Sequential decisions in a bank run model*

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Abstract

Diamond and Dybvig (JPE, 1983) and the subsequent literature modelled bank runs as a simultaneous-move game, even though empirical evidence indicates that depositors have information about others´ decision. This paper introduces explicitly sequential moves into the Diamond-Dybvig model. Depositors decide consecutively whether to withdraw their funds or continue holding balances in the bank. If agents can observe the actions of all previous depositors, I show that, contrary to Diamond and Dybvig, there are no bank runs in equilibrium. However, when only withdrawals are observed (and depositors do not know their exact position in the sequence) bank runs re-emerge as possible outcomes. I also consider a third setup in which keeping the funds in the bank is unobservable, but depositors are allowed to make this decision observable, at a cost. If the cost is moderate, then there will be no bank runs. This result suggests that allowing for communication between the bank and the depositors can help prevent bank runs.

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1 Introduction

Empirical evidence suggest that bank runs may not be driven merely by the deterioration of the fundamental variables affecting the bank. Depositors may decide to rush to the bank to withdraw their funds due to panic. For instance, Calomiris and Mason (2003) analyse econometrically the Great Depression and show that in January and February 1933 economic fundamentals leave unexplained a large part of the banking failures. Panic is a candidate to account

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for those unexplained failures. Several of the recent bank run episodes (Abacus Federal Savings Bank (USA), Bank of East Asia (Hong Kong))[1] which did not have a recognizable fundamental reason began with a false (and malicious) rumour which spread among depositors and ended up in a massive rush to the bank.

Diamond and Dybvig (1983) in their classic paper set up a model which explains how bank runs may occur even in absence of fundamental reasons. Decisions in this model and in the subsequent literature are modeled as a simultaneous-move game. However, empirical facts indicate that depositors can observe to some extent the actions of other depositors. It is enough to read descriptions of the banking panics in the nineteenth century (Sprague (1910)) or in the 1930´s (Friedman and Schwartz (1971), Wicker (2001)) which show that there were withdrawing waves. For example, banking panic episodes during the Great Depression lasted for months and withdrawals did not start at once in each panic-stricken region. Starr and Yilmaz (2007) analyze a more recent bank-run episode which affected Turkey's Islamic financial houses in 2001. They study the behavior of depositors of different size (small, medium and large) using statistical analysis. In all of the groups, depositors were responsive to their peers and to the observable behavior of depositors of other groups. Iyer and Puri (2008) examine depositor level data for a bank that faced a run in India in 2001. They show that social network effects (that is, observing what depositors to whom one is connected do) are important regarding the decision-making. This evidence suggest that sequentiality should be incorporated into models studying bank runs.

This paper introduces in an explicit manner sequentiality into the Diamond and Dybvig (1983) model. Diamond and Dybvig show in a setting with no aggregate uncertainty that if depositors decide simultaneously, then demand deposit contracts implement the efficient allocation. However, bank runs are also an equilibrium outcome. It occurs if not only the depositors with an immediate need for their funds (called impatient depositors) withdraw, but also those who do not need their money urgently (called patient depositors). Contrary to the original model, we suppose that depositors decide one-by-one and nature determines the type sequence according to which depositors make decisions. For instance, consider two patient depositors and an impatient one. There are three possible ways to arrange them in a line and nature picks one of these possibilities. Imagine that the type vector (patient,impatient,patient) is chosen. Nature first calls one of the patient depositors to decide whether she wants to withdraw or wait (that is, leave the money deposited), then the impatient one is called, and finally the other patient depositor decides. We assume that the type vector is unobservable, but, depending on the setting, decisions are completely or partially observable. Even though the number of depositors who need immediately liquidity is constant in our model, there is uncertainty about the type vector. Hence, the basic problem is how to interpret withdrawals. They

[1] For a concise description on the Abacus story, see Doug Campbell (2005). About the second run, see, for example, the following url: http://www.nytimes.com/2008/09/25/business/worldbusiness/25emerging.html.

may be due to impatient depositors in need of liquidity or patient depositors who have withdrawn in fear that the bank will not be able to pay later. In the latter case, a patient depositor has incentives to withdraw as well. Another important issue for any patient depositor[2] is that they should take into account that later-coming depositors may observe her action. Therefore, by waiting a patient agent possibly can induce subsequent patient depositors to wait as well.

As decisions are taken one-by-one, the bank accumulates information about the decisions of the depositors depending on which action is observable. The bank reveals to each depositor all the available information it has about the decision of earlier depositors. Andolfatto *et al.* (2007) used this assumption as well, and in general it is in line with the literature which considers the bank as a benevolent planner which acts in interest of the depositors.

We study three different informational scenarios. First, we show that when all previous actions (withdrawal or waiting) are revealed to depositors, then runs do not occur. Second, in the spirit of Peck and Shell (2003) we consider the case in which the observability of actions is restricted to withdrawals. In this setting depositors face uncertainty regarding their position in the type vector and run re-emerges as a possible outcome. However, we show in the third setting, that it need not be the case. We allow (but do not require) depositors to inform the later-coming depositors through the bank, at a cost, about their decision to wait. Even though waitings are unobservable, if the cost is moderate, then in the unique outcome of the game patient depositors will wait. The mere existence of the option to inform about waitings is enough to make withdrawal a dominated action for patient depositors in relevant information sets. As a consequence, patient depositors who wait do not need to inform, so no additional cost is incurred. Through this option to inform, the first best can be implemented costlessly. Hence, sequentiality is enough to eliminate bank runs and implement uniquely the *ex ante* efficient allocation when waiting is or can be made observable.

As argued before, coordination problems seem to be an important factor during financial crises. A relevant policy objective is to design optimal institutions to avoid bank runs which cannot be explained by fundamental reasons. The paper by Ennis and Keister (forthcoming) shows the difficulties of designing mechanisms which can eliminate such bank runs. These bank runs may set back considerably the financial intermediation and consequently the economic growth, as shown by Ennis and Keister (2003). Although we do not formulate concrete policy recommendations, our results indicate that policy should take into account sequentiality and the underlying information structure.

1.1 Related literature

As already mentioned, Diamond and Dybvig (1983) show that the efficient allocation can be implemented through demand deposit contracts, but bank runs are

[2]Impatient depositors always withdraw their money because of liquidity needs, so they do not care about later payments.

also an equilibrium outcome. In view of this possibility, they propose as solution adding a suspension-of-convertibility clause to the deposit contract which eliminates the run equilibrium. In a recent paper, Ennis and Keister (forthcoming) show that there are commitment problems related to the suspension of convertibility, and these problems may create incentives which cause self-fulfilling runs. Hence, the current state of art suggests that run equilibrium might be unavoidable for mechanisms that attempt to implement the efficient allocation in an environment without aggregate liquidity uncertainty. In this paper, we show that it need not be the case if we introduce sequentiality into the model.

In recent years, Green and Lin (2003) were the first to reconsider the implementation problem posed by Diamond and Dybvig (1983). They used an environment with aggregate liquidity uncertainty to show that runs do not occur necessarily. They show in a simultaneous-move setup that if depositors have information about their position in the sequence of decision-making and the deposit contracts offered by the bank are less restrictive than those applied by Diamond and Dybvig (1983), then the efficient allocation can be uniquely implemented. Subsequent papers highlighted that the result depends crucially on supposing the independent determination of each depositor's type (Andolfatto *et al.* (2007)) and showed that if types are correlated, runs reemerge as equilibrium outcomes (Ennis and Keister (2008)).

