Equilibrium and Inefficiency Properties of Interlinkage under Polyandrous Tenancy

Jaideep Roy
Department of Economics
Trinity College
Dublin 2, Ireland.

Konstantinos Serfes
Department of Economics
SUNY at Stony Brook
Stony Brook, NY 11794-4384, USA.

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Abstract

Existing literature argues that interlinked tenancy contracts are superior to contracts which provide no credit to the tenant who in turn obtains it through a professional moneylender. In this paper, we show that in economies where tenants work for more than one landlord (Polyandrous tenancy), a situation very common in many rural economies, interlinked contracts may become inferior but nevertheless constitute an equilibrium. We define two classes of credit contracts: i) Forced-credit contracts and ii) Optional-credit contracts. In the first class the landlords offer, in equilibrium, interlinked contracts which entail more credit that the socially optimal level. In the second class the landlords subsidize the loans by lending to the tenant at equilibrium interest rates which fall below the market rates, resulting in an inefficient outcome. However, under both classes of credit contracts the landlords are in a prisoners' dilemma situation.

KEYWORDS: Polyandrous tenancy, interlinkage, skills, inefficiency.

(First Draft)

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1 Introduction

In many rural economies, landlords hire tenants to carry out agricultural production. It is often the case that the tenants have insufficient capital for farming activity and the role of credit becomes vital in sustaining production. Sources of credit for such purposes are restricted to (i) landlords themselves offering interlinked tenancy-credit contracts and (ii) an informal financial trader (or moneylender) from whom an employed tenant could acquire the required capital.[1] As is widely known and discussed, for example, in [Basu (1997), Ch.14 and Ray (1998), pp.561-563], interlinked contracts are widely observed, implying that landlords are the dominant source of rural credit. In addition, the interest rate that the landlords charge is in general below the market rate [e.g. Bardhan and Rudra (1978) and Otsuka et al. (1992)]. Why?

In the past, researchers viewed interlinkage as a form of exploitation of less powerful tenants by more powerful landlords [see for example Bhaduri 1973, 1977]. More recently, Basu et al. (2000) write, "any attempt to answer this fundamental question must involve the identification and analysis of the factors that create *conditions for superiority* of interlinked contracts over non-interlinked ones." In particular one needs to see why a landlord may do strictly better with interlinkage. There are several answers to this. In a model without uncertainty and with no liability limitations on part of the borrower, Newberry (1975) shows that interlinkage is not necessary for achieving efficiency. With uncertainty, Ray and Sengupta (1989) show that if sufficiently many variables are observable, superiority of interlinked contracts may not hold if the landlord can impose nonlinear contracts. Strict optimality of interlinkage may arise out of moral hazard as in Braverman and Stiglitz (1982) or out of adverse selection as in Banerji (1995). Basu et al. show that in a sequential game where the landlord moves first and the tenant has limited liability, interlinkage becomes superior.

In all the papers mentioned above, it is assumed implicitly that a tenant once hired by some landlord cannot work for some other landlord during the same cropping period. In this case (monogamous tenancy), reasons behind superiority of the interlinkage institution seem well understood by now. However, monogamous tenancy is not the only institution observed

[1] These days, a tenant may also obtain credit from some formal or institutional lenders like government banks, commercial banks and credit bureaus. Special banks are also emerging in many less developed countries to meet especially the needs of rural production. Our money lender can in general be any credit institution, formal or informal.

in reality. As clear from Bell (1977), *polyandrous tenancy* where a tenant works for several landlords at the same time is also widespread. For example, the following table, which is reproduced from Bell, shows the distribution of tenants by the number of landlords under which they worked simultaneously in Parneo, India.

No. of landlords	1	2	3	4	5	6	7	8
No. of tenants	7	6	4	2	1	0	0	1

Out of the 21 tenants in this sample, only 7 were in a monogamous tenancy. The remaining 14 had more than one landlord, and the average number of landlords per tenant was 2.5.[2] Young and Burke (2001) also document that in contemporary Illinois agriculture, it is common for farmers to contract with several different landlords. The study of interlinked contracts, therefore, will not be complete unless we also investigate their properties in a setting where a tenant works for more than one landlord.[3] Our modeling framework is a very natural one, where the tenant has to allocate his limited time between the different landlords and the provision of capital by any one of them in the form of interlinked contracts has a dual role: One is to provide the necessary capital for the production in his land and second to increase the tenant's marginal productivity of effort so that the tenant will optimally shift more of his limited resources towards that land. Our principal finding is that *while interlinkage indeed arises in equilibrium, it may fail to be Pareto efficient as it has widely been thought of.*

We formulate a *moral hazard* model with two landlords who lease their lands to one tenant. The tenant decides how to allocate his limited time between the two lands and whether to acquire credit (if needed) from a professional moneylender. Each landlord offers a *take-it-or-leave-it* interlinked contract which specifies: i) the fixed-rent that the tenant has to pay (tenancy contract) and ii) the interest rate charged by the landlord for every unit of capital that the tenant borrows from him (credit contract). The tenant has three choices: a) rejects the interlinked contract, or b) accepts the interlinked contract, or c) accepts the tenancy contract while he rejects the credit one. In the latter case, he obtains credit from a professional money lender at the prevailing market rate. Clearly, if the rate that a landlord charges is lower than the market rate, then the tenant is better-off accepting the credit

[2] In a personal communication, Clive Bell informed us that Polyandrous tenancy is widespread in India today as well.

