# Capital Market Imperfections as an Origin of Barriers to Capital Accumulation and Low TFP

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Abstract

We propose a theory were capital market imperfections endogenously generate low TFP and barriers to capital accumulation. We assume that countries are identical but they di¤er in their ability to enforce loan contracts and we show that, in the presence of asymmetric information, countries with low enforcement use ine¢cient technologies in equilibrium. Our …ndings thus formalize the view that asymmetric information problems in the capital markets are more severe in poor than rich countries. Our theory can easily be amended so that poor countries not only have low aggregate TFP but that they are particularly ine¢cient in the production of investment goods. As a result, these countries are characterized by a high relative price of investment goods and a low real investment rate. Our theory also suggests that entrepreneurs have a vested interest in maintaining a status quo with low enforcement since this allows them to extract rents from the factor services they hire.

Keywords: Capital market imperfections; Aggregate productivity; Price of capital; Distortions; Capital accumulation

JEL classi…cation numbers: E13; G14; O11; O40

## 1 Introduction

The large cross-country di¤erences in per capita income have attracted a great deal of research. The evidence indicates that poor countries are characterized by low total factor productivity (see Hall and Jones (1999), Prescott (1998)) and high barriers to capital accumulation (see Jones (1994), Restuccia and Urrutia (2001)). Moreover, barriers to capital accumulation and TFP are negatively

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correlated across countries, suggesting that these observations may not be independent phenomena. In this paper, we propose a theory were capital market imperfections endogenously generate low TFP and barriers to capital accumulation. Our theory is motivated by evidence suggesting that capital markets tend to perform worse in poor than in rich countries and that indicators of financial development are positively and robustly correlated with productivity and investment rates across countries (see Levine (1997) for a survey).

We develop a framework where capital market imperfections (CMI) are at the origin of cross-country differences in TFP. We assume that countries are identical but they differ in their ability to enforce contracts and show that, in the presence of asymmetric information, countries with low enforcement use inefficient technologies in equilibrium. On the other hand, when enforcement is sufficiently high only the high productivity technologies are operated in equilibrium. Our findings thus formalize the view that asymmetric information problems in the capital markets are more severe in poor than rich countries, as emphasized by some early development economists (see, for instance, McKinnon, Shaw). In our theory, entrepreneurs need external funds in order to operate a productive technology and the financing of these activities is complicated by two problems. First, entrepreneurial projects can either be of low or high quality (productivity) and the quality of these projects is not observed by lenders. Second, there is limited enforcement since entrepreneurs can commit, at most, to pay a fraction $\phi$ of the resources they have after production has taken place. In equilibrium, entrepreneurs form coalitions as an incentive-compatible mechanism for allocating resources to their most productive use. The way to provide incentives for low quality entrepreneurs to reveal their type critically depends on the enforcement parameter $\phi$. When high quality entrepreneurs can commit to make a sufficiently high side payment so that low quality entrepreneurs reveal their type, the low productivity technology is not used in equilibrium. This way of providing incentives, however, may not be feasible when enforcement is low. In this case, entrepreneurs with low quality projects report their type only if they are assigned resources to operate their technology which, in turn, leads to low TFP.

Our theory shows that CMI may play an important role in understanding the positive correlation between the real investment rate and the level of per capita income across countries. In a recent study, Hsieh and Klenow (2002) argue that this correlation is due to the fact that poor countries are plagued by low efficiency in the production of investment goods. Low productivity in the investment sector leads, in turn, to a high relative price of capital (in terms of consumption goods) and to a low real investment rate. Hsieh and Klenow conclude that we need a theory not only to explain low productivity in poor countries, but to explain their low productivity in the production of investment goods. Our paper points that CMI can be an important element of this theory. We assume that entrepreneurs produce an intermediate good that is used in the consumption and investment goods sector. The financing of entrepreneurial production (intermediate goods production) is subject to enforcement and asymmetric information problems. If the expenditure share of intermediate goods in production is higher in the investment than in the consumption goods sector, our theory implies that countries with low enforcement not only have low aggregate TFP but that they are particularly inefficient in the production of investment goods. As a result, poor countries are characterized by a high relative price of investment goods and a low real investment rate.

While we do not model the reasons for why enforcement differs across countries, our theory does offer some interesting clues. We show that entrepreneurs make positive profits if and only if enforcement is limited and that entrepreneurial profits, relative to GDP, decreases with enforcement. This finding is explained as follows. When enforcement is limited, the aggregate supply of intermediate goods is constrained which leads to a high relative price of intermediate goods. This, in turn, implies that factor services are more productive in the intermediate goods sector than in the consumption and investment goods sectors. Since we assume that entrepreneurial coalitions act competitively, the price of factor services is driven by the rate of return of these factors in the consumption and investment goods sector. Then, limited enforcement implies that entrepreneurs extract rents from the factor services hired. When enforcement is not limited, marginal productivity of factor services are equated across all sectors and entrepreneurial coalitions can not extract rents. Since we assume that entrepreneurs operate a constant returns to scale technology, and that entrepreneurial coalitions act competitively, it follows that entrepreneurs make zero profits when enforcement is perfect. Our theory does suggest that entrepreneurs may have a vested interest in maintaining a status quo with low enforcement. For a political economy theory of technological change see Krusell and Ríos-Rull (1996).

Laporta et. al. (1998) present evidence that countries differ substantially on the legal protection of investors and in the quality of law enforcement. They conclude that richer countries have higher quality of law enforcement and higher accounting standards. Rajan and Zingales (1998) use cross-country cross-industry data to document that industries that are more dependent on external financing tend to have relatively higher growth rates in countries that have more developed financial markets. This finding seems supportive of the idea that some sectors in the economy are more affected by CMI than others. Interestingly, Rajan and Zingales found that Machinery is one of the sectors in the economy that relies more heavily on external financing.

Our paper contributes to the literature that investigates the quantitative impact of barriers to capital accumulation by providing a rationale for why these barriers exist (see for instance, Chari et. al. (1997), Parente and Prescott (1994), Parente et. al.(2000)). We view our contribution as complementary to the line of inquiry advocated in Parente and Prescott (1999, 2000). These authors argue that a theory of TFP is crucial for understanding the economic development problem. They build a theory where specialized suppliers of inputs to a particular production process have a vested interest in protecting their monopoly rents and block the adoption of more advanced technologies. We obtain similar results but in a framework without monopoly type of arrangements. There is a large literature discussing how financial intermediaries can improve resource allocation in economies with asymmetric information (see, for instance, Bencivenga and Smith (1991), Boyd and Prescott (1986), and Levine (1997) for a survey). A contribution of our paper is to study how enforcement problems affect the optimal way of providing incentives when dealing with imperfect information problems.