Andolfatto *et al.* (2007) change the Green and Lin setup by assuming that the bank not only observes but reveals to each depositor the actions of earlier depositors. This is a key element in our first setting as well. However, they maintain the environment with aggregate liquidity uncertainty and they suppose that the payment to those who withdraw is the outcome of a lottery. Our paper is set in an environment without aggregate liquidity uncertainty because of the results by Ennis and Keister (forthcoming) who show the possibility of runs. We think that it is important to show in this simpler setup that bank runs are not unavoidable. We think also that it is more natural to assume that depositors who decide to withdraw have a fair notion about the amount they will get, that is why in our model, similarly to Diamond and Dybvig (1983), all depositors who withdraw receive a constant amount fixed in the deposit contract (unless the bank has run out of funds) instead of facing a lottery. These differences imply that our analysis is substantially different from theirs. The way the two papers arrive at the solution testifies it well. Andolfatto *et al.* (2007) use a backward induction argument to show the no-run result and the analysis does not use the fact that depositors know what has happened. The latter is crucial in our case, and our result in the first setting rests heavily on depositors being able to find out whether previous actions have been truthful or not. For example, whereas in Andolfatto *et al.* (2007) the last depositor always acts truthfully, in our setup it might not be the case if this last depositor infers that earlier there have been patient depositors who have withdrawn. Another difference between the Green-Lin family of models and ours is that although in our model depositor´s type determination is not independent, it does not affect our no-run result.

Our second setting uses Peck and Schell´s (2003) assumption that only withdrawals are observable. Apart from this assumption, their model is different, because they assume aggregate liquidity uncertainty, use contracts in the spirit of Green and Lin and depositors play a simultaneous-move game without any information being revealed to them about other depositors´ decision. In our model, as a consequence of the unobservability assumption, the information the bank can reveal to depositors is the number of previous withdrawals. Depositors cannot be sure of their position in the line and this uncertainty is enough to have bank runs in equilibrium.

Our third setting departs from the literature because we allow (but do not require) depositors to inform the bank, at a cost, about their decision to wait. Therefore, through this new action (called reporting) depositors can make observable their waiting. If they report the decision to wait to the bank, the bank will reveal it to later-coming depositors. In the unique outcome patient depositors will wait without reporting, so the efficient allocation can be implemented uniquely and at no cost. The possibility of reporting can be seen as the possibility of a richer communication between the bank and the depositors. This result is supported by findings of Iyer and Puri (2008) who analyse a micro data set on a bank in India which has been run. They show that the longer and deeper the bank-depositor relationship is, the less likely are depositors to run. The possibility of reporting may be interpreted as sign of a deep bank-depositor relationship.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 discusses the outcome when all previous actions are observable. Section 4 studies what happens if depositors know only about previous withdrawals. In section 5, we discuss the consequences when depositors are allowed but not obliged to report if they wait. Section 6 concludes.

2 The model

2.1 Environment and depositors

There are three periods (T=0,1,2) and a single homogeneous good. Consider a finite number ($n > 2$) of depositors. Each depositor is endowed with 1 unit of the good in period 0. In period 0 each depositor is identical, and faces a privately observed, uninsurable risk of being impatient (*imp*) or patient (*pat*). Thus, the type set is $\boldsymbol{\theta} = \{imp, pat\}$ and θ_i is depositor $i's$ realized type. Nature chooses a constant number $p \in [2, n-1]$ which determines the number of the depositors who are patient.[3] The rest of depositors is impatient. The number of patient and impatient depositors is common knowledge.

Denote by (c_1, c_2) the consumption bundle of an depositor in the two periods. We use the following utility function

$$u(c_1, c_2, \boldsymbol{\theta}_i) = u(c_1 + \boldsymbol{\theta}_i c_2),$$

[3] If everybody is of either type, then our problem becomes irrelevant. If there is only one patient agent, then the first-order conditions (to be derived later) imply that being truthful is a dominant strategy for her.

where $\boldsymbol{\theta}_i$ is a binomial random variable with support $\{0,1\}$. Types are privately learnt in period 1. After realization of types, if $\theta_i = 0$, then the depositor is impatient caring only about consumption in the period 1, otherwise she is patient. The utility function, $u : R^2_{++} \to R$ is twice continuously differentiable, increasing, strictly concave, satisfies the Inada conditions and the relative risk-aversion coefficient $-cu''(c)/u'(c) > 1$ for every c. depositors are expected utility maximizers.

There is a constant-return-to-scale productive technology with the following returns:

T=0	T=1	T=2
-1	0	R
-1	1	0

with $R > 1$, so depositors have to make a decision between $(0, R)$ and $(1, 0)$ in period 1. The long-term return, R, is constant.

2.2 The first best and the bank

If a planner could observe each depositor's type and assign an allocation based on these types, then the resulting first-best allocation would solve

$$\begin{array}{l} \max\ (n-p)u(c_1) + pu(c_2) \\ s.t. \\ (n-p)c_1 + [pc_2/R] = n \end{array}$$

In the formulation of the problem we imposed the optimality condition that the $n-p$ impatient depositors consume only in period 1, whereas the patient depositors consume only in period 2 to earn the return on the deposit.

This problem yields the solution

$$u'(c_1^*) = Ru'(c_2^*),$$

which implies $R > c_2^* > c_1^* > 1$.

The rationale for a bank is the implementation of the first best. The bank pooles the resources and offers a simple demand deposit contract which specifies paying c_1^* to the withdrawing depositors. The bank has to pay to withdrawing depositors immediately c_1^* (unless it has run out of funds) and cannot make depositors wait and condition payment on information which is not available at the time the depositor wants to withdraw. A bank working this way respects the sequential service constraint. depositors who have waited receive a pro rata share of the funds which were not withdrawn but were augmented by the productive technology. Formally, we define the period-2 consumption as

$$c_2(\eta) = \begin{cases} \max\left\{0, \frac{R(n-(n-\eta)c_1^*)}{\eta}\right\} \text{ if } \eta > 0 \\ 0 \text{ if } \eta = 0 \end{cases},$$

where η is the number of depositors who wait in period 1. As usual in the literature, depositors are isolated and no trade can occur among them in period 1.

Since the optimal payments in both periods are readily established by the parameters, obtaining the first best depends only on the actions of the patient depositors, since impatient depositors always withdraw in period 1. Hence, we focus on the decision of the patient depositors, since the only source of a run is their possible miscoordination in the first period.

2.3 Decision, information and runs

The basic actions for any depositor in the first period are withdrawal (wi) and waiting (wa). In one of the setups we will allow one more action, reporting a waiting (r). Throughout the paper we consider pure-strategy equilibria. We do not consider mixed equilibria or partial withdrawal, because ex post these actions are not efficient.[4]

We allow the bank to share the information it has with the depositors if it helps to prevent runs. This is in line with the assumption that the bank maximizes the expected utility of the depositors. We view the bank as a programmed machine which given the parameters calculates c_1^* and then provides the depositors with the available information and serves them if they withdraw, excluding the possibility that the bank gives misinformation.

