Managing a Partnership Efficiently*

Nataliya Kuribko†

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Abstract

The focus of this paper is on how control rights should be allocated to ensure a successful partnership in a dynamic environment. Control rights can be viewed as an equity share for joint-ventures and R&D consortia, as patent breadth for cumulative innovation, or as the probability that a firm wins a patent race.

Several partners control a multistage project. Each partner has private cost of completing each stage. I describe the set of dynamic contracts that ensures efficient completion of the project in the absence of commitment. I find that in most cases, control rights must be dynamic for the duration of the project. For any stage, the optimal control right allocation depends on the abilities of the partners in the current stage and the sensitivity of future performance to current cost. In an optimal contract, higher expected efficiency of a partner increases his control rights, while higher sensitivity of future expected performance to his current cost diminishes his control right. In the absence of a technology spillover, partners invest at the socially optimal level.

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†Department of Economics, Duke University, Box 90097, Durham, NC 27708-0097. Tel: (919) 491 9063. E-mail: nk13@duke.edu. URL: http://www.duke.edu/∼nk13

1 Introduction.

The focus of this paper is on how control should be allocated to ensure successful partnership in a dynamic environment. Partnerships are widespread. Firms realize gains from cooperation by forming strategic alliances, R&D consortia, joint ventures, or by entering licensing and joint-patenting agreements, etc.

However, potential benefits to partnership are not always realized, because of the unstable character of many joint ventures and R&D consortia. Although these partnerships formed as long termed relationships, Harrigan (1988) found that only 45% of inter-firm R&D networks are successful. Furthermore, Kogut (1988) found a mortality rate of 20% with a peak in years 5 and 6 after the formation of an R&D cooperation.

There may be various reasons for prematurely dissolved partnership, such as risk of sharing proprietary knowledge, (mis)allocation of control, and a variety of different strategic objectives (Hagedoorn (1995), Dussauge and Garette (1999); Harrigan, (1995),(1988); Hladik (1985); Nooteboom (1999)). I focus on optimal allocation of control rights. Following existing literature, I model control rights as a probability with which a partner gets the project in case the partnership dissolves prematurely.

Previous studies (Myerson and Satterthwaite (1983), Cramton et al. (1987), Fieseler et al. (2003)) explored the problem of prematurely dissolved partnership in a static environment. However, empirical studies stress important dynamic aspects to partnerships. In particular, the motives of a company can change over time due to developments in the company itself, its environment and changes within the partnership (Harrigan (1988)).

My analysis incorporates the following salient features of a dynamic environment. First, partnerships are often formed to complete multiple stage projects, with companies differing in *abilities* for each stage. For instance, one company may be better at research, the other at development. Second, the cost realized at a given stage may provide information about the player's abilities in the future stages. I will refer to this phenomenon as *information effect*. Third, companies may invest in the technologies, which improves their abilities for certain stages.

An example of such partnership may be AIM alliance, created by Motorola, IBM and Apple to develop PowerPC microprocessor. To achieve the goal, partners had to complete several stages, such as designing a template of the microprocessor chip, developing the microprocessor based on the template, manufacturing the chip and then distributing the final product.

There is information effect between stages of designing a template and developing the microprocessor based on the template. Motorola was successful at developing the template. Therefore, one would expect it to be more successful at designing the microprocessor from this template.[1]

Several partners control a multistage project. Each partner has private cost of completing each stage. Partners do not commit to participate in the partnership and can end the partnership after any stage. I describe the set of dynamic contracts, which assign the completion of each stage to the most efficient partner, while ensuring the participation of all parties.

[1] However, it was IMB, which improved on Motorola template and designed the microprocessor.

I find that the critical feature of these contracts is flexibility of control rights. Even if initially partners are symmetric, the partnership may not be sustainable for a fixed control right distribution, unless there is no information effect.

Optimal control right allocation depends on agents' abilities and information effects for a current stage. If there is no information effect, as possibly between stages of manufacturing and distributing in AIM alliance example, then the partner who is expected to be better at the completing the stage, should hold more control.

Consider a party with strong information effect. An example might be a start-up company, for which current performance provides a lot of information about future performance. Then joining the partnership is a win-win situation for such partner. If it turns out to be inefficient, it will just pay to a more efficient partner, much less that it would cost it to develop the stage for himself. If it turns out to be very efficient, it will be rewarded with higher control rights. Therefore, it can be given lower control rights in a successful partnership and still be willing to participate.

The definition of partnership can be applied to a broader category than joint ventures and R&D consortia. A cumulative innovation, in which innovators build on each other's discoveries, can be described as a partnership. In this case, the government, or patent office, acts as a coordinator, influencing control rights by adjusting patent breadth, i.e. the extent to which other firms can infringe on the patent. In this paper, I show that there exists a patent breadth for which initial and sequential innovators enter *ex-post* licensing agreement. Once they enter the agreement, they are willing to coordinate their future cooperation through joint-patenting or other kinds of agreements that influence the level of control over future innovation. For this patent breadth, subsequent innovators would partially finance the research of the initial innovator, even if the initial innovator eventually owns the patent. An example of this finding is companies providing grants for university researchers.

In this paper, I design mechanisms that are incentive compatible, interim individually rational, ex-post budget-balanced, which allocate the task at each stage to the partner with the lowest cost. My analysis draws from a static model of partnership developed by Cramton et al. (1987).

Myerson and Satterthwaite (1983) considered an important special case of a static partnership, where ex-ante the property rights belong to only one agent. Cramton, Gibbons and Klemperer (1987) extended this analysis to situations where each agent owns a fraction of a project, and where agents have symmetric independent private values. McAfee (1992) provided simple mechanisms for allocating the assets without information on the distribution of valuations of the asset or the level of risk aversion of the partners. Fieseler, Kittsteiner, and Moldovanu (2003), Kittsteiner (2000), Jehiel and Pauzner (2001) relaxed the assumption of independent values to consider the case where the values for the asset are correlated. A unifying framework for the static analysis of partnership is presented by Moldovanu (2002). Morgan (2004) compares the mechanisms on the basis of fairness, when mechanism designer has no information about the distribution of valuations of the agents.

While the studies described above have relied on a static environment, the present paper focuses on the dynamic aspect of partnership.

Dynamic aspects of partnership have been studied in a range of papers, but the perspectives differ markedly from the one adopted in this paper. Rozenkranz and Shmitz (2003) analyzed the organization of dynamic R&D alliances between two parties each of which chooses an investment level, and decides whether to disclose know-how to each other. They compared the performance of vertical and horizontal structures for different distributions of control rights over the critical resource, given that surplus from collaboration is split according to the Nash bargaining solution. Noldeke and Schmidt (1998) analyzed sequential investment decision under various ownership structures. They assumed that bargaining, which takes place under symmetric information, always results in an efficient use of the asset, no matter how ownership rights are allocated.

Yildirim (2004) studied an allocation of a task in the multi-task project, where a buyer and sellers identities were defined at the beginning.

There exists a vast literature on joint-ventures, but its mainly focus is on cooperation among partners that compete in the output market.

The model is presented in Section 2. A possible implementation mechanism by means of handicapped auction is suggested in Section 3. In Section 4, I show that in the absence of technology spillover, companies invest at the socially-efficient level under partnerships. In Section 5, I apply the model to the cases of an R&D consortium and cumulative innovation process.

2 The Model.

There is a pool of risk-neutral players indexed by i: $i \in \mathcal{N}$, where $\mathcal{N} = \{1, \ldots, n\}$. Players control a common project, which consists of T sequential tasks. At the end of the game, the project is worth V, where V is commonly known. Player i has private information θ_i regarding the cost of completing the task for each stage. Type θ_i is drawn independently across players from distribution F_i^t on the support $[0, \bar{\theta}]$ with positive continuous density; this is common knowledge. F_i^t reflects the ex-ante efficiency (or ability) of player i in period t.

Players learn their abilities over time. For each player, a type drawn in a given period affects how his type is distributed next period, $F_i^t(\theta; \theta_i^{t-1})$, i.e. current cost provides information about next period efficiency. $(\theta_1^0, \ldots, \theta_n^0)$ are commonly known. For simplicity, the current type does not affect how future types are distributed beyond the following period.

I assume that players' utilities are linear in money and assets. I also abstract from limited liability concerns: each player is endowed with sufficient funds, so that any required transfer is feasible.

If the players are unable to reach an agreement in any period, the partnership is dissolved at that point. Each player gets the project with probability equal to his current control rights and must complete the rest of the tasks by himself. In other words, I assume that partners do not cooperate after dissolution of the partnership.[2]

Control rights can be viewed as an equity share for joint-ventures and R&D consortia, as patent breadth for cumulative innovation, or as the probability that a firm wins a patent

[2]There are different ways to model what happens if partners do not reach an agreement. I chose the above representation, because it corresponds to the largest set of efficient partnerships.

race, etc. In the last period, control rights determine the share players get in the project. For every other period, control rights determine the payoffs that player get if the partnership is to dissolve this period.

It is important for control rights to be dynamic over time, because players are not committed to participate in the partnership for the whole duration of project. By reallocating control rights, the contract affects outside payoffs, thus creating the right incentive for each player to join the partnership.

I assume that the value of the asset, V, is sufficiently large to be worth completing the task in each period, so it does not pay to wait another period for lower cost.

An allocation mechanism $\langle \boldsymbol{p}^t, \boldsymbol{s}^t, \boldsymbol{t}^t \rangle_{t=1}^T$ defines probability of completing period task t, allocation of controls for period $t+1$, and transfers, for each period t from 1 to T.

Suppose the partnership is enforceable. Then, according to the Coase Theorem, players would always form partnerships before learning their types. This result does not depend on allocation of initial control rights.

However, if the partnership is not enforceable, then players might defect on the partnership after learning their types. I am looking for incentive-compatible, interim individually-rational and ex-post efficient mechanisms. A mechanism is *ex-post efficient* if the task is assigned to a player with the lowest type:

$$p_i^t(\boldsymbol{\theta}) = \begin{cases} 1, & \text{if} \quad \theta_i^t \leq \min\{\theta_1^t, \dots, \theta_n^t\}, \\ 0, & \text{otherwise.} \end{cases}$$

If such a mechanism exists, the partnership is referred to as *sustainable*. If there is no mechanism designer (government, central authority or intermediate) that can undertake the computational burden, then the allocation can be achieved by the appropriate efficient bidding game (Cramton, et al 1987.). I propose such an auction in Section 3.

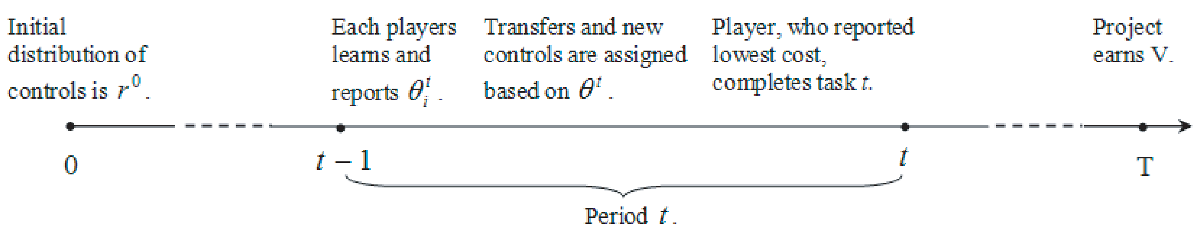

Figure 1: Timeline.