## 2 The Model

Short preview of our point in relation to previous literature

Economists do not agree on what is the key factor explaining the observed huge differences in standards of living across rich and poor countries. The wide literature devoted to understanding cross-country income differences can be divided in two different branches: While one branch emphasizes the importance of 'barriers to capital accumulation', the other branch emphasizes the importance of TFP differences across countries. In this paper, we propose a theory that integrates both approaches. In our theory, capital market imperfections endogenously generate both barriers to capital accumulation and low TFP. As a result, capital market imperfections can generate large income differences.

The literature on barriers to capital accumulation is built around a simple variation of the neoclassical growth model. Output $(Y)$ is produced according to the technology $Y = A_i K^{\alpha} L^{1-\alpha}$; where $A_i$ is a country specific parameter indicating TFP of country $i$ and $(K; L)$ are the capital and labor inputs. It is also assumed that capital is accumulated according to $K_{t+1} = (1 - \delta)K_t + \frac{X_t}{q_i}$; where $X_t$ denotes investment at date $t$ and $q_i$ is a country specific parameter indicating the size of barriers to capital accumulation. In the equilibrium of the model economy, $q_i$ is also the price of investment goods in terms of consumption. This observation is important because there is evidence of substantial differences in the relative price of capital across rich and poor countries, with the price of capital being about 5 or 6 times higher in poor than in rich countries. Given this evidence, the following question naturally arises: can differences in barriers to capital accumulation account for the large income differences across countries?

To this end, consider the US as a benchmark country for measuring income differences and assume that $q_{US} = 1$. In this way, $q > 1$ indicates the presence of a 'barrier to capital accumulation' of bigger magnitude than in the U.S. An implication of the neoclassical growth theory is that income ratio between country $j$ and the U.S. is given by

$$\frac{y_{US}}{y_j} = \left(\frac{A_{US}}{A_j}\right)^{\frac{1}{1-\alpha}} q_j^{\frac{\alpha}{1-\alpha}}. \tag{1}$$

If there are no TFP differences across countries $(A_{US} = A_j)$ then relative income differences depend only in the relative size of barriers to capital accumulation. Using NIPA data, the parameter $\alpha$ can be identified (calibrated) with the share of capital income in national income, which gives a value of $\alpha = 1/3$ and an exponent of $q$ of $.5$ in equation (1). A barrier of 4 in country $j$ will imply that the US has an income that is twice as big. This figure, though big, is small in the context of development since the ratio of income of the richest to the poorest countries is in the order of 30.

In accounting for larger income differences, one approach in the literature has been to consider a broader notion of capital such as organizational capital (Parente and Prescott, 1994) or human capital (Chari et. al., 1997). With a broader notion of capital, a larger capital income share is justified and barriers can now have a large effect (notice that the exponent of q in equation (1) is increasing in $\alpha$). A problem with this approach, however, is that we only have direct measures of barriers to physical capital accumulation but not to other forms of capital. Another approach, pioneered by Prescott (1998) and followed by Parente and Prescott (1999, 2000) is to argue that understanding income differences requires building a theory of TFP differences (e.g. the ratio $A_{US}/A_j$).

In our paper, we build a theory of TFP and barriers to capital accumulation that has the potential for generating large income differences across countries. Our model economy is built around a simple disaggregation of the neoclassical aggregate production technology. We assume that there are three sectors in the model economy producing consumption, capital, and intermediate goods according to

$$
\begin{aligned}
C &= A_c \left[K_c^{\alpha} L_c^{1-\alpha}\right]^{1-\mu_c} Z_c^{\mu_c} \\
X &= A_x \left[K_x^{\alpha} L_x^{1-\alpha}\right]^{1-\mu_x} Z_x^{\mu_x} \\
Z &= A_z K_z^{\alpha} L_z^{1-\alpha}
\end{aligned}
$$

where $Z$ denotes production of intermediate goods and $(Z_c; Z_x)$ denote intermediate goods inputs used in the production of consumption and investment goods, respectively. In our economy, the production of intermediate goods is organized by entrepreneurs that have limited resources to finance production. As a result, they need to raise external funds in the capital market. But this is complicated by the fact that they have (ex-post) private information about the productivity parameter $A_z$ and that there are some limits to the full enforcement of loan contracts.

We will assume that countries differ in their capacity to enforce loan contracts. Countries with low enforcement will produce low amounts of intermediate goods $Z$ for two reasons: First, low enforcement will directly limit the amount of resources devoted to the production of $Z$. Second, in the presence of asymmetric information, entrepreneur with low productivity will operate their projects if enforcement is sufficiently low. As a result, capital market imperfections not only (inefficiently) restrict the amount of resources used in the production of intermediate goods but distort the assignment of resources across entrepreneurs. As a result, low enforcement leads to low TFP in the production of intermediate goods and low production of intermediate goods, in turn, cause low TFP in the consumption and capital goods sector. Moreover, if the share of intermediate goods in the production of capital goods is higher than in the production of consumption goods (e.g. $\mu_x > \mu_c$), then low enforcement of loan contracts will also lead to a high price of capital relative to consumption goods.

Agents

The economy is populated by infinitely lived households that make consumption and savings decisions as in the standard Ramsey growth model. The economy is also populated by two period lived overlapping generations of entrepreneurs. Entrepreneurs are endowed with $\varepsilon$ units of labor in their first period of their lives and with an investment project. At age 2, entrepreneurs invest in their projects, receive the proceeds from their investments, consume, and die. Entrepreneurs use their labor income and external funds in order to finance the investment project. The financing of investment projects is complicated by asymmetric information and limited enforcement of loan contracts. We assume limited enforcement of loan contracts since entrepreneurs can only commit to pay back a certain fraction of the resources they hold by the end of the period. Project potential is private information of the entrepreneur. In particular, we assume that projects can be of high or low productivity and that the fraction of low productivity projects is constant over time. For simplicity, we assume that there is no population growth. We normalize the mass of infinitely lived households by 1 and the size of each cohort of entrepreneurs by $\mu$. We denote by $\nu$ the fraction of projects of low quality.

Production

At each point in time, there are three produced goods: consumption, capital, and an intermediate good. Intermediate goods are produced by entrepreneurs. An entrepreneur with a project of quality $i = \{h, l\}$, born in period $t-1$, and that uses $K_{zt}$ units of capital and $L_{zt}$ units of labor in his investment project obtains an amount $Z_t = A_i K_{zt}^{\alpha} L_{zt}^{1-\alpha}$ of intermediate goods, where i can take the values $\{h, l\}$ representing low and high quality projects, respectively. We assume that productivity increases with the quality of the project, that is $A_h > A_l$.