The sequence of decision ($\theta^n = (\theta_1, ..., \theta_n)$) is exogenously determined in the following way. The number of patient depositors (p) is known and nature chooses at random p depositors in the line who will be patient (that is, her $\theta_i = 1$) . The remaining depositors will be impatient. There are $\binom{n}{p}$ lines of length n with p patient depositors, so these are the possible type vectors (or alignments). Each possible alignment has the same probability, $\frac{1}{\binom{n}{p}}$. This assumption is the least informative possible, reflecting that we do not have a solid knowledge about the order in which depositors go to the bank. Since our results do not depend on the distribution of alignments, this exogeneity assumption is not crucial. Neither the depositors nor the bank know the alignment, they only know n and p.

Let $\theta_1^{i-1} \in \Theta_1^{i-1}$ denote the partial type vector starting with depositor 1 up to depositor $i-1$, and let $\theta_{i+1}^n \in \Theta_{i+1}^n$ stand for a feasible continuation type vector after depositor i. Thus, $\theta_1^{i-1} = (\theta_1, ..., \theta_{i-1})$ and $\theta_{i+1}^n = (\theta_{i+1}, ..., \theta_n)$.

Each depositor decides only once. This assumption is in line with the literature, but clearly it is not an innocuous one. Experiments (Garratt and Keister (2008)) show that depositors are more likely to withdraw when given multiple opportunities to do so, and in the real life depositors who wait may reconsider their decision.

As a tie-breaking rule, we suppose that a patient depositor who is indifferent between withdrawing and waiting will withdraw. This assumption is not crucial for the argument.

[4]If there is a run, then the efficient action is to withdraw all the money with probability one. Otherwise, a patient agent should leave all the money with probability one.

We define bank runs in a broad sense. We interpret a patient depositor withdrawing in the first period as a (partial) bank run.

We will work with three different information setup. First, we will impose a direct revelation mechanism, so that depositors have to report their decision. Therefore, the bank has the exact history of the decisions, since both actions are observable. This mechanism is the same as in Green and Lin (2003), but in our setup depositors get to know the history. It can be considered as the full information benchmark case. In the second setup we follow Peck and Shell (2003) who claim that it is more natural to think that only those depositors contact the bank who want to withdraw, so only withdrawals are observable. In the last information setup patient depositors may report that they have waited and this report will be seen by later-coming depositors. The reason of choosing these setups is that the first two relate closely to existing papers with a different modelling choices (simultaneous-move games with complex contracts) and lead to different conclusions regarding the possibility of bank runs. The third setup allows us to bridge the gap between the two results by showing that by changing slightly and in a plausible way the game in the second setup we obtain the same conclusion as in the full-information case. In principle, nothing prevents depositors to report their decision to wait to the bank.

3 All previous decisions are observable

In this setup we require that depositors state their action to the bank, so waitings are observable. The bank shares all the available information it has with the depositors, so each depositor knows the exact history of actions. This direct revelation mechanism has been applied by Green and Lin (2003) in an environment with complex contracts and simultaneous decision-making, in which depositors have some notion about their position in the line. We use a simple contract and depositors know exactly what happened before in a sequential decision-making setup.

3.1 The leading example

To get the intuition of what happens consider the following informal example with four depositors.[5] The interesting case is that with three patient depositors and when all have to wait to make waiting worthwhile. Hence, there are four possible alignments. Suppose that $u(c_2(\eta = 3)) > u(c_1^*) > u(c_2(\eta \leq 2)$ and $3c_1^* < 4$, so patient depositors at position 1,2 and 3 would only want to wait if all the other patient depositors wait. The optimal decision for a patient depositor in the last position is easy. When she observes a history with 2 withdrawals she withdraws, otherwise she waits.

[5]The most simple example is that of three agents with two impatient agents, where both have to wait to make waiting worthwhile. Coordination in that setup is easy and does not give the flavour of the argument we will use in the general case.

Any history containing two waitings induces a patient depositor to wait. A patient depositor observing only a waiting knows that she is in position 2 and by waiting she can induce the last patient depositor to wait. Hence, when observing only a waiting, waiting is the best response for a patient depositor. Consequently, if a patient depositor observes a waiting followed by a withdrawal, then she knows that the depositor at position 2 must have been an impatient depositor, so by waiting she can induce the last patient depositor to wait. A patient depositor observing nothing knows that she is the first in the line. By waiting she induces the other patient depositors to wait according to the previous results, so for a patient depositor in position 1 the best response is to wait. As a consequence, if a patient depositor observes a withdrawal, then she knows that it must have been an impatient depositor. Each later-coming patient depositor observing this withdrawal will come to the same conclusion. Then, the best response when observing a withdrawal is to wait, because the subsequent patient depositors will know that nobody lied. Hence, the patient depositor at position 3 will wait, because this way she induces the last patient depositor to wait as well. Thus, waiting is best response for a patient depositor when observing

- nothing,
- a withdrawal,
- a waiting,
- (waiting, withdrawal),
- (withdrawal, waiting),
- any history containing two waitings.

As the game unfolds for a patient depositor no information set may emerge to which withdrawal is the best response. Consequently, there will be no runs.

Before going to the general model, let us consider the importance of the order. In the reasoning we used the exact order of moves to get the result. Since the bank observes the exact history, this information is available in the economy. If only aggregate numbers of waitings and withdrawals without order were observable, then we do not get this result.[6] In the Appendix A, we show how run may emerge when only unordered aggregates are observed. Knowing the exact order of previous actions helps to verify the truthfulness of the history.

3.2 The general case

Each depositor entering the bank may choose either to wait (wa) or withdraw (wi). Denote by ω_i and η_i the number of withdrawals and waitings in the history of depositor in position i. Let $h_i \in H_i$ be the history observed by depositor in position $i \in \{1, 2, ..., n\}$, where H_i is the set of feasible histories.

[6] Smith and Sorensen (1998) shows in more detail the difficulties of this approach.

Therefore, it contains all the possible permutations of $\omega_i \in \{0,1,2,...i-1\}$ withdrawals and $\eta_i \in \{0,1,2,...p-1\}$ waitings such that $\omega_i + \eta_i = i-1$. Denote by $\omega \in \{0,1,2,...n\}$ the total number of withdrawals in period 1. The total number of waitings is given by $\eta = n - \omega$.

The utility of a patient depositor is

$$u_i(a_i \mid \theta_i = p, h_i) = \begin{cases} u_i(c_1^*) \text{ if } a_i = wi \text{ and } y_i \geq c_1^*, \\ u_i(y_i) \text{ if } a_i = wi \text{ and } c_1^* > y_i \geq 0, \\ u_i(c_2(\eta)) \text{ if } a_i = wa, \end{cases}$$

where y_i is the funds the bank has when depositor i arrives, and $c_2(\eta)$ is defined as before. When depositor i waits, she does not know her payoff, because it depends on what subsequent depositors do.

For simplicity, assume the extreme case that waiting is an optimal decision if all patient depositors wait, so

$$u(c_2(\eta = p)) > u(c_1^*) > u(c_2(p > \eta)).$$

A pure strategy for depositor i is a map $\mathbf{s}_i : \boldsymbol{\theta}_i \times H_i \to \{wi, wa\}$. Hence, depositors have to specify what to do when being of either type at a given position and observing all possible histories compatible with that position. Let $\mathbf{s}_i^j = (\mathbf{s}_i, \mathbf{s}_{i+1}, ..., \mathbf{s}_j)$ denote the strategies of depositors beginning with depositor i up to depositor j. Notationally, $\mathbf{s}_i$ denotes the strategy, while s_i will stand for the play implied by $\mathbf{s}_i$. Hence, $h_i = (s_1, s_2, ..., s_{i-1})$.

