \alpha\Pi_{l=0}^{k-2}\left(\frac{k-l}{k-l-(1-\beta)}\right) = \frac{1}{\beta}\hat{e}_{k-1}^{(k-1)}(ID) & , \quad t = k \\ \beta & , \quad t \in \{k+1,..., N\} \end{cases} \tag{16}$$

where $\alpha = A/k$.

Parameter α here corresponds to the average level of effort expended in every period under the interim deadline during periods $1,..., k$. I provide the proof for Proposition 1 in Appendix

II. The choice of A is equivalent to the choice of α. I assume that goal $A = k$[19] (or $\alpha = 1$), because α is a common coefficient for all effort from period 1 to k. I call this interim deadline *natural.* First, I consider the agent's behavior under the *natural* interim deadline and then discuss how the choice of α (or A) affects the agent's behavior and welfare later in this section.

Definition 4: *An interim deadline $ID = (A, k)$ is called **natural** when goal A is chosen such that the agent is restricted to expend at least the same amount of total effort by period k that a time-consistent agent would expend.*

The interim deadline imposed in period k does not affect the agent's behavior in periods $k+1$ to N, thus, I focus only on the first k periods in the project further in this subsection. In the first period, the agent expends $\hat{e}_1^{(1)}(ID) = \beta\frac{k}{k-(1-\beta)}$, which is greater than what the agent would expend without the imposed deadline $\left(\hat{e}_1^{(1)}(ND) = \beta\right)$. However, $\hat{e}_1^{(1)}(ID) < 1 = \alpha$ for any $k \geq 2$. This means that the agent still procrastinates at the beginning of the project. On the other hand, the agent has to satisfy the deadline (expend on average $\alpha = 1$ level of effort per period during periods $\{1,...,k\}$), thus, expending a lower level of effort than 1 at the beginning of the project means that she must invest greater effort ($> \alpha$) in the periods close to k. In other words, the agent continues to procrastinate and then accumulates effort just prior to the deadline. Note that the expended effort in period $t \in \{1,...,k-1\}$ is the multiplication of $\beta\alpha$ and t constants: $\frac{k}{k-(1-\beta)}, \frac{k-1}{k-1-(1-\beta)}, \ldots , \frac{k-t+1}{k-t+1-(1-\beta)}$. Each of these constants is greater than 1, because $\frac{k-l}{k-l-(1-\beta)} > 1$ for any $l \in \{1,...,k-1\}$. Precisely, the effort expended in period $t+1$ is equal to the effort expended in period t, but multiplied by $\frac{k-t}{k-t-(1-\beta)} > 1$. Thus, in every subsequent period, the agent expends more effort than in the prior period.

The agent's present bias parameter β continues to play the main role in the procrastination

[19]When A is set to be equal to k, the agent is restricted to expend at least 1 effort in every period on average before the deadline. Thus, on average, the total effort expended coincides with what the time-consistent agent would expend (first-best behavior). Note, when A is chosen to be equal to k, the deadline constraint binds in period 1, because $\beta < 1$.

process before the deadline. When $\beta \to 1$ (agent is close to time-consistent), the effort expended in every period tends to 1. In case $\beta \to 0$, the agent expends 0 effort in every period before k and $\alpha k = A$ effort in period k, which is the most inefficient behavior from the "long-run" perspective.

The interim deadline $ID = (k, k)$ increases the expended effort in total from period 1 to period k according to the first-best: the agent now expends $\sum_{t=1}^{k} \hat{e}_t^{(t)}(ID) = k$ effort in total by period k instead of $\sum_{t=1}^{k} \hat{e}_t^{(t)}(ND) = \beta k$. However, the interim deadline allows the agent to redistribute her effort across periods $\{1,..., k\}$ according to the time-inconsistent preferences. At every period $t \in \{1,..., k\}$, the agent's optimal intertemporal strategy satisfies the F.O.C.s:

$$\hat{e}_n = \frac{1}{\beta}\hat{e}_t \quad , n \in \{t+1,...,k\} \tag{17}$$

That is, the agent always expends less (times β) effort today than she will in future periods closer to the deadline (period k). In the current period, she plans to expend equal effort in every future period. However, she does less when the next period comes and again postpones some effort to future periods. If the interim deadline $ID = (k, k)$ is imposed at a relatively later period, it increases the total effort expended relatively more. However, the later interim deadline gives more space for the agent to procrastinate. If k increases, the agent expends less effort in the first period and accumulates a relatively greater degree of effort is necessary in periods closer to the deadline. When $k \to \infty$ (assuming $N \to \infty$, too), the agent expends $\hat{e}_1^1(ID) \to \beta$ in period 1, which is equal to what the time-inconsistent agent under no deadline would expend. If k decreases, the agent expends a relatively higher (closer to 1, as the time-consistent agent would expend) level of effort in period 1 and postpones relatively less effort for future periods.

Coming back to the effect of A on the agent's behavior, the deadline binds if A satisfies (13). If the opposite, the interim deadline $ID = (A, k)$ does not affect the agent's behavior in period 1. Thus, the agent behaves according to the optimal intertemporal strategy when No Deadline is imposed:

$$\left\{\hat{e}_t^{(1)}(ND)\right\}_{t=1}^{N} = \{\beta,1,...,1\} \tag{18}$$

As a result, the agent expends $\hat{e}_1^{(1)}(ID) = \hat{e}_1^{(1)}(ND) = \beta$ in period 1 and moves to the second period. In period 2, the agent still faces the interim deadline constraint, but now she has already expended $\hat{e}_1^{(1)}(ID) = \beta$ effort in period 1, and the interim deadline constraint changes to:

$$\sum_{t=2}^{k} e_t \geq A - \beta \tag{19}$$

The expression $A - \beta$ can be denoted as a new interim deadline *goal* $A' = A - \beta$ and the problem comes down to the previous one in period 1. If the deadline binds in period 2, then the agent behaves according to (16) in periods $\{2,...,k\}$, but with new $A' = A - \beta$ and $k' = k - 1$. If the deadline again does not bind in period 2, then the agent behaves according to the optimal *intertemporal* strategy in period 2 under No Deadline and moves to period 3, and so on. Therefore, if the interim deadline does not bind for the first n periods and binds in period $n + 1$, the agent behaves as a procrastinator during these n periods and then according to (16) with $A' = A - n\beta$ and $k' = k - n$:

$$\hat{e}_t^{(t)}(ID) = \begin{cases} \beta = \hat{e}_t^{(t)}(ND) & , \quad t \in \{1,...,n\} \\ \\ \beta\alpha' \Pi_{l=0}^{t-n-1} \left(\frac{k'-l}{k'-l-(1-\beta)} \right) & , \quad t \in \{n+1,...,k-1\} \\ \\ \alpha' \Pi_{l=0}^{k'-2} \left(\frac{k'-l}{k'-l-(1-\beta)} \right) = \frac{1}{\beta} \hat{e}_{k-1}^{(k-1)}(ID) & , \quad t = k \\ \\ \beta & , \quad t \in \{k+1,..., N\} \end{cases} \tag{20}$$

The resulting behavior is equivalent to the situation in which the agent behaves under the interim deadline $ID = (A', k')$ but with permutations: the agent procrastinates n periods, then expends greater effort from period $n+1$ to k, and continues to procrastinate from period $k+1$ till period N, which is equivalent to expending greater effort from period 1 to $k' = k - n$ and procrastinating from period $k'+1$ till period N. Thus, further in the paper, I abstract from the cases when the interim deadline does not bind in period 1 and assume A satisfies the condition (13).[20]

Sophisticated Agent The sophisticated agent (SA) is aware of her time inconsistency and correctly predicts her future behavior. In period $t \leq k$, the agent solves the UMP by backward induction and considers future efforts as functions of the effort choice for the current period. Because the sophisticated agent correctly estimates her present bias, she correctly predicts her future behavior and follows her optimal *intertemporal* strategy from the beginning of the projects. In other words, her optimal intertemporal strategy does not change from period to period:

[20]In the general case with $\delta < 1$, the agent would prefer to expend effort closer to the end of the project. However, the time-consistent agent would change the behavior and expend less effort at the beginning of the project and more effort in the final periods before the deadline.

$$\hat{e}_t^{(t)}\left(ID\right) \in \arg\max_{e_t} \left\{ -\frac{e_t^2}{2} + \beta \left[-\sum_{n=t+1}^{N} \frac{e_n^2(e_t)}{2} + \sum_{n=1}^{N} e_n(e_t) \right] \right\} \tag{21}$$

$$s.t.: \quad \sum_{\tau=t}^{k} e_\tau \geq A - \sum_{n=1}^{t-1} \hat{e}_n^{(n)}$$

$$\left\{ \hat{e}_n^{(n)} \right\}_{n=1}^{t-1} \quad are \quad given$$

As a result, the sophisticated agent's optimal intertemporal strategy in period t consists of $\hat{e}_t^{(t)}\left(ID\right)$ and $\left\{ \hat{e}_n \left(\hat{e}_t^{(t)} \left(ID\right) \right) \right\}_{n=t+1}^{k}$. The problem is complicated and there is no analytical solution, however, the resulting agent's action profile can be described as follows:

Proposition 2: *If the sophisticated agent faces the binding interim deadline $ID = (A, k)$ in period t and has to expend at least $A' = A - \sum_{n=1}^{t-1} e_n$ effort till period k, she expends $\omega_t^{SA} A'$ effort in the current period t. The agent expends less effort than the time-consistent agent would expend and postpones some amount of effort to future periods (share ω_t^{SA} is lower than $\frac{1}{k-t+1}$). The share ω_t^{SA} can be found by:*

$$\omega_t^{SA} = \underbrace{\Phi\left(\Phi\left(\Phi\left(...\Phi\left(1\right)...\right)\right)\right)}_{k-t \quad times} \tag{22}$$

$$where \quad \Phi(x) = \frac{1}{1 + \frac{1}{x(1-(1-\beta)x)}} \tag{23}$$

I provide the proof for Proposition 2 in Appendix III. In period 1, the agent expends $\hat{e}_1^{SA}(ID) = \omega_1^{SA} A$ and then, in period 2, she expends $\hat{e}_2^{SA}(ID) = \omega_2^{SA}(1-\omega_1^{SA})A$. Thus, in period $n \leq k$, the agent expends $\hat{e}_n^{SA}(ID) = \omega_n^{SA}(1-\omega_{n-1}^{SA})...(1-\omega_2^{SA})(1-\omega_1^{SA})A$. However, this is correct if the interim deadline binds in period 1. In other words, the sophisticated agent would affect her future behavior by the choice of the current effort when the *goal* A is greater than the total effort expended under no deadline:

$$k\beta < A \tag{24}$$

Because the agent perfectly anticipates her future behavior, the deadline does not bind in every period if it does not bind in the first period. If the deadline binds and A satisfies equation (24), the agent affects the behavior of her future selves by expending effort according to Proposition 2. However, the sophisticated agent would expend $e_1 = \beta$ if $\hat{e}_1^{SA}(ID) = \omega_1^{SA} A \leq \beta$. Similar to the naïve agent case, the *goal* in the second period adjusts to $A' = (A - \beta)$ and the situation repeats: the agent behaves as though she is facing the interim deadline $ID = (A', k-1)$ and expends $e_2 = \beta$ if $\hat{e}_2^{SA}(A',k-1) \leq \beta$. As a result, the sophisticated agent expends $e_t = \beta$ while the effort calculated according to Proposition 2 is less than or equal to β. Because $A > k\beta$, the average effort per period that the agent has to expend is increasing in every subsequent period and there is a period $n+1$ when the agent switches her behavior to conform with Proposition 2:

$$\hat{e}_t^{SA}(ID) = \begin{cases} \beta & , \quad t \in \{1,...,n\} \\ \left[\omega_{t-n}^{SA} \Pi_{l=1}^{t-n-1}\left(1-\omega_l^{SA}\right)\right](A - n\beta) & , \quad t \in \{n+1,...,k\} \\ \beta & , \quad t \in \{k+1,...,N\} \end{cases} \tag{25}$$

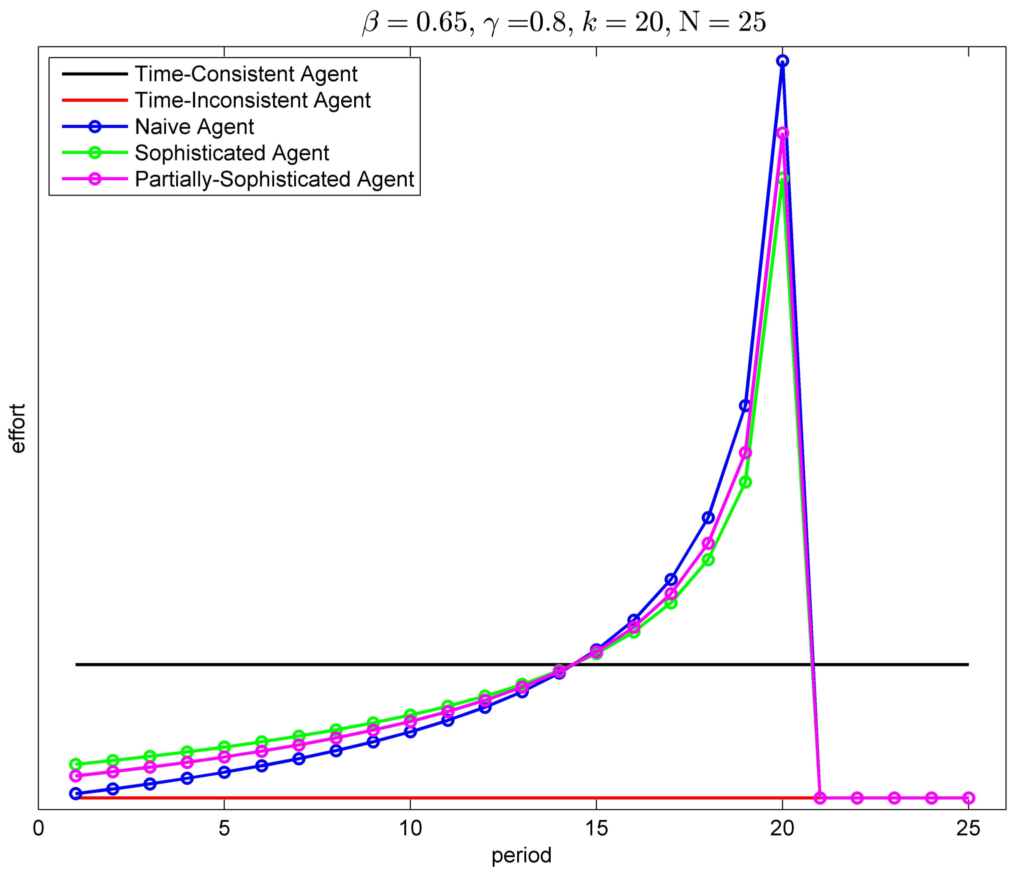

Figure 1.1: The comparison of behavior for different types of agent under the same interim deadline $ID = (20, 20)$ with present bias parameter $\beta = 0.65$ and sophistication level $\gamma = 0.8$ for the partially-sophisticated agent. The black and red lines present the behavior of time-consistent and time-inconsistent agents. The blue line presents the naïve agent's behavior, the green presents the sophisticated agent's behavior, and the magenta line presents the partially-sophisticated agent's behavior.

As a result, the sophisticated agent does better than the naïve agent under the same interim deadline $ID = (A, k)$. Because the agent anticipates the effect of her current choice of expenditure of effort on future behavior, she postpones relatively less effort to later periods than does the naïve agent.

Figure 1.1 presents a comparison of the behavior of different agent types over the entire project, given that the other parameters and interim deadline are fixed. The green line with cycles describes the sophisticated agent's behavior. The blue line depicts the behavior of the naïve agent under the same interim deadline and with the same present bias parameter β. The sophisticated agent invests more effort in the earlier periods and less in the later periods. While both agents expend more effort than without any interim deadline (red line on the graph), their behavior significantly differs from that of the time-consistent agent (black line on the graph). Thus, the sophisticated agent effectively uses the interim deadline to affect her future selves and performs better overall than the naïve agent. However, the sophisticated agent still procrastinates and postpones effort to the final periods before the deadline.

Partially-Sophisticated Agent The partially-sophisticated agent (PSA) is aware of her time inconsistency but predicts her future behavior incorrectly. She believes that her future selves will behave according to the present bias parameter $\bar{\beta} = 1 - \gamma(1 - \beta)$. Thus, the partially-sophisticated agent would behave similarly to a sophisticated and solve the problem by backward induction, but with a slightly different algorithm for calculating coefficient ω_t^{PSA} in period t. The agent's belief about how her current choice of effort affects future decisions depends on her sophistication level γ. This affects both her choice regarding the current effort and her plan for future efforts. In period $t \leq k$, the partially-sophisticated agent solves for $e_t^{(t)}(ID, \gamma)$:

$$\hat{e}_t^{(t)}\left(ID, \gamma\right) \in \arg\max_{e_t} \left\{ -\frac{e_t^2}{2} + \beta \left[-\sum_{n=t+1}^{N} \frac{e_n(e_t, \gamma)}{2} + \sum_{n=1}^{N} e_n(e_t, \gamma) \right] \right\} \tag{26}$$

$$s.t.: \quad \sum_{\tau=t}^{k} e_\tau \geq A - \sum_{n=1}^{t-1} \hat{e}_n^{(n)}$$

$$\left\{ \hat{e}_n^{(n)} \right\}_{n=1}^{t-1} \quad are \;\; given$$

As a result, the partially-sophisticated agent's choice of current effort depends on her effort history, interim deadline, and sophistication level γ. Then, the partially-sophisticated agent's optimal *intertemporal* strategy in period t consists of the current period effort $\hat{e}_t^{(t)}(ID)$ and her effort plan on future periods $\left\{\hat{e}_n\left(\gamma, \hat{e}_t^{(t)}(ID)\right)\right\}_{n=t+1}^{k}$. Because the agent's beliefs about her future behavior are incorrect, she expends $\hat{e}_t^{(t)}(ID)$ in the current period, but her optimal *intertemporal* strategy in the next period does not coincide with the current one. The resulting agent's action profile can be described as follows:

Proposition 3: *If the partially-sophisticated agent faces the binding interim deadline* $ID = (A, k)$ *in period* t *and has to expend at least* $A' = A - \sum_{n=1}^{t-1} e_n$ *effort till period* k*, she expends* $\omega_t^{PSA} A'$ *effort in the current period* t*. There* ω_t^{PSA} *can be found as:*

$$\omega_t^{PSA} = \Phi\left(\underbrace{\bar{\Phi}\left(\bar{\Phi}\left(...\bar{\Phi}(1)...\right)\right)}_{k-t-1 \quad times}\right) \tag{27}$$

$$where \quad \bar{\Phi}(x) = \frac{1}{1 + \frac{1}{x\left(1-\left(1-\bar{\beta}\right)x\right)}} \tag{28}$$

The function $\Phi(x)$ is the same as for the sophisticated agent and is defined by equation (23). Since $\beta < \bar{\beta} < 1$, the partially-sophisticated agent expends a lower share of A' compared to the sophisticated agent under the same interim deadline. In period t, the partially-sophisticated agent believes she will expend $\bar{\beta}$ effort in every subsequent period under no deadline. Thus, the deadline binds in period 1 if:

$$\beta + (k-1)\bar{\beta} < A \tag{29}$$

Because the agent has incorrect beliefs about her future behavior, the deadline may not bind in the first n periods and binds from period $n+1$:

$$\begin{cases} \beta + (k-n)\bar{\beta} \geq A - (n-1)\beta \\ \\ \beta + (k-n-1)\bar{\beta} < A - n\beta \end{cases} \tag{30}$$

From the period when the interim deadline begins to bind, the partially-sophisticated agent behaves according to Proposition 3 if her calculated effort is greater than β. Similar to a sophisticated agent, the deadline might not affect the agent's behavior during the first m periods:

$$\hat{e}_t^{PSA}(ID) = \begin{cases} \beta & , \quad t \in \{1,...,m\} \\ \\ \left[\omega_{t-m}^{PSA} \Pi_{l=1}^{t-m-1} \left(1 - \omega_l^{PSA}\right)\right] (A - m\beta) & , \quad t \in \{m+1,...,k\} \\ \\ \beta & , \quad t \in \{k+1,...,N\} \end{cases} \tag{31}$$

The magenta line in Figure 1.1 presents the behavior of the partially-sophisticated agent in comparison to the naïve and sophisticated agents. As expected, the partially-sophisticated agent performs somewhere in between naïve and sophisticated agents. The partially-sophisticated agent uses the interim deadline to affect her future selves as a sophisticated agent does, but due to her incorrect estimation of present bias, she does it less efficiently than a sophisticated agent.

For all types of time-inconsistent agents, the interim deadline significantly affects the agent's behavior. However, the agent behaves differently from a time-consistent agent, and postpones effort to the final periods before the deadline. Obviously, from the "long-run" perspective, such behavior is not optimal. Thus, while the interim deadline increases the total amount of effort expended, the effect on the agent's welfare is not clear.

1.5 Optimal Natural Interim Deadline

The interim deadline in period k does not affect the agent's behavior in periods $k+1$ to N. For periods 1 to k, the agent expends a greater degree of effort in each period than she would with no deadline. However, the agent continues to procrastinate and postpones a certain amount of effort to future periods according to the present-bias parameter β. As a result, the interim deadline in period k increases the effort spent on the project in periods 1 to k, while an accumulated greater amount of effort is expended in the final periods before the deadline. The effect of the imposed interim deadline on the agent's welfare is ambiguous. The results I provide in this and the next sections are for the agent's behavior under the *natural* interim deadline $ID = (k, k)$. Nevertheless, results hold for the general case and I consider the optimal choice of *goal* A in the Discussion section.

In the past, it was generally agreed that deadlines help agents to minimize or avoid procrastination and therefore increase agent welfare, because procrastination lowers it. As discussed above, an imposed deadline increases effort expended in the periods before the period k and accumulates the effort in the last periods before the period when imposed. On the one hand, the increase in effort helps to overcome procrastination and positively affects the agent's welfare due to greater effort expended on the project. I call this effect the "increase in effort" effect. Under the *natural* interim deadline the "increase in effort" would only positively affect the agent's welfare if the increase in effort were the same for every period {1,...,k}. On the other hand, the agent distributes the effort across periods not equally: less than optimally in the beginning and greater than optimally in the final periods before period k. Because the agent is restricted to expend, on average, $\alpha = 1$ effort in every period, unequal distribution of effort across periods {1,...,k} decreases the agent's welfare. I call this the "redistribution of effort" effect.

The interim deadline $ID = (k, k)$ increases the total effort expended from $\sum_{t=1}^{N} e_t = \beta N$ to $\sum_{t=1}^{N} e_t = \beta N + (1 - \beta)k$. During periods {1,...,$k$}, the agent expends k effort, and it would be optimal to expend 1 effort per period from the "long-run" perspective. However, the agent procrastinates and due to the "redistribution of effort" effect expends less effort in earlier periods and more effort in later periods. This effect does not affect the reward function, because the agent expends the same effort in total, but it does change the total costs. While less effort in the earlier periods decreases costs in those periods, greater effort in later periods increases costs relatively more because the cost function is convex.

Proposition 4: *There is a generically*[21] *unique period* $\hat{k}$ *such that the agent's behavior induced according to Proposition 1, 2, 3 (depending on her type) under the natural interim deadline* $ID = (\hat{k}, \hat{k})$ *yields the highest agent welfare (among all possible natural interim deadlines* $ID = (k, k)$, $k \in \{1, ..., N\}$*). The number of this period* $\hat{k}$ *depends only on the present-bias parameter* β *and the sophistication level* γ *(for partially-sophisticated agents).*

[21]There is a set of measure zero of specific values of present bias parameter β, such that there are two *natural* interim deadline in periods k and $k + 1$ which maximize the agent's welfare.

I provide the proof in Appendix IV, and here I describe the general intuition and logic. Generally, the later interim deadline helps the agent to expend greater effort overall. Thus, it would be optimal to impose the interim deadline in the last period if the agent distributes effort equally across all periods. From one side, the increase in reward function ("increase in effort" effect) is the same when the interim deadline is moved from the first period to the second or from the 11th to the 12th. However, the agent accumulates greater effort for the final period before the deadline for the later interim deadline. In other words, the "redistribution of effort" effect increases in k. The logic is that it is profitable to postpone the interim deadline to the later periods while the "increase in effort" effect is greater than the "redistribution of effort" effect. Because the first effect is constant in k and the second is increasing in k, there is $k = \hat{k}$ after which the later interim deadline decreases the agent's welfare. This *natural* interim deadline $ID = (\hat{k},\hat{k})$ can be described as one under which the agent's welfare is higher than with the interim deadline imposed in periods $\hat{k} - 1$ or $\hat{k} + 1$:

$$\begin{cases} U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k},\hat{k})\right\}_{t=1}^{N}\right) - U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}+1,\hat{k}+1)\right\}_{t=1}^{N}\right) > 0 \\ U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}-1,\hat{k}-1)\right\}_{t=1}^{N}\right) - U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k},\hat{k})\right\}_{t=1}^{N}\right) < 0 \end{cases} \tag{32}$$

Any *natural* interim deadline $ID = (k,k)$ affects the agent's behavior only in periods $\{1,...,k\}$, but in later periods, the agent expends effort equal to β. Thus, the optimal period $\hat{k}$ does not depend on the length of the project N[22] and inequalities (32) contain only parameter β and $\hat{k}$. Therefore, the optimal interim deadline $ID = (\hat{k},\hat{k})$ is defined only by the present-bias parameter β. However, there is a set of measure zero of specific values of present bias parameter β, such that $U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k},\hat{k})\right\}_{t=1}^{N}\right) = U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}+1,\hat{k}+1)\right\}_{t=1}^{N}\right)$. In that case, both *natural* interim deadlines in period $\hat{k}$ and period $\hat{k}+1$ maximize the agent's welfare. These

[22] If N is large enough. If the optimal $\hat{k}$ according to (32) is greater than N, then it is optimal to impose the interim deadline in the last period or $\hat{k} = N$.