Consumption goods can be produced by firms. Firms combine capital, labor, and intermediate goods according to the c.r.s. technology

$$C_t = F(K_{c,t}, Z_{c,t}, L_{c,t}) = A_c(K_{c,t}^{\alpha_c} L_{c,t}^{1-\alpha_c})^{1-\mu_c} Z_{c,t}^{\mu_c}, \qquad (2)$$

where $C_t$ denotes the consumption goods produced by firms and the $(K_{c,t}, L_{c,t}, Z_{c,t})$ represents the capital, labor, and intermediate goods inputs, respectively. Similarly, capital goods are produced according to the c.r.s. technology

$$X_t = G(K_{x,t}, Z_{x,t}, L_{x,t}) = A_x(K_{x,t}^{\alpha_x} L_{x,t}^{1-\alpha_x})^{1-\mu_x} Z_{x,t}^{\mu_x}, \qquad (3)$$

where $X_t$ denotes production of capital goods and $(K_{x,t}, L_{x,t}, Z_{x,t})$ represents the capital, labor, and intermediate goods inputs in the capital goods sector, respectively.

We assume that firms in the capital and consumption goods sectors behave competitively (take prices as given). Our assumptions imply that firms will make zero profits in equilibrium. For simplicity, and w.l.o.g., we normalize the number of firms in each sector to 1. As a result, the aggregate capital stock in the economy at date t ($K_t$) and the aggregate labor supply ($L_t$) satisfy

$$
\begin{aligned}
L_t &= L_{xt} + L_{ct} + L_{zt}; \\
K_t &= K_{xt} + K_{ct} + K_{zt};
\end{aligned}
$$

where $L_{zt}$ and $K_{zt}$ represent the aggregate labor and capital used by entrepreneurs to produce intermediate goods.

We also assume that

$$\mu_x > \mu_c \tag{A1}$$

so that the capital goods sector is more intensive in the use of intermediate goods. Later on we will assume that the constant $A_x$ vary across countries.

We assume that capital depreciates at a rate $\delta$ so that the aggregate capital stock $K_t$ satisfies the law of motion

$$K_{t+1} = (1 - \delta)K_t + X_t; \tag{4}$$

where $X_t$ denotes the time t production of capital goods.

Households

The representative household behaves competitively taking prices as given. Households save by holding capital, which can be rented to firms in the consumption, capital, and intermediate goods sectors. Since in equilibrium the return of capital will be equated across sectors, we can write the household's decision problem without being specific about how capital is allocated across the consumption, capital, and intermediate goods sectors. The representative household then chooses sequences $\{c_t; k_{t+1}; x_t\}_{t=0}^{\infty}$ of consumption, capital holdings, and purchases of capital in order to solve

$$\text{Max} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}; \tag{5}$$

$$\text{s.t.} \tag{6}$$

$$c_t + q_{x,t}x_t \leq w_t + k_t r_t; \tag{7}$$

$$x_t = k_{t+1} - k_t(1 - \delta); \tag{8}$$

$$c_t; k_t \geq 0 \text{ for all } t \geq 0; \text{and } k_0 \text{ given,} \tag{9}$$

where $\beta \in (0; 1)$ is the discount factor and $\sigma > 0$. Notice that we denote the date t relative price of capital in terms of the contemporaneous consumption good by $q_{x;t}$. Similarly, the date t rental price of capital ($r_t$) and the rental price of labor services ($w_t$) are expressed in terms of the contemporaneous consumption good.

The date t consumption/savings decision is governed by the Euler equation

$$q_{x;t}U_{c_t} = \beta U_{c_{t+1}}[q_{x;t+1}(1 - \delta) + r_{t+1}]: \tag{Euler}$$

The Euler equation, the budget constraint, the transversality condition, and the initial level of capital holdings fully characterized the solution to the household's problem.

Firms

Firms hire capital and labor services and purchase intermediate goods in order to maximize profits. The decision problem of the representative firm in sector $j$; where $j$ stands for the consumption or investment good sector, is given by

$$\operatorname{Max}\ q_j A_j (K_{j;t}^{\alpha_j} L_{j;t}^{1-\alpha_j})^{1-\gamma_j} Z_{j;t}^{\gamma_j} - w_t L_{j;t} - r_t K_{j;t} - q_{z;t} Z_{j;t}; \qquad (10)$$

where $q_{z;t}$ represents the date $t$ price of intermediate goods in terms of the contemporaneous consumption goods, the date $t$ relative price of capital in terms of the contemporaneous consumption good is denoted by $q_j = \begin{cases} 1 \text{ if } j = c \\ q_x \text{ if } j = x \end{cases}$: Optimality conditions then imply

$$w_t = q_j A_j (1-\alpha_j)(1-\gamma_j)\left(\frac{K_{j;t}}{L_{j;t}}\right)^{\alpha_j(1-\gamma_j)}\left(\frac{Z_{j;t}}{L_{j;t}}\right)^{\gamma_j}; \qquad (11)$$

$$r_t = q_j A_j \alpha_j (1-\gamma_j)\left(\frac{K_{j;t}}{L_{j;t}}\right)^{\alpha_j(1-\gamma_j)-1}\left(\frac{Z_{j;t}}{L_{j;t}}\right)^{\gamma_j}; \qquad (12)$$

$$q_{z;t} = q_j A_j \gamma_j \left(\frac{K_{j;t}}{L_{j;t}}\right)^{\alpha_j(1-\gamma_c)}\left(\frac{Z_{j;t}}{L_{j;t}}\right)^{\gamma_j - 1}: \qquad (13)$$

Notice that the marginal product of each of the three production inputs (expressed in terms of consumption goods) is equated across the consumption and capital goods sectors.

Entrepreneurs

Entrepreneurs are assumed to be risk neutral and to consume by the end of their second period of life. Each period a new generation of entrepreneurs is born. Entrepreneurs born in period $t-1$ invest in period $t$: These entrepreneurs start period $t$ with $\eta_t$ units of capital, where $\eta_t = w_{t-1}\varepsilon / q_{x;t-1}$ since entrepreneurs born in period $t-1$ have labor income of $w_{t-1}\varepsilon$ and buy capital at a price, in terms of consumption goods, of $q_{x;t-1}$. In order to invest an amount $I_t > \eta_t$; entrepreneurs need to resort to external financing. But loans to entrepreneurs are complicated by the fact that the type of entrepreneurs is only known to themselves and by enforcement problems. In the next section, we describe how financial intermediaries deal with the presence of asymmetric information and limits to enforcement in the financial market.