[3] We take the institution of polyandrous tenancy as exogenously given and we do not attempt in this paper to formalize and model the forces that led to its existence. This is a topic for future research.

contract. We show that it is each landlord's dominant strategy to offer interlinked contracts which entail interest rates that are lower than the market ones. Moreover this outcome is inefficient. In particular, the two landlords are in a prisoner's dilemma.

Braveman and Stiglitz show that, under some conditions, the landlord has an incentive to subsidize the loans to encourage the tenant to become indebted to him. An indebted tenant would increase his effort in order to avoid the possibility of not repaying the landlord. In such a setting interlinkage shifts the utility possibilities schedule outward. Our framework also predicts that each landlord, in equilibrium, will subsidize the loans to the tenant, but the reason behind this is different. A landlord would subsidize loans purely to increase the marginal productivity of the tenant's effort in his land *relative* to that in the other landlord's land. When the tenant's time constraint is binding, the competition between the landlords in the form of low interest rates is wasteful as it leads to *excessive* borrowing. This inefficiency, however, is a trap since it is both landlords' dominant strategy to offer lower rates.

The question which immediately arises is why would the landlords hire the same tenant, given the inefficiency which will arise, when in any market there are usually several available tenants? Although we do not fully address this question, we provide a stylized model which justifies the existence of polyandrous tenancy. We assume that a market consists of two landlords who own lands of equal productivity and two tenants where one is more skilled than the other and the time constraint is binding. Each tenant can work either for none, one or both landlords. Using a variant of the *multiple partners assignment* game [see Sotomayor (1999)] we find conditions under which in a stable equilibrium the high skilled tenant is working for both landlords and the low skilled one is unemployed.

The rest of the paper is structured as follows. Section 2 describes the environment in details. In section 3 we study the problem faced by the tenant under the assumption that if a contract is interlinked it comes with a forced-credit clause. In section 4 we focus our attention on the game played between the two landlords and section 5 proves the main results. Section 6 presents the analysis under the assumption that only optional-credit contracts are viable. The paper concludes in section 7.

2 The description of the model

Consider a market which consists of two landlords (1 and 2 and indexed by i) and one landless and assetless tenant (T). All parties are assumed to be risk neutral over and above their subsistence payoff which is set equal to zero. The landlords lease their lands to the tenant who supplies effort to produce output. We normalize the price of output to one. The level of output produced on landlord i's land is given by the production function $y_i = \varepsilon_i f(e_i, k_i)$, where y_i, e_i, and k_i denote the levels of output, effort and capital on land i. Uncertainty is entered multiplicatively through the term ε_i with $E\varepsilon_i = 1$. Since our players are risk neutral in the remaining of the paper we assume that all payoffs are in expected terms without explicitly stating it. The total cost of effort $C(e_1, e_2)$, is assumed (for simplicity) to be separable in the effort levels in the two farms and linear, i.e., $C = a(e_1 + e_2)$, where $a > 0$ is a parameter which measures the marginal cost of effort (or its inverse $1/a$ is the skill of the tenant). We assume that there is an upper bound M to the total amount of effort that the tenant can supply, i.e., $e_1 + e_2 \leq M$. For example, this can be thought of as the number of hours per week that the tenant can devote in the management of the two lands and in the supervision and monitoring of the unskilled workers, or the intensity of effort. The tenant obtains the necessary for the production capital either from a moneylender or from the landlords in the form of an interlinked contract. Each landlord can offer credit to the tenant at terms different than the market rate, which we assume to be r. Let $\hat{r}_1$ and $\hat{r}_2$ denote the interest rates that landlord 1 and 2 charge respectively. Hence, for every unit of capital that the tenant borrows at the beginning of production he must pay $(1 + \hat{r}_i)$, $i = 1, 2$ to landlord i when production is over. If the tenant instead borrows from the moneylender, he must repay $(1 + r)$. If the tenant acquires credit from landlord i he can neither use it on the other land, nor indulge in lending activities. However, the tenant is free to keep capital idle. Also, let β_1 and β_2 denote the fixed-rent that the tenant has to pay to landlord 1 and 2 respectively. We assume that only the landlords offer contracts and have all the bargaining power.