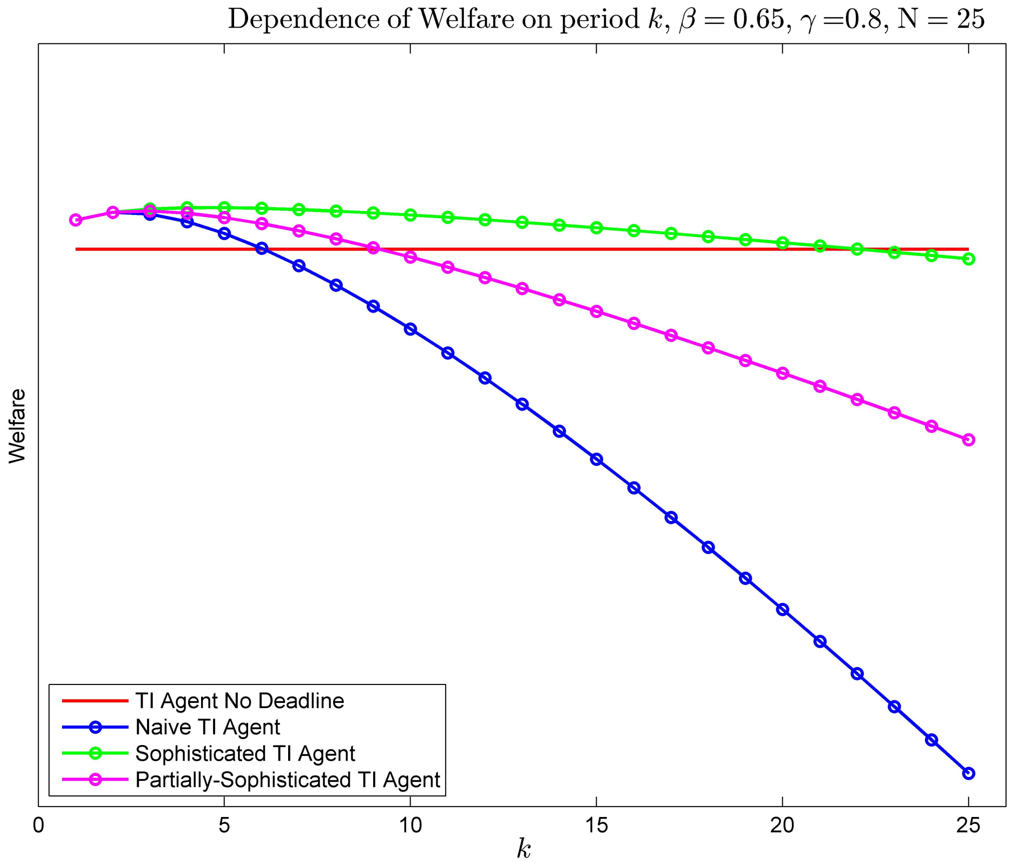

Figure 1.2: The dependence of the agent's utility level on the period k when the *natural* interim deadline $ID = (k,k)$ is imposed. The blue, green, and magenta lines present the dependence for naïve, sophisticated, and partially-sophisticated agents respectively. The red line presents the utility level of the time-inconsistent agent (regardless of the agent's type) pursuing the project under no deadline.

cases do not affect the results and conclusions of this paper and it can be assumed that the agent prefers the earlier deadline when she is indifferent between these two.

Figure 1.2 presents the dependence of the agent's welfare on the period k when the interim deadline $ID = (k,k)$ is imposed for all three agent types. Similar to Figure 1.1, the other parameters are fixed.[23] The red line shows the agent's welfare when no deadline is imposed

[23] The meaning of the present bias parameter β is chosen such all the necessary effects are observed on the graph for the 25-period project for all types of agents. Empirical studies provide mixed results of estimations of the present-bias parameter β. Laibson, Repetto, and Tobacman (2007) estimate that the parameter β is equal to 0.50, while DellaVigna and Paserman (2005) find $\beta = 0.9$. Paserman (2008) estimates $\beta = 0.5$ for

and the agent expends β effort in every period. The blue, green, and purple lines present how the welfare of the naïve, sophisticated, and partially-sophisticated time-inconsistent agent correspondingly depends on the choice of k when the agent behaves under the *natural* interim deadline $ID = (k,k)$.

Moving the interim deadline from the first period to the second significantly increases the agent's welfare, while setting it in later periods increases welfare less and less and eventually begins to decrease it. Further relocation of the *natural* interim deadline to later periods only decreases the agent's welfare. The interim deadline can even cause lower welfare than in a situation without any deadline. In Figure 1.2, this occurs after the intersections with the red line (in different periods for different types). These intersections of colored lines with the red line are defined by inequalities similar to (32) and depend only on the agent's present-biased parameter β. I discuss this specifically in Appendix IV.

As a result, when the agent's present-bias parameter β is known, it is always possible to impose a *natural* interim deadline $ID = (\hat{k},\hat{k})$ that will maximize the agent's welfare. Such $\hat{k}$ can be found from the inequalities (32). In case $\hat{k} \geq N$, it is optimal to impose the interim deadline in the last period N.

1.6 Self-Imposed Interim Deadline

The agent sets the *natural* interim deadline $ID = (k,k)$ before the project starts (in period 0) and then has to satisfy the deadline constraint. The agent has no possibility to change, adjust, or cancel the interim deadline in later periods during the project. In this framework, the naïve agent believes she will behave according to current preferences and has no incentive to impose any restrictions on her future self, while the sophisticated and partially-sophisticated

low-income workers and $\beta = 0.9$ for high-income workers. For the exhaustive review on the topic of measuring time preferences, see Cohen, Ericson, Laibson, and White (2020).

agents do. Thus, the naïve agent would choose not to impose any interim deadline.[24]

In contrast to the naïve agent, the sophisticated agent anticipates her future behavior and would prefer to use a self-imposed deadline to affect her future self. Since the sophisticated agent correctly predicts her future behavior, she can set the interim deadline optimally and imposes it according to the previous section. Therefore, the most interesting case is when the agent is partially-sophisticated and underestimates her present bias. She is aware of her time inconsistency and would use a self-imposed deadline to affect her future self, however, she cannot use the interim deadline as optimally as a sophisticated agent can.

In period 0, the partially-sophisticated agent with sophistication level γ believes she will behave as a sophisticated agent with a present bias parameter equal to $\bar{\beta}^{PSA} = 1 - \gamma(1 - \beta^{PSA})$. Thus, the partially-sophisticated agent would set her self-imposed deadline equal to the optimal interim deadline for the sophisticated agent with $\beta^{SA} = \bar{\beta}^{PSA}$. Because the optimal period for the *natural* interim deadline is increasing in β for all types of the time-inconsistent agent and $\bar{\beta}^{PSA} > \beta^{PSA}$, the partially-sophisticated agent would set the interim deadline later than the optimal one. As a result, depending on parameters β^{PSA} and γ, the agent sets her self-imposed deadline such that it might increase or decrease her welfare.

Figure 1.3 presents the regions of pairs of parameters (β^{PSA}, γ) for the partially-sophisticated agent where the self-imposed deadline (which is set by the agent with these parameters) would increase or decrease the agent's welfare. The white region presents the pairs of parameters with which the agent sets the self-imposed deadline such it will decrease her welfare. Respectively, the blue region presents the pair of parameters with which the agent sets the self-imposed deadline so that it will increase her welfare.

[24]Strictly speaking, in period 0, the naïve agent is indifferent between not imposing the interim deadline and any *natural* interim deadline because she believes she will behave as a time-consistent agent. I assume that the agent prefers not to impose the interim deadline if she is indifferent between imposing the interim deadline or not.

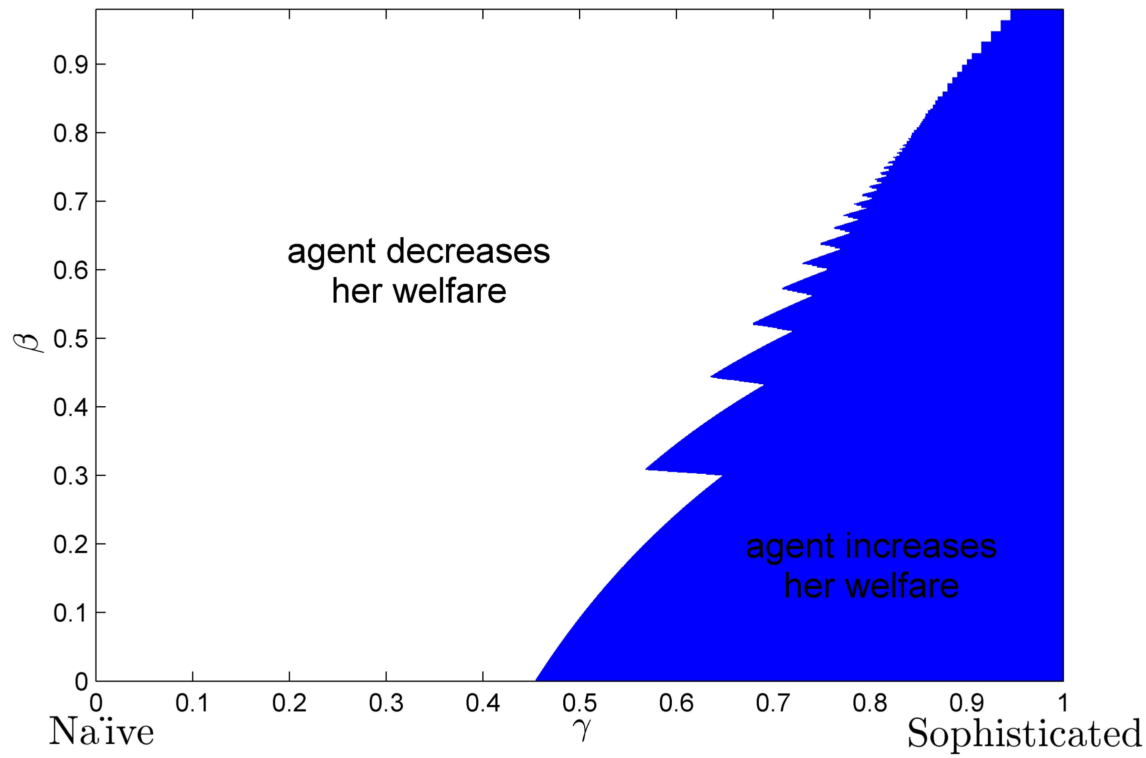

Figure 1.3: The regions of pairs $\beta - \gamma$ with which the agent would increase or decrease her welfare by setting the self-imposed deadline. The white area corresponds to the parameters' pairs with which the agent would decrease her welfare by setting the self-imposed deadline, while she would increase her welfare with parameters from the blue area.

The agent who is relatively less present-biased (with high present bias parameter β, the top half of Figure 1.3) would decrease her welfare by setting an interim deadline, while the agent who is relatively more present-biased (with low present bias parameter β, the bottom half of Figure 1.3) would increase her welfare, given the same sophistication level. However, the effect exists only for the relatively high sophistication level (the right half of Figure 1.3) and the agent with a relatively low high sophistication level (the left half of Figure 1.3) would decrease her welfare by setting a self-imposed deadline regardless her present bias parameter β. In other words, even for agents with high sophistication levels, the self-imposed deadline is a useful instrument only for agents with serious self-control problems. The agents who are close to time-consistent may suffer from setting the self-imposed deadline. The intuition here is in the fact that the same underestimation of present bias leads to different delays in the interim deadline compared to the optimal deadline for partially-sophisticated agents with different present bias parameters. The optimal interim deadline for the agent with a lower present bias parameter is in the earlier period, while for the agent with a higher present bias parameter, it is in the later period. When setting a self-imposed deadline, the agent with a lower present bias parameter sets the deadline relatively closer to the optimal one, as the agent with a higher present bias parameter does. As a result, the large delay in the imposed deadline leads to high induced costs in the final periods before the deadline which outweigh all benefits of setting a deadline for the agent with a higher present bias parameter. The small delay in the imposed deadline for the agent with a lower present bias parameter decreases the efficiency of setting a deadline, but benefits stay higher than costs. Thus, the partially-sophisticated agent with a larger present bias parameter β suffers relatively more from setting the self-imposed deadline.

1.7 Discussion

1.7.1 Choice of Goal A

Next, I consider how the choice of *goal* A ($\neq k$) affects the agent's behavior and welfare. When interim deadline $ID = (\alpha k{,}k)$ is imposed in period k, the agent's welfare is:

$$U_0\left(\left\{\hat{e}_t^{(t)}(\alpha k, k)\right\}_{t=1}^{N}\right) = \sum_{t=1}^{k}\left[\alpha\hat{e}_t^{(t)}(k{,}k) - \frac{\left(\alpha\hat{e}_t^{(t)}(k{,}k)\right)^2}{2}\right] + (N-k)\frac{\beta(2-\beta)}{2} \tag{33}$$

Taking the first-order condition, the optimal $\hat{\alpha}$ as a function of k is defined by:

$$\hat{\alpha}(k) = \frac{k}{\sum_{t=1}^{k} \left[\hat{e}_t^{(t)}(k,k)\right]^2} < 1 \tag{34}$$

The optimal $\hat{\alpha}$ characterizes how inefficiently the agent redistributes efforts across periods. By the definition of the interim deadline, $\sum_{t=1}^{k} \hat{e}_t^t(k,k) = k$. It is optimal to distribute effort equally across periods with a convex cost function and expend 1 effort in each period (as the time-consistent agent would do). Thus, the maximum $\hat{\alpha}$ is for the time-consistent agent and equal to 1, while $\hat{\alpha}$ is increasing in β (a higher β corresponds to a less present-biased agent) for the time-inconsistent agent. Plugging the optimal $\hat{\alpha}(k)$ back into the agent's welfare (33):

$$U_0\left(\left\{\hat{e}_t^{(t)}(\hat{\alpha}k,k)\right\}_{t=1}^{N}\right) = \frac{1}{2}\hat{\alpha}(k)k + (N-k)\frac{\beta(2-\beta)}{4} = \frac{1}{2}\tilde{A}(k) + (N-k)\frac{\beta(2-\beta)}{4} \tag{35}$$

Where $\tilde{A}(k) = \hat{\alpha}(k)k$. The optimal *goal* or $\hat{\alpha}$ is a function of the chosen period k and the problem comes down to the choice of k. Thus, both parameters of optimal interim deadline $ID = (\hat{\alpha}\hat{k}, \hat{k})$ are defined by the agent's present-bias parameter β, because the optimal *timing* $\hat{k}$ is defined by β.

However, the choice of α according to equation (34) does not guarantee that the interim deadline will bind in the first periods. For the naïve agent, this may occur when β is low and the agent postpones too much effort for the last periods, that $\hat{\alpha}$ is low enough and $\tilde{A}$ does not satisfy condition (13). On the other hand, the optimal $\hat{k}$ is increasing in β[25] and

[25]When $\beta \to 1$, the agent is close to time-consistent and postpones less effort for future periods. Thus, the variance of effort across periods $\{1,...,k\}$ is low and it is optimal to move k to later periods. When $\beta \to 0$,

the optimal period $\hat{k}$ is earlier for the lower present-bias parameter β. Thus, $\hat{\alpha}$ must be even lower to create the situation in which the deadline does not bind in the first period with earlier $\hat{k}$. For the sophisticated and partially-sophisticated agents, the deadline constraint always binds if it binds for the naïve agent. The sophisticated agent would affect her future selves if the *goal* A is lower or equal to the total effort under no deadline: $k\beta \geq A$. The partially-sophisticated agent believes she will behave according to the present-bias parameter $\bar{\beta}$ in future periods, thus, she would affect her future selves if $\beta + (k-1)\bar{\beta} < A$. Because $k\beta < \beta + (k-1)\bar{\beta} < \beta + (k-1)$, the deadline constraint binds for sophisticated and partially-sophisticated agents when it binds for the naïve agent. In case, when the interim deadline does not bind in the first several periods, the agent behaves according to equations (20), (25), and (31) as was described in the previous section.

In real life, it is easy to imagine a situation in which the period for the interim deadline is fixed or predefined. For example, the end of a semester for students or pre-specified deadlines in worker contracts. In that case, only the choice of *goal* A can affect the agent's welfare. When the period k is lower than the optimal period $\hat{k}$, it is optimal to set α according to (34), and no improvement is possible. However, when the interim deadline is imposed in period $k > \hat{k}$, α can be chosen such that the deadline would not bind for the first $(k - \hat{k})$ periods. Thus, the agent will behave according to (20), (25), or (31), and her behavior will be equivalent to her behavior under the interim deadline in period $\hat{k}$. For example, the deadline constraint (13) binds for any α such that $(1-\alpha)k < (1-\beta)$. In the opposite case, the naïve agent expends $e_1^{(1)}(ID) = \beta$ and moves to the next period. In the second period, the condition on α changes to $(1-\alpha)k < 2 * (1-\beta)$ and so on with condition $(1-\alpha)k < n * (1-\beta)$ in the nth period. Thus, to induce the naïve agent's behavior equivalent to that under the interim deadline in period $\hat{k}$, α can be chosen such that the deadline does not bind in the first $(k - \hat{k})$ periods and binds in period $(k - \hat{k} + 1)$:

the agent postpones almost all effort to the future and it is optimal to set $k = 1$, otherwise, the agent would invest 0 effort in periods $\{1,...,k-1\}$ and A effort in period k.

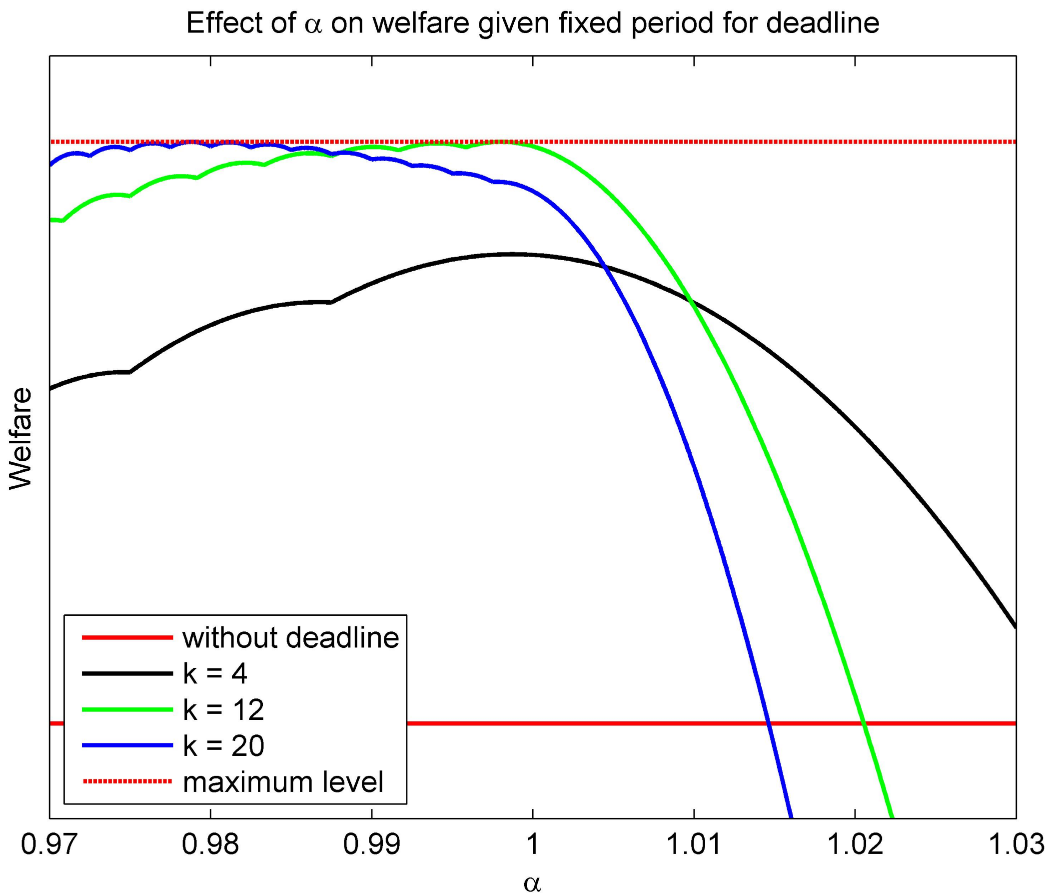

Figure 1.4: The dependence of the naïve agent's welfare on the value of parameter α for different fixed periods for the interim deadline. The black, green, and blue lines present the dependence for cases when the interim deadline is imposed in periods 4, 12, and 20 respectively. The red line presents the utility level of the time-inconsistent agent (regardless of the agent's type) pursuing the project under no deadline, while the dotted red line corresponds to the maximum possible utility level.

$$\begin{cases} (1-\alpha)k \geq (k-\hat{k})(1-\beta) \\ (1-\alpha)k < (k-\hat{k}+1)(1-\beta) \end{cases} \tag{36}$$

Figure 1.4 shows how the naïve agent's welfare depends on the parameter α in cases with different periods when the interim deadline is imposed. The red line at the bottom shows the welfare when there is no deadline imposed and the agent expends β effort in every period.

The green line demonstrates how the naïve agent's welfare depends on α when the interim deadline is imposed in period $\hat{k}$ ($\hat{k} = 12$ with chosen parameters). Finally, black and blue lines present the dependence of the agent's welfare on α when the interim deadline is imposed in an earlier ($k = 4$) or a later ($k = 20$) period than $\hat{k}$.

It is optimal to set $\alpha = \hat{\alpha}$ when the interim deadline is imposed in period $\hat{k}$ or earlier, and making it higher or lower meaning cannot increase the agent's welfare (green and black lines on Figure 1.4). However, it is possible to increase the agent's welfare by inducing the agent's procrastination in the first several periods with lower α according to (36) when the interim deadline is imposed in period $k > \hat{k}$. The blue line in Figure 1.4 illustrates this case: the interim deadline is imposed in period $k = 20$ and it is optimal to set α such the deadline constraint would not bind for the first eight periods. Then the agent behaves in the same way as under the interim deadline imposed in period $\hat{k} = 12$ from the 9th period till the 20th period, which results in the greatest welfare. The periodic waves in Figure 1.4 show when parameter α becomes too small such the condition $(1 - \alpha)k < n * (1 - \beta)$ does not hold and the deadline constraint does not bind for one more period.

When the interim deadline is imposed in period $\hat{k}$ or before it, decreasing α such that the deadline constraint would not bind in the first period yields lower welfare. However, the case when the interim deadline is set after $\hat{k}$ can be reduced to the case with optimal $\hat{k}$ by choosing a lower α: after procrastinating in several periods (several waves to the left from $\alpha = 1$) the maximum point of the blue line on Figure 1.4 yields the same welfare as the maximum point of the green line. As a result, it is always possible to find the optimal $\hat{\alpha}(k)$ given that the interim deadline is imposed in period k.

Proposition 5: *There exists a unique goal $\hat{A}(k)$ (or $\hat{\alpha}(k)$) such that, given fixed timing for the interim deadline, k, the interim deadline with goal $\hat{A}(k)$ maximizes the agent's welfare.*

The value of goal $\hat{A}(k)$ depends only on the present-bias parameter β and on period k when the interim deadline is imposed (and sophistication level γ for the partially-sophisticated agent).

The fact that $\hat{A}(k)$ depends only on β and k strictly follows from the equation (34) and inequalities (36), (30), and because the number of periods that the sophisticated and partially-sophisticated agents procrastinate are defined by functions $\Phi(\cdot)$ and $\bar{\Phi}(\cdot)$. Consequently, combining this fact with Proposition 4, the optimal period and the corresponding optimal *goal* depend only on the present-bias parameter β.

Because the agent's behavior when the interim deadline is imposed in period k and binds is equivalent to her behavior when the interim deadline is imposed in period $k+n$ and does not bind for the first n periods (with specifically chosen *goal* A), the optimal interim deadline which maximizes the agent welfare is not unique. There exists the earliest period k^* such that the interim deadline $ID = (\hat{A}(k^*), k^*)$ maximizes the agent's welfare. As discussed above, the same welfare maximum is achievable by choosing optimal *goal* $A = \hat{A}(k^* + m)$ when the interim deadline is imposed in period $k^* + m \leq N$. Based on simulations, the earliest period k^* either coincides with the period $\hat{k}$ for the optimal *natural* interim deadline $ID = (\hat{k}, \hat{k})$, or equal to the next one, $k^* = \hat{k} + 1$, depending on the present bias parameter β. It is never optimal to impose the interim deadline in an earlier period than k^* (and $\hat{k}$) because it would yield lower welfare.

While the agent's behavior under interim deadline $ID = (\hat{A}(k^*), k^*)$ is equivalent to the behavior under any later interim deadline $ID = (\hat{A}(k^* + m), k^* + m)$ when the standard discount factor δ is assumed to be equal to 1, the situation differs when $\delta < 1$. With $\delta < 1$, the agent strictly prefers to invest greater effort later (or closer to the delayed rewards). Thus, only the latest interim deadline is optimal, $ID = (\hat{A}(N), N)$. In other words, it is optimal to set the interim deadline in the last period and adjust the *goal* such that the agent would procrastinate during the optimal number of periods at the beginning of the project.

As a result, it is optimal to set the interim deadline in the last period (N) of the project for all types of agents. The optimal deadline induces behavior as follows. The agent procrastinates at the beginning of the project (expends β effort in earlier periods) and postpones significant effort for later periods. Starting from some period ($N - k^* + 1$), the deadline binds and the agent begins to expend greater effort in each subsequent period. However, while it is optimal to impose the deadline in the last period for all types, the optimal *goal* $\hat{A}(N)$ (and the resulting performance) is different for different types: the lowest for naïve and the highest for sophisticated. On the other hand, imposing the same deadline on all types leads to different welfare for different types. The sophisticated agent always does better than naïve and partially-sophisticated and naïve always does worse than other types.