Figure 2 describes timing of the game. The initial partnership is characterized by the control allocation $(r_1^0, \dots, r_n^0)$. In the beginning of each period, player i learns his type. After that each player reports his cost for a given stage θ_i^t. If any player does not participate, the partnership is dissolved and each player gets the project with probability equal to his control right.

Otherwise, based on this information and reported history, $\boldsymbol{\theta}^t = \{(\theta_1^1, \dots, \theta_n^1), \dots, (\theta_1^t, \dots, \theta_n^t)\}$, the period t task is allocated with probability $p^t(\boldsymbol{\theta}^t) = (p_1^t(\boldsymbol{\theta}^t), \dots, p_n^t(\boldsymbol{\theta}^t))$, and the players are assigned new control rights $s^t(\boldsymbol{\theta}^t) = (s_1^t(\boldsymbol{\theta}^t), \dots, s_n^t(\boldsymbol{\theta}^t))$ and transfers $t^t(\boldsymbol{\theta}^t) =$

$(t^t_1(\boldsymbol{\theta}^t), \dots, t^t_n(\boldsymbol{\theta}^t))$ for a given period. I require that these allocations balance: $\sum_i p^t_i(\boldsymbol{\theta}) = 1$, $\sum_i s^t_i(\boldsymbol{\theta}) = 1$ and $\sum_i t^t_i(\boldsymbol{\theta}) = 0$ for any $\boldsymbol{\theta} \in [0, \bar{\theta}]^n$ and $t = \{1, T\}$.

Each player discounts the future by δ, $\delta \in [0, 1]$.

To simplify notations, set $\boldsymbol{r}^t \equiv \boldsymbol{s}^{t-1}$ for $t > 1$, i.e. players start the period with control allocation $\boldsymbol{r}^t$ and by the end of the period are assigned new control rights $\boldsymbol{s}^t$. Also, define:

$$\operatorname{E}\theta^t_i \equiv \operatorname*{E}_{\boldsymbol{\theta}^t} \theta^{t+1}_i \qquad\qquad \operatorname{E}\theta^\tau_i(\theta^t_i) \equiv \operatorname*{E}_{\boldsymbol{\theta}^\tau|\theta^t_i = x} \theta^\tau_i(x)$$

$$\operatorname{E}\theta^t_{min} \equiv \operatorname*{E}_{\boldsymbol{\theta}^t} \min\{\theta^t_1, \dots, \theta^t_n\} \qquad\qquad \operatorname{E}\theta^\tau_{min}(x) \equiv \operatorname*{E}_{\boldsymbol{\theta}^\tau|\theta^t_i = x} \min\{\theta^\tau_1(x), \dots, \theta^\tau_n(x)\}$$

When every player participates in the mechanism, the player i's payoff is equal to the discounted next period payoff minus expected cost plus expected transfers. Therefore, the payoff function for player i in period t is

$$U^t_i(\theta^t_i) = \delta \operatorname{E} U^{t+1}_i(\theta^t_i) - \theta^t_i H^t_i(\theta^t_i) + T^t_i(\theta^t_i),$$

where $H^t_i(x) \equiv \prod_{k \neq i}(1 - F^t_k(x; \theta^{t-1}_k))$ is the probability that $x \leq \min \boldsymbol{\theta}^t_{-i}$.

For the mechanism to be *incentive compatible*, the following constraint must hold for each player in every period:

$$U^t_i(\theta^t_i) \geq \delta \operatorname{E} U^{t+1}_i(x|\theta^t_i) - \theta^t_i H^t_i(x) + T^t_i(x) \quad \forall i \in \mathcal{N}, \quad t = 1, \dots T, \quad x, \theta^t_i \in [0, \bar{\theta}].$$

If the player refuses to participate, he gets the project with probability equal to his control right and has to complete all stages by himself. For the mechanism to be interim *individually rational*, each player must prefer participating in the mechanism to owning the project with probability equal to his control right each period.

$$U^t_i(\theta^t_i) \geq r^t_i\Big(\delta^{T-t}V - \theta^t_i - \sum_{\tau=t+1}^{T} \delta^{\tau-t} \operatorname{E}\theta^\tau_i(\theta^t_i)\Big) \qquad \theta^t_i \in [0, \bar{\theta}] \text{ and } \forall i \in \mathcal{N}, \quad t = 1, \dots T$$

I limit attention to incentive compatible, individually-rational and ex-post budget-balanced mechanisms, which allocate the task ex-post efficiently each period. According to the Revelation Principle (Myerson, 1979), I can restrict attention to incentive-compatible mechanisms without loss of generality.

The worst-off type of player i in period t is the type who has the smallest gain from joining the partnership this period, i.e.

$$\tilde{\theta}^t_i = \operatorname*{argmin}_{\theta} \Big\{U^t_i(\theta) - r^t_i\Big(\delta^{T-t}V - \theta^t_i - \sum_{\tau=t+1}^{T} \delta^{\tau-t} \operatorname{E}\theta^\tau_i(\theta^t_i)\Big)\Big\} \qquad \theta^t_i \in [0, \bar{\theta}].$$

To avoid the complication of looking for global extremum, I assume that each player's utility is convex at his worst-off type. The assumption holds automatically if types are independent across periods.

Lemma 1. *Define* Ψ_i^t *as*

$$\Psi_i^t(\theta_i^t) \equiv r_i^t - H_i^t(\theta_i^t) + \sum_{\tau=t+1}^{T} \delta^{\tau-1}\big[r_i^t \frac{\partial \operatorname{E} \theta_i^\tau({\theta_i}^t)}{\partial \theta_i^t} - \frac{\partial \operatorname{E} \theta_{min}^\tau(\theta_i^t)}{\partial \theta_i^t}\big].$$

The worst-off type of player i *in period* t *is*

$$\tilde{\theta}_i^t(r_i^t) = \begin{cases} x, & \text{if} \quad \Psi_i^t(x) = 0 \text{ and } \quad x \in [0, \bar{\theta}]; \\ 0, & \text{if} \quad \Psi_i^t(x) > 0 \qquad \forall x \in [0, \bar{\theta}]; \\ \bar{\theta}, & \text{if} \quad \Psi_i^t(x) < 0 \qquad \forall x \in [0, \bar{\theta}]. \end{cases}$$

If $\left.\frac{\partial \Psi_i^t(x)}{\partial x}\right|_{x=\tilde{\theta}_i^t(r_i^t)} > 0$*, then* $\frac{\partial \tilde{\theta}_i^t(r_i^t)}{\partial r_i^t} \leq 0.$

Note, that if types are independent across periods, the worst off type simplifies to $\tilde{\theta}_i^t(r_i^t) = (H_i^t)^{-1}(r_i^t)$, and depends on parameters of the current period only. For this case, if player i owns the project then he does not gain anything by participating in the mechanism when he has the lowest possible cost, i.e $\tilde{\theta}_i^t(1) = 0$. On the other hand, if player i has no control over the project, he would not benefit from partnership when his cost is at maximum: $\tilde{\theta}_i(0) = \bar{\theta}$.

In general, if a player's type is lower than worst-off, he is more likely to complete the task when he joins the partnership. Therefore, he needs to be compensated to induce him to report his type truthfully. If a player's type is higher than worst-off, he has an incentive to understate his type. The worst-off type expects to complete the task with the probability equal to his control right, therefore he is not compensated, which makes him the worst-off type.

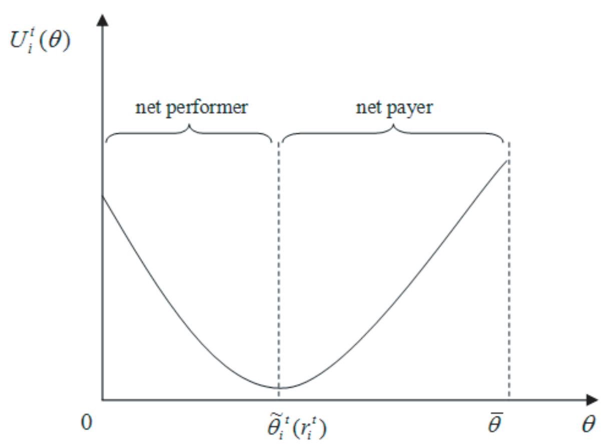

Figure 2: Worst off type for player i. Independent types.

The situation becomes more complex for correlated types. On the one hand, a higher type draw would decrease future reservation utility making players more willing to join the partnership. This *information effect on reservation utility* is reflected by sensitivity of future expected cost to current type, i.e. $\sum_{\tau=t+1}^{T} \delta^{\tau-1} r_i^t \frac{\partial \operatorname{E} \theta_i^\tau({\theta_i}^t)}{\partial \theta_i^t}$.

On the other hand, a higher type draw would decrease the value of the partnership. This *information effect on gain from cooperation* is reflected by sensitivity of future expected cost to current type, i.e. $\sum_{\tau=t+1}^{T} \delta^{\tau-1} \frac{\partial \operatorname{E} \theta^{\tau}_{min}(\theta^t_i)}{\partial \theta^t_i}$. The control right the player has over the project determines which effect is stronger.

To illustrate this point, suppose a player has no control. Then his reservation utility is not affected, and having a higher type discourages the player from joining the partnership. If the player has full control in the partnership, then the first effect dominates and the player would have a stronger incentive to seek the partnership.

Lemma 2. *The payoff from participating in the mechanism for player i in period t is at least*

$$U^t_i(\theta^t_i) = r^t_i \delta^{T-t} V - r^t_i \tilde{\theta}^t_i - \int\limits_{\tilde{\theta}_i}^{\theta^t_i} H^t_i(\theta^t_i) dx + \sum_{\tau=t+1}^{T} \delta^{\tau-t} \Big(\operatorname{E} \theta^{\tau}_{min}(\tilde{\theta}^t_i) - r^t_i \operatorname{E} \theta^{\tau}_i(\tilde{\theta}^t_i) - \operatorname{E} \theta^{\tau}_{min}(\theta^t_i) \Big).$$

Lemma 2 shows that by participating in the partnership, a player gets his reservation utility at the worst-off type plus information rent plus his contribution to the partnership at his realized type adjusted with respect to his worst-off type.

The next lemma demonstrates that the expected payoff of the player is increasing in the control he has in the partnership at the beginning of the period.