The contract that landlord i offers to the tenant is denoted by, $\mathcal{C}_i = \langle \beta_i, \hat{r}_i \rangle$. This contract two components: a) a tenancy offer and b) a credit one (this is essentially a non-linear credit contract reminiscent of the one studied in Braveman and Stiglitz, see section E in that paper). If $\hat{r}_i$ is less that the market rate r, then the contract is interlinked and the tenant is better off borrowing funds at the landlord's rate.[4] Otherwise, that is if $\hat{r}_i \geq r$, the credit offer has no force since the tenant can now borrow at the market rate and the $\mathcal{C}_i$ reduces to a standard fixed-rent tenancy contract.

Assumptions and notation: We make the following assumptions regarding the produc-

[4] As Basu (1997) writes "an interlinked deal is one in which two or more *independent* exchanges are simultaneously agreed upon."

tion function. i) $f(0,k) = f(e,0) = 0$, ii) f is continuously differentiable in both arguments and strictly concave iii) $f_e \equiv \partial f/\partial e > 0$ and $f_k \equiv \partial f/\partial k > 0$, iv) $f_{ee} \equiv \partial^2 f/\partial e \partial e < 0$, $f_{kk} \equiv \partial^2 f/\partial k \partial k < 0$, $f_{ek} \equiv \partial^2 f/\partial e \partial k > 0$ and $f_{ke} \equiv \partial^2 f/\partial k \partial e > 0$ and v) $f_{eek} \equiv \partial^3 f/\partial e^2 \partial k < 0$ and $f_{kke} \equiv \partial^3 f/\partial k^2 \partial e < 0$.[5]

The tenant's payoff function (if he borrows from the two landlords) is,

$$\Pi_T = f(e_1, k_1) + f(e_2, k_2) - ae_1 - ae_2 - \beta_1 - \beta_2 - (1+\hat{r}_1)k_1 - (1+\hat{r}_2)k_2, \tag{1}$$

while the landlords' payoff functions are,

$$\begin{aligned} \Pi_1 &= [f(e_1, k_1) - ae_1 - (1+\hat{r}_1)k_1] + (\hat{r}_1 - r)k_1 \text{ and} \\ \Pi_2 &= [f(e_2, k_2) - ae_2 - (1+\hat{r}_2)k_2] + (\hat{r}_2 - r)k_2. \end{aligned} \tag{2}$$

Each landlord's profit is the sum of the fixed rent that he receives from the tenant $\beta_i = [f(e_i, k_i) - ae_i - (1+\hat{r}_i)k_i]$ plus the gain (loss) from lending money. We assume for simplicity that the tenant's opportunity cost of accepting each contract is zero.[6]

The above environment induces the following game G played between the two landlords and their common tenant.

Stage 1. The two landlords independently and simultaneously make take-it-or-leave-it offers to the tenant. Each offer is a contract as specified above.

Stage 2. Given the two contracts on offer, the tenant decides whether to accept them or not, and if he accepts them upon the level and the allocation of his effort and capital between the two lands to maximize his profits.

[5] For example, a Cobb-Douglas production function which exhibits non-increasing returns to scale ($y = e^{\alpha}k^{\beta}$, with $\alpha > 0, \beta > 0$ and $\alpha + \beta < 1$) satisfies all of the above assumptions.

[6] This is a common-agency situation which has been studied in various other contexts. For example, Bernheim and Whinston (1986) study a common agency where the agent takes a single action which induces a probability distribution over outcomes that generate profits to each of the J principals. In our case, the agent works for two principals but takes independent actions (i.e., how much effort to put in each land) for each one of them and payoff to each principal depends solely upon the action taken by the agent for that particular principal. As a result, there is a fundamental difference between Bernheim and Whinston and our case in the way reservation incomes (or opportunity costs) are modelled. While in Bernheim and Whinston, the sum of payoffs received by the agent from each principal must add up to his reservation income (which if not met, the agent shirks and each principal receives his or her own reservation income), in our case, reservation income of the agent is tied independently to each principal. Precisely why we prefer calling it the opportunity cost of working with a given principal. This opportunity cost of a single contract comes from sources exogenous to our model and our purpose is not to address the issue of how opportunity costs of accepting a given contract are determined.

We look for a subgame perfect equilibrium of G. Before we proceed, we present the socially optimal levels of effort and capital.

2.1 Social optimum

In this subsection, we find the socially efficient level of production which will be used as a benchmark case.