1.7.2 Generalization

One of the main limitations of this paper is the specific choice of cost and reward functions. However, it can be shown that the results obtained for the naïve agent are valid for the larger class of the reward and cost functions $R(\cdot)$ and $c(\cdot)$. Because I focus on the effects of the deadline on the agent's behavior and welfare and chose the reward function such that the agent cannot affect her future self through it, the general agent's UMP in period t would look like:

$$\max_{\{e_l\}_{l=t}^{k}} \left\{ -c\left(e_l\right) - \beta \sum_{l=t+1}^{k} c\left(e_l\right) + \beta \sum_{l=t}^{k} u\left(e_l\right) \right\} \tag{37}$$

Where $c(\cdot)$ is strictly increasing, strictly convex, and $c(0) = 0$; $R(e_1,...,e_N) = \sum_{t=1}^{N} u(e_t)$; $u(\cdot)$ is increasing, concave, $u(0) = 0$, and $u'(0) > c'(0)$. The problem is equivalent to:

$$\max_{\{e_l\}_{l=t}^{k}} \left\{ -f\left(e_l\right) - \beta \sum_{l=t+1}^{k} f\left(e_l\right) + \beta \sum_{l=t}^{k} e_l \right\} \tag{38}$$

Where the new "cost function" $f(\cdot)$ is chosen such that:

$$f'(\cdot) = \frac{c'(\cdot)}{u'(\cdot)}, \quad f(0) = 0 \tag{39}$$

Under the assumptions on $c(\cdot)$ and $u(\cdot)$, the function $f(\cdot)$ is monotone, strictly increasing, and strictly convex. Thus, the solution exists and is unique. One further assumption on function $f(\cdot)$ is needed to ensure that the results are valid in the general case:

$$\beta f'(e_t) = f'(\beta e_t) \tag{40}$$

Then the naïve agent's behavior under the binding *natural* interim deadline is:

$$\hat{e}_t^{(t)} = \begin{cases} \beta \left(f'\right)^{-1}(1) \frac{\Pi_{l=0}^{t-1}(k-l)}{\Pi_{l=1}^{t}(k-(l-\beta))} & , 1 \leq t < k \\ \left(f'\right)^{-1}(1) \frac{\Pi_{l=0}^{k-2}(k-l)}{\Pi_{l=1}^{k-1}(k-(l-\beta))} & , t = k \\ \left(f'\right)^{-1}(\beta) & , k < t \leq N \end{cases} \tag{41}$$

For the naïve agent's behavior according to (41), all the propositions hold, because the difference is in constants. Generalization of the results for sophisticated and partially-sophisticated agents is complicated and requires further research. However, their behavior is similar to that of the naïve agent and the simulations suggest that it is reasonable to believe that the results would hold for the general case.

1.7.3 Limitations

Another limitation is in the assumption that the agent has to satisfy the deadline constraint. Several papers consider penalty functions as an incentive for the agent to meet the deadline (e.g., Ariely & Wertenbroch, 2002; El-Tannir, 2019). In other words, if the agent expends less overall effort than A by the end of period k, she faces the additional costs $D(\cdot)$. In this paper, the assumption that the agent has to satisfy the deadline constraint is the extreme case when the additional cost function $D(\cdot)$ instantly grows to an incomparable level to the costs if the deadline is not met. In my research, the general additional cost function would only complicate the first-order condition and create an additional trade-off for the agent postponing effort to later periods. However, it is enough to assume that $D'(\cdot)$ when the deadline is missed (the marginal costs of missing the deadline) is greater than $c'(e_k)$ (the marginal costs in the period where the higher effort is accumulated) to induce the same agent behavior.[26]

An additional open question is who sets the exogenous interim deadline to maximize the agent's welfare. In this paper, I focus on the agent's behavior and resulting welfare, leaving aside the discussion of who this entity might be. However, for the exogenous interim deadline case, it is the social planner who is informed about the agent's self-control problem. As a real-life example, this might be a schoolteacher who aims to find a good balance between giving too much homework and keeping students engaged in the study process or parents

[26] Under the assumption that $D(\cdot)$ grows faster than $c(\cdot)$.

who care about their children and push them to do exercises. These examples can motivate subsequent research when the social planner is aware of the agent's self-control problems but cannot correctly estimate her present bias. Another interesting extension would be to consider the principal-agent problem. The principal aims to maximize the total effort expended and so imposes an interim deadline. However, the principal can be not fully informed about the agent's present bias, can estimate it incorrectly, or has only restricted options for setting a deadline. This setup is closer to the examples when the agent is an employee and the principal is an employer who suggested a contract to an agent which she can accept or reject.

All the limitations and possible extensions I discussed above are potential spaces for subsequent research.

1.8 Conclusion

In this paper, I study how the design of exogenous and self-imposed interim deadlines affect an agent's behavior and welfare. The agent can be one of three types based on how she understands her present bias: naïve, sophisticated, and partially-sophisticated.

Firstly, I consider the situation in which an interim deadline is imposed exogenously with the purpose of maximizing the agent's welfare, and when the agent is restricted to meeting the deadline. I find that the agent continues to procrastinate from the beginning of the project to the deadline, and postpones her efforts to the final periods before the deadline. A later interim deadline increases her overall expended effort, while allowing the accumulation of greater effort in the final periods, and can decrease the agent's welfare. Thus, an interim deadline may increase or decrease the agent's welfare depending on the design. Further, I find that a unique interim deadline exists that maximizes the agent's welfare and that the design of this deadline depends only on the level of the agent's present bias (and sophistication level for the partially-sophisticated agent). Under the same interim deadline, the naïve agent would postpone more effort to future periods than would sophisticated and partially-sophisticated agents. Thus, the naïve agent's welfare will be the lowest. The sophisticated agent would act more like a time-consistent agent than would naive and partially-sophisticated agents and would gain the greatest welfare. The partially-sophisticated agent will fall in between naïve and sophisticated agents. As a result, the optimal deadline for the naïve agent is in an earlier period of the project, for the sophisticated agent, it is in a later period, and for the partially-sophisticated, it is in between.

Second, I study the behavior of a present-biased agent under a self-imposed interim deadline. The naïve agent has no incentive to impose any restrictions on herself and does not set a self-imposed deadline. While a sophisticated agent always sets the self-imposed deadline optimally, the partially-sophisticated agent imposes the deadline such it increases or decreases her welfare depending on the combination of the agent's present bias and sophistication level. An agent with a low sophistication level ($\gamma \lesssim 0.5$) decreases her welfare by setting a self-imposed deadline for all possible β. However, given the same high sophistication level ($\gamma \gtrsim 0.5$), the agent with a relatively high present bias parameter (close to the time-consistent agent) would decrease her welfare while the agent with a relatively low present bias parameter (far from the time-consistent agent) would increase her welfare by setting a self-imposed deadline.

The results contribute to the existing economic literature on the topics of behavioral economics, time-inconsistent preferences, and deadlines. To my knowledge, this is the first theoretical paper that considers how an interim deadline affects the agent's effort choice across several periods and impacts the resulting welfare. An additional novel result is about how the partially-sophisticated agent would affect her welfare by using a self-imposed deadline. The paper's findings are useful for the understanding of how deadlines affect our behavior and welfare.

1.9 Appendix I

For the naïve agent, the interim deadline $ID = (A,k)$ binds in period t' if the optimal intertemporal strategy without a deadline does not satisfy the deadline constraint:

$$\sum_{t=t'}^{k} \hat{e}_t(ND) = \beta + (k - t') < A - \sum_{t=1}^{t'-1} e_t \tag{42}$$

On average, the agent has to expend $\alpha' = \frac{A'}{k-(t'-1)} = \frac{A-\sum_{t=1}^{t'-1} e_t}{k-(t'-1)}$ during periods $\{t',...,k\}$. However, the agent expends less than average in period t' because her optimal intertemporal strategy in period t' has to satisfy first-order condition $\beta e_{t'} = e_t$, $t \in \{t'+1,...,k\}$. Thus, the agent has to expend on average in periods $\{t'+1,...,k\}$ more than on average in periods $\{t',...,k\}$ and the deadline constraint also binds in period $t'+1$ as well:

$$\sum_{t=t'+1}^{k} \hat{e}_t(ND) = \beta + (k - t' - 1) < A - \sum_{t=1}^{t'} e_t \tag{43}$$

To show this formally, I rewrite the inequalities (42) and (43) in average terms. Because the inequality (42) is for $k - t' + 1$ periods, and inequality (43) is for $k - t'$ periods, they can be rewritten as:

$$\begin{gathered} \frac{\beta+(k-t')}{k-t'+1} < \frac{A-\sum_{t=1}^{t'-1} e_t}{k-t'+1} \\ \frac{\beta+(k-t'-1)}{k-t'} < \frac{A-\sum_{t=1}^{t'} e_t}{k-t'} \end{gathered} \tag{44}$$

The right side of the bottom inequality becomes greater than the right side of the top inequality due to the first-order conditions. However, the left side obviously becomes lower since $\beta < 1$. Thus, the inequality (43) always holds, when the inequality (42) holds, and the deadline constraint binds in every subsequent period (till period k) if it binds in the current period.

When the interim deadline binds or does not for the sophisticated and partially-sophisticated agents is discussed in the corresponding parts of the Analysis section. However, the agents behave according to Proposition 2 and Proposition 3 when the deadline binds. As discussed in Appendix III, the agent then expends less effort than she has to on average during the remaining periods before the deadline. Precisely, the agent's behavior is defined by function $\Phi(x)$[27]. Because $x(1-(1-\beta)x) < x$ for all $x \in (0,1]$, function $\Phi(x)$ is such that if the agent has to expend at least A total effort during the remaining n periods and interim deadline binds, the agent expends less than $\frac{A}{n}$ effort in the current period. Thus, the deadline binds in the next period and in all subsequent periods (till period k).

1.10 Appendix II: Proof of Proposition 1

Proof. I prove Proposition 1 (equation (16)) using the induction method. Under the imposed deadline (A, k), in period t, the agent is considered to be her current self and behaves according to the optimal intertemporal strategy $\left\{\hat{e}_\tau^{(t)}(ID)\right\}_{\tau=t}^{N}$. This strategy can be found as a solution to the corresponding agent's UMP at period t:

$$\left\{\hat{e}_n^{(t)}(ID)\right\}_{n=t}^{N} = \arg\max_{\{e_n\}_{n=t}^{N}} \left\{-\frac{e_t^2}{2} + \beta\left[-\sum_{n=t+1}^{N}\frac{e_n^2}{2} + \sum_{n=1}^{N} e_n\right]\right\} \tag{45}$$

[27] For a formal definition, see Proposition 2

$$s.t.: \quad \sum_{n=t}^{k} e_n \geq A - \sum_{n=1}^{t-1} e_n$$

$$\{e_n\}_{n=1}^{t-1} \ \ are \ \ given$$

<u>Base</u> Requires the agent to behave according to (16) in period $t = 1$:

$$\hat{e}_1^{(1)}(ID) = \beta\alpha \frac{\Pi_{l=0}^{t-1}(k-l)}{\Pi_{l=1}^{t}\left(k-(l-\beta)\right)}\Bigg|_{t=1} = \beta\alpha \frac{k}{k-(1-\beta)} \tag{46}$$

In period 1, the agent's UMP (45) is:

$$\max_{\{e_n\}_{n=1}^{N}} \left\{ -\frac{e_1^2}{2} + \beta \left[-\sum_{n=2}^{N} \frac{e_n^2}{2} + \sum_{n=1}^{N} e_n \right] \right\} \tag{47}$$

$$s.t.: \quad \sum_{n=1}^{k} e_n \geq A$$

The deadline constraint binds by the assumption and the first-order conditions give:

$$
\begin{cases}
e_t = \frac{1}{\beta}e_1 & , \quad t \in \{2,...,k\} \\
\\
\sum_{n=1}^{k} e_n = A \\
\\
-e_t + 1 = 0 & , \quad t \in \{k+1,..., N\}
\end{cases}
\tag{48}
$$

Then the solution for the UMP (46) is:

$$
\hat{e}_t^{(1)}(ID) =
\begin{cases}
\beta\frac{A}{k-(1-\beta)} & , \quad t = 1 \\
\\
\frac{A}{k-(1-\beta)} & , \quad t \in \{2,...,k\} \\
\\
1 & , \quad t \in \{k+1,..., N\}
\end{cases}
\tag{49}
$$

This is the optimal intertemporal strategy at period 1 and the agent expends $\hat{e}_1^{(1)}(ID)$ in the first period:

$$
\hat{e}_1^{(1)}(ID) = \beta\frac{A}{k-(1-\beta)} = \beta\frac{A}{k}\frac{k}{k-(1-\beta)} = \beta\alpha\frac{k}{k-(1-\beta)} \tag{50}
$$

Assumption Next step is to assume that (16) is correct for all periods from 1 to some $t' < k$:

$$e_t^{(t)}(ID) = \beta\alpha\frac{\Pi_{l=0}^{t-1}(k-l)}{\Pi_{l=1}^{t}\left(k-(l-\beta)\right)}\ ,\quad t\in\{1,...,t'\} \tag{51}$$

<u>Induction Step</u> A further step is to show that (16) will be correct for $t = t'+1 < k$. In period $t'+1$, the agent faces the following UMP:

$$\max_{\{e_n\}_{n=t'+1}^{N}} \left\{-\frac{e_{t'+1}^2}{2} + \beta\left[-\sum_{n=t'+2}^{N}\frac{e_n^2}{2} + \sum_{n=1}^{N} e_n\right]\right\} \tag{52}$$

$$s.t.:\quad \sum_{n=t'+1}^{k} e_n \geq A - \sum_{n=1}^{t'} e_n$$

$$\{e_n\}_{n=1}^{t'}\ \ are\ \ given\ \ according\ \ to\ \ (51)$$

The deadline constraint binds and the first-order conditions give:

$$\begin{cases} e_t = \frac{1}{\beta}e_{t'+1} & ,\quad t\in\{t'+2,...,k\} \\ \sum_{n=t'+1}^{k} e_n = A - \sum_{n=1}^{t'} e_n & \\ -e_t + 1 = 0 & ,\quad t\in\{k+1,...,N\} \end{cases} \tag{53}$$

Then the deadline equality in (53) can be rewritten as:

$$\hat{e}_{t'+1}^{(t'+1)}(ID) = \frac{\beta\alpha}{k-(t'+1-\beta)}\left(k-\sum_{t=1}^{t'}\left[\beta\frac{\Pi_{l=0}^{t-1}(k-l)}{\Pi_{l=1}^{t}\left(k-(l-\beta)\right)}\right]\right) \tag{54}$$

After reduction to a common denominator, the denominator in the expression for $\hat{e}_{t'+1}^{(t'+1)}(ID)$ transforms to $\Pi_{l=1}^{t'+1}\left(k-(l-\beta)\right)$. The nominator then transforms to $\Pi_{l=0}^{t'}(k-l)$ and the optimal action for the current period $t'+1$ is:

$$e_{t'+1}^{(t'+1)}(ID) = \beta\alpha\frac{\Pi_{l=0}^{(t'+1)-1}(k-l)}{\Pi_{l=1}^{t'+1}\left(k-(l-\beta)\right)} \tag{55}$$

Then the agent's optimal intertemporal strategy at period $t'+1$ is:

$$\hat{e}_{t'+1}^{(t'+1)}(ID) = \begin{cases} \beta\alpha\frac{\Pi_{l=0}^{(t'+1)-1}(k-l)}{\Pi_{l=1}^{t'+1}(k-(l-\beta))} & , \quad t=t'+1 \\ \frac{1}{\beta}\hat{e}_{t'+1}^{(t'+1)}(ID) = \alpha\frac{\Pi_{l=0}^{(t'+1)-1}(k-l)}{\Pi_{l=1}^{t'+1}(k-(l-\beta))} & , \quad t\in\{t'+2,...,k\} \\ 1 & , \quad t\in\{k+1,...,N\} \end{cases} \tag{56}$$

In period $t'+1$, the agent expends $\hat{e}_{t'+1}^{(t'+1)}(ID)$, which satisfies (16). However, this works only for $t'+1<k$. When $t'+1=k$, the agent has to expend exactly the planned effort in period $t'=k-1$ to meet the deadline.

Period k To meet the deadline, in period k, the agent must expend effort at the same level as she planned in the previous period. In other words, the agent does not procrastinate only in period k. In period $k-1$, the agent's optimal intertemporal strategy is:

$$\hat{e}_t^{(k-1)}(ID) = \begin{cases} \beta\alpha \frac{\Pi_{l=0}^{(k-1)-1}(k-l)}{\Pi_{l=1}^{k-1}(k-(l-\beta))} & , \quad t = k-1 \\ \frac{1}{\beta} e_{k-1}^{(k-1)}(ID) = \alpha \frac{\Pi_{l=0}^{(k-1)-1}(k-l)}{\Pi_{l=1}^{k-1}(k-(l-\beta))} & , \quad t = k \\ 1 & , \quad t \in \{k+1, ..., N\} \end{cases} \tag{57}$$

That is, in period $k-1$, the agent planes to expend $\hat{e}_k^{(k-1)}(ID)$ in period k. Therefore, the agent expends exactly $\hat{e}_k^k(ID) = \hat{e}_k^{(k-1)}(ID)$ in period k. Thus, the agent behaves according to (16) in period k.

□

1.11 Appendix III: Proof of Proposition 2

Proof. In period t, the agent has to expend $A' = A - \sum_{n=1}^{t-1} e_n$ effort during the remaining periods when facing the interim deadline $ID = (A, k)$. Thus the agent solves the following maximization problem in period t:

$$\max_{\{e_n\}_{n=t}^k} \left\{ -\frac{e_t^2}{2} + \beta \left[-\sum_{n=t+1}^{k} \frac{e_n^2}{2} + \sum_{n=t}^{k} e_n \right] \right\} \tag{58}$$

$$s.t. : \quad \sum_{n=t}^{k} e_n \geq A'$$

Because the agent is sophisticated and correctly anticipates her future behavior, in period t, she knows how her future selves will behave in future periods. For future periods $\{t+1,...,k\}$, the agent will have to expend $(A' - e_t)$. Denote by ω_n^{t+1} the part of total effort left $(A' - e_t)$ after expended effort e_t in period t which agent will expend in period $n \in \{t+1,...,k\}$. Then the effort expended by the agent in the future period n can be rewritten as $\omega_n^{t+1}(A' - e_t)$ and the agent UMP is:

$$\max_{\{e_n\}_{n=t}^{k}} \left\{ -\frac{e_t^2}{2} + \beta \left[-\frac{(A'-e_t)^2}{2} \sum_{n=t+1}^{k} \left(\omega_n^{t+1}\right)^2 + A' \right] \right\} \tag{59}$$

The first-order condition gives the effort expended by the agent in period t:

$$e_t = A' \frac{\beta \sum_{n=t+1}^{k} (\omega_n^{t+1})^2}{1 + \beta \sum_{n=t+1}^{k} (\omega_n^{t+1})^2} = A' \frac{1}{1 + \frac{1}{\beta \sum_{n=t+1}^{k} \left(\omega_n^{t+1}\right)^2}} \tag{60}$$

Then the ω_t^t is defined and shares for future periods can be recalculated:

$$\begin{cases} \omega_t^t = \frac{1}{1+\frac{1}{\beta \sum_{n=t+1}^{k} \left(\omega_n^{t+1}\right)^2}} \\ \omega_n^t = (1-\omega_t^t)\omega_n^{t+1} \qquad , n \in \{t+1,...,k\} \end{cases} \tag{61}$$

The sum of squared future shares can be rewritten as:

$$\sum_{n=t+1}^{k}\left(\omega_n^{t+1}\right)^2=\left(\omega_{t+1}^{t+1}\right)^2+\left(1-\omega_{t+1}^{t+1}\right)^2\sum_{n=t+2}^{k}\left(\omega_n^{t+1}\right)^2$$

And using the first part of the equation (61) but for the next period:

$$\sum_{n=t+1}^{k}\left(\omega_n^{t+1}\right)^2=\left(\omega_{t+1}^{t+1}\right)^2+\left(1-\omega_{t+1}^{t+1}\right)\frac{\omega_{t+1}^{t+1}}{\beta}$$

Plugging this relation into the equation (61), the share ω_t^t can be represented as a function of share ω_{t+1}^{t+1}:

$$\omega_t^t\left(\omega_{t+1}^{t+1}\right)=\frac{1}{1+\frac{1}{\omega_{t+1}^{t+1}\left(1-(1-\beta)\omega_{t+1}^{t+1}\right)}} \tag{62}$$

Or

$$\omega_t^t\left(\omega_{t+1}^{t+1}\right)=\Phi\left(\omega_{t+1}^{t+1}\right) \tag{63}$$

Where

$$\Phi(x) = \frac{1}{1 + \frac{1}{x(1-(1-\beta)x)}} \tag{64}$$

Because $x(1-(1-\beta)x) < x$ for all $x \in (0,1]$, the agent expends less effort than the time-consistent agent would do. That is, in period t, the agent expends $e_t < \frac{A'}{k-t+1}$ and postpones some amount of effort to future periods.

□

1.12 Appendix IV: Proof of Proposition 4

Proof. To show that such an interim deadline exists and is generically unique, I consider how the reward and total cost functions change when the principal imposes the interim deadline $ID = (k+1, k+1)$ instead of the interim deadline $ID = (k,k)$. When the deadline is imposed one period later, the total expended effort increases by $1-\beta$, because the agent is restricted to expend $\beta N + (1-\beta)(k+1)$ instead of $\beta N + (1-\beta)k$. Then, the increase in total expended effort does not depend on period k and is constant when moving the interim deadline from the 10th period to the 11th or from the 110th period to the 111th. Thus, moving the interim deadline from period k to $k+1$ increases the agent's welfare only through the reward function $R\left(\sum_{t=1}^{N} e_t\right)$ and this increase in reward function does not depend on k.[28]

The total cost function $C\left(\{e_t\}_{t=1}^{N}\right) = \sum_{t=1}^{N} c(e_t)$ is a convex function, because $c(\cdot)$ is a convex function. Thus, in contrast to the increase in the reward function, the increase in costs is growing in k when the interim deadline is moved from period k to $k+1$. Therefore, given the linearly increasing reward function and convex increasing cost function, there is an intersection between reward and cost functions when the interim deadline is moved further (increasing

[28]Because the reward function is linear.

total effort). When the increase in the total cost function is lower than the increase in the reward function, it is optimal to move the interim deadline to the next period. However, when they become equal, the total cost function grows faster and it is not profitable to move the interim deadline to the next period. This intersection of reward and total cost functions is unique, because the agent's present-bias parameter β defines the distribution of total effort across periods according to Proposition 1, or Proposition 2, or Proposition 3. However, k is an integer, thus this intersection might lie between periods $\hat{k}$ and $\hat{k}+1$ such that the agent's welfare under interim deadline in period $\hat{k}$ equals the agent's welfare under interim deadline in period $\hat{k}+1$. Then, both *natural* interim deadlines in period $\hat{k}$ and in period $\hat{k}+1$ maximize the agent's welfare. This happens only for specific values of the present bias parameter β and the set of these values has measure zero. Thus, the interim deadline which maximizes the agent welfare is generically unique.

From the "long-run" perspective, the interim deadline $ID = (\hat{k},\hat{k})$ is optimal when this deadline causes the highest possible utility level. This period $\hat{k}$ can be characterized as follows: the agent's behavior under the interim deadline $ID = (\hat{k},\hat{k})$ yields greater welfare than the agent's behavior under interim deadlines $ID = (\hat{k}+1,\hat{k}+1)$ and $ID = (\hat{k}-1,\hat{k}-1)$:

$$\begin{cases} U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k},\hat{k})\right\}_{t=1}^{N}\right) - U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}+1,\hat{k}+1)\right\}_{t=1}^{N}\right) > 0 \\ U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k}-1,\hat{k}-1)\right\}_{t=1}^{N}\right) - U_0\left(\left\{\hat{e}_t^{(t)}(\hat{k},\hat{k})\right\}_{t=1}^{N}\right) < 0 \end{cases} \tag{65}$$

Because the interim deadline in period k does not affect the agent's behavior in periods $\{k+1,..., N\}$, the agent behaves identically in periods $\{k+2,...,N\}$ in all three cases. Thus, the inequalities (65) can be rewritten as following:

$$\begin{cases} \sum_{t=1}^{\hat{k}+1}\left[\hat{e}_t^{(t)}(\hat{k},\hat{k})-\frac{\left(\hat{e}_t^{(t)}(\hat{k},\hat{k})\right)^2}{2}\right]-\sum_{t=1}^{\hat{k}+1}\left[\hat{e}_t^{(t)}(\hat{k}+1,\hat{k}+1)-\frac{\left(\hat{e}_t^{(t)}(\hat{k}+1,\hat{k}+1)\right)^2}{2}\right]>0 \\ \sum_{t=1}^{\hat{k}+1}\left[\hat{e}_t^{(t)}(\hat{k}-1,\hat{k}-1)-\frac{\left(\hat{e}_t^{(t)}(\hat{k}-1,\hat{k}-1)\right)^2}{2}\right]-\sum_{t=1}^{\hat{k}+1}\left[\hat{e}_t^{(t)}(\hat{k},\hat{k})-\frac{\left(\hat{e}_t^{(t)}(\hat{k},\hat{k})\right)^2}{2}\right]<0 \end{cases} \tag{66}$$

Or

$$\begin{cases} \sum_{t=1}^{\hat{k}+1}\left[\left(e_t^{(t)}(\hat{k}+1,\hat{k}+1)\right)^2-\left(e_t^{(t)}(\hat{k},\hat{k})\right)^2\right]>2(1-\beta) \\ \sum_{t=1}^{\hat{k}}\left[\left(e_t^{(t)}(\hat{k},\hat{k})\right)^2-\left(e_t^{(t)}(\hat{k}-1,\hat{k}-1)\right)^2\right]<2(1-\beta) \end{cases} \tag{67}$$

Under the *natural* interim deadline, the agent's behavior in periods 1,...,k is defined by the period k and present bias parameter β according to Proposition 1, Proposition 2, and Proposition 3 (and on sophistication level γ for the partially-sophisticated agent). Thus, the conditions (67) depends only on present-bias parameter β, because $e_t^{(t)}(\hat{k},\hat{k})$ and $e_t^{(t)}(\hat{k}-1,\hat{k}-1)$ are the functions of $\hat{k}$ and β (and on sophistication level γ for partially-sophisticated agent). Therefore, the agent's present-bias parameter β defines the period $\hat{k}$ for the optimal interim deadline among all interim deadlines $ID = (k, k)$. In other words, regardless of the length of the project (assuming it is long enough), there is a period $\hat{k}$ such that the agent's welfare is higher when the agent unrestrictedly procrastinates after this period $\hat{k}$ than when the *natural* interim deadline is moved to the next period.

□

Moving the *natural* interim deadline in further periods only decreases the agent's welfare.