Lemma 3. *The expected payoff from participating in the mechanism for player i in period t before he learns his type is*

$$\begin{aligned} \operatorname{E} U^t_i(\theta^t_i) = \delta^{T-t} r^t_i V - \operatorname{E} \theta^t_{min} - \tilde{\theta}^t_i r^t_i + \int\limits_0^{\tilde{\theta}^t_i} H^t_i(x) dx \\ + \sum_{\tau=t+1}^{T} \delta^{\tau-t} \Big\{ \operatorname{E} \theta^{\tau}_{min}(\tilde{\theta}^t_i) - r^t_i \operatorname{E} \theta^{\tau}_i(\tilde{\theta}_i{}^t) - \operatorname{E} \theta^{\tau}_{min} \Big\} \end{aligned}$$

For a static game, according to Makowski and Mezzetti (theorem 3.1) (1994) there exists an ex-post budget-balancing, outcome efficient, interim individually rational, Bayesian incentive compatible mechanism if and only if the surplus generated by a partnership is greater or equal to the sum of expected utilities. A similar result holds for this dynamic game. This leads to the following lemma.

Lemma 4. *The partnership is sustainable if and only if in every period control rights are allocated in such a way that:*

$$\begin{aligned} \Phi^t(\boldsymbol{r}^t) \equiv \sum_{\tau=t+1}^{T} \delta^{T-\tau} \Big\{ (n-1) \operatorname{E} \theta^{\tau}_{min} + \sum_i \big[r^t_i \operatorname{E} \theta^{\tau}_i (\tilde{\theta}_i{}^t(r^t_i)) - \operatorname{E} \theta^{\tau}_{min}(\tilde{\theta}^t_i(r^t_i)) \big] \Big\} \\ + (n-1) \operatorname{E} \theta^t_{min} + \sum_i \Big\{ \tilde{\theta}^t_i(r^t_i) r^t_i - \int\limits_0^{\tilde{\theta}^t_i(r^t_i)} H^t_i(x) dx \Big\} \geq 0 \end{aligned}$$

The payoffs are linear in shares and transfers. Therefore, once the initial task can be allocated efficiently, all subsequent tasks can be allocated efficiently as well by assigning proper control rights to partners.

I will refer to a period t partnership for which worst-off types are equal across the players as *a central partnership* $\boldsymbol{r}^{t*}$:

$$\boldsymbol{r}^{t*} \quad \text{is s.t.} \quad \tilde{\theta}_i^t(r_i^{t*}) = \tilde{\theta}_j^t(r_j^{t*}) \qquad \forall i, j \in \mathcal{N}.$$

The following lemma provides the expression for control rights as a function of the players' characteristics.

Lemma 5. *Define* γ_i^t *as*

$$\gamma_i^t(\tilde{\theta}^t) \equiv \frac{H_i^t(\tilde{\theta}^t) + \sum_{\tau=t+1}^{T} \delta^{\tau-1} \frac{\partial \operatorname{E} \theta_{min}^{\tau}(\tilde{\theta}^t)}{\partial \tilde{\theta}^t}}{1 + \sum_{\tau=t+1}^{T} \delta^{\tau-1} \frac{\partial \operatorname{E} \theta_i^{\tau}(\tilde{\theta}^t)}{\partial \tilde{\theta}^t}}.$$

Then, player i*'s control over central partnership in period* t *is*

$$\begin{cases} r_i^{t*} \in [0, \gamma_i^t(\bar{\theta})] & \text{if} \quad \tilde{\theta}^t = \bar{\theta}; \\ r_i^{t*} = \gamma_i^t(\tilde{\theta}^t) & \text{if} \quad \tilde{\theta}^t \in (0, \bar{\theta}); \\ r_i^{t*} \in [\bar{\theta}, \gamma_i^t(0)] & \text{if} \quad \tilde{\theta}^t = 0. \end{cases}$$

For independent types the above expression simplifies to $r_i^{t*} = H_i^t(\tilde{\theta}^t)$, which is the probability that player i performs at the partnership when he is at his worst-off type. Figure 2 demonstrates the difference between independent and correlated types.

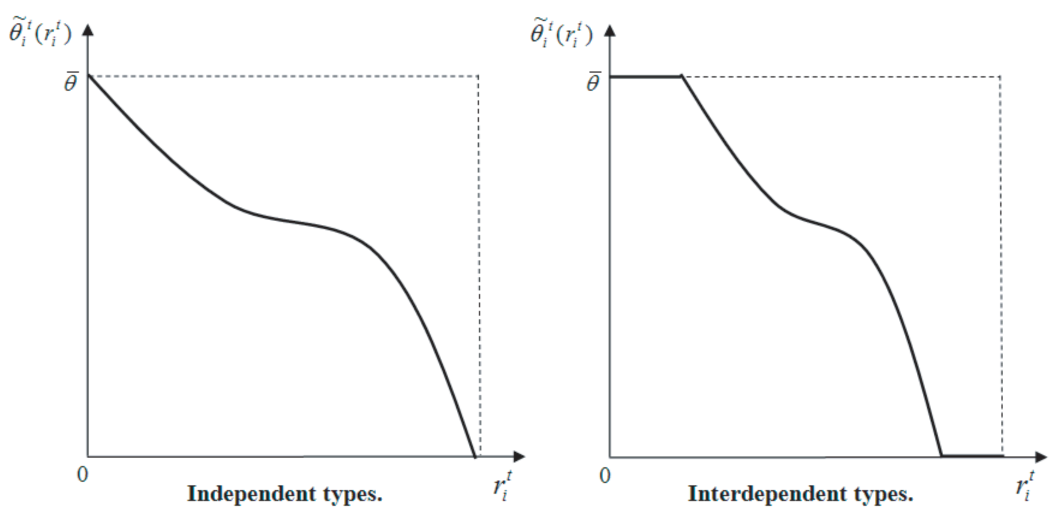

Figure 3: Worst-off type as a function of control right.

For a symmetric one-period model, Cramton et al. (1987) showed that the set of partnerships that can be sustained efficiently is centered around the equal-share partnership. The next proposition shows that in general an efficient allocation of task is guaranteed by assigning future controls s.t. worst-off types are equal.

Proposition 1. *The central partnership is always sustainable. All sustainable partnerships belong to a non-empty convex subset allocated around the central partnership.*

Corollary 1. *For a partnership to be sustainable in every period t, the control rights allocation must belong to a non-empty convex subset of sustainable partnerships allocated around central partnership $\boldsymbol{r}^{t*}$.*

Intuitively, for independent types, equality of worst-off types implies that when each player is at his worst-off type, he is assigned the task with the same probability as he gets the project if he defects. For correlated types, this probability is adjusted for which information effect is stronger at player's worst-off type.

Corollary 1 shows that it is important for control rights to adjust as the project progresses. Consider the R&D consortia example. Since the relative efficiency of performing a given task varies across the members of a consortium (as they often come from different industries), their worst-off type changes as the project moves forward. Therefore, for an R&D consortium to operate efficiently, members need to have the flexibility to choose their involvement in the project, i.e. membership level, during the life of the project.

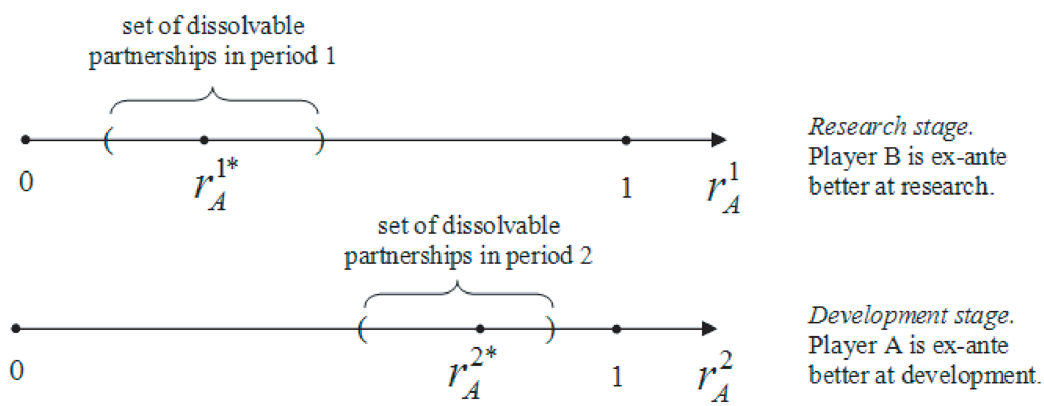

Figure 4: Allocation of control rights over time. Independent types.

Figure 4 demonstrates this statement for the two player case, when player types are independent across periods. Player B is ex-ante better at research, therefore he holds higher control in the partnership at stage 1. Player A is better at development, therefore control in the the central partnership should be redistributed in his favor for the second stage. The figure depicts the case, when there is no fixed control allocation which belongs to the sets of sustainable partnerships for period 1 and 2. Therefore, this partnership is sustainable only if control rights are flexible.

This also gives theoretical support for the empirical evidence that the departure of members does not always signify past poor performance Evan and Olk (1990). Once members cannot contribute to the project, they leave. For example, Browning et al. (1995) have reported that "several founding members left SEMATECH because of the great distance between their primary research focus and the consortiums."

For the example of cumulative innovation, Proposition (1) implies that there always exists a patent breadth for which initial and sequential innovators enter an *ex-post* licensing agreement. Once they enter the agreement, they are willing to coordinate their future cooperation through joint-patenting or other kind of agreements that influence the level of control over innovation.

For independent types, equality of the worst-off types implies higher control rights for players who are more efficient ex-ante. Suppose player i is more efficient in a way that $F_i^t(\theta) \geq F_j^t(\theta)$ for $\forall \theta \in (0, \bar{\theta})$. Then the player is more likely to be assigned the task. Therefore, when worst-off types are the same, player i has higher control in the partnership. Proposition 2 states this finding.

Proposition 2. *Suppose types are independent across periods. If player i is ex-ante more efficient than player j in period t, i.e. $F_i^t(\theta) \geq F_j^t(\theta)$ for $\forall \theta \in [0, \bar{\theta}]$, then player i has higher control rights over the central partnership $\boldsymbol{r}^{t*}$.*

Propositions (1) and (2) imply that the types reported in period t do not influence the allocation of control rights for period $t+1$, if types are independent. It is the relative ex-ante efficiency which drives control right allocation. This effect might be viewed as the cost-economizing rationale for a company to join the partnership.

For correlated types, control right allocation over a central partnership in period t is influenced by the following effects:

1. Higher ex-ante efficiency drives up initial control rights.
2. Higher information effect on reservation utility, i.e. sensitivity of future expected cost to current type, decreases control rights.
3. Higher information effect on gain from cooperation, i.e. sensitivity of future minimum expected cost to current type, increases control rights.

For the first effect, the intuition is similar to the independent case. The ex-ante more efficient player expects to perform with higher probability. To be willing to join the partnership, he should not expect to perform much more often under the partnership than alone.

If the future expected cost of the player become more sensitive to the next period type, he gains more from joining the partnership. In case of a good draw, he will complete the task and be compensated. In case of a bad draw, he will lose less under partnership than if he were to complete the project by himself.

Similar logic applies for the third effect. If the player's type has a higher effect on the minimum expected cost, the partnership provides less "hedging" for the player. Since the gain from partnership decreases, the player needs to be assigned higher initial control rights to be willing to join the partnership.