# 3 Entrepreneurial Coalitions

Entrepreneurs need external financing but their ability to raise funds is complicated by two capital market imperfections: First, there is a limit to how much entrepreneurs can commit to pay back once the returns of the project are realized. Second, the ability of entrepreneurs is not known by the lenders. Following Boyd and Prescott, we assume that entrepreneurs form coalitions that raise funds from households and organize production among its members. We assume that there is a large number of coalitions and that these coalitions are formed before entrepreneurs learn their type. Financial coalitions raise external funds at the market interest rate and announce production contracts for its members. Production contracts are given by a pair of expenditures (resources used in production) and payment schedules, one for each type of agents. Payments are constrained by enforcement problems: We assume that entrepreneurs can commit to pay at most a fraction $\phi < 1$ of output.

In order to simplify the presentation of the problem faced by the entrepreneurial coalition, it is convenient to use a notation that abstracts from the decision of how to divide total production expenditures between the capital and labor input. To this end, we define output per unit of expenditure as

$$B_i \equiv \max_{K,N} \left\{ q_z A_i K^{\alpha_z} N^{1-\alpha_z} \right\}$$
$$s.t.\ rK + wN \leq 1.$$

It is easy to show that $B_i = A_i (\frac{\alpha}{r})^{\alpha} (\frac{1-\alpha}{w})^{1-\alpha}$ where $i = \{h, l\}$.

Timing of events

The timing of events is as follows:

1. Entrepreneurial coalitions are formed. Coalitions obtain funds and announce investment contracts for each ability type. Contracts are represented by $\{(I_l, L_l), (I_h, L_h)\}$, where $I$ represents the level of expenditure (e.g. the value of resources used in production) and $L$ the payment that entrepreneur contracts to do at the end of the period. Entrepreneurs join a financial coalition by putting their net worth as equity.

2. Entrepreneurs learn their ability.

3. They report their type to the coalition and hire capital and labor with the resources received from the coalition.

4. Production takes place. Entrepreneurs sell the output of intermediate goods, if any, make payments to the coalition, and consume.

Discussion: Ex-ante vs Ex-post information

NOTE: Argue that ex-post information (relative to contracting time) allows for existence of equilibria. If we assume ex-ante information (relative to contracting time) and free entry in the intermediation sector the equilibria does not exist. A way of modeling ex-ante information, would be to focus in efficient contracts (that unfortunately can not be decentralized). The results should be the same.

Entrepreneurs' consumption

Consider an entrepreneur of type $i$: The entrepreneur obtains an output of intermediate goods worth $q_z B_i I_i$ in terms of consumption goods and pay an amount $L_i$ to the coalition. The entrepreneur's consumption is thus given by

$$c_i^e = q_z B_i I_i - L_i = y_i I_i - L_i; \qquad (14)$$

where $y_i = q_z B_i$ denotes the value of output per unit of expenditure in projects of quality $i$. Entrepreneurs' expected consumption when they enter the financial coalition (before knowing their ability) is thus

$$\nu c_l^e + (1 - \nu) c_h^e. \tag{15}$$

Enforcement and Incentive Compatibility

We assume that coalitions have a limited ability to enforce repayments by entrepreneurs. Loan repayment is constrained by

$$L_i \leq \phi q_z B_i I_i = \phi y_i I_i. \qquad \text{(Enforcement)}$$

Since ability type is not publicly observed, contracts are specified so that entrepreneurs report their true type. The following incentive compatibility constraints guarantees that it is in their best interest to report their type truthfully

$$c_i^e = y_i I_i - L_i \geq (1 - \phi) y_i I_{\tilde{i}}; \qquad \text{(incentive compatibility (IC))}$$

for $i$ and $\tilde{i} \in \{l, h\}$. Notice that the maximum punishment that an entrepreneur can receive for lying is equal to a fraction $\phi$ of the gross output of the project.

Feasibility

We assume that Financial Coalitions are sufficiently large so that, as a result of the law of large numbers, a fraction $\nu$ of its members are endowed with projects of low quality. Financial Coalitions obtain funds from two sources: contributions from its members and external funds from its non-members. Because the financing problem is intra-period, the opportunity cost of funds is given by 1. Expenditures are constrained by

$$\nu I_l + (1 - \nu) I_h = E + \eta; \tag{16}$$

where $E$ denote funds raise from households. Payments collected at the end of the period should satisfy

$$E \leq \nu L_l + (1 - \nu) L_h. \qquad \text{(feasibility)}$$

Entrepreneurial Coalition's problem

The objective of Financial Coalitions is to maximize expected consumption of its members by choosing $\{(c_l^e; I_l; L_l); (c_h^e; I_h; L_h); E\}$ in order to solve

$$\begin{aligned} & \text{Max } \nu c_l^e + (1 - \nu) c_h^e \\ & \text{s.t. (enforcement)(IC)(feasibility)} - (16). \end{aligned}$$

Contracts have to be incentive, resource, and enforcement feasible. Notice that Entrepreneurial Coalitions take prices of intermediate goods and factor services as given. Before solving the Entrepreneurial Coalition's problem we specify the market clearing conditions.

Market Clearing

In equilibrium the following markets need to clear for all $t \geq 0$:

1. Labor market

$$L_{ct} + L_{xt} + L_{zt} = 1 + \mu\varepsilon;$$

where $L_{zt}$ denotes the labor used in the production of intermediate goods which satisfies

$$L_{zt} = \frac{(1-\alpha)}{w_t}\mu\nu I_{lt} + (1-\nu)I_{ht}.$$

2. Capital market

$$k_t + \mu\eta_t = K_{xt} + K_{ct} + K_{zt};$$

where $K_{zt}$ denotes the capital used in the production of intermediate goods which satisfies

$$K_{zt} = (\frac{\alpha}{r_t})\mu\nu I_{lt} + (1-\nu)I_{ht}.$$

3. Intermediate goods

$$Z_{ct} + Z_{xt} = Z_t = \mu(\nu B_l I_{lt} + (1-\nu)B_h I_{ht}).$$

4. Consumption goods

$$c_t + \mu c_t^e = C_t;$$

where entrepreneurial consumption $c_t^e$ is defined in expression (14) and $C_t$ is defined in expression (2).

5. Investment goods

$$X_t = k_{t+1} - (1-\delta)k_t + \mu\left[\eta_{t+1} - (1-\delta)\eta_t\right];$$

where $X_t$ denotes production of investment goods, $k_t$ denotes household's holdings of capital goods in $t$.

## 3.1 Full Information

It is convenient to start by considering the case where entrepreneurs' type is known. In this case, there are no truth telling constraints in the maximization problem of the coalition and the allocation of expenditures is only limited by enforcement and resource feasibility problems.