Moreover, every time the deadline is moved to the next period will decrease welfare even more due to the convex cost function. Since the increase in costs is growing with k there is a period $\underline{k}$ such that the *natural* interim deadline imposed in any later period leads to lower agent's welfare than under no deadline. In other words, any *natural* interim deadline $ID = (k,k)$ imposed in period $k > \underline{k}$ decreases the agent's welfare (compared to the situation when the agent behaves under no deadline). This period $\underline{k}$ can be described with the following inequalities:

$$\begin{cases} U_0\left(\left\{\hat{e}_t^{(t)}(\underline{k},\underline{k})\right\}_{t=1}^{N}\right) - U_0\left(\left\{\hat{e}_t^{(t)}(ND)\right\}_{t=1}^{N}\right) > 0 \\ U_0\left(\left\{\hat{e}_t^{(t)}(\underline{k}+1,\underline{k}+1)\right\}_{t=1}^{N}\right) - U_0\left(\left\{\hat{e}_t^{(t)}(ND)\right\}_{t=1}^{N}\right) < 0 \end{cases} \tag{68}$$

Using the same logic as in proof, the inequalities can be rewritten as:

$$\begin{cases} \sum_{t=1}^{\underline{k}} e_t^{(t)}(\underline{k},\underline{k}) < \underline{k}(2-2\beta+\beta^2) \\ \sum_{t=1}^{\underline{k}+1} e_t^{(t)}(\underline{k}+1,\underline{k}+1) > (\underline{k}+1)(2-2\beta+\beta^2) \end{cases} \tag{69}$$

Thus, similar to $\hat{k}$, the period $\underline{k}$ is defined only by the present bias parameter β (and sophistication level γ for partially-sophisticated agent). The period $\underline{k}$ is increasing in β and $\underline{k}^{NA} < \underline{k}^{PSA} < \underline{k}^{SA}$. This period and the corresponding *natural* interim deadline $ID = (\underline{k},\underline{k})$ show how much someone can push the agent to expend more effort without making the agent worse off. This can be interesting in the setup in which the person who imposes an interim deadline is the principal. The principal uses the interim deadline to maximize the agent's welfare, however, for example, the agent then has the outside option and the principal cannot push the agent to expend as much effort as the principal wishes her to.

2 Optimal Deadlines for Parallel Projects

2.1 Introduction

Deadlines are common in modern life: students' homework, preparing for exams, and job contracts all involve deadlines. Often, we restrict our future selves by setting deadlines on purpose to increase our performance or to achieve a certain goal. In real life, we focus on different projects and goals in real life, striving to be successful in all directions and/or aspects of our lives. Thus, the question of how to impose deadlines optimally in the context of parallel projects is important and relevant in the modern world.

In hindsight, many who procrastinated would have preferred to stick to their original project plan to produce better outcomes. In this paper, I analyze the behavior of an agent (procrastinator) who pursues two parallel projects under interim deadlines. I study how the timing of a single interim deadline in one project affects the optimal timing for the interim deadline in a parallel project. I also investigate how the optimal timing of deadlines depends on the agent's present bias. In real life, different situations can be considered procrastination. In this paper, I study the behavior of an agent with time-inconsistent preferences to model the procrastinator. Specifically, I study the behavior of a present-biased agent with quasi-hyperbolic preferences.

A procrastinator invests less effort than she planned to invest. People who are aware that they experience self-control problems often use different tools, such as deadlines, commitment contracts, and self-control penalties[29] to overcome procrastination. I describe and analyze the present-biased agent's behavior under interim deadlines and shed new light on how best to impose interim deadlines in the context of parallel projects. I investigate the effect of the timing chosen for an interim deadline imposed in one project on the agent's behavior and optimal timing for the interim deadline in the other project. In my research, I leave aside other instruments and incentives to overcome procrastination. I consider interim deadlines to

[29] See Giné et al. (2010); Houser et al. (2018); Trope and Fishbach (2000).

be a unique instrument that can be used to improve the agent's performance and welfare. In my model, the deadline specifies the goal and the timing (period).[30] To meet the deadline, the agent must achieve the set goal by this period.

Procrastination is a widespread problem that is commonly connected with and explained by time-inconsistent preferences. However, as Ericson and Laibson (2019) mention, there is no precise and standard definition of procrastination. They suggest the following definition: *"we will define someone as procrastinating if, when completing a costly task, there is a delay that appears suboptimal from their own perspective."*[31] In other words, people prefer to put substantially less effort into tasks, goals, or projects than they have planned. This choice is optimal at each moment of time but suboptimal from the perspective of the overall performance.

In general, the present-biased agent can be one of three types depending on how she understands her present bias: naïve, sophisticated, and partially-sophisticated. In this paper, I study the behavior of the naïve agent and discuss how the results would be different for sophisticated and partially-sophisticated agents in the Discussion section. The naïve agent is not aware of her time inconsistency and does not realize that she will experience self-control problems in the future (believes she is time-consistent), while sophisticated and partially-sophisticated agents are aware. The sophisticated agent correctly knows her present bias and fully predicts her future behavior. The partially-sophisticated agent underestimates her present bias and cannot correctly predict her future.

In this paper, I investigate how an interim deadline imposed in one project affects the optimal timing of an interim deadline in a parallel project. I vary the timing for the interim deadline in one project from the first period to the last. Then I compare the resulting optimal timing

[30]For the formal definition, see Definition 2.
[31]Ericson and Laibson (2019)

for the interim deadline in the parallel project with the optimal timing if there were no interim deadline in the first project (I denote this timing as t^*). The results suggest that imposing an interim deadline in one project may shift the optimal timing of the interim deadline in the parallel project to earlier or later. When the interim deadline in the first project is imposed before t^*, the optimal timing for the interim deadline in the second project shifts to later. In contrast, the optimal timing is shifted earlier when the interim deadline in the first project is imposed later than t^*. As a result, the pair of optimal timings for interim deadlines in both projects is not necessarily unique. The symmetric pairs of timings are optimal at the same time when one interim deadline is imposed later than the other interim deadline in the parallel project.

Additionally, I study how the size of the effect and the uniqueness of the optimal pair of timings depend on the agent's present bias. The pair of optimal timings is unique for agents with relatively lower present bias parameters (those who experience more self-control problems) and for the agents who are almost time-consistent. In other words, it is optimal to impose interim deadlines with the same timing in both projects. However, there is a mass of agents with a relatively high present bias parameter (those with mild-to-moderate self-control problems) for whom it is optimal to impose the interim deadline in one project later than the interim deadline in the other project, and for whom the pair of optimal timings is not unique. The distance between optimal timings for interim deadlines also depends on the agent's present bias. Generally, the distance increases with present bias when it is optimal to impose interim deadlines in the parallel projects in the same period. However, it is optimal to impose deadlines in the last periods when the agent is close to time-consistent ($\beta = 1$). Thus, the distance between optimal timings moves to zero when present bias approaches zero.

Time-inconsistent preferences is an important direction in economic literature (Ericson and Laibson (2019)). The topic arises from the phenomenon in which people change their choices from a fixed choice set in time (e.g., one thought yesterday that he would go to the gym today, but today he prefers to stay at home). Economists observe how agents prefer different actions or choices at different points in time in the data or in the behavior of experiment participants (e.g., Augenblick & Rabin, 2019; O'Donoghue & Rabin, 1999a). This phenomenon can be explained by the concept of time-inconsistent preferences or, in other words, when an agent's preferences differ at some different points in time (e.g., Ainslie, 1975, 2012; Ainslie & Haslam, 1992; Laibson, 1997; Loewenstein & Prelec, 1992; O'Donoghue & Rabin, 1999a, 1999b; Strotz, 1955). Nevertheless, the phenomenon can also be explained by time-consistent preferences, according to several concepts. Examples include the concept of a myopic agent (e.g., Frederick, 2005; Gabaix & Laibson, 2017; Kőszegi & Szeidl, 2013; Steele & Josephs, 1990) or the concept of temptation (e.g., Bernheim & Rangel, 2004; Dekel, Lipman, & Rustichini, 2001, 2009; Gul & Pesendorfer, 2001, 2004; Laibson, 2001; Lipman, Pesendorfer, et al., 2013; Noor, 2007, 2011). In this paper, I focus on time-inconsistent preferences, leaving aside the other concepts and models (for a comprehensive literature review of different concepts and models, see Ericson and Laibson (2019)).

The results contribute to the existing literature on behavioral economics, time-inconsistent preferences, and deadlines. To my knowledge, this is the first theoretical paper that considers how an interim deadline in one project affects the optimal timing for the interim deadline in another project in the context of parallel projects. This paper characterizes the optimal pair of timings for interim deadlines in both projects that maximize the agent's welfare. The paper also presents how the effects and the distance between optimal timings depend on the agent's present bias. The findings of the paper are useful for understanding how deadlines affect an agent's behavior and welfare.

The remainder of the paper is organized as follows. Section 2 describes related literature and contextualizes this paper in existing research. Section 3 lays out the model of effort choice under the interim deadline. Section 4 analyzes the agents' behavior and the resulting agent welfare under deadlines in different periods. Section 5 provides the results on the optimal pair of interim deadlines. Section 6 discusses generalizations, limitations, and possible extensions of the research. Section 7 concludes.

2.2 Related Literature

Deadlines are among the commonly used tools to overcome procrastination and are widely studied in experimental literature. While some papers have shown a positive effect of deadlines on performance (e.g., Ariely & Wertenbroch, 2002; Herweg & Müller, 2011; Knowles, Servátka, Sullivan, & Genç, 2017), other experiments document no effect (e.g., Bisin & Hyndman, 2020) or even negative effects (e.g., Burger et al., 2011). The main concern I raise in this paper is that imposing deadlines in one project will affect not only decisions on effort spent on that project, but also on all other activities in the same time period. Indeed, conducting the experiment on how imposed deadlines affect performance and manipulating deadline settings can lead not only to variations in behavior inside the experiment, but to changes in behavior outside the experiment as well. For example, Burger et al. (2011) conducted an experiment asking participants (students) to spend at least 75 hours on studying over a five-week period. The treatment was a restriction for students to spend at least an additional 15 hours by the end of each week: 15 hours by the end of the first week, 30 hours by the end of the second week, etc. In these conditions, students who were pushed to smooth their effort across a five-week period could redistribute their other tasks and activities during this period. Consequently, the results were able to reflect not only the effect of deadlines on procrastination but also time flexibility across different tasks for a particular student. Therefore, studying the effects of deadlines by considering one task, goal, or project could lead to overestimating the effect of deadlines on procrastination. In this study, I suggest a model that aims to describe the effects of interim deadlines in a multiple-project context and to find the optimal design of interim deadlines for parallel projects.

Few studies consider multiple goals in the topic of deadlines and/or time-inconsistent preferences. Bisin and Hyndman (2020) conduct a field experiment and compare the effects of final deadlines (I define final deadlines as those imposed at the end of a project/the available time left to achieve the goal) in a single task and repeated tasks settings. They find that present-biased students successfully manage to self-control in repeated task settings, while there is no effect of deadlines on the completion rates of experiment tasks. Herweg and Müller (2011) present a model of effort choice and extend it to a multiple-task case. They consider two-period settings with a final deadline for one task at the end of the first period and a final deadline for the other task at the end of the second period. Their model shows that it is optimal to invest only in the task with a shorter final deadline in the first period for present-bias agents. In other words, it is optimal to approach tasks sequentially, not simultaneously. However, it is important to underline that this result is only relevant under specific assumptions on utility and cost functions (e.g., additive utility function). In contrast, I model projects that are being pursued simultaneously. Ballard, Vancouver, and Neal (2018) study agents' effort choice across two goals by conducting an experiment and estimating their model's parameters on data. They focus on time pressure and distance to the goal deadline. In contrast to my idea, their model does not imply time-inconsistent preferences. Additionally, the authors did not consider the question of optimal deadlines, but state that it would be an important extension of their model to develop a scheduling algorithm that helps people to allocate their time more efficiently.

In the context of time-inconsistent preferences, researchers commonly consider three types of agents: naïve agents, sophisticated agents, and partially naïve (partially sophisticated) agents. Naïve agents are unaware of their time inconsistency and always think that they will behave according to their current preferences. In contrast, sophisticated agents are correctly informed about their present bias and consequently would prefer to impose restrictions on their future themselves. Partially-naïve (partially-sophisticated) agents are aware of their time inconsistency, but they underestimate their present bias. For economic reasons, it is important to understand the agents' type, since that would significantly affect the predictions of their behavior.

Several studies have aimed to identify the type of agents and/or to distinguish and describe the behavior of different types (e.g., Ariely & Wertenbroch, 2002; Bisin & Hyndman, 2020; Burger et al., 2011). Ariely and Wertenbroch (2002) find evidence of partially naïve agents by comparing the effects of self-imposed vs exogenously imposed deadlines on performance. In their experiment, self-imposed deadlines resulted in lower performance than evenly-spaced exogenous ones, which is interpreted as evidence of agents' underestimation of present bias (that the agents are partially-naïve). Nevertheless, self-imposed deadlines could remain optimal in a multiple-project context. In the experiment, this extension makes it challenging to exclude the possibility that agents are sophisticated. Bisin and Hyndman (2020) document partially naïve agents at the deadline setting stage in their field experiment. They find no effect of deadlines on performance in single task treatment, while the participants in the repeating task treatment successfully manage to self-control. Burger et al. (2011) conduct field and online experiments on self-control and find that interim deadlines lead to even lower completion rates (contrary to their expectations) in their study. They also report that the degree of present bias is low across the participants, which could be interpreted as evidence for agents being time-consistent. However, in theory, deadlines only negatively affect time-consistent agents. While most of these studies document widespread present-bias problems and strong demand for commitment, the effect of deadlines on performance remains ambiguous and highly dependent on settings.

In this study, I show that it is not correct to generalize the results from a single-project environment to a multiple-project environment using an example with two parallel complementary projects. I develop an appropriate model that accounts for all the issues described above, and I find the optimal timing for interim deadlines in the context of procrastination, time-inconsistent preferences, and parallel complementary projects. This study can contribute to the broad literature on time-inconsistent preferences and provide a more precise algorithm for effort allocation choice and effort commitment in multiple goals/tasks/projects context.

2.3 Model

2.3.1 Agent

The agent has to perform two projects for several (N) periods. In each period $t \in \{1,...,N\}$, she chooses an effort level $e_{it} \geq 0$ for project $i \in \{1,2\}$ that she expends in the current period. In period t, she also forms an effort plan for the future periods: $e_{in} \geq 0$, $n \in \{t+1,...,N\}$, $i \in \{1,2\}$. However, the agent is not obliged to follow her plan when future periods arrive. The expended effort levels e_{it} in the current period are associated with the immediate costs $c(e_{1t},e_{2t})$ and the agent's reward after the project is over (after period N). The cost function is assumed to be the same for every period and is taken in a quadratic form:

$$c(e_t) = \frac{e_{1t}^2 + e_{2t}^2}{4} \tag{70}$$

The convex cost function represents the fact that the agent becomes tired when she works more hours on a given day. When switching to another project, the agent switches to different "type" task and faces the same initial marginal costs of expending effort. The quadratic form is the simplest and most popular for modeling convex costs. The agent is rested and faces the same marginal cost at the beginning of any period.

The agent is rewarded after the end of the project in period $N+1$ according to the reward function $R\left(\sum_{t=1}^{N} e_{1t}, \sum_{t=1}^{N} e_{2t}\right)$. The reward function is assumed to be complementary, so an increase in effort spent on one project increases the marginal return from expending effort on the second project:

$$R\left(\sum_{t=1}^{N} e_{1t}, \sum_{t=1}^{N} e_{2t}\right) = \sqrt{\sum_{t=1}^{N} e_{1t} \sum_{t=1}^{N} e_{2t}} \tag{71}$$

I study the agent's behavior with time-inconsistent preferences with quasi-hyperbolic discounting or so-called (β, δ) – preferences (Laibson, 1997; O'Donoghue & Rabin, 1999a; O'Donoghue & Rabin, 1999b).[32] Precisely, the agent's intertemporal preferences at the chosen period $t \in \{1,..., N\}$ can be represented by the intertemporal utility function U_t:

$$U_t = u_t + \beta \left[\sum_{n=t+1}^{N+1} \delta^{n-t} u_n\right] \tag{72}$$

u_t represents the agent's instantaneous utility from period t; $\delta \in (0,1]$ – is a standard discount factor; and $\beta \in (0,1]$ – is a present bias parameter. The present bias parameter here is crucial and represents the time inconsistency in the model. In any fixed period t, the agent weights

[32] (β, δ) – preferences in period t can be presented as the following utility function:

$$U_t = u_t + \beta\delta u_{t+1} + \beta\delta^2 u_{t+2} + \beta\delta^3 u_{t+3} + \beta\delta^4 u_{t+4} + ...$$

Here β is a present bias parameter and δ is a long-run discount factor, U_t is total utility, and u_t is the utility in period t. Under these preferences, the agent chooses the current and future consumption/effort in period t to maximize total utility U_t. Moving to period $t+1$, the agent's preferences change and can be described as:

$$U_{t+1} = u_{t+1} + \beta\delta u_{t+2} + \beta\delta^2 u_{t+3} + \beta\delta^3 u_{t+4} + ...$$

Again, the agent chooses current and future consumption/effort in period $t+1$ maximizing the total utility U_{t+1}. In period $t+1$, the intertemporal substitution between periods $t+1$ and any further period has changed compared to period t. Specifically, under the assumption $0 < \beta < 1$, the agent prefers to consume more and puts in less effort in period $t+1$ compared to her plan for the period $t+1$ in period t; in other words, the agent procrastinates.

the current period more than the future when $\beta < 1$.[33] In order to focus on the agent's procrastination problem, I abstract away from the standard exponential discounting and set $\delta = 1$. Because the agent faces the cost functions in each period $t \in \{1,..., N\}$ and encounters the reward function after the project at period $N + 1$, her preferences in period t can be represented by intertemporal utility U_t:

$$U_t = -c(e_{1t},e_{2t}) + \beta \left[- \sum_{n=t+1}^{N} c(e_{1n},e_{2n}) + R\left(\sum_{n=1}^{N} e_{1n}, \sum_{n=1}^{N} e_{1n}\right)\right] = \\ = -\frac{e_{1t}^2+e_{2t}^2}{4} + \beta \left[- \sum_{n=t+1}^{N} \frac{e_{1n}^2+e_{2n}^2}{4} + \sqrt{\sum_{n=1}^{N} e_{1n} \sum_{n=1}^{N} e_{2n}}\right] \tag{73}$$

The agent is modeled as a sequence of intertemporal selves at each period $t \in \{1,..., N\}$. At any chosen period t, the agent maximizes her intertemporal utility U_t by choosing the effort level for the current period (e_{1t},e_{2t}) and forming an effort plan for future periods, $\left\{e_{1n}^{(t)}, e_{2n}^{(t)}\right\}_{n=t+1}^{N}$, given her effort history and her beliefs about the behavior of her future selves in the next periods. The superscript denotes the period t for the intertemporal agent's self who maximizes the intertemporal utility U_t. In other words, the agent behaves according to the optimal *intertemporal* strategy $\left\{\hat{e}_{1\tau}^{(t)}, \hat{e}_{2\tau}^{(t)}\right\}_{\tau=t}^{N}$ in period t.

Definition 1: *The agent's* ***optimal intertemporal strategy*** *at period t is the profile of the optimal actions for current and future periods from the perspective of the agent's intertemporal self at period t.*

The optimal *intertemporal* strategy consists of the actions for the current and future periods

[33] β is a present bias parameter that describes how differently the agent weights the future compared to the current moment (today). $\beta = 1$ describes the time-consistent agent, while $0 < \beta < 1$ corresponds to the agent who weighs today relatively higher than the future. That agent would prefer to work less and enjoy leisure more today, and work more and enjoy leisure less tomorrow.

that maximize the agent's intertemporal utility U_t, given the agent's beliefs about the behavior of her future selves. Thus, the agent behaves according to the current optimal *intertemporal* strategy and invests $\left(\hat{e}_{1t}^{(t)}, \hat{e}_{2t}^{(t)}\right)$ into projects in every period $t \in \{1,...,N\}$. Therefore, the resultant agent's action profile during the project consists of her current actions from the optimal *intertemporal* strategies: $\left\{\hat{e}_{1t}^{(t)}, \hat{e}_{2t}^{(t)}\right\}_{t=1}^{N}$.

2.3.2 Agent Type

In this paper, I focus on the analysis of the behavior of the naïve agent.[34] The naïve agent is not aware of her time inconsistency and believes she is time-consistent. In the current period t, the agent invests only effort levels $\left(\hat{e}_{1t}^{(t)}, \hat{e}_{2t}^{(t)}\right)$ into projects. The naïve agent believes she is time-consistent and will behave according to the effort plan that maximizes her intertemporal utility U_t or, in other words, that β will be equal to 1 in all future periods. Thus, the optimal *intertemporal* strategy for the naïve agent (without a deadline) is the solution to the following utility maximization problem (UMP):

$$\left\{\hat{e}_{1\tau}^{(t)}, \hat{e}_{2\tau}^{(t)}\right\}_{\tau=t}^{N} \in \arg \max_{\{e_{1\tau}, e_{2\tau}\}_{\tau=t}^{N}} \left\{ -\frac{e_{1t}^2 + e_{2t}^2}{4} + \beta \left[-\sum_{n=t+1}^{N} \frac{e_{1n}^2 + e_{2n}^2}{4} + \sqrt{\sum_{n=1}^{N} e_{1n} \sum_{n=1}^{N} e_{2n}} \right] \right\} \quad (74)$$

$$s.t.: \quad \{e_{1n}, e_{2n}\}_{n=1}^{t-1} \ \ are \ \ given$$

[34] In general, the agent can be naïve, sophisticated, or partially-sophisticated. I discuss the behavior of other types in the Discussion section.

2.3.3 Welfare Criteria

Because the agent's preferences change over the periods, the agent evaluates her overall performance differently in different periods. The same agent's action profile gives different utility levels for different intertemporal selves during the project. Therefore, to analyze the agent's welfare, it is standard to take the so-called "long-run" preferences to evaluate the agent's performance:

$$U_0 = \left[- \sum_{t=1}^{N} c(e_{1t}, e_{2t}) + R \left(\sum_{t=1}^{N} e_{1t}, \sum_{t=1}^{N} e_{2t} \right) \right] \tag{75}$$

This approach is in line with O'Donoghue and Rabin (1999a), O Donoghue et al. (2006) and Herweg and Müller (2011). Additionally, these preferences represent the agent who considers the entire project in advance (how she evaluates her performance in a 0-period). The difference then is in multiplication by the constant β.

2.3.4 Interim Deadline

In this paper, I study the agent's behavior under exogenously imposed interim deadlines to investigate the effect of deadlines on the agent's behavior and welfare. It is possible to impose only one interim deadline for one project. I focus on the optimality of imposing the interim deadline in one project, given that the interim deadline in the second project is fixed. In all cases, the flexible interim deadline is an instrument used to maximize the agent's welfare, the agent's "long-run" utility (75).

Definition 2: *An **interim deadline** (ID) is the constraint on the agent's behavior defined by two parameters: **timing** $k \in \{1,...,N\}$ and **goal** $A \geq 0$. The agent is restricted to investing a total level of effort greater than or equal to A by the end of period k.*

In other words, if the agent faces an interim deadline $ID = (A, k)$ in project $i \in \{1,2\}$, then her intertemporal selves face the following constraint in every period 1,...,k:

$$\sum_{t=1}^{k} e_{it} \geq A \tag{76}$$

The deadline is exogenous, and the agent is informed about the interim deadline before the project starts, and she must meet it. After the interim deadline is imposed, the agent has no option to adjust or cancel it and must satisfy the deadline. Under the interim deadline, the agent can affect her future selves through her choice of the current effort, and this choice depends on her effort history.[35] After the interim deadline is met at period k, the agent behaves according to the optimal *intertemporal* strategies when no deadline is imposed.

2.4 Analysis

2.4.1 Time-Consistent Agent

The time-consistent agent behaves according to the optimal solution for the "long-run" preferences from the first period to the last, even without any interim deadlines. Indeed, the agent's preferences remain the same during projects ($\beta = 1$). Thus, in every period $t \in \{1,...,N\}$, the agent optimizes the same utility function (75):

[35] Note that different types of agents hold different beliefs about their future selves' behavior. Thus, the behavior of naïve, sophisticated, and partially-sophisticated agents will be different.

$$\max_{\{e_{1\tau},e_{2\tau}\}_{\tau=t}^{N}} \left[-\sum_{\tau=t}^{N} c(e_{1\tau}, e_{2\tau}) + R\left(\sum_{t=1}^{N} e_{1t}, \sum_{t=1}^{N} e_{2t} \right) \right] \tag{77}$$

$$\{e_{1n}, e_{2n}\}_{n=1}^{t-1} \quad are \ \ given$$

The first-order conditions are:

$$\begin{aligned} e_{1\tau} &= \sqrt{\frac{\sum_{t=1}^{N} e_{1t}}{\sum_{t=1}^{N} e_{2t}}} \ \ , \ \forall\, \tau \in \{t,...,N\} \\ e_{2\tau} &= \sqrt{\frac{\sum_{t=1}^{N} e_{2t}}{\sum_{t=1}^{N} e_{1t}}} \ \ , \ \forall\, \tau \in \{t,...,N\} \end{aligned} \tag{78}$$

As a result, the time-consistent agent implements $e_{1t} = e_{2t} = 1$ in every period.

2.4.2 Time-Inconsistent Agent Under No Deadline

The time-inconsistent agent has changing preferences over periods. Under no interim deadline, in every period t, the agent maximizes the intertemporal utility:

$$\max_{\{e_{1\tau},e_{2}\}_{\tau=t}^{N}} \left\{ -c(e_{1t}, e_{2t}) + \beta \left[-\sum_{\tau=t+1}^{N} c(e_{1\tau}, e_{2\tau}) + R\left(\sum_{t=1}^{N} e_{1t}, \sum_{t=1}^{N} e_{2t} \right) \right] \right\} \tag{79}$$

$$\{e_{1n}, e_{2n}\}_{n=1}^{t-1} \quad are \ \ given$$

The first-order conditions are:

$$
\begin{aligned}
e_{1t} &= \beta\sqrt{\frac{\sum_{t=1}^{N} e_{2t}}{\sum_{t=1}^{N} e_{1t}}} \\
e_{1\tau} &= \sqrt{\frac{\sum_{t=1}^{N} e_{2t}}{\sum_{t=1}^{N} e_{1t}}} \ , \ \forall\, \tau \in \{t,...,N\} \\
e_{2t} &= \beta\sqrt{\frac{\sum_{t=1}^{N} e_{1t}}{\sum_{t=1}^{N} e_{2t}}} \\
e_{2\tau} &= \sqrt{\frac{\sum_{t=1}^{N} e_{1t}}{\sum_{t=1}^{N} e_{2t}}} \ , \ \forall\, \tau \in \{t,...,N\}
\end{aligned}
\tag{80}
$$

Solving for the first-order conditions, the time-inconsistent agent implements $e_{1t} = e_{2t} = \beta$ in the current periods t and plans to implement $e_{1\tau} = e_{2\tau} = 1$ in every future period $\tau \in \{t + 1,...,N\}$. However, the agent faces the same problem in the next period. As a result, the agent implements only β in every period in every project. Because $\beta < 1$, the time-inconsistent agent (procrastinator) spends less effort on the project than does the time-consistent agent.