Maximizing the social surplus in this economy is equivalent to solving,

$$\begin{aligned} \max_{e_1,k_1,e_2,k_2} S &= f(e_1,k_1) + f(e_2,k_2) - ae_1 - ae_2 - (1+r)k_1 - (1+r)k_2 \\ \text{s.t.} &: \quad e_1 + e_2 \square M. \end{aligned}$$

The Lagrangian of the above problem is,

$$\begin{aligned} \mathcal{L} &= f(e_1,k_1) + f(e_2,k_2) - ae_1 - ae_2 - (1+r)k_1 - (1+r)k_2 \\ &\quad +\lambda(M - e_1 - e_2). \end{aligned}$$

The socially optimum levels of effort and capital in the two lands are the solutions to,

$$\begin{aligned} i)\ f_{e_1}(e_1,k_1) - a - \lambda &= 0,\ ii)\ f_{e_2}(e_2,k_2) - a - \lambda = 0, \\ iii)\ f_{k_1}(e_1,k_1) - (1+r) &= 0,\ iv)\ f_{k_2}(e_2,k_2) - (1+r) = 0, \\ \text{and } v)\ M - e_1 - e_2 &= 0. \end{aligned}$$

and are denoted as,

$$e_1^S(a),\ e_2^S(a),\ \ k_1^S(a) \text{ and } k_2^S(a). \tag{3}$$

3 Analysis

We first solve the tenant's problem who given the two contracts decides about the level of effort and capital in the two lands. Then, we find the equilibrium contracts that the two landlords offer.

3.1 Stage 2: The tenant's decisions

We assume that the time constraint is always binding and that the tenant borrows from the two landlords and not from the moneylender. Next, we will show that both landlords charge interest rates which are lower than the market rate and therefore the tenant will never exercise his option to borrow from the moneylender. The F.O.C. (with respect to e_1, e_2, k_1, k_2 and λ) are given below,

$$\begin{aligned} i)\ f_{e_1} - a - \lambda &= 0,\ ii)\ f_{e_2} - a - \lambda = 0,\ iii)\ f_{k_1} - (1 + \hat{r}_1) = 0 \\ iv)\ f_{k_2} - (1 + \hat{r}_2) &= 0 \text{ and } v)\ M - e_1 - e_2 = 0 \end{aligned} \tag{4}$$

The solutions to the above system are,

$$e_1^*(\hat{r}_1, \hat{r}_2, a), e_2^*(\hat{r}_1, \hat{r}_2, a), k_1^*(\hat{r}_1, \hat{r}_2, a) \text{ and } k_2^*(\hat{r}_1, \hat{r}_2, a). \tag{5}$$

Remark 1: If the effort constraint is not binding, i.e., $\lambda = 0$, then the above maximization problem can be decomposed into two independent problems with the tenant choosing the optimal levels of effort and capital in one land independently of his choices in the other. In this case, the solutions would be,

$$e_1^*(\hat{r}_1, a), e_2^*(\hat{r}_2, a), k_1^*(\hat{r}_1, a) \text{ and } k_2^*(\hat{r}_2, a). \tag{6}$$

By invoking the Implicit Function Theorem, and assuming that $\lambda > 0$ the matrix of the comparative statics is,

$$\begin{pmatrix} \frac{\partial e_1}{\partial \hat{r}_1} & \frac{\partial e_1}{\partial \hat{r}_2} & \frac{\partial e_1}{\partial a} \\ \frac{\partial e_2}{\partial \hat{r}_1} & \frac{\partial e_2}{\partial \hat{r}_2} & \frac{\partial e_2}{\partial a} \\ \frac{\partial k_1}{\partial \hat{r}_1} & \frac{\partial k_1}{\partial \hat{r}_2} & \frac{\partial k_1}{\partial a} \\ \frac{\partial k_2}{\partial \hat{r}_1} & \frac{\partial k_2}{\partial \hat{r}_2} & \frac{\partial k_2}{\partial a} \\ \frac{\partial \lambda}{\partial \hat{r}_1} & \frac{\partial \lambda}{\partial \hat{r}_2} & \frac{\partial \lambda}{\partial a} \end{pmatrix} = - \begin{pmatrix} f_{e_1e_1} & 0 & f_{e_1k_1} & 0 & -1 \\ 0 & f_{e_2e_2} & 0 & f_{e_2k_2} & -1 \\ f_{k_1e_1} & 0 & f_{k_1k_1} & 0 & 0 \\ 0 & f_{k_2e_2} & 0 & f_{k_2k_2} & 0 \\ -1 & -1 & 0 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= -\frac{1}{A}\begin{pmatrix} -cd & cd & ed & -fc & bcd-f^2c \\ cd & -cd & -ed & fc & acd-e^2d \\ ed & -ed & -ad-bd+f^2 & ef & -ebd+ef^2 \\ -fc & fc & ef & -ac+e^2-bc & -afc+e^2f \\ bcd-f^2c & acd-e^2d & -bed+f^2e & -afc+e^2f & abcd-af^2c-be^2d+e^2f^2 \end{pmatrix} \times$$

$$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad (7)$$

where $A = -acd + e^2d - bcd + f^2c > 0$ due to the strict concavity of the production functions, and $a = f_{e_1e_1}$, $b = f_{e_2e_2}$, $c = f_{k_1k_1}$, $d = f_{k_2k_2}$, $e = f_{e_1k_1} = f_{k_1e_1}$ and $f = f_{e_2k_2} = f_{k_2e_2}$.