Consumption of entrepreneurs is given by the difference between output of intermediate goods minus the cost of external funds: $c^e = (1-\nu)y_i I_i - E$. Using the feasibility constraint to substitute out for $E$ and plugging the resulting expression in the equation for consumption we obtain

$$c^e = (1-\nu)(y_i - 1)I_i + \eta.$$

The production function of intermediate goods take the form $A_i(\frac{\alpha}{r})^{\alpha}(\frac{1-\alpha}{w})^{1-\alpha}I_i.$ Because of this assumption, when entrepreneurs' type is known, it will be optimal to provide funds only to the high type. From the equation above, a necessary conditions for positive production of intermediate goods is that $y_h \geq 1$. This inequality states that the return on high quality projects be no less than the opportunity cost of funds. In general equilibrium, we shall later see, prices of intermediate goods will be such that this inequality is satisfied. We thus divide the characterization of the Full Information Contract in two cases:

Case 1: $y_h > 1$

In this case, the coalition makes a return, per unit spent, that is higher than the opportunity cost of funds. As a result, optimal investment is given by the maximum enforceable level of expenditure. The optimal amount of expenditure, $I_h^*,$ is obtained by combining the feasibility, payment, and enforcement constraints (all at equality) and is given by

$$I_h^* = \frac{\eta}{(1-\nu)}\frac{1}{[1-\phi y_h]}.$$

Notice that $I_h^*$ is finite only if $\phi y_h < 1.$ In general equilibrium, prices will adjust so that this condition holds.

Case 2 $y_h = 1.$

In this case, their return on high quality projects is equal to the opportunity cost of funds. As a result, the coalition is indifferent about how much to spend so that expenditure can take any value between 0 and the maximum enforceable level $I_h^*.$

The above discussion is summarized in the following proposition:

Proposition 1. Assume $y_h \geq 1 \geq \phi y_h$ and $A_h > A_l.$ Let $I_h^* = \frac{\eta}{(1-\nu)}\frac{1}{[1-\phi y_h]}.$ The Full Information Contract specifies $c_l^e = I_l = L_l = 0$ and for entrepreneurs with projects of high quality it specifies:

Case 1) If $y_h > 1$, then $I_h = I_h^*;$ $c_h^e = (1-\phi)(1-\nu)y_h I_h^*;$ $L_h = \phi y_h I_h^*;$ $E = (1-\nu)I_h^* - \eta.$

Case 2) If $y_h = 1;$ then $I_h \in [0; I_h^*];$ $c_h^e = (1-\phi)(1-\nu)y_h I_h;$ $L_h = \phi y_h I_h;$ $E = (1-\nu)I_h - \eta.$

## 3.2 Asymmetric Information

The full information contract is not incentive compatible under asymmetric information. While low quality entrepreneurs are assigned zero consumption under the full information contract, they can obtain a positive consumption by misreporting their type. As a result, entrepreneurs with low quality projects need to be provided incentives in order to truthfully report their type. This can be done in two ways. In principle, the "cheapest" way would be to provide a transfer $L_l < 0$ so that production decisions do not need to be distorted relative to the full information case. But this way of providing incentives may not be feasible when enforcement is low. In this case, the coalition needs to allocate resources to the low quality projects so that entrepreneurs report the truth. Below, we characterize in detail the contract under asymmetric information.

Maximizing the entrepreneurial coalitions' consumption requires full utilizing all available resources. As a result, the resource and repayment constraint bind as it is established in the next proposition.

Proposition 2.1. The resource and repayment constraint bind in an optimal contract.

Proof. Trivial.

In the next proposition we establish that the incentive compatibility of low quality entrepreneurs bind in the optimal contract. This result should be quite intituitive: the optimal contract should imitate as much as possible the full information allocation. This is done by giving low quality entrepreneurs the minimum possible resources so that they do not lie.

Proposition 2.2: $IC_l$ binds in an optimal contract.

Proof. See appendix B.

Propositon 2.2 shows that low quality entrepreneur need to receive a transfer of resources, relative to the full information case, as an incentive to report the truth. The coalition can provide incentives for low quality entrepreneurs to reveal their type in two ways. The first way consists in giving a side payment to low quality entrepreneurs after production has taken place. The second way is to give resources to low quality entrepreneurs so that they operate their technology. The crucial difference, the reader should notice, is that in the first case only high quality projects are operated. Below we consider in detail these two ways of providing incentives. Then, we focus on the conditions that make each of these ways of incentive provision optimal.

Case I: Characterizing contract when $I_l = 0$:

In this case, entrepreneurs with a low quality project receive a transfer at the end of the period that give them incentives to reveal their type. Using the incentive compatibility constraint $IC_l$ at equality, the transfer received is equal to $-L_l = (1-\phi)y_l I_l$.

Consumption of entrepreneurs is given by the difference between output of intermediate goods and the cost of funds $c^e = (1-\gamma)y_h I_h - E$. Combining this expression with the feasibility constraint we obtain

$$c^e = (1-\gamma)(y_h - 1) I_h + \eta.$$

If the return in high quality projects ($y_h$) is higher than the opportunity cost of funds (1), entrepreneurial consumption is maximized by choosing the highest feasible level of expenditure. In order to understand how this level of expenditure is determined, it is important to bear in mind that the cost of funding one unit of expenditure in high quality projects is composed of two terms. The first term is given by the opportunity cost of funds (1) and the second term is given by the cost of providing incentives to entrepreneurs with low quality projects to reveal their type. In order to report the truth, each entrepreneur with a low project should be payed $(1-\phi)y_l$ per unit invested in high quality projects. Since that there are $\frac{\gamma}{(1-\gamma)}$ entrepreneurs with bad projects per entrepreneur with good projects, the incentive cost of financing one unit of capital in high quality projects is given by $\frac{\gamma}{(1-\gamma)}(1-\phi)y_l$. As a result, the total cost of funding a high quality project is given by $1+\frac{\gamma}{(1-\gamma)}(1-\phi)y_l$ per unit of expenditure.

To find the maximum level of feasible expenditure we set $L_h = \phi y_h I_h$ and combine the feasibility and payment constraints in order to obtain an expression for the amount of funds raised from households

$$E = (1-\gamma)I_h - \eta = (-\gamma(1-\phi)y_l + (1-\gamma)\phi y_h)I_h;$$

and solving for $I_{\theta}$ we obtain

$$I_h^1 = \frac{\eta}{(1-\gamma)}\frac{1}{\left(1+\frac{\gamma}{(1-\gamma)}(1-\phi)y_l-\phi y_h\right)}. \qquad \text{(Investment Case1)}$$

Notice that $\phi y_h < 1+\frac{\gamma}{(1-\gamma)}(1-\phi)y_l$ is a necessary condition for a well defined optimal expenditure level. Otherwise, expenditure is unbounded. To understand this observation the reader should take into account that the entrepreneurial coalition can commit, at most, to repay an amount $\phi y_h$ per unit spent in good projects. When this amount is bigger than the total cost of funds, expenditure is not limited by enforcement problems (the enforcement constraint for the high type does not bind) so that the optimal expenditure level becomes infinity. In general equilibrium, however, prices will adjust so that this will not be an equilibrium outcome.