2.4.3 Time-Inconsistent Agent Under Interim Deadlines

I consider the agent's behavior under interim deadlines in both projects. Only one interim deadline can be imposed on one project. Thus, the agent faces the following constraints on her maximization problem in every period:

$$\sum_{n=1}^{k} e_{1n} \geq A$$
$$\sum_{n=1}^{m} e_{2n} \geq B \tag{81}$$

While the deadline is an instrument to increase the agent's welfare when procrastinating, in this paper, I focus on studying the optimal timings k and m, leaving aside the question of optimal *goals* A and B. For any interim deadlines in periods k and m, I set the *goals* such that the agent must invest efforts according to time-consistent behavior: $A = k$, $B = m$.[36] Thus, in period t, the agent solves the following maximization problem:

$$\max_{\{e_{1\tau}, e_2\}_{\tau=t}^{N}} \left\{ -c(e_{1t}, e_{2t}) + \beta \left[-\sum_{\tau=t+1}^{N} c(e_{1\tau}, e_{2\tau}) + R\left(\sum_{n=1}^{N} e_{1n}, \sum_{n=1}^{N} e_{2n} \right) \right] \right\} \tag{82}$$

$$s.t.: \quad \sum_{n=t}^{k} e_{1n} \geq k - \sum_{n=1}^{t-1} e_{1n}$$
$$\sum_{n=t}^{m} e_{2n} \geq m - \sum_{n=1}^{t-1} e_{2n}$$
$$\{e_{1n}, e_{2n}\}_{n=1}^{t-1} \quad are \;\; given$$

Note that the interim deadlines bind from the first period, because without deadlines, the agent would expend β effort for each project, and under deadlines, she has to expend 1 effort per period on average for each project. Thus, the constraints of the interim deadlines can be rewritten as equalities and can be used to express the efforts for the current period in terms

[36]When relaxing this assumption, the solution for agent's choice of effort will be multiplied by A/k for the first project in periods $\{1,...,k\}$ and by B/m for the second project in periods $\{1,...,m\}$ (if $A \geq 1$ and $B \geq 1$). For details and cases when $A < 1$ (or $B < 1$); see Razumovskii (2023).

of efforts for future periods:

$$\begin{cases} e_{1t} = k - \sum_{n=1}^{t-1} e_{1n} - \sum_{n=t+1}^{k} e_{1n} \\ \\ e_{2t} = m - \sum_{n=1}^{t-1} e_{2n} - \sum_{n=t+1}^{m} e_{2n} \end{cases} \tag{83}$$

The first-order conditions for $t \leq \min(k,m)$ are then:[37]

$$\begin{aligned} & e_{1t} = \beta e_{1\tau} \;\;, \; \forall \, \tau \in \{t+1,...,k\} \\ & e_{1t} = k - \sum_{n=1}^{t-1} e_{1n} - \sum_{\tau=t+1}^{k} e_{1\tau} \\ & e_{1l} = \sqrt{\frac{m+\sum_{n=m+1}^{N} e_{2n}}{k+\sum_{n=k+1}^{N} e_{1n}}} \;\;, \; \forall \, l \in \{k+1,...,N\} \\ & e_{2t} = \beta e_{2\tau} \;\;, \; \forall \, \tau \in \{t+1,...,m\} \\ & e_{2t} = m - \sum_{n=1}^{t-1} e_{2n} - \sum_{\tau=t+1}^{m} e_{2\tau} \\ & e_{2l} = \sqrt{\frac{k+\sum_{n=k+1}^{N} e_{1n}}{m+\sum_{n=m+1}^{N} e_{2n}}} \;\;, \; \forall \, l \in \{m+1,...,N\} \end{aligned} \tag{84}$$

Directly from the first-order conditions, the effort expended for the first project in periods $(1,...,k)$ and in the second project in periods $(1,...,m)$ can be found from:

[37] Note that the ID binds during all periods from the 1st to the kth (the mth) because the f.o.c.s show that the agent spends less in the current period than she has to spend on average during the remaining periods before the deadline ($e_{it} = \beta e_{i\tau}, \;\; i \in \{1,2\}$). Thus the equalities (83) hold for any t before the deadlines periods k and m.

$$
\begin{aligned}
&e_{1t} = \beta^{\frac{k-\sum_{n=1}^{t-1} e_{1n}}{k-t+\beta}} \\
&e_{1\tau} = \frac{k-\sum_{n=1}^{t-1} e_{1n}}{k-t+\beta} \quad , \; \forall\, \tau \in \{t+1,...,k\} \\
&e_{2t} = \beta^{\frac{m-\sum_{n=1}^{t-1} e_{2n}}{m-t+\beta}} \\
&e_{2\tau} = \frac{m-\sum_{n=1}^{t-1} e_{2n}}{m-t+\beta} \quad , \; \forall\, \tau \in \{t+1,...,m\}
\end{aligned}
\tag{85}
$$

While the effort expended on the project after the interim deadlines can be found from the following part of f.o.c.s:

$$
\begin{aligned}
&e_{1l} = \sqrt{\frac{m+(N-m)e_{2q}}{k+(N-k)e_{1l}}} \quad , \; \forall\, l \in \{k+1,...,N\}, \; q \in \{m+1,...,N\} \\
&e_{2q} = \sqrt{\frac{k+(N-k)e_{1l}}{m+(N-m)e_{2q}}} \quad , \; \forall\, l \in \{k+1,...,N\}, \; q \in \{m+1,...,N\}
\end{aligned}
\tag{86}
$$

These equations can be reduced to one equation of the 4th degree for e_{1l} or e_{2q}:

$$
e^4 + ae^3 + ce + d = 0 \tag{87}
$$

Where a, c, and d for $e = e_{2l}$ can be found as following:

$$
a = \frac{m}{N-m}, \quad c = -\frac{k}{N-m}, \quad d = -\frac{N-k}{N-m}
$$

The equation (87) always has at least one positive real solution and can be solved using Ferrari's solution and Cardano's method.

The equations (85) and (86) describe the agent's behavior and her plan for the future in the current period t. Thus, the agent implements only e_{1t} and e_{2t} in period t and then moves to the next period $t' = t + 1$. In period t', the agent faces the maximization problem similar to (82), however, the solution will be different from equations (85) and (86) because her preferences will have changed. According to Razumovskii (2023), the agent's behavior before the interim deadline is met in each project can be found from:

$$\begin{cases} e_{1t} = \beta \Pi_{l=0}^{t-1} \left(\frac{k-l}{k-l-(1-\beta)} \right) & , \quad t \in \{1,...,k-1\} \\ e_{1k} = \frac{1}{\beta} e_{1t} = \Pi_{l=0}^{k-2} \left(\frac{k-l}{k-l-(1-\beta)} \right) & , \quad t = k \\ e_{2t} = \beta \Pi_{l=0}^{t-1} \left(\frac{m-l}{m-l-(1-\beta)} \right) & , \quad t \in \{1,...,m-1\} \\ e_{2m} = \frac{1}{\beta} e_{2t} = \Pi_{l=0}^{m-2} \left(\frac{m-l}{m-l-(1-\beta)} \right) & , \quad t = m \end{cases} \tag{88}$$

When one interim deadline is met, the first-order conditions in period $t : m < t \leq k$ change to (w.l.o.g. I assume $m \leq k$):

$$
\begin{aligned}
&e_{1t} = \beta e_{1\tau} \ \ , \ \forall\, \tau \in \{t+1,...,k\} \\
&e_{1t} = k - \textstyle\sum_{n=1}^{t-1} e_{1n} - \sum_{\tau=t+1}^{k} e_{1\tau} \\
&e_{1l} = \sqrt{\frac{\sum_{n=1}^{N} e_{2n}}{k+\sum_{n=k+1}^{N} e_{1n}}} \ \ , \ \forall\, l \in \{k+1,...,N\} \\
&e_{2t} = \beta e_{2l} \ \ , \ \forall\, l \in \{t+1,...,N\} \\
&e_{2l} = \sqrt{\frac{k+\sum_{n=k+1}^{N} e_{1n}}{\sum_{n=1}^{N} e_{2n}}} \ \ , \ \forall\, l \in \{t+1,...,N\}
\end{aligned}
\tag{89}
$$

The current period effort e_{1t} can be found as before, according to equations (88), while effort e_{2t} can be found as $e_{2t} = \beta e_{2l}$. e_{2l} is a real positive solution of the equation (87) with parameters:

$$
a = \frac{\sum_{n=1}^{t-1} e_{2n}}{N-t+\beta}, \quad c = -\frac{k}{N-t+\beta}, \quad d = -\frac{N-k}{N-t+\beta}
$$

When both interim deadlines are met, the first-order conditions in period $t : k < t$ change to:

$$
\begin{aligned}
&e_{1t} = \beta e_{1l} \ \ , \ \forall \, l \in \{t+1,...,N\} \\
&e_{1l} = \sqrt{\frac{\sum_{n=1}^{N} e_{2n}}{\sum_{n=1}^{N} e_{1n}}} \ \ , \ \forall \, l \in \{t+1,...,N\} \\
&e_{2t} = \beta e_{2l} \ \ , \ \forall \, l \in \{t+1,...,N\} \\
&e_{2l} = \sqrt{\frac{\sum_{n=1}^{N} e_{1n}}{\sum_{n=1}^{N} e_{2n}}} \ \ , \ \forall \, l \in \{t+1,...,N\}
\end{aligned}
\tag{90}
$$

Thus, the current period effort (e_{1t}, e_{2t}) can be found by $e_{1t} = \beta / e_{2l}$ and $e_{2t} = \beta e_{2l}$. e_{2l} is a real positive solution of the equation (87) with parameters:

$$
a = \frac{\sum_{n=1}^{t-1} e_{2n}}{N - t + \beta}, \quad c = -\frac{\sum_{n=1}^{t-1} e_{1n}}{N - t + \beta}, \quad d = -1
$$

As a result, the agent's behavior before the interim deadlines are met is defined by the choice of k and m according to equations (88). However, the agent's behavior after the interim deadlines (in each project) also depends on parameters k and m, because it depends on the total sum of effort before the current period through parameters a, c, and d in equation (87).

2.5 Simulations and Results

The agent's effort choice in the periods before the interim deadlines are met (in each project) is defined by equations (88). As Razumovskii (2023) shows, the agent accumulates effort before the interim deadline and experiences increased costs (due to a convex cost function) in the last periods before the interim deadline. Figure (2.1) provides an example of the agent's

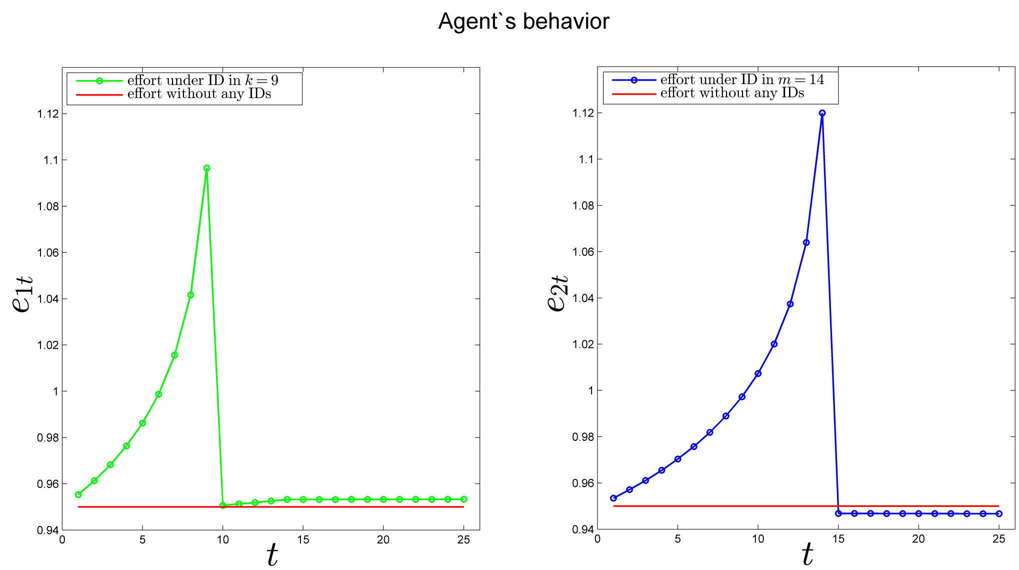

Figure 2.1

behavior with interim deadlines in periods 9 and 14 ($k = 9$, $m = 14$) when she is engaged in two 25-period projects. On the left graph of Figure (2.1), the green line shows the agent's effort on the first project for periods $t \in \{1,...,N\}$. According to equation (88), the agent postpones effort towards the deadline ($k = 9$) and accumulates effort in the last periods before the interim deadline. The blue line on the right graph presents the agent's effort for the second project. The red lines on both graphs present the agent's effort choice without any deadlines in the projects ($e_{1t} = e_{2t} = \beta$).

When neither interim deadline is met, the agent accumulates effort on projects towards the interim deadlines (k and m) according to equation (88). When one interim deadline is met (e.g., in the first project), the agent's effort in the second project is still described by the equation (88), however, the effort in the first project (where the interim deadline is already met) increases (the relatively small increase of green line on the left graph of Figure (2.1) between the 10th and the 14th periods). For these periods, the first-order condition for effort in the current period in the first project is:

$$e_{1t} = \beta e_{1\tau} = \beta\sqrt{\frac{\sum_{n=1}^{N} e_{2n}}{\sum_{n=1}^{N} e_{1n}}} \quad , \ \tau \in \{t+1,...,N\} \tag{91}$$

When the agent moves from period t to period $t' = t+1$, the first-order condition for effort in period t' in the first project changes from equation $e_{1t+1} = \sqrt{\frac{\sum_{n=1}^{N} e_{2n}}{\sum_{n=1}^{N} e_{1n}}}$ to:

$$e_{1t'} = \beta\sqrt{\frac{\sum_{n=1}^{N} e_{2n}}{\sum_{n=1}^{N} e_{1n}}} \tag{92}$$

There are several effects here. First, the agent spends less effort than she planned due to her present bias ($\beta < 1$). However, there are indirect effects as well. Because the interim deadline in the first project is already met, the agent keeps expending less than time-consistent effort on the first project due to present bias. Thus, the total sum of effort expended (and planned to be expended in future periods) on the first project, $\sum_{n=1}^{N} e_{1n}$, decreases. This indirectly increases the effort spent on the first project, because the sum of effort is in the denominator in equation (92).

Note that there is no direct effect on the total sum of effort spent (and planned for the future periods) on the second project, because the interim deadline in the second project is not met and the sum of effort between the 1st and the mth periods is fixed. The indirect (second-order) effect (caused by the decrease of e_{1t} in the total sum of effort spent on the first project) decreases the effort planned for the future periods in the second project: $e_{2\tau} = \sqrt{\frac{\sum_{n=1}^{N} e_{1n}}{\sum_{n=1}^{N} e_{2n}}}$, $\tau \in \{m+1,...,N\}$. However, the direct effect accumulates when the agent moves to later periods and it prevails over the indirect effects. Thus, the effort spent on the first project increases from the period when the interim deadline is met in the first project till the period when the interim deadline is met in the second project (the green line between the 10th and the 14th periods on the left graph of Figure (2.1)).

After the interim deadline in the second project is also met, the agent's behavior is defined by the difference between the currently spent effort on projects. Because the effort for the current period in projects needs to satisfy equality $e_{1t}e_{2t} = \beta^2$ and first-order conditions (90), the effort spent on the first project is greater, due to a later interim deadline in the second project and greater effort already spent on the second project.

According to Razumovskii (2023), there is only one unique optimal period for an interim deadline for the procrastinator pursuing one project. However, in the case of two projects, the optimal period for the interim deadline in one project depends on the period in which the interim deadline in the other project is imposed. Figure (2.2) presents the agent's welfare as a function of the periods when the interim deadlines are imposed (k and m). Fixing the interim deadline for one project, there is only one unique optimal period for the interim deadline in another project. While the figure is symmetric to the plane $k = m$, there is no unique optimal choice of periods for the interim deadline in projects. Figure (2.2) has two symmetrical maximums when the interim deadline in one project is imposed earlier and the interim deadline in another project is imposed later than would be optimal if the interim deadline can be imposed in only one of the two projects.

In Figure (2.3), the blue line presents the optimal period for the interim deadline in the second project (m) depending on the period when the interim deadline in the first project is imposed (k). The green line presents the optimal period for the interim deadline in the first (k), depending on when the interim deadline in the second project is imposed (m). The red line is a symmetric line when $k = m$. Figure (2.3) shows that the interim deadline in one project affects the optimal timing for the interim deadline in another project if it is imposed relatively closer in time. For example, it is optimal to impose the interim deadline in the

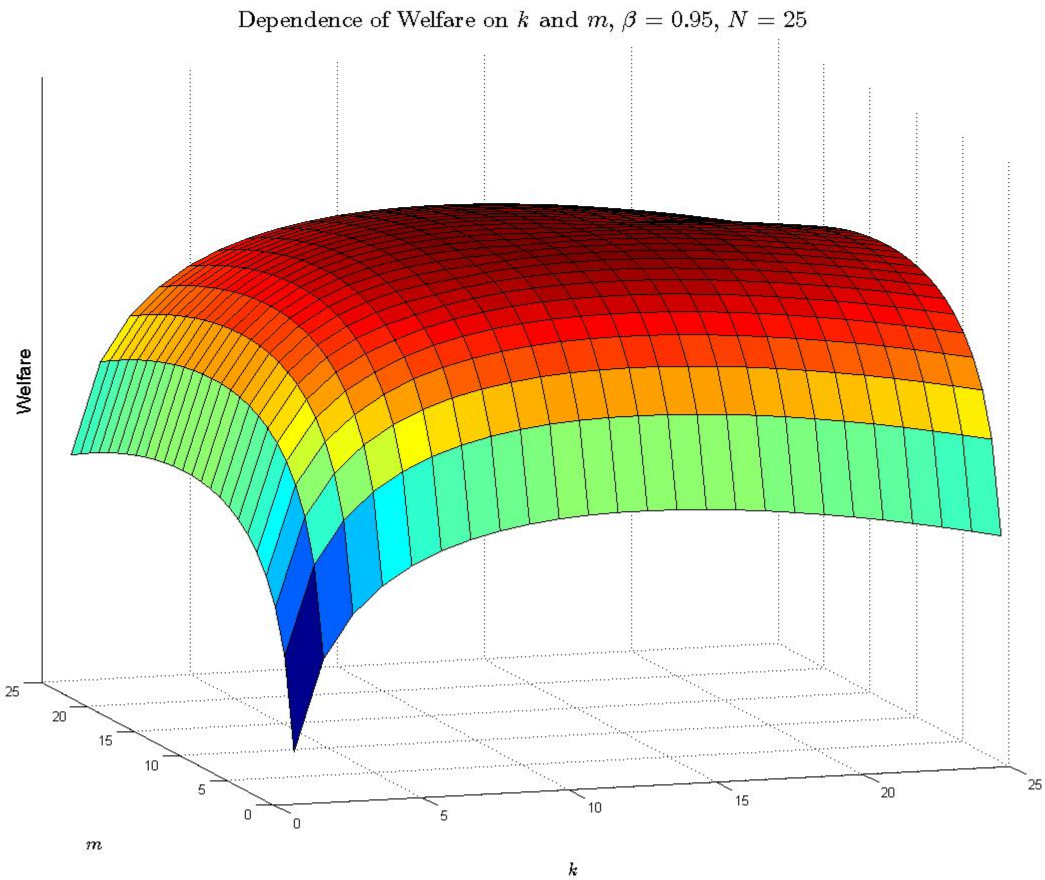

Figure 2.2

second project in the 12th period ($m = 12$) when there is no interim deadline in the first project ($k = 0$).[38] It is still optimal to set $m = 12$ when the interim deadline in the first project is imposed relatively earlier or later ($k \in \{1,...,5,19,...24\}$). However, it is optimal to move the interim deadline in the second project to the 13th period ($m = 13$) when the interim deadline in the first project is imposed closer but earlier ($k \in \{6,...,11\}$) and to the 11th period ($m = 11$) when closer but later ($k \in \{12,...,18\}$). Due to symmetry, the best response choice of the period for the interim deadline in the first project depends on the period when the interim deadline in the second project is imposed. Similarly, the optimal k moves from the 12th period to the 11th or the 13th if the chosen m is close to k.

[38]With the chosen parameters $\beta = 0.95$ and $N = 25$.

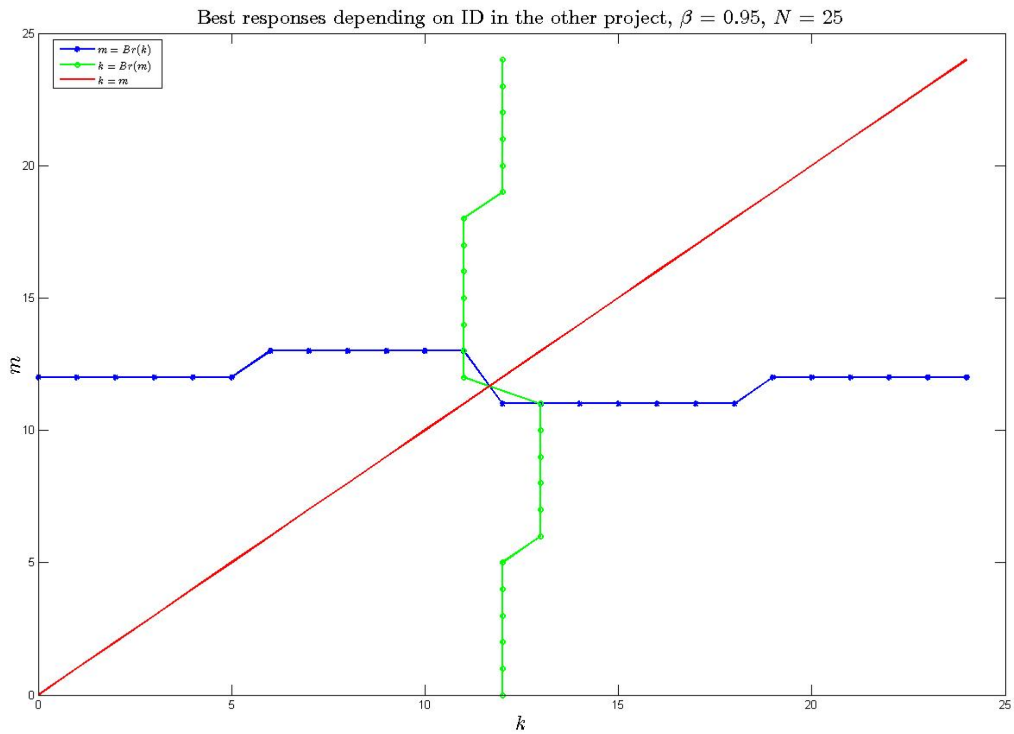

Figure 2.3: $\beta = 0.95$

Proposition: *The optimal timing for the interim deadline in one project depends on the timing of the interim deadline in the other project when the present-biased agent is pursuing two complementary projects. When the interim deadline in one project is set close to the interim deadline in the other, the optimal timing might be earlier or later than if there were no interim deadline in the parallel project.*

The intuition comes from the marginal effects. Without loss of generality, consider the case when the interim deadline in the first project is set optimally (in the example provided, $k = 12$), and there is no interim deadline in the second project ($m = 0$). Adding the interim deadline in the second project will have two direct effects: an increase in the reward function due to the increase in total effort spent in the second project, and a corresponding increase in costs. While the increase in effort is almost linear,[39] the increase in costs grows when the interim deadline is moved later, as discussed in Razumovskii (2023).

When the increase in marginal returns from the effort expended on the first project, $MR_1 = \frac{\partial R}{\partial e_{1t}} = \frac{1}{2}\sqrt{\frac{\sum_{n=1}^{N} e_{2t}}{\sum_{n=1}^{N} e_{1t}}}$, are large enough and the increase in cost is still low (earlier periods), it is optimal to move the interim deadline in the first project to a later period ($k = 12 \rightarrow k = 13$) due to higher marginal returns. Moving the interim deadline further in the second project induces another effect when the timing reaches the optimal period for the interim deadline in only one project ($t = 12$). At that point, it is optimal to move the interim deadline in the first project to an earlier period, such that it will be in an earlier period than the interim deadline in the second project ($k < m$). I denote this in the remaining text as a "switching effect". Then, according to first-order conditions after both interim deadlines are met (90), the agent expends relatively more effort on the first project than on the second. This effect keeps a high degree of total effort spent on the first project while lowering the corresponding costs (due to earlier timing for the interim deadline in the first project).

The presence and size of these effects depend on the agent's present bias parameter β. In Figure (2.3), when $\beta = 0.95$, the blue line shows that there is a marginal effect on the optimal timing for the interim deadline in the second project when the interim deadline in the first project is moved from period 5 to 6 and when moving from period 18 to 19 (optimal m changes from $m = 12$ to $m = 13$ and from $m = 11$ to $m = 12$ correspondingly). When the interim deadline in the first project is set in the 12th period ($k = 12$), it is optimal to move the interim deadline in the second project to an earlier period ($m = 11 < 12 = k$) to induce greater effort spent on the second project after both interim deadlines are met and to reduce

[39]The direct effect on effort is linear, similar to Razumovskii (2023). However, the increase in total effort in the second project has a marginal effect on the effort spent on the first project. Even if the interim deadline in the first project is fixed, there is a positive marginal effect on the effort spent after the interim deadline in the first project is met. Thus, this decreases the effort spent on the second project after both interim deadlines are met according to f.o.c.s.