This is the strategic rationale for joining the partnership. If the company's future cost is very sensitive to current performance, as may be the case for newly created firms, joining the partnership provides a backup in case of an unfavorable outcome. If, however, the company turns out to be an efficient type, it will be rewarded with higher control in the partnership later. This speaks in favor of the existing tradition of favoring well-established partners with more control.

The next proposition states that existing partners always benefit from taking a new member before learning their types. The answer does not depend on the ex-ante efficiency of the member. A prospective new member is defined as a player who does not have control over the project and does not report his type.

Proposition 3. *Adding a new member to a partnership before players learned their type is Pareto-improving. New member will be given some control over the central partnership.*

Proposition 3 also implies that it is never profitable to exclude a partner. If the partner is expected to be very inefficient, he will be assigned very low control rights in a central partnership. However, he will still have an incentive to report his type to a partnership.

If there are several partnerships in the industry that are working on the same problem, each partnership can be considered as a player. Therefore, there exists a control right allocation across the partnerships which allows them to pool their resources together to complete their goal at the minimum cost. This finding provides rationale for an industry-wide consortia.

In every period, there is some net gain for almost all sustainable partnerships. In other words, players participate in the mechanism even if total transfers sum up to some (small) negative number. The more future period exist, the higher net gain the players anticipate. This expands the set of sustainable partnerships in the initial period, which leads to the following proposition:

Proposition 4. *The set of sustainable partnerships for the whole project strictly includes the set of sustainable partnerships for any sub-project.*

As the number of tasks increases, the relative amount of private information each player holds decreases, suggesting it is very easy to achieve efficiency in early periods, and harder as time passes. In other words, in early periods it is possible for transfers to sum up to a negative number, with some gain to be used in future periods. However, this would not affect the size of the set of sustainable partnerships, since partners internalize this possibility in the first period. A sustainable partnership exists in any period, because control rights are flexible. The players expect control rights to be adjusted such that they belong to the set of sustainable partnerships.

A central partnership is never an one-owner partnership. Cramton, et al. (1987) showed that one-owner partnerships are not sustainable [3] However, this is not necessarily the case for a dynamic partnership. There exists δ^* and T^* for which the amount of private information each player holds becomes small enough to enable even a one-owner partnerships to be efficient. Result 1 states this finding.

Result 1. *There exist* (T^*, δ^*) *s.t. for any* $T > T^*$*, and* $\delta > \delta^*$ *one-owner partnership* $(r_i = 1, r_j = 0, \forall j \neq i)$ *is efficient.*

[3] For the static game, Cramton, et al. (1987) use the term *dissolvable*, since the partnership dissolves at the end of the period. For dynamic games, where the goal is to sustain the partnership, this term might have been confusing.

3 Implementation Mechanism.

This section describes a possible scheme of task allocation by means of a handicapped auction. Partners commit to participating in the auction by accepting side-payments, which may be considered as membership fees. The procedure is stated formally in Proposition 5.

Proposition 5. *The efficient mechanism can be implemented by the bidding game with the winning rule*

$$p_i^t(\theta_1^t,\dots,\theta_n^t) = \begin{cases} 1, & \text{if} \quad (b_i^t)^{-1}(\theta_i^t) \le \min\{(b_1^t)^{-1}(\theta_1^t),\dots,(b_n^t)^{-1}(\theta_n^t)\} \\ 0, & \text{otherwise} \end{cases},$$

payments $\alpha_i^t(b_1^t,\dots,b_n^t) = b_i^t - \frac{1}{n-1}\sum\limits_{j\neq i} b_j^t$,

preceded by side-payments

$$\beta_i^t = \frac{1}{n-1}\operatorname*{E}_{j\neq i}\sum_{j\neq i} b_j^t(\theta_j^t) - \delta\,\mathrm{E}\,U_i^{t+1}(\tilde\theta_i^t) + H_i^t(\tilde\theta_i^t)\tilde\theta_i^t + r_i^t\Big\{\delta^{T-t}V - \tilde\theta_i^t - \sum_{\tau=t+1}^{T}\delta^{\tau-t}t\,\mathrm{E}\,\theta_i^\tau(\tilde\theta_i^t)\Big\} + \frac{1}{n}\Phi^t(\boldsymbol{r}^t).$$

Given the above procedure, partner i's equilibrium bid is

$$b_i^t(\theta_i^t) = \int_{\tilde\theta_i^t}^{\theta_i^t}\theta\,dH_i^t(\theta) + \delta\,\mathrm{E}\,U_i^{t+1}(\tilde\theta_i^t) - \delta\,\mathrm{E}\,U_i^{t+1}(\theta_i^t) + b_i^t(\tilde\theta_i^t) - \sum_{\tau=t+1}^{T}\delta^{\tau-t}\Big\{\mathrm{E}\,\theta_{min}^\tau(\tilde\theta_i^t) - \mathrm{E}\,\theta_{min}^\tau(\theta_i^t)\Big\}.$$

On one hand, a higher bid guarantees higher transfers to the player. On the other, it increases the probability of task assignment. It is not necessarily the highest bid that wins the auction. If the types are independent, the auction "favors" with the task bidders who are ex-ante more efficient, in order to allocate it efficiently. In other words, if $H_i^t(\theta) \le H_j^t(\theta)$ for every θ, then there exists b_i and b_j s.t. $b_i \le b_j$, but bidder i is awarded the task.

If the types are correlated across the periods, the information effects influence both biding strategy and the auctioneer's decision.

Note that while the payments based on the bids do not depend on the current share, the side payments do. The side-payments are higher for players with higher control to compensate them for the fact that they are favored in the auction.

The above auction does not allocate control rights for the next period. It is not necessary, because players are willing to reallocate control rights among themselves by engaging in a free market trade over the rights, before they learn their cost for a given stage. This result follows from Coasean theorem: players will trade, because (1) there is gain to cooperating next period, and (2) at the point of trade, players do not hold any private information which can cause hold-up problem.

4 Dynamic Model with Efforts.

In this section, the players can exert effort at the beginning of each period before learning their net value to improve their chances of getting more favorable private information. For example, firms may invest in improving their technology to lower the cost of developing a new product or performing a given task for a project, or may conduct additional research to improve the value of the product.

The vector of effort exerted in period t is denoted by $\mathbf{e}^t$, where the components are the effort exerted by individual players. To save on notation, I drop the superscript t whenever it does not cause confusion. Once exerted effort is publicly observed, player i faces distribution $F_i^t(\theta \,|\, \theta_i^{t-1}, \mathbf{e})$, $i \in \mathcal{N}$. The cost of effort is given by $C^t(e_i)$. By exerting efforts in period t, player i improves the probability distribution of θ_i^t, but at a decreasing rate: $\frac{\partial F_i^t(\theta \,|\, \mathbf{e})}{\partial e_i} > 0$ and $\frac{\partial^2 F_i^t(\theta \,|\, \mathbf{e})}{\partial e_i^2} < 0$, $\forall \theta \in [0, \bar{\theta}]$.

The socially optimal level of effort in period t maximizes expected social surplus from cooperation in period t:

$$\mathbf{e}^* \in \operatorname*{argmax}_{e_1 \ldots e_n} \left\{ \left(\delta^{T-t} V - \sum_{\tau=t}^{T} \delta^{\tau-t} \operatorname{E} \theta_{min}^{\tau}(\mathbf{e}) \right) - \sum_i C_i^t(e_i) \right\}$$

Before learning his private information, player i's expected payoff from his effort is:

$$\operatorname*{E}_{\theta_i} U_i^t(\theta_i|\mathbf{e}) = \delta^{T-t} r_i^t V - \operatorname{E} \theta_{min}^t(\mathbf{e}) - r_i^t \tilde{\theta}_i^t(\mathbf{e}) + \int_0^{\tilde{\theta}_i^t(\mathbf{e})} H_i^t(x|\mathbf{e}) dx - C_i^t(e_i)$$
$$+ \sum_{\tau=t+1}^{T} \delta^{\tau-t} \left\{ \operatorname{E} \theta_{min}^{\tau}(\tilde{\theta}_i^t(\mathbf{e}))) - r_i^t \operatorname{E} \theta_i^{\tau}(\tilde{\theta}_i^{\,t}(\mathbf{e})) - \operatorname{E} \theta_{min}^{\tau}(\mathbf{e}) \right\}$$

It consists of the sum of the expected valuation of player i's assets if he refuses to cooperate, the expected valuation of asset in period t, and the maximum information rent the player can extract, net of his cost of effort. Investments affect only the expected minimum cost for this task, the information rent and the cost of effort.

I say that there is *a technology spillover* if the effort of one player improves the chances of a higher net value for the other player: $\frac{\partial F_i^t(\theta \,|\, \mathbf{e})}{\partial e_j} > 0$. Again, I assume that probability increases at a decreasing rate: $\frac{\partial^2 F_i^t(\theta \,|\, \mathbf{e})}{\partial e_j^2} > 0$. This is best illustrated by the positive effect that one firm's investment in technology may have on the technological level of another firm.

For simplicity, assume that the social surplus is quasiconcave in effort. This assumption will take away the issue of a local maximum versus a global one.

If the effort of one player does not affect the probability distribution of the other players, then the effort level of the player does not affect his maximum information rent. Therefore, players' optimization problems would be equivalent to the maximization of social surplus.

However, if there is a technology spillover, then a higher investment by the player decreases the maximum information rent he can earn. Therefore, each player's marginal payoff from investing is lower than the marginal change in social surplus. Thus, players would invest less than their socially optimal level when faced with a technology spillover. Proposition 6 formalizes this result.

Proposition 6. *Suppose that the social surplus is quasiconcave in effort levels. Then in the absence of a technology spillover, the players exert effort at the socially optimal level. If there is a technology spillover, then each player exerts effort at less than his socially optimal level.*

If the social surplus function is of a general form, then players may end up exerting effort levels which are locally optimal, while the planner can always choose efforts level that corresponds to a global maximum.

5 Discussion.

In this section, I apply the developed model to the examples of an R&D consortium and cumulative innovation process.

5.1 R&D Consortium.

Partnerships are a widespread form of cooperation in research, product development, technology transfer and marketing. Partners perform tasks within their respective fields, contribute technology, products or skills. They control interim agreements through contracts, which may specify ownership over assets, prices and transfers, project design and delivery schedule, or other terms.

R&D consortia may involve from two up to a hundred companies pooling their resources together to create a new legal entity, with members contributing capital, technology, resources or other assets.

There are several reasons why companies join the partnership. One rationale is to economize on cost of its R&D activities by sharing the costs with one or more other companies. This motivation appears to be particularly important in capital and R&D intensive industries, such as the telecom capital goods industry (Hagedoorn (1993)). The strategic motive of R&D partnerships become important in new, high-risk areas of R&D. In such cases, future performance of firm's technological capabilities remains unclear for a considerable period of time. Most companies, however, join the partnership because of both cost-economizing and strategic motives (Hagedoorn (2002)).

The degree of the firms' control over the partnership can be reflected by equity share, membership level, etc.

Realized cost today affects how costs are distributed for the next task. For example, low cost of innovating this period might imply that the firm has a highly efficient R&D division.