Henceafter, if we put $s_i = \theta_i$, then it means that depositors should act according to their type, that is, patient depositors wait and impatient ones withdraw.

Since *ex ante* depositors ignore their type and position in the line, a strategy is $\mathbf{s} = \mathbf{s}_1 \times \mathbf{s}_2 \times ... \times \mathbf{s}_n$, where $\mathbf{s}_i$ is defined as before for any $i \in [1, n]$. Before the game starts each depositor has to specify what to do in any position upon observing any possible history given their type. Being truthful means that patient depositors wait, whereas impatient ones withdraw. The first best obtains if all depositors act truthfully.

3.2.1 Alignment is public knowledge

It is instructive to see what happens if we eliminate the uncertainty of alignment. Suppose that the alignment, that is the type vector of depositors is publicly known. This setup allows a patient depositor to know how many patient depositors are in front of her, how many come later and she knows exactly at which positions they are. The most important information for a patient depositor is her relative position among the patient depositors. By eliminating the uncertainty about the alignment we may apply standard backward induction to find the best responses. We have the following result.

Proposition 1 *When the alignment is public knowledge, in the unique subgame perfect equilibrium each depositor acts truthfully.*

Proof. See Appendix B. ■

The intuition of this result is as follows. The last patient depositor's decision is straightforward. If there have been enough waitings before, so that with her waiting the period-2 payment is high enough, then she waits, otherwise she withdraws. Anticipating this decision, the next to the last patient depositor's decision is of the same nature, and by moving backwards all patient depositors' decision rule becomes clear. Given these rules, as the game unfolds the first best obtains.

3.2.2 Alignment is unknown

When alignments are not observable, depositors cannot apply the previous reasoning, because in general patient depositors will not know their relative position among the patient depositors. The nice feature of the model when the alignment is known is that you know exactly what has happened (how many patient depositors have lied) and you can predict exactly what will happen (how many later-coming depositors will wait). Therefore, patient depositors do not need beliefs. This is not true when the alignment is unknown, but still there are histories for which the best response is clear regardless of beliefs. For any patient depositor at any position,

$$BR_k(h_k \mid \eta_k \geq \eta_l - 1) = wa,$$

where k is the absolute position (and not the relative one) in the line. If the k^{th} depositor's waiting makes waiting a better choice, then a patient depositor in this position will wait. This best response can be applied only to a small subset of histories which is not sufficient to determine the equilibria of the game. In the case of histories for which the previous best responses do not apply, beliefs are crucial in finding the optimal action.

The game depositors play is one of incomplete information where beliefs are important, so the solution concept we use will be perfect Bayesian equilibrium. Let $\mu(\theta_{i+1}^{n} \mid h_i, \theta_i)$ denote depositor $i's$ belief about the continuation type vector conditional on the history and $i's$ type.[7] Our formulation of the belief is equivalent to $\mu(\theta_1^{i-1} \mid h_i, \theta_i)$, that is the belief about the type of the preceding depositors, because this belief coupled with the own type determines the belief about the type of later-coming depositors.[8]

Regarding the formation of beliefs, we will use two restrictions:

1. a waiting at any position reveals that it must have been a patient depositor, and

[7] In the sense of Harsányi, the type of an agent is being patient or impatient and the history she observes. Our abuse of equating type with only being patient or impatient does not affect the analysis, because we condition the strategy both on being patient or not and the history.

[8] In our leading example, if a patient agent at the third position believes that the last agent is an impatient one with certainty, then it is equivalent to believe that the previous agents have been patient ones.

2. if for a patient depositor the dominant strategy given history h_i is to wait, then observing a withdrawal in position $i+1$ reveals that the depositor at that position is impatient.

The first restriction eliminates the possibility of impatient depositors acting mistakenly, whereas the second one does the same with patient depositors. Since impatient depositors do not make mistakes, we have $s_i : imp \times H_i \rightarrow wi$ for all i, so impatient depositors always withdraw. Hence, we focus on the truthfulness of patient depositors' actions.

These restrictions amount to say that depositors are rational and it is common knowledge. The restrictions also show that beliefs depend on the history. These assumptions allow us to use the iterated elimination of strictly dominated strategies. It makes possible that for a subset of histories depositors can predict how later-coming depositors will behave if they choose to wait.

Let us now formulate the equilibrium solution concept we are going to use.

Definition 1 *The strategy* $\mathbf{s}$ *and the belief* μ *is a perfect Bayesian equilibrium if*

$$\sum_{\theta_{i+1}^n} \mu(\theta_{i+1}^n \mid h_i, \theta_i) u \left[c_1^*, c_2(h_i, s_i, \mathbf{s}_{i+1}^n), \theta_i\right] \geq$$
$$\geq \sum_{\theta_{i+1}^n} \mu(\theta_{i+1}^n \mid h_i, \theta_i) u \left[c_1^*, c_2(h_i, \tilde{s}_i, \mathbf{s}_{i+1}^n), \theta_i\right]$$

for all i*, and if* $\mu(\theta_{i+1}^n \mid h_i, \theta_i)$ *is consistent with Bayes' rule whenever possible.*

The difficulty lies in the fact that h_i, in general, is compatible with several θ_1^{i-1}, because any withdrawal may be due to a misrepresenting patient depositor. Given $\mathbf{s}$, using Bayes´ rule $\mu(\theta_{i+1}^n \mid h_i, \theta_i)$ determines what depositor i expects to be the total number of waitings at the end of period 1 which defines her payoff if she decides to wait.

A special case of $\mu(\theta_{i+1}^n \mid h_i, \theta_i)$ is when depositor i believes that all previous actions have been truthful. We will introduce an even stricter definition which we call truthful history.

Definition 2 *We call a history truthful, if using restrictions 1 and 2 it can be unambiguosly verified that all previous actions have been truthful.*

Formally, a truthful history is one where $h_i = \theta_1^{i-1}$.[9] It implies that $\mu(\theta_{i+1}^n \mid h_i, \theta_i) = \mu(\theta_{i+1}^n \mid \theta_1^i)$, so there are $p-(\eta_i+\theta_i)$ patient and $n-p-(\omega_i+(1-\theta_i))$ impatient subsequent depositors and any continuation alignment is equiprobable. We require that using the restrictions depositors be able to verify the truthfulness of the history. By our common knowledge assumption any depositor able

[9] We have defined strategies as waiting (wa) and withdrawal (wi) and agents are either patient (pat) or impatient (imp), so when we put $h_i = \theta_1^{i-1}$, then we translate in a straightforward manner wa into pat and wi into imp.

to verify the truthfulness of a history can be sure that all other depositors do the same when observing the same history. When we speak about a truthful history, then it is equivalent to speaking about a degenerate belief where it can be verified that all previous actions have been truthful. For example, in our leading example we have found that the best response of a patient depositor when observing a waiting is to wait, so using the restrictions a patient depositor observing the history (wa, wi) concludes that it is a truthful history, because a patient depositor in the second position would have waited. If a patient depositor observes a truthful history, then she knows her relative position among the patient depositors.

Our last definition before the main result of this section concerns implementability.