3.2 Stage 1: The game between the two landlords

Landlord $i's$, problem is,

$$\max_{\hat{r}_i} [f(e_i^*, k_i^*) - ae_i^* - (1+\hat{r}_i)k_i^*] + (\hat{r}_i - r)k_i^*, \ i = 1, 2.$$

The F.O.C. is,

$$f_{e_1}\frac{\partial e_i^*}{\partial \hat{r}_i} + f_{k_i}\frac{\partial k_i^*}{\partial \hat{r}_i} - a\frac{\partial e_i^*}{\partial \hat{r}_i} - (1+\hat{r}_i)\frac{\partial k_i^*}{\partial \hat{r}_i} - k_i^* + (\hat{r}_i - r)\frac{\partial k_i^*}{\partial \hat{r}_i} + k_i^* = 0,$$

which by using Eq.(4) simplifies to,

$$\lambda\frac{\partial e_i^*}{\partial \hat{r}_i} + (\hat{r}_i - r)\frac{\partial k_i^*}{\partial \hat{r}_i} = 0. \qquad (8)$$

$>$From Eq.(7) we know that,

$$\frac{\partial e_1^*}{\partial \hat{r}_1} = \frac{f_{e_1k_1} f_{k_2k_2}}{A} < 0 \text{ and } \frac{\partial e_2^*}{\partial \hat{r}_2} = \frac{f_{e_2k_2} f_{k_1k_1}}{A} < 0,$$

and

$$\frac{\partial k_1^*}{\partial \hat{r}_1} = \frac{-f_{e_1e_1} f_{k_2k_2} - f_{e_2e_2} f_{k_2k_2} + (f_{e_2k_2})^2}{A} < 0 \text{ and } \frac{\partial k_2^*}{\partial \hat{r}_2} = \frac{-f_{e_2e_2} f_{k_1k_1} - f_{e_1e_1} f_{k_1k_1} + (f_{e_1k_1})^2}{A} < 0,$$

since the production functions are strictly concave and $f_{k_1k_1} \equiv f_{k_2k_2}$.

Proposition 1 *Suppose that the effort constraint is binding, i.e., $\lambda > 0$. Then, it is each landlords strictly dominant strategy to offer to the tenant an interest rate that is lower than the market rate, i.e., $\hat{r}_i < r$, $i = 1, 2$.*

Proof. By differentiating (8) with respect to $\hat{r}_i$ we obtain,

$$\lambda \frac{\partial^2 e_i^*}{\partial \hat{r}_i^2} + \frac{\partial k_i^*}{\partial \hat{r}_i} + (\hat{r}_i - r)\frac{\partial^2 k_i^*}{\partial \hat{r}_i^2} = \frac{\partial k_i^*}{\partial \hat{r}_i} < 0,$$

using Eq.(7). Hence, the payoff functions are concave and by using standard fixed point arguments we can readily show that an equilibrium in pure strategies exists.

It can also be seen from Eq.(8) that,

$$\hat{r}_i = r - \lambda \frac{\partial e_i / \partial \hat{r}_i}{\partial k_i / \partial \hat{r}_i} < r, \; i = 1, 2,$$

where the RHS is independent of $\hat{r}_i$ [see Eq.(7)]. Hence, it is each landlord's dominant strategy to offer lower than the market rate. The equilibrium is unique and symmetric. ■

Clearly, if the constraint is not binding [see remark 1] then no landlord has an incentive to lower the rate from its market level. Under a binding constraint, the game played between the two landlords is a prisoner's dilemma. That is, each landlord is better off when they both offer the market rate as opposed to both offering lower rates. The efficient outcome is given in section 2.1 where the true cost of capital r is used. The landlords' competition for the tenant's time, however, distorts the true cost of funds, leading to excessive use of capital. Since the tenant always receives his reservation utility and the total surplus shrinks, it must be that the landlords' profits decrease.

As mentioned in the introduction, what remains to be studied is why in fact such an institution may exist given its unquestioned iffeciencies as shown above. The following section addresses this issue by using a stylized model a la Sotomayor (1999).

4 Existence of Polyandrous tenancy

In this section, we present a stylized 2×2 model to justify the existence of polyandrous tenancy. This is an application of the assignment game with multiple partners, Sotomayor (1999). There are two landlords ℓ_1 and ℓ_2 and two tenants t_1 and t_2. Landlords own plots of identical productivity and can hire no more than one tenant. Tenants have different (observable) skills with t_1 being more productive than t_2. Each tenant can work for either one or both landlords. A contract offered to a tenant by a landlord must give him at least his reservation utility $v = 0$. Landlords also are assumed to have zero reservation utility.