One distinguishing characteristic of a consortium is the continuous changes in membership. Unlike most conventional two-partner joint-ventures, the majority of consortia continue to operate after losing members. This example illustrates the main result of this paper, that as partners' values of the project change over time, the efficient management of a consortium requires reallocation of control. [4] It is especially important when companies come from different industries, and therefore have different areas of specialization.[5]

If partners joined the consortium because of cost-economizing motives, then the most efficient partner at a given stage has higher control in the project, while the least efficient members may have no control, i.e. they may be excluded. Intuitively, if a player is very efficient, he is more likely to be assigned a task. If his control over the project is too low, he cannot secure enough information rent to cover his opportunity costs.

If strategic rationale plays a role too, a player with technology very sensitive to currently realized costs has an additional benefit from joining the partnership - having backup in case of unfavorable outcomes. Therefore, he requires less compensation than a company with an established reputation, and therefore should be assigned less control over the partnership.

5.2 Cumulative Innovation Process.

The model can also be applied to the case of cumulative innovation. While the environment is stark, it helps to illustrate some useful insights.

Cumulative innovations, in which innovators build on each other's discoveries, provide another example of a dynamic partnership. In this setting, a problem arises when later products supplant the earlier products in the market, thus suppressing profits of the initial innovator. This is especially true for biotechnology, as well as computer software and hardware technologies. In such cases, the initial innovator may have insufficient incentive to provide the earlier products (Scotchmer (1999)). A possible solution is to give a broader patent to the earlier innovator, thus improving his bargaining position (Kitch(1977)). However, this action may in turn suppress later innovations (Merges and Nelson (1990)), (1991). Green and Scotchmer (1995) showed that the latter problem can be solved by allowing ex-ante contracting. However, ex-ante agreement requires enforceable contracting.

In this paper I demonstrate that if contracts are non-enforceable[6], firms would enter an *ex-post* licensing agreement given the right patent protection.

As an example, suppose that there are two firms innovating in the same area. The innovation process consists of two stages: an initial innovation and a subsequent innovation. Each stage is patentable.

The values of innovating may be defined as the gain from the innovation minus the cost of developing it. They are distributed independently across firms and stages according to commonly known cumulative distribution functions, $F_A^t(x)$ for player A and $F_B^t(x)$ for player

[4]Indeed, most successful horizontal consortia appear to have varying levels of membership. Deviating from this structure may create greater risk. For example, SEMATECH was originally designed as a horizontal alliance and in 1993 consisted of 11 firms and the US Department of Defence, all of whom shared cost equally. Ultimately, it was unable to function this way and eventually adopted a vertical structure. One of the stated reasons was the large diversity in areas of expertise and levels of technological sophistication among the members.(1993)

[5]For example, the members of the Plastics Recycling Foundation, which is developing technology to recycle plastic bottles, include plastics manufacturers (e.g., Du Pont, Exxon), plastic package makers (e.g., Coca-Cola, Pepsi-Cola), and plastic package users (e.g., Procter R Gamble, Kraft (1990)).

[6]That is, players can defect after learning their values privately

B, where t denotes the stage, $t = \{0, 1\}$. The value of innovating is private information to each player.

Suppose Firm A holds a patent for some basic research, which is required to develop further innovations. Its patent breadth, r_A^0, is exogenously determined by the probability of infringement.

Both firms can develop the initial innovation. After learning their values privately, the firms can enter an *ex-post* licensing agreement. According to the agreement, the license for the basic research is transferred to the firm with the highest value for the first stage innovation.

During the last stage, both firms can develop a subsequent innovation, which makes the initial product obsolete. It may be a next generation drug, a next generation computer, a new version of a software program, etc. After learning their values privately, the firms can enter an *ex-post* licensing agreement, where the license is transferred to the firm with the highest value for the last stage innovation.

If at any stage a firm refuses to enter the licensing agreement, the firm i, which does not hold the patent, undertakes some additional effort, which costs w. The effort can be additional research to differentiate the new product from the initial one so that it does not infringe on the patent for the original product; or it may be compensation to lawyers working to determine whether a new product infringes on an old one. The cost w is commonly known and set to 0 to keep the model clean.[7] If its effort is successful (which happens with probability r_i^t and is observed by all parties), Firm i develops the product and gets value $(-\theta_i)$. If not, Firm j develops an improvement and earns profit $(-\theta_j)$.

I assume that in case of disagreement, a losing party can partially recover its cost to some commonly known value.

For such an environment, I show that there exists a patent breadth for the basic innovation such that later innovators would finance basic research, even if the initial innovator is to own the patent on its invention. This situation is often observed when university researchers are funded by industry grants. Moreover, with the right patent protection, these types of projects will be undertaken whenever it is socially optimal.

6 Conclusion.

This paper explores how control should be allocated to ensure successful partnership in a dynamic environment. Previous studies focus on resolving the problem of prematurely dissolved partnership in a static setting. However, according to empirical studies, there are important dynamic aspects to partnerships.

I explored a dynamic partnership in which partners jointly control a project for multiple periods. Each partner has a private valuation for the project, which changes as time progresses. I design incentive compatible, interim individually rational, budget-balanced mechanisms, which allocate the asset each period to the partner with the highest valuation. I then examine the effort incentives of the partners.

[7] w affects disagreement payoff, and therefore bargaining power. Setting it to 0 does not influence the results in any significant way.

I find that there exists a dynamic contract, under which a partnership continues to operate until the project is completed. To ensure continued participation, control rights may need to be reallocated from stage to stage, especially when companies come from different industries, and with different areas of specialization. If firms join the partnership only because of a cost-economizing motive, the ex-ante more efficient company should have higher control. The strategic rationale comes into play when current costs provide information about future abilities. For instance, a company's future cost may be very sensitive to current performance, particularly for newly created firms. In this case, joining the partnership provides a backup in case of an unfavorable outcome, such as high cost for the future stages. If, however, the company turns out to be high performer, i.e. relatively more efficient, it will be rewarded with higher control. This speaks in favor of the existing tradition of favoring well-established companies with more control in the partnership.

All partners benefit from having new member, even though he might be very inefficient and will require some control in the partnership. This might be a rationale for industry-wide consortia.

I also demonstrate that it is suboptimal to split a project into separate sub-projects. There exists allocation of initial control rights, for which partnership would be sustainable for the whole project, but not for a separate part of the project.

Consider a case of no technology spillover. In the absence of a partnership, the industry overinvest from the social point of view, because of duplicating efforts. Members of partnership are found to invest in technology at socially efficient level. Technology spillover depresses investment level for both partners and non-cooperative players.

A Appendix.

Lemma 1. *Define Ψ_i^t as*

$$\Psi_i^t(\theta_i^t) \equiv r_i^t - H_i^t(\theta_i^t) + \sum_{\tau=t+1}^{T} \delta^{\tau-1} \big[r_i^t \frac{\partial \operatorname{E} \theta_i^\tau({\theta_i}^t)}{\theta_i^t} - \frac{\partial \operatorname{E} \theta_{min}^\tau(\theta_i^t)}{\theta_i^t} \big].$$

The worst-off type of player i in period t is

$$\tilde{\theta}_i^t(r_i^t) = \begin{cases} x, & \text{if} \quad \Psi_i^t(x) = 0 \text{ and } \quad x \in [0, \bar{\theta}]; \\ 0, & \text{if} \quad \Psi_i^t(x) > 0 \qquad \forall x \in [0, \bar{\theta}]; \\ \bar{\theta}, & \text{if} \quad \Psi_i^t(x) < 0 \qquad \forall x \in [0, \bar{\theta}]. \end{cases}$$

Proof. Expected payoff to player i when he reports x as his type in period t is

$$U_i^t(x|\theta_i^t) = \delta \operatorname{E} U_i^{t+1}(\theta_i^t; x) - \theta_i^t H_i^t(x) + T_i^t(x).$$

Reservation utility is

$$\tilde{U}_i^t(\theta_i^t) = r_i^t \Big(\delta^{T-t} V - \theta_i^t - \sum_{\tau=t+1}^{T} \delta^{\tau-t} \operatorname{E} \theta_i^\tau(\theta_i^t) \Big).$$

For the mechanism $\langle p^t, s^t, t^t \rangle$ to be *incentive compatible*, the following constraint must hold for each player in every period:

$$U_i^t(\theta_i^t) = \max_x \delta \operatorname{E} U_i^{t+1}(x|\theta_i^t) - \theta_i^t H_i^t(x) + T_i^t(x),$$
$$\text{and } \forall i \in \mathcal{N}, \quad t = 1, \ldots T, \quad x, \theta_i^t \in [0, \bar{\theta}].$$

I use standard mechanism design approach (see Fudenberg and Tirole, 1999), adjusting it for dynamics. Applying Envelope theorem on above equation implies:

$$\frac{\partial (U_i^t - \tilde{U}_i^t)}{\partial \theta_i^t} = r_i^t - H_i^t(x) + \delta \frac{\partial \operatorname{E} U_i^{t+1}(x|\theta_i^t)}{\partial \theta_i^t}. \tag{1}$$

To ensure that single-crossing property holds, have to check later that

$$\frac{\partial^2 (U_i^t - \tilde{U}_i^t)}{\partial \theta_i^t \partial x} = \frac{\partial H_i^t(x)}{\partial x} + \frac{\partial^2 \delta \operatorname{E} U_i^{t+1}(x|\theta_i^t)}{\partial \theta_i^t \partial x} < 0,$$

Evaluating $\frac{\partial U_i^t}{\partial \theta_i^t}$ at $x = \theta_i^t$ results in

$$\Psi(\theta_i^t) \equiv r_i^t - H_i^t(\theta_i^t) + \delta \frac{\partial \operatorname{E} U_i^{t+1}(\theta_i^t|\theta_i^t)}{\partial \theta_i^t}$$

Define $\tilde{\theta}_i^t$ as

$$\tilde{\theta}_i^t = \begin{cases} x, & \text{if} \quad \Psi_i^t(x) = 0 \text{ and} \quad x \in [0, \bar{\theta}]; \\ 0, & \text{if} \quad \Psi_i^t(x) > 0 \qquad \forall x \in [0, \bar{\theta}]; \\ \bar{\theta}, & \text{if} \quad \Psi_i^t(x) < 0 \qquad \forall x \in [0, \bar{\theta}]. \end{cases}$$

Assuming that $\left. \frac{\partial \Psi(\theta_i^t)}{\partial \theta_i^t} \right|_{\theta_i^t = \theta_i^t} \leq 0$, $\tilde{\theta}_i^t$ minimizes player i's utility. Therefore,

$$U_i^t(\theta_i^t) = U_i^t(\tilde{\theta}_i^t) + \tilde{U}_i^t(\theta_i^t) - \tilde{U}_i^t(\tilde{\theta}_i^t) + \int_{\tilde{\theta}_i}^{\theta_i^t} \Psi(x) dx. \tag{2}$$