Definition 3 *The first best is strongly implementable if* $\mathbf{s}_i(\theta_i, \theta_1^{i-1}) = \theta_i$ *and* $\mu(\theta_{i+1}^n \mid \theta_1^i)$ *for all* $i \in [1, n]$ *is the unique perfect Bayesian equilibrium of the game.*

If for any depositor the belief to observe the truthful history and the strategy to act truthfully is the unique perfect Bayesian equilibrium, then as a consequence the first best obtains

Proposition 2 *The first best is strongly implementable.*

Proof. See Appendix C. ■

To get the intuition behind the proof, consider the following informal analysis. A patient depositor observing $p-1$ waitings at any position knows with certainty that she is the last patient depositor, so her optimal action is to wait. Thus, at any equilibrium the strategy for a patient depositor when observing $p-1$ waitings and at most $n-p$ withdrawals should be to wait. Otherwise, she would like to deviate unilaterally, because waiting dominates withdrawal.

Consider now the history consisting of $p-2$ waitings and no withdrawals. Knowing the best response of a patient depositor observing $p-1$ waitings, a patient depositor´s optimal action is to wait. But then the history $((p-2)\ wa, wi)$ reveals that the last depositor must have been an impatient one. Therefore, a patient depositor observing this history knows that she is the $(p-1)^{th}$ patient depositor in the line and her best response is to wait, because this decision induces the last patient depositor to wait as well. We may apply the same line of reasoning to show that for any history beginning with $p-2$ waitings any subsequent withdrawal must be a truthful one. A patient depositor upon observing such a history knows exactly her relative position and she knows also what the last patient depositor will wait, so her best response is to wait. Hence, at any equilibrium the strategy for a patient depositor when observing a history which begins with $p-2$ waitings should be to wait. Otherwise, she would like to deviate unilaterally. On the other hand, whenever a patient depositor upon observing $p-2$ waitings knows that she is the $(p-1)^{th}$ patient depositor in the line[10], her best response is to wait. This is the case, because we have seen that the last patient depositor will wait upon observing $p-1$ waitings.

Consider the history consisting of $p-3$ waitings and no withdrawals. By the previous result a patient depositor´s best response when observing this history is to wait. Thus, the history $((p-3)\ wa, wi)$ reveals that the last depositor must have been an impatient one. Therefore, a patient depositor observing this history knows that she is the $(p-2)^{th}$ patient depositor in the line. If she waits, then the resulting history will have $p-2$ waitings and a patient depositor would know that she is the $(p-1)^{th}$ patient depositor in the line, and her best response would be to wait. The same argument holds for any history beginning with $p-3$ waitings and followed by at most $n-p$ withdrawals. At any equilibrium the strategy for a patient depositor when observing a history which begins with $p-3$ waitings should be to wait. Otherwise, she would like to deviate unilaterally. Furthermore, whenever a patient depositor upon observing $p-3$ waitings knows that she is the $(p-2)^{th}$ patient depositor in the line, her best response is to wait. This is the case, because the following patient depositor will observe $p-2$ waitings and will know that she $(p-1)^{th}$ patient depositor in the line, so her best response is to wait, as it will be the last patient depositor's best response.

We can continue along the same lines to show that at any equilibrium the strategy for a patient depositor when observing a history which begins with $[0, p-1]$ waitings should be to wait. The reasoning excludes the possibility of equilibria where patient depositors at the beginning of the line withdraw because they believe that later-coming patient depositors will withdraw as well. If they wait, then they can induce those later-coming patient depositors to wait as well. As the game begins, based on the best responses if the first depositor is a patient one, then she will be truthful, because she observes a truthful history. Hence, the second depositor can be sure to observe a truthful history as well, implying that she will also act truthfully. The same logic ensures that any later-coming depositor can be sure to observe a truthful history to which the best response is to be truthful, so the first best obtains.

In Appendix D, we show using our leading example, why run cannot happen in equilibrium.

3.2.3 Rationalizability

We can make our result even stronger. The uniqueness of the equilibrium was based on the idea that if for some histories waiting is the dominant strategy, then for other histories (which led up to those histories) waiting will be dominant as well. Hence, in an iterative manner we excluded strategies from the action sets of depositors. This procedure is what rationalizability would do as well.

Rationalizability does not impose that the beliefs depositors hold about each other should be equilibrium beliefs, it just states that a rational player only uses strategies which are best responses to some beliefs the depositor holds about the strategies of his opponents. It just attempts to answer the question what

[10] That is, she knows that all patient agents before her have been truthful.

can be considered rational behavior in a noncooperative strategic situation. In this sense, it is more general than the equilibrium concepts. Its main drawback is the lack of clear predictions, but in our model it is not the case.

On the other hand, rationalizability uses all the available information the game offers, in particular it uses information provided by the play leading up to the subgame starting with the depositor whose turn it is to decide. Thus, although rationalizability seems to coarsen the concept of Nash equilibrium, sometimes it helps to refine it by eliminating unreasonable equilibria as shown by the seminal papers of Bernheim (1984) and Pearce (1984). Our case is an example where rationalizability yields a clear prediction.

Rationalizability analyses use three assumptions following Pearce (1984):

1. depositors lacking an objective probability distribution over another player´s choice of strategy form a subjective prior that does not contradict any of the information at her disposal.

2. depositors are rational and maximize their expected utility.

3. The structure of the game (including the two previous assumptions) is common knowledge.

Our previous restrictions are in line with these three assumptions.

Formally, we are looking for the rationalizable strategies of depositor i after observing the history and having a conjecture $(c^i(h_i))$ about what the other depositors´ strategies are conditional on history h_i being reached. The common knowledge assumption implies that if given a history there is only one rational conjecture that the depositor who is deciding may have, then all later-coming depositors will know that at that point of the game that depositor must have had that particular conjecture.

For any $c^i(h_i) \in S_{-i}$ which represents depositor i´s conjecture about other players´ strategies, let $U_i(s_i, c^i(h_i))$ denote for each s_i the expected utility of depositor i given her conjecture. Then, i´s best response correspondence is

$$B_i(c^i(h_i)) := \arg\max_{s_i \in S_i} U_i(s_i, c^i(h_i)).$$

We want to find the sets $R_i(c^i(h_i))$ of rationalizable strategies. We look for strategies which are best responses conditional on having reached h_i and the conjectures depositor i may entertain. The sets are constructed recursively as follows. First, let $R_i^0 := S_i$ for each depositor i. Then, let

$$R_i^k(c^i(h_i)) := B_i(\Pi_{j\in[1,n]\setminus\{i\}} R_j^{k-1}(c^j(h_j))) \quad \forall i \in [1,n]\,; \forall h_i \in H_i; k = 1, 2, ...$$

Thus, $R_i^1(c^i(h_i))$ consists of depositor i´s possible best responses, conditional on being at a node in h_i and given the various conjectures that i might have about the strategies chosen by the other depositors from the sets $R_j^0 = S_j$. Then, $R_i^2(c^i(h_i))$ contains i´s possible best responses given the conjectures about the strategies chosen by other depositors from $R_j^1(c^j(h_j))$, and so on. Each depositor i has a well-defined limit set

$$R_i(c^i(h_i)) := \lim_{k\to\infty} R_i^k(c^i(h_i)) = \cap_{k=0}^{\infty} R_i^k(c^i(h_i)),$$

which is the rationalizable strategy set for depositor i.