We can have the following two cases: i) one-to-one matching, and ii) many-to-one matching. Case i) is when each landlord is matched exactly with one tenant and vice versa and case ii) when each tenant is allowed to form a partnership with more than one landlord, but not the other way around. The matrices below depict the profits that accrue to the landlords from each matching.

<u>One-to-one matching</u>

$t\backslash\ell$	ℓ_1	ℓ_2
t_1	Π_{11}	Π_{12}
t_2	Π_{21}	Π_{22}

(1)

For example, Π_{11} represents landlord 1's profit when he employs tenant 1. Since tenant 1 is more productive than tenant 2 and both lands are of the same quality, it is reasonable to assume that,

$$\Pi_{11} = \Pi_{12} > \Pi_{21} = \Pi_{22}. \tag{2}$$

<u>Many-to-one matching</u>

Now each tenant works for both landlords.

$t\backslash\ell$	ℓ_1	ℓ_2
t_1	$\bar{\Pi}_{11}$	$\bar{\Pi}_{12}$
t_2	$\bar{\Pi}_{21}$	$\bar{\Pi}_{22}$

(3)

For example, $\bar{\Pi}_{11}$ is landlord 1's profit if he hires tenant 1 who has also been hired by landlord 2. We further assume decreasing marginal productivity as a tenant works for more landlords, i.e.,

$$\Pi_{11} + \Pi_{12} > \bar{\Pi}_{11} + \bar{\Pi}_{12}, \tag{4}$$

and,

$$\Pi_{21} + \Pi_{22} > \bar{\Pi}_{21} + \bar{\Pi}_{22}. \tag{5}$$

This is consistent with our main model and can be interpreted as an upper bound on effort.

[**This part follows Sotomayor's notation and definitions**] There are two disjoint sets of players $L = \{\ell_1, \ell_2\}$ and $T = \{t_1, t_2\}$. Each player ℓ_i in L can form no more than one partnership with the players in T, but each player t_j in T can form any number of partnerships with the players in L. For each landlord-tenant pair (ℓ_i, t_j) in $L \times T$ there are two non-negative numbers Π_{ij} and $\bar{\Pi}_{ij}$ which depict the profits that ℓ_i and t_j can generate under the assumptions that t_j has been hired by one or two landlords respectively [see (1) and (3)]. Here, we differ from Sotomayor (1999) who assumes that the gain to a partnership (f_i, w_j) does not depend on the number of partnerships that f_i or w_j has formed with other players. We believe that in many economic situations our assumption is more plausible. Define the function $\alpha_{ij} : L \times T \to R_{++}$ by,

$$\alpha_{ij} = \begin{cases} \Pi_{ij}, & \text{if } t_j \text{ works only for } \ell_i \\ \\ \bar{\Pi}_{ij}, & \text{if } t_j \text{ works for both landlords} \end{cases}$$

The gain from each partnership depends on all other matchings that t_j has formed. When a partnership (ℓ_i, t_j) is formed t_j will receive a payoff $v_{ij} \geq 0$ and ℓ_i will receive a payoff $u_{ij} = \alpha_{ij} - v_{ij} \geq 0$.

Definition 2 *A feasible matching x is an 2×2 matrix x_{ij} with zeroes and ones such that $\sum_{\ell_i \in L} \sum_{t_j \in T} = 2$ and $\sum_{t_j \in T} = 1$.*

For example, consider the following 2×2 matrices,

$$x = \begin{pmatrix} 1 & 0 \\ & \\ 0 & 1 \end{pmatrix}, \ x^{'} = \begin{pmatrix} 1 & 1 \\ & \\ 0 & 0 \end{pmatrix}, \ x^{''} = \begin{pmatrix} 1 & 0 \\ & \\ 1 & 0 \end{pmatrix}.$$

Matrix x represents an outcome where ℓ_1 is matched with t_1 and ℓ_2 with t_2; $x^{'}$ an outcome where tenant 1 works for both landlords (polyandrous tenancy), while the assignment in $x^{''}$ is not allowed by our model. The set of all t_j's partners at x is denoted by $C(t_j, x)$. So, $C(t_j, x) = \{\ell_1, \ell_2\}$ denotes that tenant j is matched with both landlords under x. Similarly, $C(\ell_i, x)$ is defined for all $\ell_i \in L$ and contains no more than one element since each landlord is matched with only one tenant.