For the mechanism to be *individually rational*, the worst-off type must have a non-negative gain from participation:

$$U_i^t(\tilde{\theta}_i^t) \geq \tilde{U}_i^t(\tilde{\theta}_i^t).$$

The game is solved backwards. In the last period:

$$U_i^T(\theta_i^T; x) = S_i^T(\tilde{\theta}_i|x)V + T_i^T(\tilde{\theta}_i^T|x) - \theta_i^T H_i^T(\theta_i^T) + \int_{\tilde{\theta}_i^T}^{\theta_i^T} \theta d\, H_i^T(\theta), \tag{3}$$

where x is reported history of types for player i. Rearranging, we get

$$T_i^T(\theta_i|x) = V[S_i^T(\tilde{\theta}_i|x) - S_i^T(\theta_i|x)] + T_i^T(\tilde{\theta}_i^T|x) + \int_{\tilde{\theta}_i^T}^{\theta_i^T} \theta dH_i^T(\theta).$$

For the mechanism to be individually rational, the worst-off type must have a non-negative gain from participation. This is true when

$$T_i^T(\tilde\theta_i^T|x) \geq (r_i^T - S_i^T(\tilde\theta_i^T|x))V.$$

Minimum net expected payoff is calculated by taking expectation of (3) and plugging in minimum expected transfers for the worst-off type, and recalling that $r_i^T \equiv s_i^{T-1}(x, \boldsymbol{\theta}_{-i}^{T-1})$:

$$\mathrm{E}\, U_i^T(\theta_i^T|x) = S_i^{T-1}(x)V - \mathrm{E}\,\theta_{min}^T - \int_0^{\tilde\theta_i^T} \theta dH_i^T(\theta).$$

Solving backwards, we get that in general

$$\begin{aligned}\mathrm{E}\, U_i^{t+1}(\theta_i^{t+1}|x) = \delta^{T-t-1}S_i^t(x)V - \mathrm{E}\,\theta_{min}^{t+1} - \tilde\theta_i^{t+1}S_i^t(x) + \int_0^{\tilde\theta_i^{t+1}(S_i^t(x))} H_i^{t+1}(x)dx \\ + \sum_{\tau=t+2}^{T}\delta^{\tau-t-1}\Big\{\mathrm{E}\,\theta_{min}^{\tau}(\tilde\theta_i^{t+1}) - S_i^t(x)\,\mathrm{E}\,\theta_i^{\tau}(\tilde\theta_i{}^{t+1}) - \mathrm{E}\,\theta_{min}^{\tau}\Big\}.\end{aligned}$$

Substituting the expression for expected payoff next period into (1):

$$\Psi(\theta_i^t) \equiv \frac{\partial(U_i^t - \tilde U_i^t)}{\partial\theta_i^t} = r_i^t - H_i^t(x) + \sum_{\tau=t+1}^{T}\delta^{\tau-1}\big[r_i^{\tau-1}\frac{\partial\,\mathrm{E}\,\theta_i^{\tau}(\theta_i{}^t)}{\theta_i^t} - \frac{\partial\,\mathrm{E}\,\theta_{min}^{\tau}(\theta_i^t)}{\theta_i^t}\big],$$

and single-crossing property indeed holds:

$$\frac{\partial^2(U_i^t - \tilde U_i^t)}{\partial\theta_i^t\partial x} = \frac{\partial H_i^t(x)}{\partial x} < 0.$$

□

Lemma 2. *Payoff from participating in the mechanism for player i in period t is at least*

$$U_i^t(\theta_i^t) = r_i^t\delta^{T-t}V - r_i^t\tilde\theta_i^t - \int_{\tilde\theta_i}^{\theta_i^t} H_i^t(\theta_i^t)dx + \sum_{\tau=t+1}^{T}\delta^{\tau-t}\Big(\mathrm{E}\,\theta_{min}^{\tau}(\tilde\theta_i^t) - r_i^t\,\mathrm{E}\,\theta_i^{\tau}(\tilde\theta_i^t) - \mathrm{E}\,\theta_{min}^{\tau}(\theta_i^t)\Big).$$

Proof. For the mechanism to be individually rational, transfers to the worst-off type should be at least

$$\begin{aligned}T_i^t(\tilde\theta_i^t) \geq (r_i^t - S_i^t(\tilde\theta_i^t))\delta^{T-t}V - r_i^t\Big(\tilde\theta_i^t + \sum_{\tau=t+1}^{T}\delta^{\tau-t}\,\mathrm{E}\,\theta_i^{\tau}(\tilde\theta_i^t)\Big) \\ -\delta\sum_{\tau=t+2}^{T}\delta^{\tau-t-1}\Big\{\mathrm{E}\,\theta_{min}^{\tau}(\tilde\theta_i^{t+1}) - \delta S_i^t(\tilde\theta_i^t)\,\mathrm{E}\,\theta_i^{\tau}(\tilde\theta_i{}^{t+1}) - \mathrm{E}\,\theta_{min}^{\tau}\Big\} \\ + \tilde\theta_i^t H_i^t(\tilde\theta_i^t) + \delta\,\mathrm{E}\,\theta_{min}^{t+1} + \tilde\theta_i^{t+1}S_i^t(\tilde\theta_i^t) - \delta\int_0^{\tilde\theta_i^{t+1}(S_i^t(\tilde\theta_i^t))} H_i^{t+1}(x)dx.\end{aligned}$$

By definition, expected payoff is equal to

$$U_i^t(\theta_i^t) = -\theta_i^t H_i^t(\theta_i^t) + T_i^t(\theta_i^t) +$$

$$\delta^{T-t} S_i^t(\theta_i^t) V - \delta\,\mathrm{E}\,\theta_{min}^{t+1} - \tilde{\theta}_i^{t+1} S_i^t(\theta_i^t) + \delta \int\limits_0^{\tilde{\theta}_i^{t+1}(S_i^t(\theta_i^t))} H_i^{t+1}(x)dx$$

$$+ \delta \sum_{\tau=t+2}^{T} \delta^{\tau-t-1} \Big\{ \mathrm{E}\,\theta_{min}^{\tau}(\tilde{\theta}_i^{t+1}) - \delta S_i^t(\theta_i^t)\,\mathrm{E}\,\theta_i^{\tau}(\tilde{\theta}_i^{\,t+1}) - \mathrm{E}\,\theta_{min}^{\tau} \Big\}.$$

By plugging in the expression for $T_i^t(\tilde{\theta}_i^t)$ into (2) and using above definition, we obtain expected payoff for player i in period t for incentive-compatible, interim individually rational mechanisms:

$$U_i^t(\theta_i^t) = r_i^t \delta^{T-t} V - r_i^t \tilde{\theta}_i^t - \int\limits_{\tilde{\theta}_i}^{\theta_i^t} H_i^t(\theta_i^t)dx + \sum_{\tau=t+1}^{T} \delta^{\tau-t} \Big(\mathrm{E}\,\theta_{min}^{\tau}(\tilde{\theta}_i^t) - r_i^t\,\mathrm{E}\,\theta_i^{\tau}(\tilde{\theta}_i^t) - \mathrm{E}\,\theta_{min}^{\tau}(\theta_i^t) \Big).$$

□

Lemma 3. *Expected payoff from participating in the mechanism for player i in period t before he learns his type is*

$$\mathrm{E}\,U_i^t(\theta_i^t) = \delta^{T-t} r_i^t V - \mathrm{E}\,\theta_{min}^t - \tilde{\theta}_i^t r_i^t + \int\limits_0^{\tilde{\theta}_i^t} H_i^t(x)dx$$

$$+ \sum_{\tau=t+1}^{T} \delta^{\tau-t} \Big\{ \mathrm{E}\,\theta_{min}^{\tau}(\tilde{\theta}_i^t) - r_i^t\,\mathrm{E}\,\theta_i^{\tau}(\tilde{\theta}_i^{\,t}) - \mathrm{E}\,\theta_{min}^{\tau} \Big\}.$$

Proof. Obtained by taking expectation of payoff given in lemma 2 w.r.t. θ_i^t. □

Lemma 4. *The partnership is sustainable if and only if in every period control rights are allocated in such a way that:*

$$\Phi^t(\boldsymbol{r}^t) \equiv \sum_{\tau=t+1}^{T} \delta^{T-\tau} \Big\{ (n-1)\,\mathrm{E}\,\theta_{min}^{\tau} + \sum_i \big[r_i^t\,\mathrm{E}\,\theta_i^{\tau}(\tilde{\theta}_i^{\,t}(r_i^t)) - \mathrm{E}\,\theta_{min}^{\tau}(\tilde{\theta}_i^t(r_i^t)) \big] \Big\}$$

$$+ (n-1)\,\mathrm{E}\,\theta_{min}^t + \sum_i \Big\{ \tilde{\theta}_i^t(r_i^t) r_i^t - \int\limits_0^{\tilde{\theta}_i^t(r_i^t)} H_i^t(x)dx \Big\} \geq 0.$$

Proof. Total expected surplus generated from the partnership in period t, given that the partnership would not dissolve prematurely, is $\delta^{T-t}V - \sum_{\tau=t}^{T} \delta^{\tau-1}\,\mathrm{E}\,\theta_{min}^{\tau}$. Analogous to static case (see Makowski and Mezzetti (1994), theorem 3.1), there exists an ex-post budget-balancing, outcome efficient, interim individually rational, Bayesian incentive compatible mechanism if and only if

$$\sum_{\tau=t+1}^{T} \delta^{T-\tau} \Big\{ (n-1)\,\mathrm{E}\,\theta_{min}^{\tau} + \sum_i \big[r_i^t\,\mathrm{E}\,\theta_i^{\tau}(\tilde{\theta}_i^{\,t}(r_i^t)) - \mathrm{E}\,\theta_{min}^{\tau}(\tilde{\theta}_i^t(r_i^t)) \big] \Big\}$$

$$+ (n-1)\,\mathrm{E}\,\theta_{min}^t + \sum_i \Big\{ \tilde{\theta}_i^t(r_i^t) r_i^t - \int\limits_0^{\tilde{\theta}_i^t(r_i^t)} H_i^t(x)dx \Big\} \geq 0$$

As long as distribution of control rights in period t satisfies the above expression, the partnership will continue to operate. Therefore, the possible choice of $\{s_i^t(\boldsymbol{\theta}^t)\}_{i=1}^n$ must satisfy

$$\sum_{\tau=t+2}^{T} \delta^{T-\tau}\Big\{(n-1)\,\mathrm{E}\,\theta_{min}^{\tau} + \sum_i \left[S_i^t(\boldsymbol{\theta}_i^t)\,\mathrm{E}\,\theta_i^{\tau}(\tilde{\theta}_i^{\,t}(S_i^t(\boldsymbol{\theta}_i^t))) - \mathrm{E}\,\theta_{min}^{\tau}(\tilde{\theta}_i^t(S_i^t(\boldsymbol{\theta}_i^t)))\right]\Big\}$$

$$+ (n-1)\,\mathrm{E}\,\theta_{min}^{t+1} + \sum_i \Big\{\tilde{\theta}_i^{t+1}(S_i^t(\boldsymbol{\theta}_i^t))S_i^t(\boldsymbol{\theta}_i^t) - \int_0^{\tilde{\theta}_i^{t+1}(S_i^t(\boldsymbol{\theta}_i^t))} H_i^{t+1}(x)dx\Big\} \geq 0.$$