The utility function of impatient depositors makes it clear that their unique rationalizable strategy at any position given any history is to withdraw. We restrict our attention to patient depositors. The intuition of our result is similar to the previous one and has the following logic.

Consider any history containing $p-1$ waitings. Obviously, for a patient depositor the only rational conjecture given this history is that all patient depositors have already decided to wait. Therefore, the best response is to wait. This is our first restriction.

Given this restriction, waiting is best response to any history which contains $p-2$ waitings and depositors can make sure that no withdrawal has been made by patient depositors. (This is what we have defined earlier as truthful history with $p-2$ waitings.) This is the case, because by waiting a patient depositor knows that the last patient depositor will wait as well, so all patient depositors will enjoy the highest possible payoff. This implies the next restriction, namely that to any history starting with $p-2$ waitings the best response is to wait. Consequently, if a history starting with $p-2$ waitings and followed by a withdrawal is observed, then the last depositor must have been an impatient depositor. Hence, the previous reasoning has that the best response is to wait which is our next restriction. This logic applies to any history starting with $p-2$ waitings and followed by at most $n-p$ withdrawals.

We can repeat the same arguments with histories containing less and less waitings, and so we restrict more and more the rationalizable strategies. Thus, in the end we have that a patient depositor´s unique rationalizable strategy to any history which may emerge is to wait. We have the following result.

Proposition 3 *In the game,* $R_i(c^i(h_i)) = \theta_i$ *for* $\forall i \in [1, n]$ *and* $\forall h_i \in H_i$.

Proof. See Appendix E. ■

Corollary 1 *The first best is strongly implementable.*

3.3 Relating to the literature

An alternative interpretation[11] of the sequentiality is imagining the Diamond-Dybvig model where the depositors coordinate on a run. Running means that they form a queue at the door of the bank which will serve them in a sequential manner. Diamond and Dybvig's analysis ends there, while ours begins at this point by posing the question: what is the outcome of the game if any depositor can observe the actions of those who are in front of her and if she knows that

[11] This interpretation was suggested to me by Alfonso Rosa García.

those coming later will know what she did. If you let people decide in this situation, then our result predicts that the patient depositors will not withdraw. In turn, it means that the policies studied by Diamond and Dybvig (suspension of convertibility and deposit insurance) are not necessary to avoid the bad outcome. It is enough that depositors are aware of the fact that their decision will be observed by later-coming depositors to implement the good equilibrium.

Green and Lin obtained a no-run result with a model with huge withdrawal demand uncertainty, allowing the bank to write sophisticated contracts, but restricting the depositors´ information to have some notion about the position without knowing anything about the others´ decision. In contrast, we work with simple contracts and no demand uncertainty, but allow the bank to share available information with the depositors. This trade-off in modeling choice does not change the positive result of not having runs.

4 Only withdrawals observed

Peck and Shell (2003) assert that it is implausible that depositors contact the bank at period 1 to say that they do not want to withdraw. It is more natural to think that only those who want to withdraw will go to the bank. Thus, strategy has to be based on the number of previous withdrawals, that is, the possible strategy profile is of the form $\mathbf{s} = (\mathbf{s}_0, \mathbf{s}_2, ..., \mathbf{s}_{n-1})$ where $\mathbf{s}_i : \{imp, pat\} \times i \to \{wi, wa\}$ for $i = 0, 1, ..., n-1$ tells what action to take when being either type and observing i withdrawals. To exclude trivial cases, assume that at least two waitings are needed to make waiting a better choice.

Run after any number of withdrawals is rationalizable, because the conjecture that all previous depositors have withdrawn and that all later-coming depositors will withdraw as well does not contradict the available information any depositor has. Contrary to the previous setup there is no observable history for which a patient depositor would eliminate withdrawal, so we cannot restrict in any way the depositors´ strategy set. Given the proposed conjecture withdrawal is a rationalizable strategy for a patient depositor, so run may happen.

Proposition 4 *Run is a rationalizable outcome.*

Proof. See Appendix F. ■

It is easy to see that even if we impose equilibrium beliefs and use an equilibrium solution concept, run remains a possible outcome. Let us construct a run strategy profile for patient depositors which allows for the maximum number of withdrawals and which is an equilibrium candidate. The most obvious run strategy candidate is the one prescribing to run when observing any number of previous withdrawals. There is only one potential depositor who would like to deviate: a patient depositor who knows to be the last depositor in the line. Suppose that everybody up to the last depositor has withdrawn and the bank still has some funds. Then, in case that this depositor is patient, her optimal decision is to wait and consume more in the next period.

Therefore, our proposed run strategy is

$$\mathbf{s}_i = \begin{cases} wi \text{ if } i < n-1, \\ wa \text{ if } i = n-1 \end{cases}.$$

The game is as follows. Nature picks an alignment, players are called to decide (wait or withdraw) sequentially, and each of them observes the number of previous withdrawals. Again, the idea is that the bank knows how many depositors have withdrawn and can share this information with the subsequent depositors. Nevertheless, neither the bank, (and consequently) nor the depositors observe if a depositor decides to wait.

Proposition 5 *The proposed run strategy is a perfect Bayesian equilibrium.*

Proof. See Appendix G. ■

The intuition behind this result is easy. Since deviations from the run strategy cannot be observed, no patient depositor can induce later-coming depositors to wait by waiting. When waiting is observable, then being truthful makes possible that later-coming depositors have information about what happened before, and then these depositors will find it profitable to be truthful as well. In this setup, being truthful is not revealing, depositors do not even know their position, so it is not possible that there be enough information to eliminate run as an equilibrium outcome.

5 Reporting is allowed

Up to this point we have shown that if everybody has to report and the history is observable, then bank run does not occur. Nevertheless, by modifying the game so that only withdrawals are observable, runs may happen. A way to bridge the gulf between the results is to allow (but not to require) patient depositors to report their waiting. It is a new game, since the available actions (withdraw (wi), wait without reporting (wa), wait and report (r))[12] and the possible information sets are different. Since reporting to the bank in period 1 is not related to consumption, we allow for the possibility that it is costly.[13] Intuitively, a patient depositor would like to report, because sending this signal could induce subsequent patient depositors not to withdraw, and have a high period-2 payment.

Assume a nonnegative and uniform cost for reporting in utility terms and denote it by k. If $k > u(c_2^*) - u(c_1^*)$, then the cost is so high that it does not compensate for the potential gain in utility, so to make reporting a real option suppose the opposite. Otherwise we have the previous setup where run is an equilibrium outcome.

[12] Note that to report and withdraw does not make sense, so we do not consider it.

[13] How are reporting costs in real life? Our guess is that they are rather small as a consequence of technological advances, like Internet banking. Notice that in Green and Lin (2003) the compulsory reporting is not costly.

If we change the setup by adding additional information, then we are back to previously analyzed cases. If both the position and the alignment were known, patient depositors would know their relative position. Hence, the game would simplify to that in section 3.2.1 with the same outcome. Patient depositors would not report, because it is costly and redundant. If only the position ($i = 1, 2, ...n$) was known, a patient depositor would know exactly how many patient depositors have waited without reporting. It is simply $(i-1)-(\omega_i+\rho_i)$, that is the difference between all previous actions and all observable actions. It means that both waitings and withdrawals are observable, so we are back to section 3.2.2. with the same outcome. We do not need the costly reporting to obtain the first best, so patient depositors would not use it. Nevertheless, in this setup neither the position, nor the alignment is known. To give some flavour of this new game consider the following example.