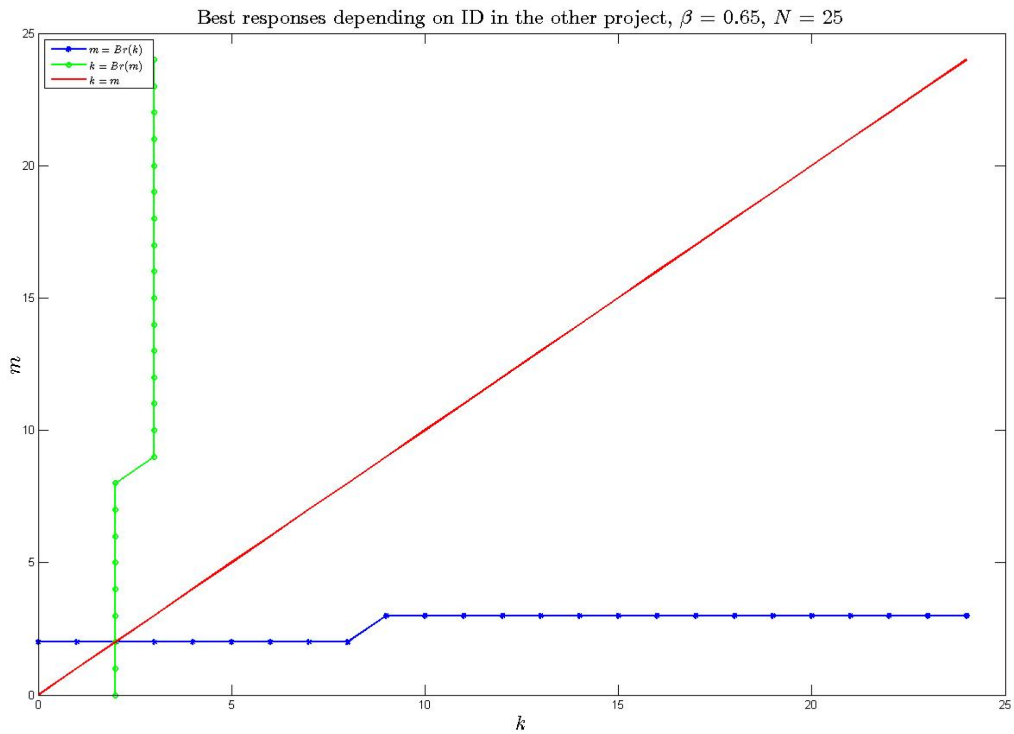

Figure 2.4: $\beta = 0.65$

the costs before the deadline ("switching effect").

Figures (2.4) and (2.5) present the best responses for the agent with different present biases. In Figure (2.4), the agent is relatively more present-biased ($\beta = 0.65$). There is only one marginal effect when the interim deadline in one project is moved from period $k = 8$ to period $k = 9$. It is optimal to move the interim deadline in the other project from period $m = 2$ to $m = 3$. In Figure (2.5), the agent is close to being time-consistent ($\beta = 0.97$), there is a cascade of marginal effects, and there is a "switching effect" in the later periods of the projects.

As a result, the optimal timing for the interim deadline in one project significantly depends on the timing for the interim deadline in the other project. Adding the interim deadline in the second project (or moving it to sooner or to later) may move the optimal timing for the

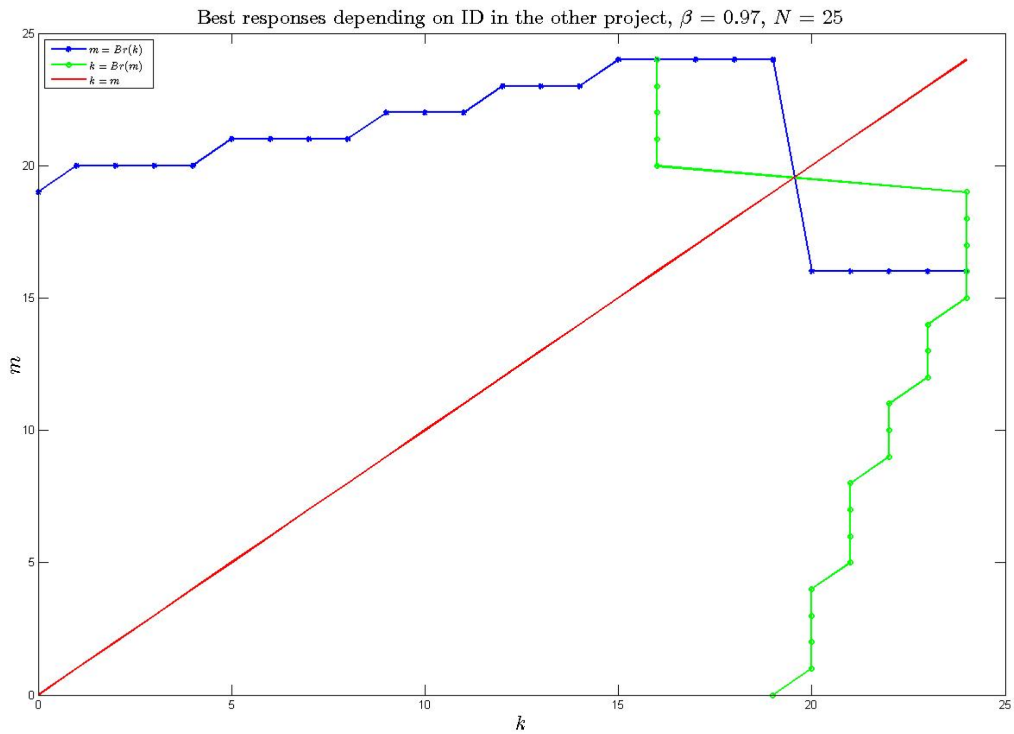

Figure 2.5: $\beta = 0.97$

interim deadline in the first project backward or forward from the previously optimal period (with no interim deadline in the parallel project) depending on the timing chosen for the interim deadline in the second project. The optimal pair of timings for both interim deadlines is not necessarily unique. For an agent with a low present bias parameter β (relatively more present-biased agent), there is a unique symmetric ($k^* = m^*$) optimal pair of timings for interim deadlines (e.g., Figure (2.4)). However, for an agent with a higher present bias parameter β (relatively less present-biased agent), the are two that are symmetric to each other optimal pairs of timings $((k_1^*, m_1^*), (k_2^*, m_2^*) : \ k_1^* = m_2^*, m_1^* = k_2^*)$ (e.g., Figures (2.3), (2.5)).

2.6 Discussion

2.6.1 Uniqueness and Distance

As discussed in the Simulations and Results section, the optimal pair of timings for the interim deadline in both projects (k,m) is not necessarily unique. For example, Figure (2.4) presents the case when the optimal pair is unique for an agent with a present bias parameter of $\beta = 0.65$, while Figure (2.5) presents the case when there are two pairs that yield maximal welfare for an agent with a present bias parameter of $\beta = 0.97$).

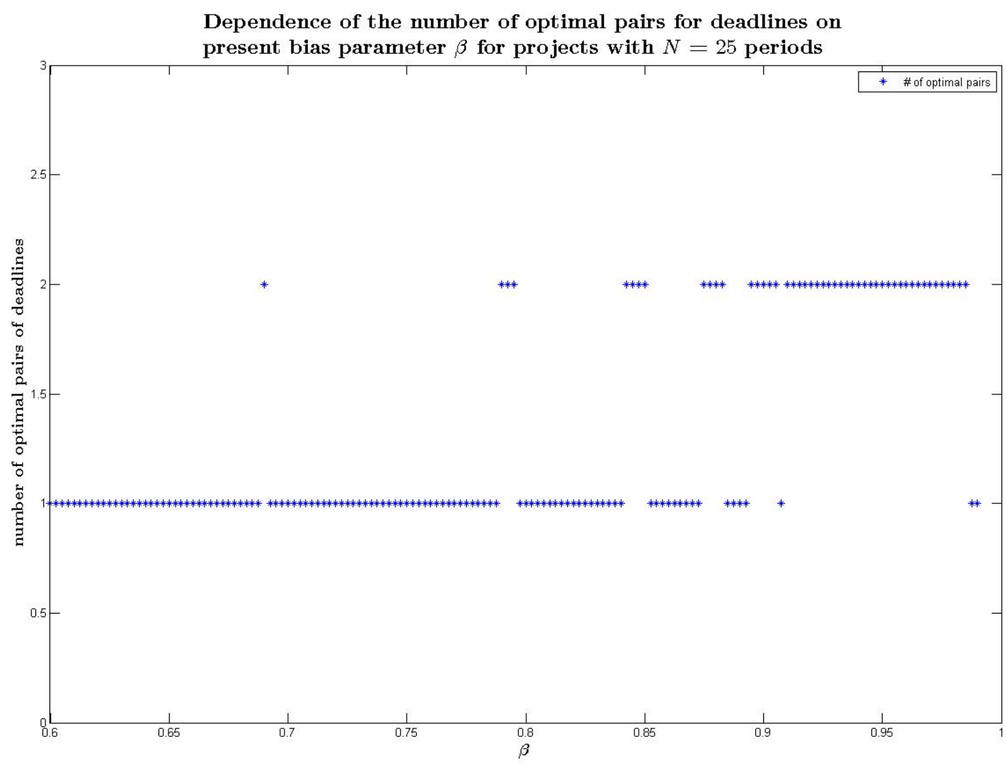

Figure 2.6

Figure (2.6) presents the dependence of the number of optimal timing pairs for interim deadlines on the agent's present bias parameter β.[40] While there is a unique optimal pair

[40]The graph presents the results for present bias parameter β from $\beta = 0.6000$ to $\beta = 0.9900$ with a 0.0025 step.

of timings for the relatively low present bias parameter and two optimal timings for the relatively high present bias parameter, the dependence is not monotone. In general, the optimal timing is later for the agent with a higher present bias parameter β (Razumovskii, 2023). Thus, for the almost time-consistent agent (β is close to 1), the optimal timings for both projects are in the last period. At the same time, the optimal timings for both projects are in the first period for the agent with a low present bias parameter. When the agent's present bias parameter increases, the optimal timing moves from the first period to the second and beyond. Because of the discrete time, it may be optimal to move only one interim deadline to the next period rather than moving both later simultaneously when increasing β. Then the uniqueness of the optimal timings is broken solely due to the discrete nature of time, and the optimal pairs of timing become $(k_1^*, q_1^*) = (t^*, t^* + 1)$ and $(k_2^*, q_2^*) = (t^* + 1, t^*)$. When the present bias parameter is increased further, the optimal pair becomes unique again. However, the optimal timings for interim deadlines in different projects are far away from each other when the present bias parameter is high enough (e.g., on Figure (2.5)).

Figure (2.7) presents how the distance between optimal timings for interim deadlines in parallel projects depends on the agent's present bias. When it becomes optimal to distinguish deadlines in time, the distance decreases with the agent's present bias (i.e., is increasing in the agent's present bias parameter β). However, it goes back to zero when the agent becomes close to time-consistent, and it is optimal to impose both interim deadlines in the last periods of the projects.

As a result, the agent who suffers relatively more from self-control problems does not need to distinguish the interim deadlines for concurrent projects in time, while the agent who is close to being time consistent would benefit from separating deadlines in time for different projects.

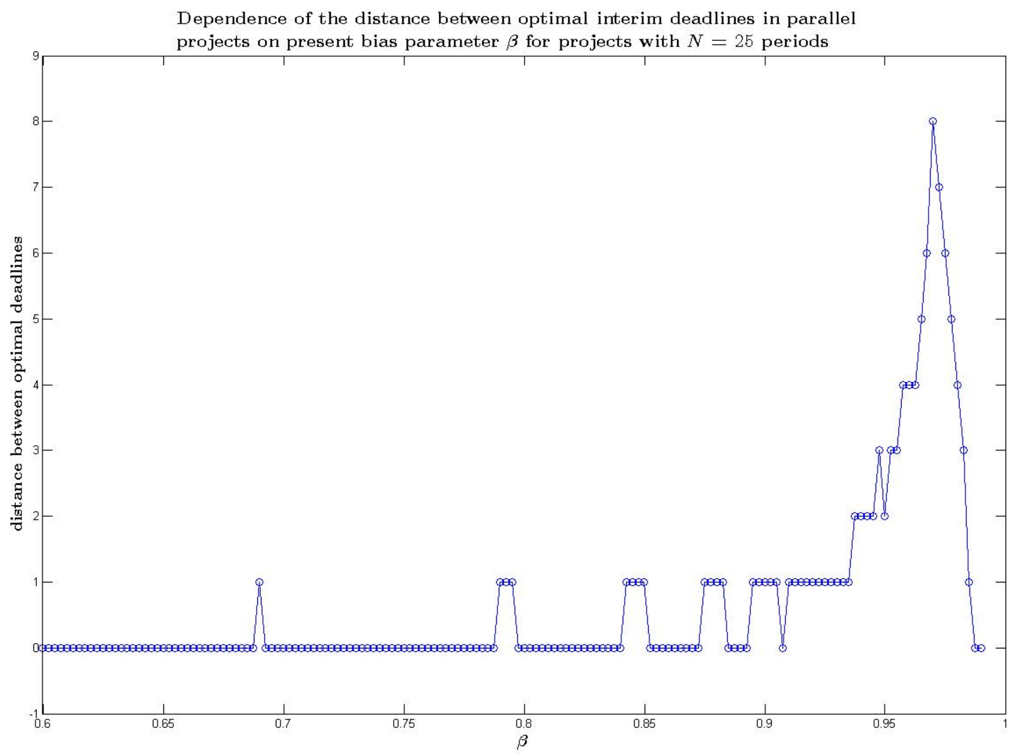

Figure 2.7

2.6.2 Agent Types

As stated in the Model section, in this paper, I focus on the behavior of the naïve agent. In general, the agent can be one of three types, based on how she understands her present bias (β): naïve, sophisticated, or partially-sophisticated. The naïve agent is not aware of her time inconsistency and believes she is time-consistent. The sophisticated agent correctly knows her present bias and fully predicts her future behavior. While the partially-sophisticated agent is aware of her self-control problems, she underestimates her present bias.

In the current period t, the agent invests only effort levels $\left(\hat{e}_{1t}^{(t)}, \hat{e}_{2t}^{(t)}\right)$ into projects. Because the naïve, sophisticated, and partially-sophisticated agents differ in terms of how they understand their present bias, they have different beliefs about their future behavior. As already discussed, the naïve agent believes she is time-consistent and will behave according to the effort plan which maximizes her intertemporal utility U_t or, in other words, that β will be equal to 1 in all upcoming next periods. Thus, the optimal *intertemporal* strategy for the naïve agent (without a deadline) is the solution to the UMP (74).

On the contrary, the sophisticated agent fully understands her present bias and correctly predicts the behavior of her future selves. In period t, she knows that she will face the same problem (74) in every future period. The partially-sophisticated agent anticipates her future behavior, however, she underestimates her present bias in period t and believes she will face a similar UMP as (74) from period $t+1$ onwards, but with the present bias parameter equal to $\bar{\beta}$ instead of β, where $\beta < \bar{\beta} < 1$:

$$\begin{gathered}\left\{\hat{e}_{1\tau}^{(t+1)}, \hat{e}_{2\tau}^{(t+1)}\right\}_{\tau=t+1}^{N} \in \\ \arg \max_{\{e_{1\tau}, e_{2\tau}\}_{\tau=t+1}^{N}} \left\{-c(e_{1t+1}, e_{2t+1}) + \bar{\beta}\left[-\sum_{n=t+2}^{N} c(e_{1n}, e_{2n}) + R\left(\sum_{n=1}^{N} e_{1n}, \sum_{n=1}^{N} e_{2n}\right)\right]\right\} \qquad (93) \\ s.t.: \quad \{e_{1n}, e_{2n}\}_{n=1}^{t-1} \ \ are \ \ given\end{gathered}$$

Thus, the partially-sophisticated agent forms the effort plan for future periods depending on her sophistication level γ.[41] As a result, the sophisticated agent would not experience the postponing effort to future periods when solving for her current period effort. Her behavior and solution in every period coincide with what she planned in the first period or in any other periods. As shown by Razumovskii (2023), the optimal timing for the interim deadline for the sophisticated agent is later than the optimal timing for the naïve agent, given that the present bias is the same. The partially-sophisticated agent predicts her future behavior incorrectly,

[41] I define the sophistication level as in Razumovskii (2023):*A* ***sophistication level*** *is the parameter* γ *which characterizes how incorrectly the agent estimates her present bias parameter* β *and is defined by:* $\gamma = \frac{1-\bar{\beta}}{1-\beta}$.

thus still postponing some effort to future periods toward the deadline as the naïve agent does. The optimal timing for partially-sophisticated agents is thus between that of the naïve agent and that of the sophisticated agent, depending on the degree of sophistication. This coincides with the optimal timing for the naïve agent when $\gamma = 0$ and for the sophisticated agent when $\gamma = 1$.

Given the description above, the best-response timings on Figures (2.3, 2.4, 2.5) for partially-sophisticated and sophisticated agents would be shifted to the later periods according to their individual degree of sophistication. However, the results remain robust, because the agent's sophistication leads only to redistribution of the effort before the interim deadline and later optimal timings for the same present bias (Razumovskii, 2023), but it does not change the nature of the effects.

2.6.3 Limitations and Possible Conjectures

Although I focus on the simplest deadline setup with only one interim deadline in each project in this paper, there is an additional question: How does the timing of the interim deadline in one project affect the optimal distribution of interim deadlines in the other project in case of several interim deadlines in the other project? The timing of the interim deadline in one project affects the optimal timing of the interim deadline in the other project when the second is imposed shortly after or shortly before the first. Thus, there should be a cascade effect on the other interim deadlines imposed on the same project. Also, the agent's sensitivity to these effects should depend on the agent's present bias.

Additionally, I focus on two complementary projects; the results should be different for substitutable projects. In my paper, the interim deadline in one project affects the optimal timing for the interim deadline in the other project only through the reward function. Thus, complementarity of the projects is one of the crucial assumptions in the model. Alternatively, one may think about the agent's time endowment per period (day) as a resource that the agent can use to invest in projects. Then the interim deadline in one project decreases the time available to be spent on the parallel project, and consequently on the optimal timing for the interim deadline.

The other limitation is the number of possible interim deadlines in the projects. Increasing the number of interim deadlines to be imposed could change the result. This is a direction for my subsequent research and extensions of this study. Firstly, I plan to consider a setup in which the final deadlines (the deadlines in the Nth period) are fixed for both projects according to the zero-period plan (what the agent believes she will do when the projects are in the future) and to find an optimal design for interim deadlines in this case. This setup can represent a situation in which a worker signs a contract with a manager before starting projects according to the worker's current preferences. The contract specifies the total effort to be invested in the projects (final deadlines). However, the worker knows that she will procrastinate and can impose interim deadlines to overcome procrastination. Secondly, I intend to investigate the optimal design of deadlines, increasing the number of costly interim deadlines in projects. On the one hand, an additional interim deadline helps to smooth the effort between periods and reduce procrastination. However, it may be costly to impose an additional deadline. Therefore, there is a trade-off between the positive effect of an additional deadline and the costs of imposing it.

Finally, I plan to add uncertainty to the model. The deadlines themselves could be considered a commitment to spend some effort on pursuing a goal, task, or project. Most people are accustomed to having plans for their days from waking up to going to bed. However, we commonly encounter unexpected events and sudden changes in our plans, which are costly to ignore. Consequently, imposing a deadline can cause large losses of welfare by lowering flexibility in the future. In this context, procrastination appears to be a force that incentivizes one to impose relatively strict interim deadlines, while uncertainty is a force that incentivizes one to impose relatively flexible deadlines. Several recent studies consider deadlines and uncertainty together. El-Tannir (2019) considers the mean-variance contract design with stochastic project duration. The author calculated optimal deadlines, leaving aside time-inconsistent preferences and considering only one project. Ballard et al. (2018) investigate the multiple-goal pursuit model (MGPM) (Vancouver, Weinhardt, & Schmidt, 2010) and study how agents allocate effort across two goals with different deadlines. The authors conducted four experiments and aimed to explain how agents choose which goal to pursue in the current time, depending on distances to deadlines and the probability of goal completion. Their model explains the complex behavior of participants more precisely than the previous version. However, the study does not raise the question of deadline optimality or time-inconsistent preferences. Thus, my conjecture research is a logical addition to the existing literature.

2.7 Conclusion

In this paper, I raise the question of optimal deadline design in a multiple-project environment. I consider a naïve agent with time-inconsistent preferences who pursues two parallel and complementary projects. I investigate the spillover effects of deadlines on the effort spent on projects, the agent's welfare, and the optimal timings for interim deadlines.

The results suggest that the optimal timing for the interim deadline in one project significantly depends on the timing for the interim deadline in the other project. Adding the interim deadline in the first project may move the optimal timing for the interim deadline in the second project to earlier or later from the previously optimal period (without an interim deadline in the concurrent project), depending on the timing chosen for the interim deadline in the first project. The optimal pair of timings for both interim deadlines is not necessarily unique. For relatively more present-biased agents (low β) and for almost time-consistent agents, there is a unique pair of optimal timings, and it is optimal to impose interim deadlines in the same period. However, there are less present-biased (high β) agents, for whom it is optimal to delay the interim deadline in one of the projects, and there are two symmetric pairs of optimal timings. The distance between optimal timings for interim deadlines also depends on the agent's present bias in this case. If it is optimal to distinguish the interim deadlines for parallel projects in time, the distance between them is increasing in the agent's present bias parameter β. The distance increases until the agent is close to being time-consistent and then drops to zero, because it becomes optimal to impose both interim deadlines in the final period.

The results contribute to the existing economic literature on the topics of behavioral economics, time-inconsistent preferences, and deadlines. To my knowledge, this is the first theoretical paper that considers the spillover effects of an interim deadline in the context of two concurrent projects. The paper's findings are useful for understanding how deadlines affect our behavior and welfare.

3 Removing the Toll Barrier

Co-authored with Misha Gipsman[42] and Artyom Jelnov[43]

Letter from merchant to the Tzar and the decree back (1708, Russia):
"Tzar, a merciful sovereign, I'm a merchant in Voronezh...
And before I went to other cities to buy goods, and now, my lord, I wish to travel to other cities for the trade ... but still, without your great sovereign's decree, I dare not go. I beg Your Majesty, have mercy, on your command let me go from Voronezh... Your lowest slave, a Voronezh trading suburb man, Timofey Potapov".
The text of the decree: "In the cities stewards and captain and government officials and atamans in the Cossack towns, and elders, and in towns, and villages, and bridges, and on crossings who manage it, are ordered to allow him, Timofey and his workers to travel everywhere without detention".

3.1 Introduction

Neither regulators nor rent-seekers possess absolute power. It is not uncommon for regulators to struggle to eliminate predatory organizations or, at times, to abandon such efforts altogether. An example is the collection of tolls and control of city entry during the medieval period. Historically, the outcome of conflicts between the central authority (a King) and lords was often uncertain, as even a King was not all-powerful. The central question we address is whether a Lord would voluntarily remove trade barriers when faced with the threat of conflict from the King, or instead, choose to restrict trade.

Merchants voyaging to a city's market commonly faced 'middle barriers', e.g., some could be natural monopolies because of geographical location, such as river crossings or narrow

[42]Ariel University, Israel. E-mail address: mishag@ariel.ac.il
[43]Ariel University, Israel. E-mail address: artyomj@ariel.ac.il

mountain passes. At these strategic points, a 'Lord of the Crossing'[44] controlled access, collecting taxes, and deciding who could pass. However, this Lord not only extracted merchant incomes but also hindered trade with the destination city, thereby diminishing the wealth of the central authority (the King). Consequently, the King was incentivized to initiate conflict with the Lord to eliminate such trade barriers. However, success in this endeavor was far from guaranteed.

The tension between rent-seeking gatekeepers and central authorities is ancient. The Book of Nehemiah (5th century BCE): "may letters be given to me to the governors beyond the river, that they escort me through." The pattern persists: the Grosse Ile Bridge to Detroit, the Ambassador Bridge on the US-Canada border, and digital store platforms as tolls represent the same structure. The "Tollbooth Problem" in essential facilities doctrine: regulatory intervention is required to mitigate rent extraction at non-bypassable nodes.

Our model utilizes historical context to illustrate contemporary situations analogously. Legitimate barriers are well-known in the industrial organization literature (see Tirole, 2015). Today, 'Lords' can represent licensing bodies, criminals, corrupt officials, or oligarchs who extort cashflow, while the 'King' symbolizes central law enforcement authorities like government, police, prosecutors, etc.

We explore a model in which the Lord prefers to maintain a monopoly by allowing only one merchant to access the market in the absence of the King's involvement. However, the Lord voluntarily relaxes the barrier under threat of conflict with the King, permitting a few merchants to enter the market, though not entirely free of charge. As a result, the threat of a conflict itself increases the Social Welfare generated on the market by at least 56% of what is theoretically possible at the pre-conflict stage. However, the King's probability of winning the conflict and successfully removing the barriers is low. The probability is increasing with the number of existing merchants. However, it reaches only 1/5 when this number tends to infinity. Therefore, while the presence of the King and the conflict itself do not fully remove

[44] The term 'Lord of the Crossing' is borrowed from George R. R. Martin's *"A Song of Ice and Fire"*, metaphorically representing a monopoly over a critical route.

the trade barriers, the threat of a conflict significantly increases the generated welfare even before the results of the conflict.

In addition, we study the case of multiple Lords on one road. Despite the previous case, the monopoly occurs with the probability of 2/3. The first Lord allows only two merchants to cross his land. The second Lord would charge an additional toll and reduce the market to a monopoly. The King then prefers to involve himself in the conflict with the second Lord. As a result, the King's probability of winning a conflict is higher and is equal to 1/3. However, in the case of winning, the King removes only the second Lord's barrier, and only two merchants can reach the market.

The situation changes rapidly when each Lord controls his own road, and merchants can choose between different roads. The Lords compete in tolls to attract more merchants to use their roads. As a result, the barriers are removed voluntarily, with all merchants reaching the market, and the King does not involve any Lord in the conflict. Thus, a higher number of merchants and a higher number of Lords located on the same road slightly increase the probability of the King winning, while not leading to unrestricted trade or a high probability of removing trade barriers. Only competition between several trade roads leads to removing the barriers and free trade. In the historical context, only a significant improvement of the infrastructure allowed free trade.

The rest of the paper is organized as follows. Section 2 describes related literature and contextualizes this paper in the existing research. Section 3 lays out the model of the Lord's choice of the toll payment and the conflict between the King and the Lord. Section 4 analyzes the Lord's optimal choice of toll payments in case of no conflict and in case of conflict with the King. Section 5 provides the results of the model extension on the case with several Lords and roads. Section 6 discusses generalizations, limitations, and possible extensions, while Section 7 concludes.