□

Lemma 5. *Define γ_i^t as*

$$\gamma_i^t(\tilde{\theta}^t) \equiv \frac{H_i^t(\tilde{\theta}^t) + \sum_{\tau=t+1}^T \delta^{\tau-1}\frac{\partial\,\mathrm{E}\,\theta_{min}^{\tau}(\tilde{\theta}^t)}{\partial\tilde{\theta}^t}}{1 + \sum_{\tau=t+1}^T \delta^{\tau-1}\frac{\partial\,\mathrm{E}\,\theta_i^{\tau}(\tilde{\theta}^t)}{\partial\tilde{\theta}^t}}.$$

Then, player i's control over central partnership in period t is

$$\begin{cases} r_i^{t*} \in [0, \gamma_i^t(\bar{\theta})] & \textit{if} \quad \tilde{\theta}^t = \bar{\theta}; \\ r_i^{t*} = \gamma_i^t(\tilde{\theta}^t) & \textit{if} \quad \tilde{\theta}^t \in (0, \bar{\theta}); \\ r_i^{t*} \in [\bar{\theta}, \gamma_i^t(0)] & \textit{if} \quad \tilde{\theta}^t = 0. \end{cases}$$

Proof. The control right is derived from the expression for worst-off type, defined in lemma 1. The intervals correspond to the cases when the FOC is always negative, holds with equality, and is always positive. □

Proposition 1. *Central partnership is always sustainable. All sustainable partnerships belong to a non-empty convex subset allocated around the central partnership.*

Proof. Steps to show:

1. Gain from partnership is maximized when worst-off types are equal.

2. When worst-off types are equal, the gain from partnership is positive.

Step 1. For the partnership to be sustainable, the surplus should be greater than expected utilities, as shown in lemma 4:

$$\Phi(r_1, ..., r_n) = \sum_{\tau=t+1}^{T} \delta^{T-\tau}\Big\{(n-1)\,\mathrm{E}\,\theta_{min}^{\tau} + \sum_i \left[r_i^t\,\mathrm{E}\,\theta_i^{\tau}(\tilde{\theta}_i^{\,t}(r_i^t)) - \mathrm{E}\,\theta_{min}^{\tau}(\tilde{\theta}_i^t(r_i^t))\right]\Big\}$$

$$+ (n-1)\,\mathrm{E}\,\theta_{min}^{t} + \sum_i \Big\{\tilde{\theta}_i^t(r_i^t)r_i^t - \int_0^{\tilde{\theta}_i^t(r_i^t)} H_i^t(x)dx\Big\}$$

Control allocation that maximizes above surplus solves

$$\max_{r_1, r_2, ..., r_n} \Phi(r_1, r_2, \ldots, r_n) \quad \text{s.t.} \quad \sum r_i = 1.$$

Φ is concave in r_i:

$$\frac{\partial \Phi(r_1, ..., 1-\sum_{i\neq n} r_i)}{\partial r_i} = \tilde{\theta}_i^0 + \frac{\partial \tilde{\theta}_i^0(r_i))}{\partial r_i}\Big[r_i^t - H_i^t(\theta_i^t) + \sum_{\tau=t+1}^{T}\delta^{\tau-1}\big[r_i^t\frac{\partial \operatorname{E}\theta_i^\tau({\theta_i}^t)}{\partial\theta_i^t} - \frac{\partial \operatorname{E}\theta_{min}^\tau(\theta_i^t)}{\partial\theta_i^t}\big]\Big]$$

If solution for the worst-off type is interior,i.e $\tilde{\theta}_i^0(r_i)\in(0,1)$, then $\Big[r_i^t - H_i^t(\theta_i^t) + \sum_{\tau=t+1}^{T}\delta^{\tau-1}\big[r_i^t\frac{\partial \operatorname{E}\theta_i^\tau({\theta_i}^t)}{\theta_i^t} - \frac{\partial \operatorname{E}\theta_{min}^\tau(\theta_i^t)}{\theta_i^t}\big]\Big] = 0$ by definition of the worst-off type.

If the solution is corner, then $\frac{\partial \tilde{\theta}_i^0(r_i))}{\partial r_i} = 0$. Therefore,

$$\frac{\partial \Phi(r_1, ..., r_n)}{\partial r_i} = \tilde{\theta}_i^0(r_i) \geq 0, \quad \frac{\partial^2 \Phi(r_1, r_2, \ldots, r_n)}{\partial r_i \partial r_j} = 0$$

$$\frac{\partial^2 \Phi(r_1, ..., r_n)}{\partial r_i^2} = \frac{\partial \tilde{\theta}_i^0(r_i)}{\partial r_i} \leq 0$$

$\Phi(r_1, r_2, \ldots, r_n)$ is maximized s.t. $\sum r_i = 1$ when

$$\frac{\partial \Phi(r_1, ..., r_{n-1}, 1-\sum_{k\neq n} r_k)}{\partial r_i} = \tilde{\theta}_i^0(r_i) - \tilde{\theta}_n^0(r_n) = 0,$$

or, equivalently

$$\tilde{\theta}_i^0(r_i) = \tilde{\theta}^0 \quad \forall\ i.$$

Using the equation for worst-off type, we obtain that

$$\sum r_i = \sum_i \frac{H_i^t(\tilde{\theta}^t) - \sum_{\tau=t+1}^{T}\delta^{\tau-1}\frac{\partial \operatorname{E}\theta_{min}^\tau(\tilde{\theta}_i^t)}{\partial\tilde{\theta}_i^t}}{1-\sum_{\tau=t+1}^{T}\delta^{\tau-1}\frac{\partial \operatorname{E}\theta_i^\tau({\tilde{\theta}_i}^t)}{\partial\tilde{\theta}_i^t}} = 1$$

The above expression is continuous in $\tilde{\theta}^0$. Since, it is positive when $\tilde{\theta}^0 = 0$ and negative when $\tilde{\theta}^0 = 1$, there exists $\tilde{\theta} \in [0, \bar{\theta}]$, which satisfies the above equation.

Step 2. The next step is to show that $\Phi(r_1^{t*}, r_2^{t*}, \ldots, r_n^{t*}) \geq 0$. It can be rewritten as

$$\Phi(\boldsymbol{r}^{t*}) = \int_0^{\bar{\theta}}(n-1)(1-F_i^t(x))H_i^t(x)dx + \tilde{\theta}^t - \int_0^{\tilde{\theta}^t}\sum_i H_i^t(x)dx$$

$$+\sum_{\tau=t+1}^{T}\delta^{\tau-1}\Big[(n-1)\int_0^{\bar{\theta}}(1-\bar{F}_i^\tau(x))\bar{H}_i^\tau(x)dx - \int_0^{\bar{\theta}}\sum_i(1-F_i^\tau(x;\tilde{\theta}^t))(\bar{H}_i^\tau(x)-r_i^t)\Big].$$

The first term can be rewritten as

$$\int_0^{\bar{\theta}}(n-1)(1-F_i^t(x))H_i^t(x)dx + \tilde{\theta}^t - \int_0^{\tilde{\theta}^t}\sum_i H_i^t(x)dx$$

$$= \Big\{\int_0^{\bar{\theta}}(n-1)(1-F_i^t(x))H_i^t(x)dx + \tilde{x} - \int_0^{\tilde{x}}\sum_i H_i^t(x)dx\Big\} + \int_{\tilde{x}}^{\tilde{\theta}^t}(1-\sum_i H_i^t(x))dx,$$

where $\tilde{x}$ is s.t. $\sum_i H_i^t(\tilde{x}) = 1$. The term in curly brackets is increasing in x and non-negative when $x = 0$, the integral is non-negative too. The second term is non-negative:

$$\sum_{\tau=t+1}^{T} \delta^{\tau-1}\Big[(n-1)\int_0^{\bar{\theta}}(1-\bar{F}_i^\tau(x))\bar{H}_i^\tau(x)dx - \int_0^{\bar{\theta}}\sum_i(1-F_i^\tau(x;\tilde{\theta}^t))(\bar{H}_i^\tau(x)-r_i^t)dx\Big] \geq$$
$$\sum_{\tau=t+1}^{T} \delta^{\tau-1}\Big[(n-1)\int_0^{\bar{\theta}}(1-\bar{F}_i^\tau(x))\bar{H}_i^\tau(x)dx - \int_0^{\tilde{z}_i^\tau}\sum_i(\bar{H}_i^\tau(x)-r_i^t)dx\Big],$$

where $\tilde{z}_i^\tau$ is s.t. $\sum_i \bar{H}_i^\tau(\tilde{z}_i^\tau) = 1$. It is positive by the same argument as the first term. □

Proposition 2. *Suppose types are independent across periods. If player i is ex-ante more efficient than player j in period t, such that $F_i^t(\theta) \geq F_j^t(\theta)$ for $\forall\theta \in [0,\bar{\theta}]$, then player i has higher control rights over the central partnership $\boldsymbol{r}^{t*}$.*

Proof. The result follows from the fact that for independent types $r_i^{t*} = H_i^t(\tilde{\theta}^t)$. □

Proposition 3. *Adding a new member to a partnership is Pareto-improving. New member will be given some control over central partnership.*

Proof. The expected gain that new player brings to the partnership is

$$\sum_{\tau=t}^{T}\delta^{\tau-t}\,\mathrm{E}\,\theta_{min}^\tau\Big|_{n\text{ players}} - \sum_{\tau=t}^{T}\delta^{\tau-t}\,\mathrm{E}\,\theta_{min}^\tau\Big|_{n+1\text{ players}}$$

Noting that $r_{n+1}^t = 0$, player $n+1$ has to be awarded net payoff of at least

$$\mathop{\mathrm{E}}_{\theta_{n+1}} U_{n+1}^t(\theta_{n+1}) = -\,\mathrm{E}\,\theta_{min}^t\Big|_{n+1\text{ players}} - \delta\,\mathrm{E}\,\theta_{min}^{t+1}\Big|_{n+1\text{ players}} + \int_0^{\tilde{\theta}_{n+1}^t} H_{n+1}^t(x)dx + \delta\,\mathrm{E}\,\theta_{min}^{t+1}(\tilde{\theta}_{n+1}^t)$$

The derivations below show that the gain is not less than payment to a new player:

$$\sum_{\tau=t}^{T}\delta^{\tau-t}\,\mathrm{E}\,\theta_{min}^\tau\Big|_{n\text{ players}} - \sum_{\tau=t+2}^{T}\delta^{\tau-t}\,\mathrm{E}\,\theta_{min}^\tau\Big|_{n+1\text{ players}} - \int_0^{\tilde{\theta}_{n+1}^t} H_{n+1}^t(x)dx - \delta\,\mathrm{E}\,\theta_{min}^{t+1}(\tilde{\theta}_{n+1}^t))$$
$$= \Big\{\sum_{\tau=t+2}^{T}\delta^{\tau-t}\,\mathrm{E}\,\theta_{min}^\tau\Big|_{n\text{ players}} - \sum_{\tau=t+2}^{T}\delta^{\tau-t}\,\mathrm{E}\,\theta_{min}^\tau\Big|_{n+1\text{ players}}\Big\} + \int_0^{\bar{\theta}} H_{n+1}^t(x)dx$$
$$- \int_0^{\tilde{\theta}_{n+1}^t} H_{n+1}^t(x)dx + \delta\int_0^{\bar{\theta}}\bar{H}_{n+1}^{t+1}(x)dx - \delta\int_0^{\bar{\theta}}(1-F_{n+1}^{t+1}(x;\tilde{\theta}_{n+1}^t))\bar{H}_{n+1}^{t+1}(x)dx \geq 0$$