5.1 The leading example

Suppose that we have the same example as in section 3.1 with an impatient and three patient depositors and to make waiting worthwhile no patient depositor should withdraw.

Consider the observable history (r). A patient depositor observing it may have two beliefs: (i) there was also a patient depositor who waited without reporting, (ii) the observed history coincides with the true history. Clearly, if the history contained also an unobserved waiting, then for a patient depositor the best response is to wait without reporting. In the other case reporting dominates withdrawal, because the last patient depositor would observe two reports which would make her wait and the reporting depositor would have $u(c_2^*)-k > u(c_1^*)$. Therefore, a patient depositor observing a report will not withdraw. As a consequence, when observing (r, wi) depositors know that the withdrawal must have been truthful. Hence, for a patient depositor observing this history reporting dominates withdrawal. Since no patient depositor withdraws when observing (r, wi), the best response is to wait without reporting. Anticipating this decision, a patient depositor´s best response observing (r) is also to wait without reporting.

Let us see what happens if a patient depositor observes (wi). We have seen that when the history begins with a report, then given any of the possible ensuing histories later-coming patient depositors will not withdraw.[14] Consequently, for a patient depositor who observes nothing reporting dominates withdrawal, so this depositor will not withdraw. Therefore, if an observable history begins with a withdrawal, it must have been a truthful one. When observing (wi, r) reporting dominates withdrawal, since when there are two reports in any observable history, then the next patient depositor (if there is any) will wait without reporting. Again, since the unique impatient depositor has already withdrawn and no patient depositor observing (wi, r) withdraws, the best response is to

[14] A patient agent would best respond by withdrawing to an observable history (r, wi, wi), but by our previous argument it cannot arise.

wait without reporting. It implies also that when observing (wi) reporting dominates withdrawal, because the ensuing information sets surely lead to higher payoffs than c_1^*. Moreover, waiting without reporting is the best response, because when observing a withdrawal a patient depositor knows that it was done by the impatient depositor and if there are any later-coming patient depositors, then those depositors will not withdraw.

As we have seen, if a patient depositor does not observe anything, then she will not withdraw. But, will she report? No, since for a patient depositor the best response to the observable history (wi) is to wait without reporting, so the best response to observing nothing is to wait without reporting. Hence, when observing either $(\emptyset)$ or (wi) the best response is to wait without reporting, so as the game unfolds the unique outcome is the first best and patient depositors do not report.

5.2 The general case

The information set consists of the history which is observable and the own type. We denote by $H^{obs}_{\omega_j,\rho_j}$ the set of observed histories containing any permutation of $\omega_j \in \{0,1,2,...n-1\}$ withdrawals and $\rho_j \in \{0,1,2,...p-1\}$ reports.[15] Denote any generic element of this set by $h^{obs}_{\omega_j,\rho_j}$. Notice that it is possible that two (or even more) patient depositors observe the same observable history.

Due to the unobservability of waitings, an depositor observing any history in $H^{obs}_{\omega_j,\rho_j}$ does not know her position, she just knows that she is at least in position $\omega_j + \rho_j + 1$ and at most in position $\omega_j + p$. The range of possible positions is $p - \rho_j - 1$ which makes the uncertainty larger than in the first setup.

A pure strategy for an depositor is a map $\mathbf{s}(\boldsymbol{\theta}, H^{obs}) : \{imp, pat\} \times H^{obs} \rightarrow \{wi, wa, r\}$, where $H^{obs} = \times (H^{obs}_{\omega_j,\rho_j})_{\substack{\omega_j \in \{0,1,2,...n-1\} \\ \rho_j \in \{0,1,2,...p-1\}}}$ is the set of all possible observable histories. Therefore, each depositor has to specify what to do when observing any possible history and being of either type. We focus on patient depositors, because impatient depositors always withdraw.

The unobservability of waitings without reporting makes it difficult to verify whether a strategy profile is a perfect Bayesian equilibrium or not. The difficulty lies in that an depositor observing a given history generally does not know her exact position, so the calculation of the probability of possible continuation alignments becomes too demanding. Fortunately, to find the rationalizable strategies of this game is easier, so it will be our focus here.

We will modify in a natural way the restrictions on the formation of beliefs:

1´: a reporting at any position reveals that it must have been a patient depositor, and

[15] The order of actions is very important in the analysis, but this notation will prove to be convenient.

2´: if for a patient depositor given history $h^{obs}_{\omega_j,\rho_j}$ reporting dominates withdrawal, then an observed withdrawal following the history must be due to an impatient depositor.

The first restriction is equivalent to saying that impatient depositors always withdraw. The second one states that patient depositors never play dominated strategies.

To prove the main result of this section, we modify slightly the definition of truthful history.

Definition 4 *We call a history truthful, if using restrictions 1´ and 2´ it can be unambigously verified that no patient depositor has withdrawn.*

The only difference compared to the previous definition is that this one allows for unobserved waitings. We are ready to state the following result.

Proposition 6 *The unique rationalizable strategy for patient depositors is to wait.*

Proof. See Appendix H. ∎

The way to show that the unique rationalizable strategy is to be truthful will be to consider observable histories and to see which strategies can be eliminated. As a first step, patient depositors observing any history use both waiting and withdrawal as rationalizable strategy. To obtain the first restriction, consider a patient depositor observing a history with $p-1$ reports. The only rational conjecture which our depositor may hold is that she is the last patient depositor, so her unique rationalizable strategy given these histories is to wait. Given this restriction, to any truthful history with $p-2$ reports reporting dominates withdrawal for a patient depositor, because by reporting she gets $u(c_2^*)-k$ which is higher than the payoff she would get by withdrawing. Therefore, withdrawal is not rationalizable when observing any truthful history with $p-2$ reports. The easiest example of a truthful history with $p-2$ reports is the history consisting of $p-2$ reports. Since for this history reporting dominates withdrawal, a patient depositor observing $p-2$ reports followed by a withdrawal knows that the withdrawal must have been a truthful one. Consequently, this history is also a truthful one with $p-2$ reports and by the same arguments as before, for a patient depositor reporting dominates withdrawal. Along the same line of reasoning, for any history starting with $p-2$ reports and followed by at most $n-p$ withdrawals reporting dominates withdrawal for patient depositors. Hence, we have eliminated withdrawal from their action set. As a consequence, we can eliminate reporting also, because by waiting thay do not spend resources on reporting, and although the last patient depositor will not observe the waiting, but she will have a history for which reporting dominates withdrawal. The last patient depositor (without knowing the she is the last) may reason in the same way, so if a history starts with $p-2$ reports, then the unique rationalizable strategy for the last patient depositors is to wait. As a consequence, withdrawal is not rationalizable when observing any truthful history with $p-3$ reports and you can do the same analysis as in the case with $p-2$ reports. By repeating the same procedure with less and less reports, we obtain our result.

The proposition predicts a unique outcome of the game in which patient depositors do not report. The mere existence of reporting is enough to bring about the first best. We need reporting to make withdrawal a dominated action, but once it was dominated reporting becomes dominated as well. Such arguments appear in models with "money burning", e.g. Ben-Porath and Dekel (1992).