Definition 3 *A feasible outcome, denoted by $(u, v; x)$, is a feasible matching x, and an array of numbers u_{ij}, with $\ell_i \in L$ and $t_j \in C(\ell_i, x)$ and v_{ij} with $t_j \in T$ and $\ell_i \in C(t_j, x)$, such that $u_{ij} + v_{ij} = \alpha_{ij}$, $u_{ij} \geq 0$ and $v_{ij} \geq 0$.*

If $(u, v; x)$ is a feasible outcome, we say that the matching x is compatible with the payoff (u, v). Tenant j's total payoff under the outcome $(u, v; x)$ is denoted by τ_j and is: $\tau_j = \sum_{\ell_i \in C(t_j, x)} v_{ij}$. Hence, we can write,

$$\sum_{\ell_i \in L} u_{ij} + \sum_{t_j \in T} \tau_j = \sum_{(\ell_i, t_j) \in L \times T} \alpha_{ij} x_{ij}.$$

Given a feasible outcome $(u, v; x)$ define $\nu_j = \min \{v_{ij} : \ell_i \in C(t_j, x)\}$. This is the minimum payoff that tenant j receives among all his payoffs with the landlords under x. If tenant j works for only one landlord, then $\nu_j = 0$. Let $\Delta\Pi = \Pi_{i' j} - \bar{\Pi}_{i' j}$, denote the profit decrease in the $(i^{'}, j)$ partnership when tenant j works for landlord i in addition to landlord $i^{'}$. Consider a feasible outcome $(u, v; x)$. This outcome is stable if it is not blocked by any group of players. From the assumption of our model a landlord can form only one partnership and therefore he can block the current assignment by matching with a new tenant and severing his partnership with his assigned under x tenant. A tenant however can potentially block the current assignment in three different ways: i) If under x he is employed by only one landlord, he can switch landlords and still be employed by only one of them ii) he can work for one more landlord, if under x he works only for one and iii) if he works for both landlords under x he can stop working for one them. If no blocking is profitable, then we say that $(u, v; x)$ is stable.

Definition 4 *The feasible outcome* $(u, v; x)$ *is stable if: i)* $u_{ij} + \nu_j \geq \alpha_{ij} - \Delta\Pi$, *ii)* $u_{ij} + v_{ij} \geq \alpha_{ij}$ *for all* (ℓ_i, t_j), *with* $x_{ij} = 0$ *and* $x_{ij} = 1$.

Notice that this definition of stability is different from the one in Sotomayor. Essentially, the entries of the 2×2 matrix α depend not only on the specific pair (i, j) but also on x. Consequently, when x changes, due to a blocking, we move to a different matrix α [see (1) and (3)]. Consider an outcome $(u, v; x)$ where ℓ_i is not matched with t_j, that is $x_{ij} = 0$. Further assume that under x, ℓ_i receives u_{ij} and t_j receives v_{ij}. The first possible blocking is when tenant j keeps working for his assigned under x landlord but also works for landlord i. This can happen if (ℓ_i, t_j) can generate more surplus than what they destroy, i.e., $u_{ij} + 0 + \Delta\Pi < \alpha_{ij}$. The left term is the surplus that is being lost if (ℓ_i, t_j) forms and the right term is what is being created. The second blocking is when tenant j leaves his assigned under x landlord to work for landlord j. In this case the term $\Delta\Pi$ is not needed since the tenant works for one landlord before and after the blocking. Finally, a tenant may choose to work exclusively for one landlord. This can happen due to the presence of decreasing marginal productivity. In this case $x_{ij} = 1$, but the α_{ij} changes.

Definition 5 *The feasible matching* x *is optimal if* $\sum_{(\ell_i,t_j)\in L\times T} \alpha_{ij} x_{ij} \geq \sum_{(\ell_i,t_j)\in L\times T} \alpha_{ij} x'_{ij}$ *for every feasible matching* x'.

Our game differs from the one in Sotomayor in the following respect. In her paper, the number of partnerships that each player is allowed to form (capacities) are exogenously fixed. Moreover, the formation of one more partnership does not affect the gains from the previous partnerships. Our game can be viewed as an extension of Sotomayor's game where the number of partnerships that are formed is endogenously determined. Effectively, in the

2×2 game that we are analyzing, instead of having one α matrix which represents the gains (profits) from each pair, we have two [see (1) and (3)]. Sotomayor finds the optimal assignment x given one payoff matrix α, while we, on the top of that, have to choose the best x among the x's that are derived from (1) and (3). This adds one more step to Sotomayor's analysis. She showed that if $(u, v; x)$ is a stable outcome, then x is an optimal matching. Clearly, this can be applied in our game and therefore, in search of a stable outcome we can focus on an optimal assignment.