□

Proposition 4. *The set of sustainable partnerships for the whole project strictly includes the set of sustainable partnerships for any sub-project.*

Proof. The set of sustainable partnerships is defined by

$$\Phi^t(\boldsymbol{r}^t) \equiv \sum_{\tau=t+1}^{T} \delta^{T-\tau}\Big\{(n-1)\,\mathrm{E}\,\theta^{\tau}_{min} + \sum_i \big[r^t_i\,\mathrm{E}\,\theta^{\tau}_i(\tilde{\theta}_i{}^t(r^t_i)) - \mathrm{E}\,\theta^{\tau}_{min}(\tilde{\theta}^t_i(r^t_i))\big]\Big\}$$

$$+ (n-1)\,\mathrm{E}\,\theta^t_{min} + \sum_i \Big\{\tilde{\theta}^t_i(r^t_i)r^t_i - \int_0^{\tilde{\theta}^t_i(r^t_i)} H^t_i(x)dx\Big\} \geq 0$$

Splitting a project into subproject decreases the number of periods in the first term. Each period gives positive "extra" gain. Therefore, the set of sustainable partnerships shrinks when a period is removed. □

Proposition 5. *The efficient mechanism can be implemented by the bidding game with the winning rule*

$$p^t_i(\theta^t_1, \ldots, \theta^t_n) = \begin{cases} 1, & \text{if} \quad (b^t_i)^{-1}(\theta^t_i) \leq \min\{(b^t_1)^{-1}(\theta^t_1), \ldots, (b^t_n)^{-1}(\theta^t_n)\} \\ 0, & \text{otherwise} \end{cases},$$

payments $\alpha^t_i(b^t_1, \ldots, b^t_n) = b^t_i - \frac{1}{n-1}\sum_{j\neq i} b^t_j$,

preceded by side-payments

$$\beta^t_i = \frac{1}{n-1}\underset{j\neq i}{\mathrm{E}}\sum_{j\neq i} b^t_j(\theta^t_j) - \delta\,\mathrm{E}\,U^{t+1}_i(\tilde{\theta}^t_i) + H^t_i(\tilde{\theta}^t_i)\tilde{\theta}^t_i$$

$$+ r^t_i\Big\{\delta^{T-t}V - \tilde{\theta}^t_i - \sum_{\tau=t+1}^{T}\delta^{\tau-t}t\,\mathrm{E}\,\theta^{\tau}_i(\tilde{\theta}^t_i)\Big\} + \frac{1}{n}\Phi^t(\boldsymbol{r}^t).$$

Given the above procedure, partner i's equilibrium bid is

$$b^t_i(\theta^t_i) = \int_{\tilde{\theta}^t_i}^{\theta^t_i} \theta\, dH^t_i(\theta) + \delta\,\mathrm{E}\,U^{t+1}_i(\tilde{\theta}^t_i) - \delta\,\mathrm{E}\,U^{t+1}_i(\theta^t_i) + b^t_i(\tilde{\theta}^t_i) - \sum_{\tau=t+1}^{T}\delta^{\tau-t}\Big\{\,\mathrm{E}\,\theta^{\tau}_{min}(\tilde{\theta}^t_i) - \mathrm{E}\,\theta^{\tau}_{min}(\theta^t_i)\Big\}.$$

Proof. To show that $b^t_i(\theta^t_i)$ is an equilibrium bid, suppose that all players except player i follow this strategy. Then best-response for player i is

$$b^t_i(\theta^t_i) = \underset{b}{\mathrm{argmax}}\,\delta\,\mathrm{E}\,U^{t+1}_i\big((b^t_i)^{-1}(b))\big) - \theta^t_i H^t_i\big((b^t_i)^{-1}(b)\big) + b - \frac{1}{n-1}\underset{j\neq i}{\mathrm{E}}\sum_{j\neq i} b^t_j(\theta^t_j).$$

By taking first order condition, setting it to 0 at θ^t_i, and solving differential equation, we get:

$$b^t_i(\theta^t_i) = \int_{\tilde{\theta}^t_i}^{\theta^t_i} \theta\, dH^t_i(\theta) + \delta\,\mathrm{E}\,U^{t+1}_i(\tilde{\theta}^t_i) - \delta\,\mathrm{E}\,U^{t+1}_i(\theta^t_i) + b^t_i(\tilde{\theta}^t_i) - \sum_{\tau=t+1}^{T}\delta^{\tau-t}\Big\{\,\mathrm{E}\,\theta^{\tau}_{min}(\tilde{\theta}^t_i) - \mathrm{E}\,\theta^{\tau}_{min}(\theta^t_i)\Big\}.$$

The constant, $b^t_i(\tilde{\theta}^t_i)$, influences side payments only and therefore can be set to 0.

The side-payments are s.t.

$$
\begin{aligned}
&-\theta_i^t H_i^t(\theta_i^t) + \int_{\tilde{\theta}_i^t}^{\theta_i^t} \theta \, dH_i^t(\theta) + \delta \operatorname{E} U_i^{t+1}(\tilde{\theta}_i^t) + \\
&b_i^t(\tilde{\theta}_i^t) - \sum_{\tau=t+1}^{T} \delta^{\tau-t}\Big\{ \operatorname{E}\theta^\tau_{min}(\tilde{\theta}_i^t) - \operatorname{E}\theta^\tau_{min}(\theta_i^t)\Big\} - \frac{1}{n-1} \operatorname*{E}_{j\neq i} \sum_{j\neq i} b_j^t(\theta_j^t) \geq \\
&r_i^t \delta^{T-t} V - r_i^t \tilde{\theta}_i^t - \int_{\tilde{\theta}_i}^{\theta_i^t} H_i^t(\theta_i^t) dx + \sum_{\tau=t+1}^{T} \delta^{\tau-t}\Big(\operatorname{E}\theta^\tau_{min}(\tilde{\theta}_i^t) - r_i^t \operatorname{E}\theta_i^\tau(\tilde{\theta}_i^t) - \operatorname{E}\theta^\tau_{min}(\theta_i^t)\Big).
\end{aligned}
$$

$$
\beta_i^t \geq \frac{1}{n-1} \operatorname*{E}_{j\neq i} \sum_{j\neq i} b_j^t(\theta_j^t) - \delta \operatorname{E} U_i^{t+1}(\tilde{\theta}_i^t) + H_i^t(\tilde{\theta}_i^t)\tilde{\theta}_i^t + r_i^t\Big\{ \delta^{T-t}V - \tilde{\theta}_i^t - \sum_{\tau=t+1}^{T} \delta^{\tau-t} t \operatorname{E}\theta_i^\tau(\tilde{\theta}_i^t)\Big\}.
$$

Note that $\sum_i \beta_i^t \geq -\Phi^t(\boldsymbol{r}^t)$, which is non-positive. Therefore there exists equilibrium side-payments, which add up to 0. For example, as given in the proposition:

$$
\begin{aligned}
\beta_i^t = \frac{1}{n-1} \operatorname*{E}_{j\neq i} \sum_{j\neq i} b_j^t(\theta_j^t) &- \delta \operatorname{E} U_i^{t+1}(\tilde{\theta}_i^t) + H_i^t(\tilde{\theta}_i^t)\tilde{\theta}_i^t \\
&+ r_i^t\Big\{ \delta^{T-t}V - \tilde{\theta}_i^t - \sum_{\tau=t+1}^{T} \delta^{\tau-t} t \operatorname{E}\theta_i^\tau(\tilde{\theta}_i^t)\Big\} + \frac{1}{n}\Phi^t(\boldsymbol{r}^t).
\end{aligned}
$$

□

Proposition 6. *Suppose that the social surplus is quasiconcave in effort levels. Then in the absence of technology spillover, the players exert effort at the socially optimal level. If there is technology spillover, then each player exerts effort at less than his socially optimal level.*

Proof. Social planner maximizes social surplus

$$
\max_{e_1 \ldots e_n} \Big\{ \delta^{T-t} V - \sum_{\tau=t}^{T} \delta^{\tau-t} \operatorname{E}\theta^\tau_{min}(\mathbf{e}) - \sum_i C_i^t(e_i) \Big\}.
$$

First order conditions are

$$
-\sum_{\tau=t}^{T} \delta^{\tau-t} \frac{\partial \operatorname{E}\theta^\tau_{min}(e_1 \ldots e_n)}{\partial e_i} - \frac{\partial C_i^t(e_i)}{\partial e_i} = 0 \quad \forall i \in \mathcal{N}
$$

Each partner maximizes his expected payoff:

$$
\begin{aligned}
\max_{e_i} \delta^{T-t} r_i^t V - \operatorname{E}\theta^t_{min}(\mathbf{e}) - r_i^t \tilde{\theta}_i^t(\mathbf{e}) + \int_0^{\tilde{\theta}_i^t(\mathbf{e})} H_i^t(x|\mathbf{e}) dx \\
+ \sum_{\tau=t+1}^{T} \delta^{\tau-t}\Big\{ \operatorname{E}\theta^\tau_{min}(\tilde{\theta}_i^t(\mathbf{e}))) - r_i^t \operatorname{E}\theta_i^\tau(\tilde{\theta}_i{}^t(\mathbf{e})) - \operatorname{E}\theta^\tau_{min}(\mathbf{e})\Big\} - C_i^t(e_i)
\end{aligned}
$$

Applying Envelope Theorem, the first order condition for player i is

$$-\sum_{\tau=t}^{T}\delta^{\tau-t}\frac{\partial\,\mathrm{E}\,\theta^{\tau}_{min}(e_1\ldots e_n)}{\partial e_i}+\int_0^{\tilde{\theta}^t_i(\mathbf{e})}\theta\,d\frac{\partial H^t_i}{\partial e_i}(\theta|\mathbf{e})-\frac{\partial C^t_i(e_i)}{\partial e_i}=0\quad\forall i\in\mathcal{N}\tag{4}$$

It coincides with the FOCs for the social surplus maximization if there is no technology spillover, i.e. $\frac{\partial H^t_i}{\partial e_i}(\theta|\mathbf{e})=0$ and therefore an increase in efforts of a player does not affect his information rent.

If there is technological spillover, then the integral in equation (4) is positive. Given the assumptions about the effect of efforts on the distributions, $\mathrm{E}\,\theta_{min}$ is convex in efforts. Therefore, the partners would underinvest if there is a technology spillover. □

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