3.2 Literature Review

Trade barriers imposed by rent-seekers, from medieval toll collectors to modern corrupt officials, pose persistent economic challenges.

McAfee, Mialon, and Williams (2004) examine a variety of entry barriers. We focus on rent collectors who establish “pass protection” for productive agents, a phenomenon dating back to ancient times. Middleton (2005) documents how tolls have persisted at the same trade locations since the Roman Empire, with only institutional legitimisation changing over time. In the economic history literature, the Commercial Revolution is understood as an institutional transformation: Greif (1993); Greif and Laitin (2004) show how endogenously merchant coalitions constrained ruler predation, while Mokyr (2017) emphasises the role of competitive markets in sustaining economic growth.

Our study examines conflicts between central authorities and vassals (“Lords”). Konrad and Skaperdas (2005, 2012) show that centralised protection minimises conflict costs, while Grechenig and Kolmar (2011) demonstrate that state enforcement reduces reliance on private protection. Skaperdas and Vaidya (2019) note that effective governance mitigates trade disruptions. Bjorvatn and Naghavi (2011) show that resource rents can promote regime stability by raising conflict costs; this is partially reflected in our finding that Lords voluntarily relax barriers under credible threats.

Analogous mechanisms appear in other settings. Patent licensing studies (Kamien & Tauman, 1986; Katz & Shapiro, 1986) examine strategic interactions in which rights holders restrict licenses while collecting fees. Shleifer and Vishny (1993) models officials granting market access in exchange for bribes, explaining key corruption mechanisms. Hillman (1982); Hillman and Ursprung (1988) show that domestic pressures drive protectionist policies, while Grossman and Helpman (1994) model politicians aligning with interest groups.

We integrate a market-entry model with the Tullock contest framework (Tullock, 1984; Tullock & Rowley, 2005). Nitzan (1994) surveys rent-seeking contest models. Van Long (2013) provides a unified framework showing Tullock contests and first-price auctions as special cases. For additional surveys, see Congleton, Grofman, and Voigt (2018); Congleton and Hillman (2015); Congleton, Hillman, and Konrad (2008); Hillman (2019). We adopt Nti (1999)'s asymmetric prize valuation model, which is well-suited to our setting, where the King and the Lord value winning differently. Klunover (2023) shows that asymmetry in a contest can intensify competition.

Building on the literature on rent-seeking and market access, we analyse how the threat of conflict by authorities affects the trade barriers. We show that such threats induce only partial toll reduction even when complete removal maximises social welfare. We further quantify the welfare gains from credible threats. We show that competition among routes dominates direct intervention by a central authority in eliminating barriers. The analysis delineates the scope and limits of central authority in reducing rent-seeking.

3.3 Model

N identical merchants seek to arrive at a market located in the Destination city. To reach the market, they need to pass the road, which is a unique way to the Destination city. The merchants maximize their income and compete in quantities. The demand in the Destination city is given by price per unit $p(Q) = A - BQ$, where Q is the quantity, and A and B are positive constants. The cost of producing, delivering,[45] and placing q_i units on the market for merchant $i \in \{1,...,N\}$ is $c(q_i) = Cq_i$, where C is a common positive constant for all merchants.

In the Middle of the trade road is a barrier: a gate under the control of the "Lord of the Crossing" (hereafter L) that cannot be bypassed. L sets and collects a toll $T \in \mathbb{R}_{\geq 0}$ from each merchant. Thus, the merchants have to pay the toll T to cross the gate and reach the Destination city. L maximizes his income $I_L = Tm$, where $m \in \{1,...,N\}$ is the number of merchants who decided to pay the toll T and reach the market.

There is a "King" (hereafter K) in the Destination city who is interested in maximizing the

[45] Excluding the discussed later toll T.

social welfare, the surplus generated on the market. Social Welfare (SW) is defined as the sum of Customers' Surplus and the merchants' revenue on the market (excluding the toll payments T):

$$SW = CS + \sum_{i=1}^{m}(\pi_i - T) \tag{94}$$

Given the set toll T, K can initiate a conflict with L to remove the trade barrier. We model the conflict with the simple version of the Tullock contest: K and L simultaneously choose efforts e_K and e_L, respectively, and their probabilities to win are $P_K = \frac{e_K}{e_K+e_L}$ and $P_L = \frac{e_L}{e_K+e_L}$. If K wins, L is not allowed to set any toll (obtains zero), and all N merchants reach the market.[46] In contrast, L collects toll T from each of m merchants when winning the conflict. Figure (3.1) represents the model.

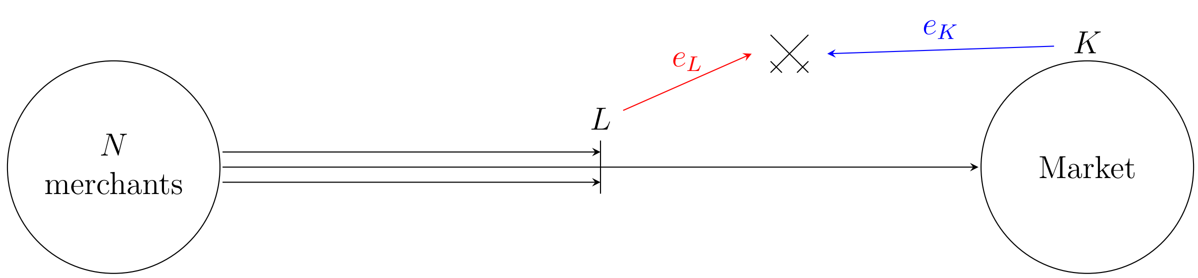

Figure 3.1: Basic Model

The timing is as follows. In the first stage, L announces the toll T and commits to it. The merchants and K observe the toll. Given the toll T and the competition on the market, m is derived from T.[47] Then K and L enter the Tullock contest. If K wins, the toll is not implemented, and all N merchants cross the Middle. If L wins, the proposed T is implemented.

The interaction among all sides is modeled in three stages by a backward analysis. First, we consider the merchants' competition and analyze how the number of merchants on the market (m) and the social welfare depend on the given toll T. Second, we calculate the optimal

[46] Note that the conflict is about the right to collect tolls only. In this model, we do not consider any other privileges of a feudal lord.

[47] We show how exactly in the Analysis section.

toll T for L and the corresponding m and Social Welfare when there is no conflict with K. Last, we analyze how L adjusts the trade barrier (toll T) under the threat of conflict with K, which is the main goal of the paper. We recalculate the optimal proposed T for L, the corresponding number of merchants on the market m, and the corresponding social welfare, taking into account the future conflict with K. Additionally, we discuss the probabilities of winning for K and L.

3.4 Analysis

3.5 Merchants' Competition

The total quantity Q can be found as $Q = \sum_{i=1}^{m} q_i$. Therefore, given the toll T, the merchant i's profit is defined by:

$$\pi_i = pq_i - Cq_i - T = \left(A - B\sum_{i=1}^{m} q_i - C\right) q_i - T \tag{95}$$

Then, the merchant i's participation constraint (PC) is:

$$\pi_i \geq 0 \Leftrightarrow \left(A - B\sum_{i=1}^{m} q_i - C\right) q_i \geq T \tag{96}$$

And the merchant i's profit maximization problem (PMP) is:

$$\max_{q_i} \left\{ \left(A - B\sum_{j=1}^{m} q_j - C\right) q_i - T \right\} \tag{97}$$

$$s.t., \quad \left(A - B\sum_{i=1}^{m} q_i - C\right) q_i \geq T$$

By arriving at the market, merchants compete in quantities, and the symmetric equilibrium for competition of merchants on the market is characterized by:

$$q_i = q^* = \frac{A - C}{B(m+1)}, \qquad p^* = \frac{A + mC}{m+1} \tag{98}$$

Consequently, the profit π_i of a merchant i on the market and the total merchants' profit $\pi = \sum_{i=1}^{m} \pi_i$ in the symmetric equilibrium are:

$$\pi_i = \frac{(A - C)^2}{B(m+1)^2} - T \tag{99}$$

$$\pi = m\pi_i = m\left(\frac{(A - C)^2}{B(m+1)^2} - T\right) \tag{100}$$

Given the profit π_i and the participation constraint (96), the number of merchants in the market is defined by:

$$m^* \leq \frac{A - C}{\sqrt{BT}} - 1 \tag{101}$$

The merchants come to the market until they make a non-negative profit, then:

$$m^* = \left\lfloor \frac{A - C}{\sqrt{BT}} - 1 \right\rfloor \tag{102}$$

While $m \in \mathbb{N}$, it is decreasing in toll T. The CS can be calculated as the gray area below the demand line in Figure (3.2):

$$CS = \frac{m^* q^* (A - p(m^* q^*))}{2} = \frac{(m^* q^*)^2 B}{2} = \frac{(m^*)^2 (A - C)^2}{2B(m^* + 1)^2} \tag{103}$$

And the total Social Welfare is:

$$SW = \frac{(A - C)^2}{2B} \frac{m^*(m^* + 2)}{(m^* + 1)^2} \tag{104}$$

Note, the Social Welfare is decreasing in m^* and, consequently, is decreasing in T. Thus, the

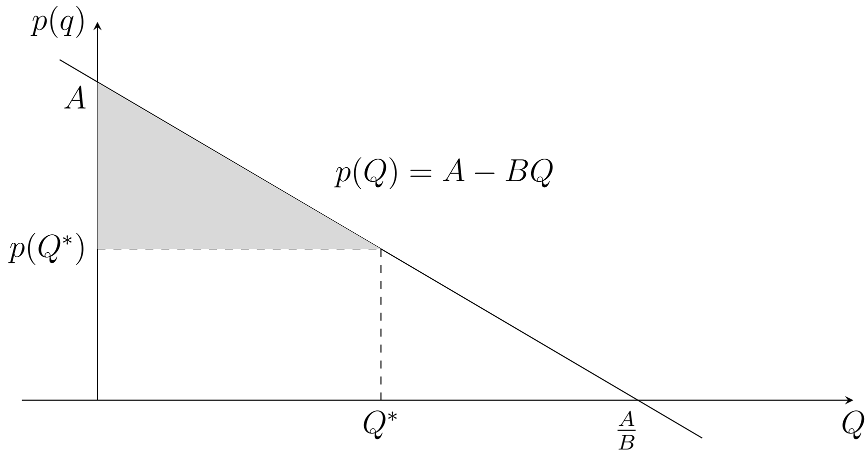

Figure 3.2: Customers Surplus

King K would prefer the Lord L to set a lower toll T to increase the Social Welfare.

3.6 Lord's Optimal T

The Lord L maximizes his income I_L by choosing the toll T:

$$\max_T \left\{ Tm^*(T) = T \left\lfloor \frac{A-C}{\sqrt{BT}} - 1 \right\rfloor \right\} \tag{105}$$

Given the number of merchants on the market, m, the Lord L would maximize his income by setting the highest possible toll T according to PC (96). From the merchant i's profit (99):

$$T^*(m) = \frac{(A-C)^2}{B(m+1)^2} \tag{106}$$

The L's PMP can be rewritten then in terms of the number of merchants in the market, because, L chooses the number of merchants m indirectly through PC (96):

$$\max_T \{Tm^*(T)\} = \max_m \{mT^*(m)\} = \max_m \left\{ m\frac{(A-C)^2}{B(m+1)^2} \right\} \tag{107}$$

The objective function is decreasing in m. Thus, L sets the toll T such that only one merchant crosses his land:

$$m^* = 1, \quad T^*(m^*) = \frac{(A-C)^2}{4B} \quad \Rightarrow \quad q^*(m^*) = \frac{A-C}{2B}, \quad p^*(m^*) = \frac{A+C}{2}$$

As a result, the Lord L uses the toll T to extract all the generated profit on the market by merchants. Therefore, he restricts the number of merchants to maximize the total generated profit. The total profit is then maximized when there is a monopoly on the market. Thus, L induces a monopoly on the market when there is no threat of conflict from the King.

The Social Welfare, then, is at its minimum, since it is increasing in m according to equation (104):

$$SW^* = \frac{3(A-C)^2}{8B}$$

Next, we analyze how the Lord L would adjust his toll T compared to the case above when he is involved in a Tullock contest with King K.

3.7 King's Intervention

The King K maximizes the social welfare (104).[48] Note that SW_m is increasing in m. Thus, the K's objective function is at the maximum when $m = N$. Thus, the King's value of winning is the difference between social welfare when N merchants enter the market and when m do so:

$$V_K = SW_N - SW_m = \frac{N(N+2)(A-C)^2}{2B(N+1)^2} - \frac{m(m+2)(A-C)^2}{2B(m+1)^2} \tag{108}$$

The Lord collects the toll $T = \frac{(A-C)^2}{B(m+1)^2}$ in case of winning and collects 0 in case of losing the

[48]Even if the King is not benevolent, he may be interested in enhancing the tax collected from the trade.

conflict. Therefore, the Lord's value of winning the conflict is:

$$V_L = mT^*(m) = \frac{m(A-C)^2}{B(m+1)^2}, \tag{109}$$

where $T^*(m)$ is a maximal toll such that m merchants pass the Lord's land according to equation (106).

Note, it is easy to verify that $V_L > V_K$. In the Tullock contest, the players choose effort according to their valuations:

$$\frac{e_L}{V_L} = \frac{e_K}{V_K}$$

Therefore, the Lord always chooses a greater effort and has a higher probability of winning. Further, by Nti (1999), the effort choices and probabilities of winning are:

$$\begin{cases} e_L = \frac{V_L^2 V_K}{(V_L+V_K)^2} \\ e_K = \frac{V_K^2 V_L}{(V_L+V_K)^2} \end{cases} \tag{110}$$

and

$$\begin{cases} P_K = \frac{V_K}{V_K+V_L} \\ P_L = \frac{V_L}{V_L+V_K} \end{cases} \tag{111}$$

As a result, the expected income of the Lord L before entering the Tullock contest, if m merchants cross the Middle, is:

$$E_L(m) = P_L V_L - e_L = \frac{V_L^3}{(V_K+V_L)^2} \tag{112}$$

Thus, the L's PMP can be rewritten as follows:

$$\max_m \{E_L(m)\} = \max_m \left\{ \frac{4(A-C)^2}{B} \frac{\left[\frac{m}{(m+1)^2}\right]^3}{\left[\frac{N(N+2)}{(N+1)^2} - \frac{m^2}{(m+1)^2}\right]} \right\} \tag{113}$$

where L chooses m indirectly through the PC (96).

Proposition 1. *Under the threat of the conflict with the King, the Lord relaxes the trade barrier and sets the toll T such that three ($m^* = 3$) merchants when $N \in \{3,4\}$ or two ($m^* = 2$) merchants when $N > 4$ cross his land.*

We provide the proof of Proposition 1 at the Appendix I. Thus, L is willing to relax the trade barrier to decrease the K's incentive to invest in the conflict. As a result, the Social Welfare increases regardless the results of the conflict. Assuming $N > 4$, the Lord L proposes the toll $T^* = \frac{(A-C)^2}{9B}$ so $m^* = 2$ merchants can cross his land. The minimal SW then even if L wins the conflict is:

$$SW(m^* = 2) = \frac{4}{9}\frac{(A-C)^2}{B} \tag{114}$$

The SW equals to $\frac{1}{2}\frac{(A-C)^2}{B}$ at maximum when $m = N \to \infty$, and equals to $\frac{3}{8}\frac{(A-C)^2}{B}$ at minimum when $m = 1$. Thus, the threat of the conflict increases the SW at least on $5/9 \simeq 56\%$ of what is possible even if $m = N \to \infty$.

While the King's value of winning the conflict is increasing in N and the Lord sets the toll such that only 2 merchants cross his land (assuming $N > 4$), the King's probability of winning is lower than the Lord's even when $N \to \infty$:

$$P_K(m)|_{N\to\infty} = 1 - \left.\frac{2\frac{m}{(m+1)^2}}{\frac{N(N+2)}{(N+1)^2} - \frac{m^2}{(m+1)^2}}\right|_{N\to\infty} = \frac{1}{2m+1} \tag{115}$$

Thus, the King's probability of winning the conflict at maximum is equal to $1/5$ since $m^* = 2$ when $N \to \infty$. Thus, a greater number of merchants presented on the market does not lead to the removal of the trade barriers.

As a result, the threat of the conflict increases the Social Welfare at least by 56% (even if L wins the conflict) of what is possible, while the probability of K winning and the barriers are removed stays below 20%.

3.8 Multiple Lords

Next, we consider the model extension with several Lords presented. There are l Lords presented on one road. Every merchant has to cross all the Lords and pay tolls to every Lord to reach the Destination City. Figure 3.3 presents the described structure.

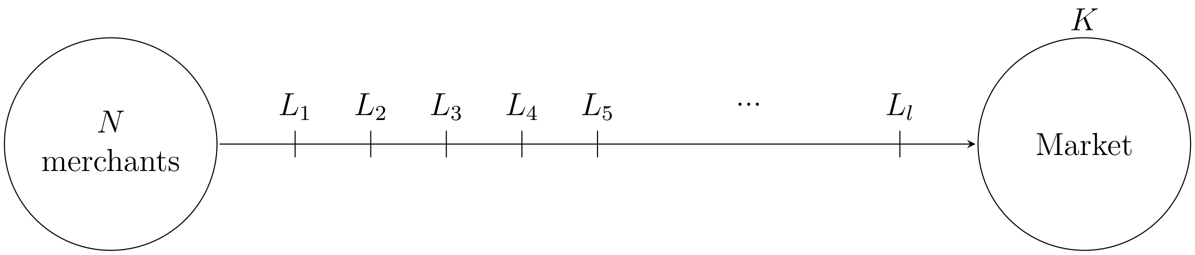

Figure 3.3: One road and l Lords

The timing is the following. First, the Lords sequentially announce the tolls T_1, T_2,..., T_l on the merchants' way to the market and thus define $\{m_1,m_2,...,m_l\}$ where m_i is a number of merchants who can cross the first i Lords' lands (we denote Lord i by L_i). Each Lord L_i observes all the previous tolls $\{T_1,..T_{i-1}\}$ before making a decision on his own toll T_i. Second, in case the King is present, the King decides on which Lord to involve in the conflict. First, we assume that the King can fight with one Lord only. Later in this section, we consider the case when the King can involve Lords in conflict sequentially if winning. Then the King and the chosen Lord enter the Tullock contest, and only the chosen Lord cannot collect the toll in case the King wins. Lastly, the merchants reach the market and pay tolls to the remaining Lords.

Without conflict, the 1st Lord (L_1) collects all the surplus by allowing only one merchant to cross his land. Indeed, the merchants cross the lands sequentially, and the other Lords are then indifferent between setting an additional toll or allowing merchants to cross their lands for free. As a result, the case is reduced to the previous one with one Lord on the road.

When the King is present, the 1st Lord has incentives to behave differently from the one-Lord case. If the first Lord allows more than one merchant to cross his land ($m_1 > 1$), other Lords would prefer to collect the additional tolls while the total toll allows more than one merchant to cross their lands. As a result, the King prefers to involve the last Lord in the conflict, the one who restricts the trade to a monopoly. Thus, the 1st Lord does not face the probability of being involved in a conflict.

Proposition 2. *If multiple Lords are present on the same road, the first Lord (L_1) sets the toll so that only two merchants cross his land ($m_1 = 2$), the second Lord reduces the trade to monopoly ($m_2 = 1$), and the King involves the second Lord in the conflict and wins with a probability of 1/3.*

We provide the proof of Proposition 2 in Appendix II. As a result, the presence of additional Lords on the same road eliminates the probability of the total removal of trade barriers. The King involves the second Lord in the conflict and removes his barrier with a probability of 1/3. Thus, two merchants reach the market only with a probability of 1/3, and the monopoly case occurs with a probability of 2/3.

The assumption that the King can involve only one Lord in conflict is crucial. Next, we consider the case where the King can sequentially involve the Lords in a conflict in reverse order (starting from the closest Lord to the market). Moreover, we assume hereafter that in each conflict a new King is born, and he is myopic (seeks to optimize his utility in the current period only).

Specifically, we consider the following settings. There is an unlimited number of Lords on the road. Starting from the first Lord on the merchant's way to the market, the Lords decide sequentially on their tolls. According to the discussion above, the Lords would reduce the number of merchants until some Lord L_l minimizes the number of merchants to 1. Without loss of generality, we assume that the Lords after L_l do not impose any tolls and that they do not exist on the road. Next, the King attacks the Lords in reverse order, starting with the Lord L_l, by involving them in the Tullock contest, as in the basic model. The King can attack the next Lord only in the event of winning the conflict against the previous one (by the previous King). Every conflict is considered separately, where the King independently decides on the effort in each conflict and does not take into account the potential value of winning future conflicts.

In this case, the first Lord prefers to set the toll such that only one merchant crosses his land. Otherwise, the next Lord would impose an additional toll that restricts the trade to a monopoly. The overall expected income for the first Lord then decreases.

Proposition 3. *If multiple Lords are present on the same road and the King involve them in conflicts sequentially, the first Lord (L_1) sets the toll so that only one merchant crosses his land ($m_1 = 1$).*

We provide the proof of Proposition 3 in Appendix III. The general intuition behind the proof is as follows. If the first Lord does not restrict trade to a monopoly, there will always exist a pre-last Lord. We then show that, for this pre-last Lord, it is always profitable to become the last one (what he obviously can do, since he decides on his toll before the next one) and impose a monopoly, rather than allowing the next Lord to do so.

Multiple Roads

The case when each Lord controls his own road to a destination point is almost trivial. There are $l < N$ Lords between merchants and the Destination City. Each Lord controls his own road to the market, so merchants can choose one of the Lords to reach the Destination City. The Lords then compete in tolls T_l between each other to attract merchants. Figure 3.4 presents the described structure.

As a result, the Lords set tolls equal to zero, and all the merchants reached the market. Indeed, the merchants choose the road with the Lord who set the lowest toll $\underline{T}_i : \ \underline{T}_i \leq T_j, \ \ \forall j \in \{1,...,l\}$. Assuming $\underline{T}_i > 0$, there is a profitable deviation for all other Lords to set their toll equal to $\underline{T}_i - \epsilon$. Thus, all Lords would set toll equal to 0 in the equilibrium, and all the merchants would reach the market.

The King then has no incentive to participate in conflict since all the merchants reach the market and there are no trade barriers, which maximizes Social Welfare. Note that it is enough to have at least $l = 2$ Lords with roads, so this equilibrium exists.

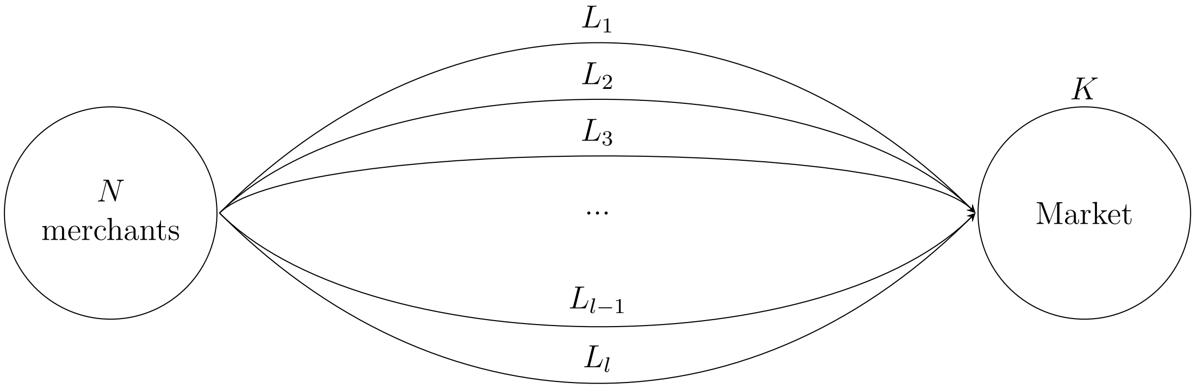

Figure 3.4: l Lords and l roads

3.9 Discussion

Our research focused on the political interests of authorities when no one has a monopoly on power. This approach complements the historical explanation that guilds sought to monopolize markets and actively lobbied for commercial reforms (see Casson & Casson, 2013; Ogilvie, 2011). Furthermore, it aligns with the historical spread of the Royal Charters and landlords' self-limitation. The demonstrated importance of infrastructure and market-trade capacity constraints offers a nuanced political economy perspective for institutional views on development.

Further investigations might focus on dynamic and repeated games with different market conditions and products. An ongoing question is under what circumstances the Lord of the Crossing transits into a local market founder and makes a rollback to predatory formation.

3.10 Conclusion

This paper examines the removal of trade barriers imposed by rent collectors (Lords) on trade routes, under the threat of conflict with the central authority (King). With no threat of a conflict, the rent collector seeks to maximize his income by setting tolls so that only one merchant can access the destination market, inducing a monopoly as a result. Opposite, the King prefers to maximize Social Welfare by enabling as many merchants as possible to reach the market.

We found that the Lord adjusts his behavior based on the King's willingness to invest in conflict. Without conflict, the Lord maintains a monopoly, but when faced with the threat of conflict, the Lord lowers his toll, allowing a few merchants to pass. As a result, the Social Welfare is increased by at least 56% of what is theoretically possible at the pre-conflict stage. The King's chances of winning the conflict and removing the barriers remain low and never exceeds 1/5. While the probability of the full removal of the trade barrier is not high, the threat of the conflict itself significantly increases the generated Social Welfare.

We then extend the model to multiple Lords and roads. When several Lords are on the same route, and the King can involve only one Lord in a conflict, a monopoly emerges with a 2/3 probability. In this case, the first Lord allows only two merchants, and the second imposes an additional toll, reducing trade to a monopoly. The King, seeing a higher chance of success, engages the second Lord in the conflict. However, even if the King wins, only two merchants reach the market. Under the relaxed assumption that the King can involve only one Lord in a conflict, the first Lord prefers to induce a monopoly, and the case reduces to one with only one Lord on the road.

The situation changes dramatically when each Lord controls an independent road, and merchants can choose from multiple options. Here, the Lords compete by adjusting tolls to attract merchants, leading to the voluntary removal of barriers and enabling all merchants to access the market. No conflict is needed, and even the King's involvement becomes unnecessary. While an increased number of merchants and Lords slightly improves the King's chances of success, it does not result in unrestricted trade or the removal of all barriers. Only when multiple roads compete does free trade emerge.