Notice that external funding is positive when $I_h > \frac{\eta}{1-\gamma}$, which holds true if $\phi y_h > \frac{\gamma}{(1-\gamma)}(1-\phi)y_h$.

It should be said that when prices are such that $y_h = 1$ the optimal level of expenditure is not unique and is given by $I_h \in [0; I_h^1]$. In general equilibrium, investment will be such that the market for intermediate goods clears.

Case 2. Characterizing contract when $I_l > 0$

In this case, entrepreneurs with a low quality project receive an amount of resources that give them incentives to reveal their type. This amount is determined from the incentive compatibility constraint $IC_l$ at equality, which is given by $y_l I_l = (1-\phi)y_l I_h$ and implies that $I_l = (1-\phi)I_h$. Consumption of entrepreneurs is then given by the difference between output of intermediate goods and the cost of funds $c^e = [(1-\gamma)y_h + \gamma(1-\phi)y_l]I_h - E$; where we have made used of the relation $I_l = (1-\phi)I_h$. Using the feasibility constraint we obtain $E = [1 - \nu + \nu(1 - \phi)]I_h - \eta$, which substituted in the equation defining consumption gives

$$c^e = (1 - \nu + \nu(1 - \phi))\,(\hat{y} - 1)\, I_h + \eta,$$

where $\hat{y} \equiv \frac{(1-\nu)y_h + \nu(1-\phi)y_l}{1-\nu+\nu(1-\phi)}$. Notice that $\hat{y}$ represents the average return per unit of expenditure in a good quality project. The denominator of $\hat{y}$ is the aggregate expenditure per unit of investment in a good project. In effect, spending one unit in each good project requires an aggregate expenditure of $1 - v$ in good projects (since the fraction of high quality entrepreneurs is given by $1 - \nu$) and an aggregate expenditure of $\nu(1 - \phi)$ in bad projects (since the fraction of bad projects is given by $\nu$ and each bad entrepreneur invests $(1 - \phi)$ per unit spent in good projects). The numerator of $\hat{y}$, in turn, represents the aggregate output per unit of expenditure in good projects.

When the return per unit of expenditure in high quality projects ($\hat{y}$) is higher than the opportunity cost of funds (1), entrepreneurial consumption is maximized by choosing the highest feasible level of expenditure. To find the maximum level of feasible expenditure we combine the feasibility, the payment, and enforcement constraints for high quality projects, all at equality, in order to obtain

$$I_h^2 = \frac{\eta}{(1-\nu)}\left[\frac{1}{1 + \frac{\nu(1-\phi)}{(1-\nu)} - \phi y_h}\right]. \qquad \text{(Investment Case 2)}$$

Notice that the maximum feasible level of expenditure is well defined (e.g. $I_h^2 \in R_+$) only if $\phi y_h < 1 + \frac{\nu(1-\phi)}{(1-\nu)}$. This condition is quite intuitive: The total cost of financing one unit of expenditure in a good project with external funds is composed of the opportunity cost of funds (1) and an incentive cost of $\frac{\nu(1-\phi)}{1-\nu}$. The incentive cost arises from the fact that there are $\frac{\nu}{1-\nu}$ bad projects per good project and each bad project receives an amount of expenditure equal to $1 - \phi$ of the expenditure in a good project. The entrepreneurial coalition can commit, at most, to repay an amount $\phi y_h$ per unit spent in good projects. When this amount is bigger than the total cost of external financing, expenditure and entrepreneurial consumption are unbounded. In general equilibrium, however, prices will adjust so that this will not be an equilibrium outcome.

Notice that external financing is positive as long as $I_h^2 > \frac{\eta}{1-\nu+\nu(1-\phi)}$, which holds true as long as $\hat{y} \geq 1$ and $\phi > 0$. It should be said that when prices are such that $y_h = 1$ the optimal level of expenditure is not unique and is given by $I_h \in [0, I_h^2]$. In general equilibrium, expenditures are such that the market for intermediate goods clears.

The next proposition establishes that the optimal way to provide incentives depends on the value of $y_l$.

**Proposition 2.3.** (a) If $1 > y_l$, then it is optimal to provide incentives as in Case 1 so that the low productivity technology is not used: $I_h = I_h^1, I_l = 0$.

(b) If $1 < y_l$; then it is optimal to provide incentives as in Case 2 so that the low productivity technology is used: $I_h = I_h^2$ and $I_l = (1 - \phi) I_h^2 > 0$:

Proof. Denoting consumption in case 1 and 2 by $c_1^e$ and $c_2^e$ it is easy to show that $c_1^e > c_2^e$ iff $\frac{(1-\gamma)(y_h - 1)}{1 + \frac{\gamma}{(1-\gamma)}(1-\phi) y_l - \phi y_h} > \frac{(1 - \gamma + \gamma(1-\phi))(y_h - 1)}{1 + \frac{\gamma}{(1-\gamma)}(1-\phi) - \phi y_h}$: Using the definition $y_h$, we can show that the numerator (denominator) of the ratio in the LHS is bigger (smaller) than the numerator (denominator) of the ratio in the RHS if and only if $1 > y_l$. QED.

Proposition 2.3 establishes that when $y_l > 1$ the low productivity technology is operated under the optimal contract (Case 2). This results is quite intutitive: when $y_l > 1$ the low productivity technology is profitable and the optimal way to provide incentives to low quality entrepreneurs to reveal their type is to assign them resources to operate their technology. On the contrary, when $y_l < 1$ the low productivity technology is not profitable and it is not operated in equilibrium.

It should be clear that whether the low productivity technology is profitable or not (e.g. $y_l > 1$ or $y_l < 1$) depends on general equilibrium prices $(q_z; w; r)$: In the next section of the paper we characterize how the general equilibrium value of $y_l$ depends on the enforcement parameter $\phi$:

# 4 Aggregate Implications of Limited Enforcement

In this section we study how limited enforcement affects equilibrium allocations. We show that in general equilibrium the way to provide incentives for low quality entrepreneurs to reveal their type crucially depends on the enforcement parameter $\phi$: In particular, low quality projects are operated only if enforcement is sufficiently low. Moreover, with imperfect enforcement entrepreneurs are able to extract rents from the factors of production that they hire. We also study how the price of capital is affected by the enforcement parameter $\phi$: The analysis focus in steady state equilibria and consists in a comparative statics exercise.

In the previous section we show that the optimal way to provide incentives depend on general equilibrium prices (whether $y_l$ is lower or bigger than 1): We now argue that in the presence of perfect enforcement ($\phi = 1$); the low productivity technology will not be used in the production of intermediate goods.