Theorem 6 *Consider a 2×2 landlord-tenant market. If $\bar{\Pi}_{11} + \bar{\Pi}_{12} > \Pi_{11} + \Pi_{22} = \Pi_{12} + \Pi_{21}$, then polyandrous tenancy will occur in any stable outcome.*

Proof. We have to find the optimal matrix x as it was defined in definition 4. The feasible matrices are:

$$x = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; x^{''} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; x^{'''} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \; x^{''''} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}.$$

Clearly under the assumption of this theorem $x^{'''}$ is the optimal matrix. The tenant with the higher productivity works for both landlords and this is a stable outcome. ■

How can one find a stable outcome $(u, v; x)$? Under the assumption of the above theorem tenant 1 works for both landlords. But what can we say about the distribution of the surplus? This depends on the bargaining power distribution among the players. Let's assume that the landlords have all the bargaining power. In this case in a stable outcome no tenant is overpaid [this is similar to the minimum price equilibrium in Roth and Sotomayor (1990)]. First of all it is clear that tenant 2 who is not employed receives $v_2 = 0$. Now let's look at the one-to-one matching (which is not optimal overall) case and assume that the optimal assignment is,

$$x = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Under x, t_1 must receive $v_1 = 0 + (\Pi_{12} - \Pi_{22}) > 0$, that is landlord 1 pays tenant 1 up to landlord 2's incremental profit if he hires tenant 1 instead of his assigned tenant 2. If now tenant 1 works for both landlords he must receive at least $(\Pi_{12} - \Pi_{22})$. Since the landlords will not pay him more than that the payoffs under,

$$x = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix},$$

are, $v_2 = 0$, $v_1 = (\Pi_{12} - \Pi_{22})$, $u_1 + u_2 = \bar{\Pi}_{11} + \bar{\Pi}_{12} - (\Pi_{12} - \Pi_{22})$, with $u_1 \geq \Pi_{11} - (\Pi_{12} - \Pi_{22})$ and $u_2 \geq \Pi_{22}$. Observe, that in equilibrium, eventhough the landlords have all the bargaining power the high skilled tenant receives rents. Without any further assumptions on the relative power between the two landlords we cannot predict how $u_1 + u_2$ will be divided between them.

Example 1

♦ One-to-one matching

$t\backslash\ell$	ℓ_1	ℓ_2
t_1	10	10
t_2	5	5

.

♦ Many-to-one matching

$t\backslash\ell$	ℓ_1	ℓ_2
t_1	8	8
t_2	4	4

.

If tenant 1 works only for one landlord he generates a surplus equal to 10, while if he works for both he generates a surplus equal to 16. Notice that this example satisfies the condition of theorem ??? since $8 + 8 > 10 + 5$. Thus, the optimal matrix is,

$$x = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix},$$

and polyandrous tenancy prevails. The payoffs are: $v_1 = 5$, $v_2 = 0$, $u_1 + u_2 = 16 - 5 = 11$, with $u_1 \geq 5$ and $u_2 \geq 5$.

Example 2

♦ One-to-one matching

$t \backslash \ell$	ℓ_1	ℓ_2
t_1	10	10
t_2	5	5

.

♦ Many-to-one matching

$t \backslash \ell$	ℓ_1	ℓ_2
t_1	7	7
t_2	4	4

.

Notice that this example does not satisfy the condition of theorem ??? since $7+7 < 10+5$. Thus, in this case there are two optimal matrices, i.e.,

$$x = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \text{ and } x^{'} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and polyandrous tenancy does not prevail. The payoffs are: $v_1 = 5$, $v_2 = 0$, $u_1 = 5$ and $u_2 = 5$.

The message of this paper is not that polyandrous tenancy is inefficient. Polyandrous tenancy is an efficient institution and it can become more efficient in the absence of interlinkage. In example 1, for instance, in the absence of interlinked contracts we could have,

$t \backslash \ell$	ℓ_1	ℓ_2
t_1	9	9
t_2	4	4

,

that is polyandrous tenancy would still be optimal but the surplus would increase.

5 Conclusion

In this paper we argue that while interlinkage may enhance efficiency under monogamous tenancy, this desired efficiency property of interlinking contracts may not always hold when we focus our attention on polyandrous tenancy. As we explained, with polyandrous tenancy and the fact that the total available effort (or hours of work) on part of the tenant is limited, as is the case in reality, the landlords enter into wasteful competition for the tenant's effort through over investment of capital. An important aspect of our results is that such inefficiencies of interlinkage are more likely to occur if the tenant is high skilled. This may be disturbing because one could argue that it is relatively difficult for low skilled tenants to obtain employment to begin with and thus if there is polyandrous tenancy it must be the case that most of the tenants in this institution are high skilled. And thus it may not seem unrealistic to wonder if polyandrous tenancy is itself an inefficient institution given that landlords may not always be able to coordinate their decisions. It would certainly be an interesting line of future research to study an environment where the choice between different tenurial institutions is endogenously determined.

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