This indicates that encouraging competition among rent collectors on different routes is more effective than engaging in direct confrontation with them. It highlights the increased social welfare benefits resulting from the central authority's or predators' investment in infrastructure, particularly the construction of new routes.

3.11 Appendix I: Proof of Proposition 1

Proof. Directly by (112), the expected profit for the Lord as a function of m is:

$$E_L(m) = \frac{4(A-C)^2}{B} \frac{\left[\frac{m}{(m+1)^2}\right]^3}{\left[\frac{N(N+2)}{(N+1)^2} - \frac{m^2}{(m+1)^2}\right]^2} = \frac{4(A-C)^2}{B} \frac{m^3}{\left[\frac{N(N+2)}{(N+1)^2}(m+1)^3 - m^2(m+1)\right]^2} = \\ = \frac{4(A-C)^2}{B} \frac{m^3}{\left[2m^2+3m+1-\frac{(m+1)^3}{(N+1)^2}\right]^2} \tag{116}$$

Thus,

$$sign[\frac{\partial E_L(m)}{\partial m}] = \\ = sign[3\left[\frac{N(N+2)}{(N+1)^2}(m+1)^3 - m^2(m+1)\right] - 2m\left[3\frac{N(N+2)}{(N+1)^2}(m+1)^2 - 3m^2 - 2m\right]]$$

By rearranging terms,

$$3\left[\frac{N(N+2)}{(N+1)^2}(m+1)^3 - m^2(m+1)\right] - 2m\left[3\frac{N(N+2)}{(N+1)^2}(m+1)^2 - 3m^2 - 2m\right] = \\ 3\frac{N(N+2)}{(N+1)^2}(m+1)^2(1-m) + 3m^3 + m^2 = \\ 3\left[1-\frac{1}{(N+1)^2}\right]\left[m+1-m^3-m^2\right] + 3m^3 + m^2 = \\ \frac{3(m+1)^2(m-1)}{(N+1)^2} - 2m^2 + 3m + 3 \tag{117}$$

By direct substitution, and by $1 > \frac{N(N+2)}{(N+1)^2}$, this expression is positive for $m = 1$ and $m = 2$. The Lord would adjust the number of merchants he allowed to cross the Middle to decrease the probability of losing the conflict. Note, that the expression (117) for $m = 3$ can be rewritten as follows:

$$3\frac{N(N+2)}{(N+1)^2}(m+1)^2(1-m)+3m^3+m^2\bigg|_{m=3} = 90-96\frac{N(N+2)}{(N+1)^2} \tag{118}$$

However, as $N \geq m$, the $\frac{\partial E_L(m)}{\partial m}\Big|_{m=3} \leq 0$ for $m = 3$. Therefore, $E_L(m)$ has to have a maximum at $m = 3$ or $m = 2$ since $m \in \{1,...,N\}$:

$$E_L(m=2) = \frac{4(A-c)^2}{B}\frac{8}{9\left[5-\frac{9}{(N+1)^2}\right]^2}$$

$$E_L(m=3) = \frac{4(A-c)^2}{B}\frac{27}{16\left[7-\frac{16}{(N+1)^2}\right]^2}$$

Comparing these two expressions:

$$\frac{8}{9\left[5-\frac{9}{(N+1)^2}\right]^2} > \frac{27}{16\left[7-\frac{16}{(N+1)^2}\right]^2}$$

$$\Longleftrightarrow$$

$$128(7(N+1)^2-16)^2 > 243(5(N+1)^2-9)^2$$

$$\Longleftrightarrow$$

$$197(N+1)^4-6802(N+1)^2+13085 > 0$$

$$\Longleftrightarrow$$

$$N \in (-\infty,\,-6.70)\cup(-2.43, 0.43)\cup(4.70,\,+\infty)$$

Therefore, $E_L(m)$ is maximized at $m = 3$ when $N \in \{3,4\}$ or $m = 2$ when $N > 4$, since $N \in \mathbb{N}$ and $N \geq m$.[49] Thus, even under the threat of conflict with the King, the Lord allows a limited number of merchants to cross his land. □

[49]Note, that $E_L(m) \sim 1/m^3$ for $m > 3$ and there is no other extremum for a greater total number of merchants N.

3.12 Appendix II: Proof of Proposition 2

Proof. Assuming L_1 sets the toll such that he allows $m_1 > 1$ merchants to cross his land: $T_1 = T(m_1 > 1)$. Then, it is profitable to charge additional tolls for the subsequent Lords. Thus, there is a i'th Lord ($i \geq 1$) who restricts trade from $m_{i-1} \leq m_1$ to $m_i = 1$ and induces monopoly. The King then prefers to involve the i'th Lord in the conflict. Indeed, the highest incentive for the King to involve another Lord in the conflict would be when the first Lord allows two merchants to cross his land ($m_1 = 2$) and the second Lord induces a monopoly.[50] Note that the second Lord L_2 collects a higher toll than the first Lord L_1 in this case:

$$T_1(m_1 = 2) = \frac{1}{9}\frac{(A-C)^2}{B} < \frac{5}{36}\frac{(A-C)^2}{B} = T(m = 1) - T_1 = T_2(m_2 = 1) \tag{119}$$

The King then chooses which Lord to involve in the conflict. The King's value of winning the conflict is positive when he involves L_2 since two merchants will reach the market instead of one in case the King wins. However, the King's value of winning the conflict is zero when he involves L_1. Indeed, only one merchant then reaches the market in case the King wins since $T_2 > T_1$, and the merchant's participation constraint is not met for $m = 2$. As a result, the King does not attack the 1st Lord. Similarly, if $i > 2$, the *total* tolls paid by a merchant while passing L_{i-1} is $T(m_{i-1})$. Since $T(m_{i-1}) \leq T(2)$, by(119), this total toll is lower than required by L_i. Thus, the King always involves the last Lord, who reduces the trade to one merchant in the conflict.

The King's value of winning the conflict against the i'th Lord is:

$$V_K = SW(m_{i-1}) - SW(1) = \frac{(A-C)^2}{B}\left[\frac{m_{i-1}(m_{i-1}+2)}{2(m_{i-1}+1)^2} - \frac{3}{8}\right] =$$

$$= \frac{(A-C)^2}{B}\frac{(m_{i-1}+3)(m_{i-1}-1)}{8(m_{i-1}+1)^2} \tag{120}$$

[50]The King then has the lowest value of winning against the last lord (L_2) and the highest possible value of winning against another Lord (L_1) since the first Lord decreases the number of merchants from N to 2.

The i'th Lord's value of winning the conflict against the King is:

$$V_{L_i} = m_i T_i(m_i) = m_i(T(m_i = 1) - T(m_{i-1})) = \frac{(A-C)^2}{B}\left[\frac{1}{4} - \frac{1}{(m_{i-1}+1)^2}\right] =$$

$$= \frac{(A-C)^2}{B}\frac{(m_{i-1}+1)^2 - 4}{4(m_{i-1}+1)^2} = \frac{(A-C)^2}{B}\frac{(m_{i-1}+3)(m_{i-1}-1)}{4(m_{i-1}+1)^2} \tag{121}$$

Note, the King's value of winning a conflict is twice less than the i'th Lord's ($V_{L_i} = 2V_K$). Then, the King wins the conflict with a probability $P_K = \frac{V_K}{V_K + V_{L_i}} = 1/3$. Thus, the first Lord's expected revenue is:

$$E_{L_1}(m_1) = \frac{(A-c)^2}{B}\left[\frac{2}{3}\frac{1}{(m_1+1)^2} + \frac{1}{3}\frac{m_{i-1}}{(m_1+1)^2}\right] \tag{122}$$

The first Lord's expected revenue is decreasing in m_1, and he sets the toll such that only two merchants cross his land ($m_1 = 2$),[51] since $m_{i-1} \leq m_1$. As a result, the first Lord's expected revenue in case he does not induce a monopoly is:

$$E_{L_1}|_{m_1=2} = \frac{4}{3}T_1(m_1 = 2) = \frac{4}{3}\frac{1}{9}\frac{(A-C)^2}{B} \tag{123}$$

Another option for the first Lord L_1 is to set $T_1 = T(m_1 = 1)$ and induce a monopoly. The expected income can be calculated then according to equation (116):

$$E_{L_1}|_{m_1=1} = \frac{4}{\left[6 - \frac{8}{(N+1)^2}\right]^2}\frac{(A-C)^2}{B} \tag{124}$$

Compare the L_1's expected revenues in these two cases, the 1st Lord always prefers to set toll $T_1 = T(m_1 = 2)$, since $\left[6 - \frac{8}{(N+1)^2}\right]^2 > 27$ for any $N \geq 3$. The second Lord then reduces the trade to a monopoly, and the King involves him in conflict with a probability of winning equal to 1/3. □

[51] Now we consider the case when the first Lord does not induce a monopoly, $m_1 > 1$.

3.13 Appendix III: Proof of Proposition 3

Proof. First, we assume that the first Lord L_1 does not restrict the trade to a monopoly. Therefore, there are two closest to the market Lords such that:

- The pre-last Lord L_{l-1} reduces the number of merchants to some number k: $N \geq k > 1$.
- The last lord L_l reduces the number of merchants from k to 1 (monopoly).

First, the King involves the closest Lord L_l in the conflict. Then, the King's and Lord L_l's values are:

$$V_K = SW|_{m=k} - SW|_{m=1} = \frac{(A-C)^2}{2B}\left[\frac{k(k+2)}{(k+1)^2} - \frac{3}{4}\right] = \frac{(A-C)^2}{2B}\left[\frac{(k+1)^2-4}{4(k+1)^2}\right]$$

$$V_{L_l} = T|_{m=1} - T|_{m=k} = \frac{(A-C)^2}{B}\left[\frac{1}{4} - \frac{1}{(k+1)^2}\right] = \frac{(A-C)^2}{B}\left[\frac{(k+1)^2-4}{4(k+1)^2}\right]$$

Thus,

$$P(L_l \; wins \; K) = 2/3 \quad and \quad P(K \; wins \; L_l) = 1/3 \tag{125}$$

Next, in case of K won, the next King involves the Lord L_{l-1} in the conflict. The Lord L_{l-1} reduces the number of merchants from some n ($N \geq n \geq k$) to k, the King's and Lord's values of winning the conflict are:

$$V_{L_{l-1}} = k(T|_{m=k} - T|_{m=n}) = \frac{(A-C)^2}{B}k\left[\frac{1}{(k+1)^2} - \frac{1}{(n+1)^2}\right] = \frac{(A-C)^2}{B}k\left[\frac{n(n+2)-k(k+2)}{(n+1)^2(k+1)^2}\right]$$

$$V_K = SW|_{m=n} - SW|_{m=k} = \frac{(A-C)^2}{2B}\left[\frac{n(n+2)}{(n+1)^2} - \frac{k(k+2)}{(k+1)^2}\right] = \frac{(A-C)^2}{2B}\left[\frac{n(n+2)-k(k+2)}{(n+1)^2(k+1)^2}\right]$$

Thus,

$$P(L_{l-1}\ wins\ K) = \frac{2k}{2k+1} \quad and \quad P(K\ wins\ L_{l-1}) = \frac{1}{2k+1} \tag{126}$$

Then, the expected income of L_{l-1} can be found from:

$$E[I_{L_{l-1}}] = \frac{2}{3}\frac{(A-C)^2}{B}\left[\frac{1}{(k+1)^2} - \frac{1}{(n+1)^2}\right] + \frac{1}{3}\frac{2k}{2k+1}\frac{(A-C)^2}{B}k\left[\frac{1}{(k+1)^2} - \frac{1}{(n+1)^2}\right] =$$

$$= \frac{(A-C)^2}{3B}\left[\frac{(n+1)^2-(k+1)^2}{(k+1)^2(n+1)^2}\right]\left(2+k\frac{2k}{2k+1}\right) = \frac{(A-C)^2}{B}\frac{2}{3}\left[\frac{(n+1)^2-(k+1)^2}{(n+1)^2(2k+1)}\right]$$

Since $k \leq n$, the expected income of the Lord L_{l-1} is decreasing in k. Therefore, the pre-last Lord always prefers to restrict the trade to a monopoly and allow only one merchant to cross his land. Then the Lord L_{l-1} becomes the last one, and the same logic is implemented for the previous Lord L_{l-2}. As a result, the first Lord L_1 set the toll, such that only one merchant crosses his land. □

References

Ainslie, G. (1975). Specious reward: a behavioral theory of impulsiveness and impulse control. *Psychological bulletin*, *82*(4), 463.

Ainslie, G. (1991). Derivation of "rational" economic behavior from hyperbolic discount curves. *The American Economic Review*, *81*(2), 334–340.

Ainslie, G. (1992). *Picoeconomics: The strategic interaction of successive motivational states within the person*. Cambridge University Press.

Ainslie, G. (2012). Pure hyperbolic discount curves predict "eyes open" self-control. *Theory and Decision*, *73*(1), 3–34.

Ainslie, G., & Haslam, N. (1992). Hyperbolic discounting.

Ainslie, G., & Herrnstein, R. J. (1981). Preference reversal and delayed reinforcement. *Animal Learning & Behavior*, *9*, 476–482.

Akerlof, G. A. (1991). Procrastination and obedience. *The American Economic Review*, *81*(2), 1–19.

Ariely, D., & Wertenbroch, K. (2002). Procrastination, deadlines, and performance: Self-control by precommitment. *Psychological science*, *13*(3), 219–224.

Augenblick, N., & Rabin, M. (2019). An experiment on time preference and misprediction in unpleasant tasks. *Review of Economic Studies*, *86*(3), 941–975.

Ballard, T., Vancouver, J. B., & Neal, A. (2018). On the pursuit of multiple goals with different deadlines. *Journal of Applied Psychology*, *103*(11), 1242.

Bernheim, B. D., & Rangel, A. (2004). Addiction and cue-triggered decision processes. *American economic review*, *94*(5), 1558–1590.

Bisin, A., & Hyndman, K. (2020). Present-bias, procrastination and deadlines in a field experiment. *Games and economic behavior*, *119*, 339–357.

Bjorvatn, K., & Naghavi, A. (2011). Rent seeking and regime stability in rentier states. *European Journal of Political Economy*, *27*(4), 740–748. DOI: 10.1016/j.ejpoleco.2011.05.007

Burger, N., Charness, G., & Lynham, J. (2011). Field and online experiments on self-control. *Journal of Economic Behavior & Organization*, *77*(3), 393–404.

Cao, D., & Werning, I. (2016). *Dynamic savings choices with disagreements* (Tech. Rep.). National Bureau of Economic Research.

Casson, M., & Casson, C. (2013). *The entrepreneur in history: From medieval merchant to modern business leader*. Palgrave Macmillan. DOI: 10.1057/9781137305824

Chung, S.-H., & Herrnstein, R. J. (1967). Choice and delay of reinforcement 1. *Journal of the experimental analysis of behavior*, *10*(1), 67–74.

Cohen, J., Ericson, K. M., Laibson, D., & White, J. M. (2020). Measuring time preferences. *Journal of Economic Literature*, *58*(2), 299–347.

Congleton, R. D., Grofman, B. N., & Voigt, S. (2018). *The Oxford Handbook of Public Choice, Volume 2*. Oxford University Press. Retrieved from `https://books.google.co.il/books?id=j8p-DwAAQBAJ`

Congleton, R. D., & Hillman, A. L. (2015). *Companion to the Political Economy of Rent Seeking*. Elgar. Retrieved from `https://books.google.co.il/books?id=DEzfBgAAQBAJ`

Congleton, R. D., Hillman, A. L., & Konrad, K. A. (2008). *40 Years of Research on Rent Seeking 2: Applications: Rent Seeking in Practice*. Springer Berlin Heidelberg. Retrieved from `https://books.google.nl/books?id=PCcs-Bo-uf4C`

Dekel, E., Lipman, B. L., & Rustichini, A. (2001). Representing preferences with a unique subjective state space. *Econometrica*, *69*(4), 891–934.

Dekel, E., Lipman, B. L., & Rustichini, A. (2009). Temptation-driven preferences. *The Review of Economic Studies*, *76*(3), 937–971.

DellaVigna, S., & Paserman, M. D. (2005). Job search and impatience. *Journal of Labor Economics*, *23*(3), 527–588.

El-Tannir, A. (2019). Optimal project deadlines for mean-variance incentive contract designs. *Computers & Industrial Engineering*, *137*, 106018.

Ericson, K. M., & Laibson, D. (2019). Intertemporal choice. In *Handbook of behavioral economics: Applications and foundations 1* (Vol. 2, pp. 1–67). Elsevier.

Fischer, C. (2001). Read this paper later: procrastination with time-consistent preferences.

Journal of Economic Behavior & Organization, *46*(3), 249–269.

Frederick, S. (2005). Cognitive reflection and decision making. *Journal of Economic perspectives*, *19*(4), 25–42.

Gabaix, X., & Laibson, D. (2017). *Myopia and discounting* (Tech. Rep.). National bureau of economic research.

Giné, X., Karlan, D., & Zinman, J. (2010). Put your money where your butt is: a commitment contract for smoking cessation. *American Economic Journal: Applied Economics*, *2*(4), 213–235.

Goldman, S. M. (1979). Intertemporally inconsistent preferences and the rate of consumption. *Econometrica: Journal of the Econometric Society*, 621–626.

Goldman, S. M. (1980). Consistent plans. *The Review of Economic Studies*, *47*(3), 533–537.

Grechenig, K. R., & Kolmar, M. (2011). The state's enforcement monopoly and the private protection of property. *U.S. Constitutional Law: Rights & Liberties eJournal*.

Greif, A. (1993). Contract enforceability and economic institutions in early trade: The Maghribi Traders' Coalition. *American Economic Review*, *83*(3), 525. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=buh&AN=9307235343&site=ehost-live

Greif, A., & Laitin, D. D. (2004). A theory of endogenous institutional change. *American Political Science Review*, *98*(4), 633–652. DOI: 10.1017/S0003055404041395

Grenadier, S. R., & Wang, N. (2007). Investment under uncertainty and time-inconsistent preferences. *Journal of Financial Economics*, *84*(1), 2–39.

Grossman, G. M., & Helpman, E. (1994). Protection for Sale. *American Economic Review*, *84*(4), 833-850. Retrieved from https://ideas.repec.org/a/aea/aecrev/v84y1994i4p833-50.html

Gul, F., & Pesendorfer, W. (2001). Temptation and self-control. *Econometrica*, *69*(6), 1403–1435.

Gul, F., & Pesendorfer, W. (2004). Self-control and the theory of consumption. *Econometrica*, *72*(1), 119–158.

Harris, C., & Laibson, D. (2012, 12). Instantaneous Gratification *. *The Quarterly Journal of Economics*, *128*(1), 205-248. Retrieved from https://doi.org/10.1093/qje/qjs051 DOI: 10.1093/qje/qjs051.

Herweg, F., & Müller, D. (2011). Performance of procrastinators: On the value of deadlines. *Theory and Decision*, *70*(3), 329–366.

Hillman, A. L. (1982). Declining industries and political-support protectionist motives. *The American Economic Review*, *72*(5), 1180–1187.

Hillman, A. L. (2019). *Public Finance and Public Policy: A Political Economy Perspective on the Responsibilities and Limitations of Government*. Cambridge University Press. Retrieved from https://books.google.co.il/books?id=unWIDwAAQBAJ

Hillman, A. L., & Ursprung, H. W. (1988). Domestic politics, foreign interests, and international trade policy. *The American Economic Review*, 729–745.

Houser, D., Schunk, D., Winter, J., & Xiao, E. (2018). Temptation and commitment in the laboratory. *Games and Economic Behavior*, *107*, 329–344.

Hsiaw, A. (2013). Goal-setting and self-control. *Journal of Economic Theory*, *148*(2), 601–626.

Hyndman, K., & Bisin, A. (2022). Procrastination, self-imposed deadlines and other commitment devices. *Economic Theory*, *74*(3), 871–897.

Kamien, M. I., & Tauman, Y. (1986). Fees versus royalties and the private value of a patent. *The Quarterly Journal of Economics*, *101*(3), 471–491. Retrieved 2023-05-11, from http://www.jstor.org/stable/1885693

Katz, M. L., & Shapiro, C. (1986). Technology Adoption in the Presence of Network Externalities. *Journal of Political Economy*, *94*(4), 822–841. Retrieved from https://doi.org/10.1086/261409 DOI: 10.1086/261409

Kaur, S., Kremer, M., & Mullainathan, S. (2015). Self-control at work. *Journal of Political Economy*, *123*(6), 1227–1277.

Klunover, D. (2023). Asymmetric technologies in contests. *European Journal of Political Economy*, *78*, 102337. DOI: 10.1016/j.ejpoleco.2022.102337

Knowles, S., Servátka, M., Sullivan, T., & Genç, M. (2017). Deadlines, procrastination, and forgetting in charitable tasks: A field experiment.

Konrad, K. A., & Skaperdas, S. (2005). The market for protection and the origin of the state. *Economic Theory*, *50*, 417-443.

Konrad, K. A., & Skaperdas, S. (2012). The market for protection and the origin of the state. *Economic Theory*, *50*(2), 417–443. Retrieved 2023-05-12, from http://www.jstor.org/stable/41486016

Kőszegi, B., & Szeidl, A. (2013). A model of focusing in economic choice. *The Quarterly journal of economics*, *128*(1), 53–104.

Laibson, D. (1994). Essays in hyperbolic discounting. *Ph.D. dissertation, MIT*.

Laibson, D. (1996). *Hyperbolic discount functions, undersaving, and savings policy.* National bureau of economic research Cambridge, Mass., USA.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *The Quarterly Journal of Economics*, *112*(2), 443–478.

Laibson, D. (2001). A cue-theory of consumption. *The Quarterly Journal of Economics*, *116*(1), 81–119.

Laibson, D., Repetto, A., & Tobacman, J. (2007). *Estimating discount functions with consumption choices over the lifecycle.* National Bureau of Economic Research Cambridge, Mass., USA.

Lipman, B. L., Pesendorfer, W., et al. (2013). Temptation. In *Advances in economics and econometrics: Tenth world congress* (Vol. 1, pp. 243–288).

Loewenstein, G., & Prelec, D. (1992). Anomalies in intertemporal choice: Evidence and an interpretation. *The Quarterly Journal of Economics*, *107*(2), 573–597.

McAfee, R. P., Mialon, H. M., & Williams, M. A. (2004). What is a barrier to entry? *The American Economic Review*, *94*(2), 461–465. Retrieved 2023-05-11, from http://www.jstor.org/stable/3592928

Middleton, N. (2005). Early medieval port customs, tolls and controls on foreign trade. *Early Medieval Europe*, *13*(4), 313–358. Retrieved from https://

onlinelibrary.wiley.com/doi/abs/10.1111/j.1468-0254.2005.00161.x DOI: https://doi.org/10.1111/j.1468-0254.2005.00161.x

Mokyr, J. (2017). *A culture of growth: The origins of the modern economy.* Princeton: Princeton University Press.

Nitzan, S. (1994). Modelling rent-seeking contests. *European Journal of Political Economy*, *10*, 41-60.

Noor, J. (2007). Commitment and self-control. *Journal of Economic theory*, *135*(1), 1–34.

Noor, J. (2011). Temptation and revealed preference. *Econometrica*, *79*(2), 601–644.

Nti, K. O. (1999). Rent-seeking with asymmetric valuations. *Public Choice*, *98*(3-4), 415–430. DOI: 10.1023/a:1018391504741

O'Donoghue, T., & Rabin, M. (1999a). Doing it now or later. *American economic review*, *89*(1), 103–124.

O'Donoghue, T., & Rabin, M. (1999b). Incentives for procrastinators. *The Quarterly Journal of Economics*, *114*(3), 769–816.

O Donoghue, T., Rabin, M., et al. (2006). Incentives and self-control. *Econometric Society Monographs*, *42*, 215.

Ogilvie, S. (2011). *Institutions and European Trade.* Cambridge University Press. DOI: 10.1017/cbo9780511974410

Paserman, M. D. (2008). Job search and hyperbolic discounting: Structural estimation and policy evaluation. *The Economic Journal*, *118*(531), 1418–1452.

Peleg, B., & Yaari, M. E. (1973). On the existence of a consistent course of action when tastes are changing. *The Review of Economic Studies*, *40*(3), 391–401.

Phelps, E. S., & Pollak, R. A. (1968). On second-best national saving and game-equilibrium growth. *The Review of Economic Studies*, *35*(2), 185–199.

Pollak, R. A. (1968). Consistent planning. *The Review of Economic Studies*, *35*(2), 201–208.

Razumovskii, A. (2023). Interim deadline for procrastinators. *CERGE-EI Working Paper Series*, *769*.

Shleifer, A., & Vishny, R. W. (1993). Corruption. *The quarterly journal of economics*, *108*(3), 599–617.

Skaperdas, S., & Vaidya, S. (2019). Why did pre-modern states adopt big-god religions? *Public Choice*, *182*, 373 - 394.

Steele, C. M., & Josephs, R. A. (1990). Alcohol myopia: Its prized and dangerous effects. *American psychologist*, *45*(8), 921.

Strotz, R. H. (1955). Myopia and inconsistency in dynamic utility maximization. *The review of economic studies*, *23*(3), 165–180.

Tirole, J. (2015). Market failures and public policy. *American Economic Review*, *105*(6), 1665-82. Retrieved from https://www.aeaweb.org/articles?id=10.1257/aer.15000024 DOI: 10.1257/aer.15000024

Trope, Y., & Fishbach, A. (2000). Counteractive self-control in overcoming temptation. *Journal of personality and social psychology*, *79*(4), 493.

Tullock, G. (1984). Long-run equilibrium and total expenditures in rent-seeking: A comment. *Public Choice*, *43*, 95-97.

Tullock, G., & Rowley, C. K. (2005). *The Rent-seeking Society, v.5.* Liberty Fund.

Vancouver, J. B., Weinhardt, J. M., & Schmidt, A. M. (2010). A formal, computational theory of multiple-goal pursuit: Integrating goal-choice and goal-striving processes. *Journal of Applied Psychology*, *95*(6), 985.

Van Long, N. (2013). The theory of contests: A unified model and review of the literature. *European Journal of Political Economy*, *32*, 161–181. DOI: 10.1016/j.ejpoleco.2013.07.006

Yaari, M. E. (1977). Consistent utilization of an exhaustible resource, or, how to eat an appetite-arousing cake. *Center for Research in Mathematical Economics and Game Theory Research memorandum*, *26*.