Proposition 3.1 If enforcement is sufficiently high ($\phi$ close to 1), then the low productivity technology will not be used in equilibrium and the optimal contract is characterized by Case 1 in the previous section of the paper. Moreover, if enforcement is perfect ($\phi = 1$); entrepreneurs do not collect rents in the production of intermediate goods ($y_h = 1$).

Proof. From Proposition 2.3, we know that it suffices to show that $y_l < 1$ when $\phi = 1$: First, notice that $y_h < 1$ cannot hold in general equilibrium because there would not be production of intermediate goods and output will be equal to 0: Second, $y_h > 1$ cannot hold in equilibrium when enforcement is perfect (Á = 1): Otherwise, the production of intermediate goods would be unbounded ($I_h^1$ = 1 ) and we would contradict market clearing in the intermediate goods market. As a result, in equilibrium, prices of intermediate goods will adjust so that $1 = y_h$: It follows that $1 = y_h > y_l$; where the last inequality follows from $A_h > A_l$. Then, Proposition 2.3 implies that the low productivity technology is not used in the production of intermediate goods. By continuity, we know that $y_l > 1$for Á close to 1 so that the low productivity technology is not used when enforcement is almost perfect. QED.

We now …nd restrictions in the parameter space so that if enforcement is su¢ciently low, the low productivity technology will be used in equilibrium and entrepreneurs will make positive pro…ts. We restrict the analysis to the case ± = 1 (capital depreciates in one period) and ¹$_c$ = ¹$_x$ because it greatly simpli…es the algebra. The qualitative results, of course, do not depend on these restrictions.

We …rst show that if there is no enforcement (Á = 0); we can …nd ¹¤ < 1 so that if ¹ = ¹$_c$ = ¹$_x$ > ¹¤ the low productivity technology is used in equilibrium. Intuitively, we …nd conditions so that intermediate goods are su¢ciently scarce for having an equilibrium where low quality entrepreneur make positive pro…ts ($y_l > 1$): From Proposition 2.3 it will then follow that the low productivity technology is used in equilibrium.

Lemma 1. Let ± = 1 and ¹ = ¹$_c$ = ¹$_x$: Then, there exist ¹¤ < 1 so that if ¹ > ¹¤ the low productivity technology is used in equilibrium in the absence of enforcement ( Á = 0) and entrepreneurial coalition make positive pro…ts.

Proof. See appendix.

The result in Lemma 1 is quite intuitive: as the share of intermediate goods (¹) in the production technologies increases, intermediate goods are increasingly scarce and the pro…tability of entrepreneurial production increases. For ¹ suf-…ciently high, low quality projects become pro…table and they will be used in equilibrium.

NOTE: COMMENT ON HOW ¹¤ depends on the technological distance between $A_l$ and $A_h$:

Lemma 2. Let ± = 1 and ¹ > ¹¤: Then, entrepreneurial pro…ts ($y_l$ and $y_h$) decrease with enforcement.

Proof. See appendix.

Naturally, for ¹ …xed, an increase in enforcement reduces the scarcity of intermediate goods and entrepreneurial pro…ts (per unit of expenditure in production) decrease. Using Lemma 1 and Lemma 2 we can establish the following proposition.

Proposition 3.2. Let $\delta = 1$ and $\mu > \mu^*$. Then, there exist $\phi^* < 1$ so that the low productivity technology is used in equilibrium if and only if $\phi < \phi^*$.

We know consider economies with a two final goods sectors ($\mu_c \neq \mu_x$) and find conditions for which an increase in enforcement leads to a lower price of capital in terms of consumption.

Proposition 3.3. Let $\delta = 1$ and $\mu_c \neq \mu_x$. Then, the relative price of investment goods decreases with the level of enforcement $\phi$ when assumption A1 is satisfied.

Proof. See appendix.

# 5 National Income Accounting

In this section we compute the NIPA of our model economy. The economy is composed of three sectors. In Table 1 we compute the value added in each of the sectors in the economy.

Table 1: NIPA

| Sector | Consumption | Investment | Intermediate |
|---|---|---|---|
| Sales | $C$ | $q_x X$ | $q_z Z$ |
| (minus) Purchases | $-q_z Z_c$ | $-q_z Z_x$ | 0 |
| Value added | $C - q_z Z_c$ | $q_x X - q_z Z_x$ | $q_z Z$ |
| wages | $w L_c$ | $w L_x$ | $w L_z$ |
| return to capital | $r K_c$ | $r K_x$ | $r K_z$ |
| profits | 0 | 0 | $\pi$ |

As the table shows, (gross) National Income in the economy is given by

$$NI = w(L_c + L_x + L_z) + r(K_c + K_x + K_z) + \pi;$$

where $r$ is the gross return to capital (includes depreciation $q_x\delta$) and $\pi$ are profits in the intermediate good sector, which can be positive in equilibrium. National Income equals aggregate value added in the economy which, in turn, is equal to GDP

$$NI = VA = GDP = C + q_x X.$$

Notice that is not clear whether profits received by entrepreneurs are a payment to capital or labor services. We follow the practice of Cooley and Prescott (1995) and Golin (2003) in assuming that the share of capital income in GDP is the same as the contribution of capital income to entrepreneurial profits. As a result, the capital income share in the economy can be computed as follows

$$\alpha = \frac{rK + \alpha\pi}{GDP} = \frac{rK}{GDP - \pi}.$$

## 6 Numerical Experiment

TBW

## 7 References


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# 8 Appendix A: Optimal Contract as a LPP

$$
\begin{aligned}
& \mathrm{Max}_{L_l, L_h, I_l, I_h} \; \nu[y_l I_l - L_l] + (1-\nu)[y_h I_h - L_h] \\
L_l & \leq \phi y_l I_l \\
L_h & \leq \phi y_h I_h \\
y_l I_l - L_l & \geq (1-\phi) y_l I_h \\
y_h I_h - L_h & \geq (1-\phi) y_h I_l \\
\nu I_l + (1-\nu) I_h & = \eta + E \\
E & = \nu L_l + (1-\nu) L_h \\
E, I_l, I_h & \geq 0.
\end{aligned}
$$

Notice that sign of $L_l$ and $L_h$ are unrestricted.

# 9 Appendix B: Proofs of propositions

Proof of Proposition 2.1

Proof. The proof proceeds by contradiction and assume that $IC_l$ does not bind at the optimal contract, e.g. $y_l I_l - L_l > (1-\phi) y_l I_l$. Then we can either reduce $I_l$ or increase $L_l$ by a sufficiently small amount without violating $IC_l$ and find an alternative contract that delivers higher consumption. We divide the analysis in 2 cases:

Case 1: Suppose $I_l > 0$. Then we can decrease $I_l$ by a small amount $\varepsilon > 0$, which allow us to increase $I_h$ by an amount $\frac{\nu}{1-\nu}\varepsilon$ (using feasibility). In order to be sure that the enforcement constraint of the low type is satisfied $L_l \leq \phi y_l I_l$, we decrease $L_l$ by an amount $\phi y_l \varepsilon$. We can also increase $L_h$ by an amount $\phi \frac{\nu}{1-\nu}\varepsilon y_h$ (notice that the enforcement constraint is still satisfied and that the same applies to $IC_h$). Then, consumption of high (low) quality entrepreneurs increase (decrease) by

$$\Delta c_h^e = (1-\phi)\frac{\nu}{1-\nu}\varepsilon y_h;$$
$$\Delta c_l^e = -(1-\phi)\varepsilon y_l;$$

so that aggregating over types we obtain a change in expected consumption of

$$\Delta c^e = -\nu\Delta c_l^e + (1-\nu)\Delta c_h^e = \varepsilon\nu(1-\phi)(y_h - y_l) > 0;$$

which contradicts that $IC_l$ does not bind at the optimal contract.

Case 2: Suppose $I_l = 0$. Then we can increase $L_l$ by $\varepsilon > 0$ sufficiently small so that $IC_l$ still holds. Resource feasibility then implies that investment in high quality projects can be increased by an amount

$$\Delta I_h = \frac{\nu\varepsilon}{(1-\nu)}.$$

Notice that the increase in $I_h$ adds slack into the enforcement constraint and IC associated to high quality projects. Aggregate consumption changes as follows

$$\Delta c^e = \nu\varepsilon[-1 + y_h] > 0,$$

if $y_h > 1$: Notice that if $y_h = 1$ we can assume w.l.o.g. that $IC_l$ binds (the optimal amount of consumption is not affected by whether $IC_l$ binds or not).

Proof of Lemma 1,2, and Proposition 3.3. TBW

# 10 Appendix C

A steady state equilibrium $\{w; r; q_x; q_z; K_c; L_c; Z_c; K_x; L_x; Z_x; L_z; K_z; I_l; I_h; E; \eta\}$ can be solved as a system of 17 equations in 17 unknowns : From household problem we obtain 1 equation

$$q_x = \beta\{q_x(1-\delta) + r\}:$$

From firms' in the consumption and investment goods sectors we obtain 6 equations

$$w_t = q_j A_j (1-\alpha_j)(1-\mu_j)\left(\frac{K_{j;t}}{L_{j;t}}\right)^{\alpha_j(1-\mu_j)}\left(\frac{Z_{j;t}}{L_{j;t}}\right)^{\mu_j}; \quad (17)$$

$$r_t = q_j A_j \alpha_j (1-\mu_j)\left(\frac{K_{j;t}}{L_{j;t}}\right)^{\alpha_j(1-\mu_j)-1}\left(\frac{Z_{j;t}}{L_{j;t}}\right)^{\mu_j}; \quad (18)$$

$$q_{z;t} = q_j A_j \mu_j \left(\frac{K_{j;t}}{L_{j;t}}\right)^{\alpha_j(1-\mu_c)}\left(\frac{Z_{j;t}}{L_{j;t}}\right)^{\mu_j - 1}: \quad (19)$$

From Financial Coalitions' problem we obtain 7 equations

$$I_\theta = I(1+r; w; q_x; q_z) \text{ for } \theta \in \Theta \text{ (2 equations)}$$
$$E; \eta; K_Z; L_z; \text{ as functions of } (1+r; w; q_x; q_z)$$

where we use the net worth formula to get rid of this variable. We also have the following equilibrium relationship between aggregate variables

$$\begin{aligned} L_c + L_x + L_z &= L + \mu\psi \\ Z_c + Z_x &= Z = \mu\nu B_l l_l + \mu(1-\nu)B_h l_h \\ \delta(K_c + K_x + K_z) &= A_x \left[K_x^{\alpha} L_x^{1-\alpha}\right]^{1-\mu_x} Z_x^{\mu_x}. \end{aligned}$$

Notice that we do not need to compute consumption nor use the market clearing condition for consumption goods because by Walras Law this market will clear. Also, notice that capital used in the production of intermediate goods is equal to $K_z = E + \mu\eta$.

Algorithm to solve for equilibrium:

1. Guess 4 unknowns: $\frac{K_c}{L_c}$, $\frac{Z_c}{L_c}$; $L_c; L_x$.
2. Use the firms' FOC in the consumption goods sector to obtain $q_z; w; r$.
3. Obtain $q_x$ from households' Euler equation: $q_x = \frac{r}{1/\beta - (1-\delta)}$.
4. From firms' FOC in the investment good sector (equations 19 and 20) obtain the follo

$$\begin{aligned} \frac{}{L_x} &= \frac{\alpha_x}{1-\alpha_x}\left(\frac{w}{r}\right) \\ \frac{Z_x}{L_x} &= \frac{\mu_x}{(1-\alpha_x)(1-\mu_x)}\left(\frac{w}{q_z}\right). \end{aligned}$$

5. Use the previous ratios and the guess for $L_x$ in order to compute $K_x$ and $Z_x$.
6. Set the demand of intermediate goods as $Zd = Z_x + Z_c$.
7. Compute $\eta = \frac{w\psi}{q_x}$;set $R = r + (1-\delta)q_x$; and use values of $(q_x; q_z; Z_d)$ to solve Financial Coalitions' problem in order to obtain: $\{l_i; L_i; c_i^e\}_{i\in\{l;h\}}$ ,$E; K_z; Y_z$ (production of intermediate goods); $L_z$ (labor demand in the intermediate goods sector).
8. Check the following four equations:

$$\begin{aligned} \delta(K_x + K_c + K_z) &= A_x\left[K_x^{\alpha} L_x^{1-\alpha}\right]^{1-\mu_x} Z_x^{\mu_x} \\ Z_c + Z_x &= \mu\nu B_l l_l + \mu(1-\nu)B_h l_h \\ q_x A_x (K_x^{\mu_x} Z_x^{1-\mu_x})^{\alpha} L_x^{1-\alpha} = wL_x + rK_x + q_z Z_x & \quad \text{(zero profits)} \\ L_c + L_x + L_z &= 1 + \mu\psi. \end{aligned}$$

   If the four equations are satisfied, we have found an equilibrium. Otherwise, go to (1). Note: we can replace the zero profit condition for any of the FOC of firms in the investment goods sector (so far we have used only two ratios of the three first order conditions!).

9. Once the unknowns are obtained, we can compute $c^e$ and use feasibility in the consumption goods sector to obtain household consumption of market products.