A direct consequence of the proposition is the following corollary.

Corollary 2 *The first best is strongly implementable.*

The possibility of reporting can be seen as a metaphor of richer communication between the bank and its depositors. While the no-run result by Green and Lin (2003) rests on complex contracts, our no-run result impinges on the possibility of richer communication. As already noted, this result is in line with the findings of Iyer and Puri (2008) which state that the longer and deeper the relation ship between a depositor and the bank, the less likely is that depositor to run.

6 Conclusion

Most of the literature on bank runs uses a simultaneous-move approach to model the depositors´ decision. In contrast, we model it using a sequential focus. We find that in an environment in which each previous action is observable, the coordination problem pointed out by Diamond and Dybvig does not emerge. When we restrict the observable information to withdrawals, then runs occur. Nonetheless, they disappear if we allow depositors to report the bank their decision to wait. Besides the no-run result, this last case is interesting because no reports are made to the bank. The mere existence of reports is enough to obtain the first best. In the first two setups, we prove our result both by using an equilibrium solution (concretely, perfect Bayesian equilibrium) and rationalizability, while for the last setup we use only rationalizability.

Although we do not study explicitly policy issues, our results have a clear policy message. Sequentiality matters in depositors´ decision-making, so it must be taken into account when designing the optimal policy.

Our results rest heavily on the concept of the bank as a benevolent institution which serves the depositors; an assumption adopted by much of the literature. When taking into account that the bank possibly follows self-interest as well (see Andolfatto and Nosal (2008)), then the potential agency problems may question our results, although competition in the banking sector may mitigate these problems.

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8 Appendix

8.1 Appendix A

Consider the same example, but now the previous actions are unordered. The optimal decision for a patient depositor in the last position is as before. When she observes 2 withdrawals and a waiting she withdraws, otherwise she waits. We focus on patient depositors at the first three positions. Consider the following strategies

$$s_3(wi, wi) = s_3(wa, wi) = s_2(wi) = s_1(\emptyset) = wi,$$
$$s_3(wa, wa) = s_2(wa) = wa,$$

where the subscript denotes the position in the line and in the brackets there are the unordered previous actions. Do these strategies form a run equilibrium?

Notice that given these strategies as the game unfolds, the information set (wa, wi) cannot emerge. Therefore, the unique candidate for profitable unilateral deviation is the information set in which a patient depositor does not observe anything, that is the depositor at the first position in the line. The deviation consists in waiting instead of withdrawing. There are 3 alignments which begin with a patient depositor. If a patient depositor at this information set waits, then in two of the three cases in which the second depositor is a patient one the first best is achieved. This is the case, because this second depositor will wait inducing the third patient depositor to follow suit as well. Nevertheless, when the second depositor is the impatient one, then the strategies imply that the later-coming patient depositors withdraw. For the parameters $R = 1,1$, $\gamma = 6$ and the utility function $u(c) = c^{1-\gamma}/(1-\gamma)$,[16] the proposed strategy profile is an equilibrium. The utility of withdrawing is $-0,149$, while by waiting the expected utility for a patient depositor at the first position is $-0,205$, so a patient depositor at the first position would withdraw.

8.2 Appendix B

Proof. The last patient depositor´s best response in any known alignment is to wait if and only if with her waiting η is so high that $c_2(\eta) > c_1^*$. Denote the minimum η for which it is true by $\tilde{\eta}$, It is given by $c_2(\tilde{\eta}) > c_1^* \geq c_2(\tilde{\eta} - 1)$, and notice that $\tilde{\eta} \leq p$. If the last patient depositor observes at least $\tilde{\eta} - 1$ waitings, then she will wait, otherwise she withdraws. Note that there is no uncertainty here. Now consider the next to the last patient depositor. Knowing what the last patient depositor will do, her best response is to wait if she observes at least $\tilde{\eta} - 2$ waitings, otherwise she withdraws. By following the same line of argument,

[16]For this utility function we have

$$c_1^* = \frac{4}{1 + 3R^{\frac{1-\gamma}{\gamma}}}, c_2^* = R^{\frac{1}{\gamma}} c_1^*.$$

it is easy to write in general the best response of any patient depositor:

$$BR_\kappa = \begin{cases} wa \text{ if } \eta_\kappa \geq \tilde{\eta} - (p + 1 - \kappa) \\ wi \text{ otherwise} \end{cases},$$

for $\kappa \in [1, p]$ where the subscript κ denotes the κ^{th} patient depositor in the line.

Now consider how the game unfolds. The first patient depositor waits, because $0 \geq \tilde{\eta} - p$. The second patient depositor also waits, because $1 \geq \tilde{\eta} + 1 - p$, and so on. In the end, all patient depositors will wait yielding $\eta = p$, so the first best obtains. To get this result we need less than knowing with certainty the alignment. It is enough that patient depositors know their position among the patient depositors. ■

8.3 Appendix C

The proof has two parts. First, we show that the best response when observing a truthful history is to be truthful. In the second step, we argue that as the game unfolds, only truthful histories are observed, so these best responses lead to the first best.

Denote by $H^{tr}(\eta)$ the set of truthful histories which contain η waitings (and any $\hat{\omega} \in [0, n-p]$ withdrawals). Notice that characterizing features of the set are the number of waitings and truthfulness, but not the position of the depositor observing any element of the set. It is in line with our previous finding that not the absolute position, but the relative position among the patient depositor is what really matters for a patient depositor.

Lemma 1 *Assume that once an element in $H^{tr}(\hat{\eta})$ is reached all subsequent depositors will act truthfully, that is, $s_i(\theta_i, h^{tr}(\eta_i \geq \hat{\eta})) = \theta_i$ for $i = \hat{\eta} + \hat{\omega} + 1, ..., n$, where $h^{tr}(\eta_i \geq \hat{\eta}) \in H^{tr}(\eta_i \geq \hat{\eta})$. Then, for the set of truthful histories which contain $\hat{\eta} - 1$ waitings (and any $\hat{\omega} \in [0, n-p]$ withdrawals), we have $s_{\hat{\eta}+\hat{\omega}}(\theta_{\hat{\eta}+\hat{\omega}}, h^{tr}(\eta - 1)) = \theta_{\hat{\eta}+\hat{\omega}}$.*

Proof. The lemma assumes that once a truthful history containing $\hat{\eta}$ waitings and at most $n - p$ withdrawals is reached, for any possible continuation alignment later-coming patient depositors will wait. Therefore, the only equilibrium strategy when observing a truthful history with $\hat{\eta} - 1$ waitings is to act truthfully. If a patient depositor observes $h^{tr}(\hat{\eta}-1) \in H^{tr}(\hat{\eta}-1)$, then by waiting she will cause a history which belongs to $H^{tr}(\hat{\eta})$. By our assumption, all subsequent depositors will be truthful, so the first best obtains yielding the highest obtainable payoff to the patient depositors. Since any truthful history is equivalent to a degenerate belief, given such a history the unique perfect Bayesian equilibrium strategy is to be truthful, since there is no unilateral profitable deviation. ■

The previous induction step can be used repeatedly.

Corollary 3 *Assume that for the set of truthful histories which contain $\hat{\eta}$ waitings (and any $\hat{\omega} \in [0, n-p]$ withdrawals) we have $s_i(\theta_i, h^{tr}(\eta_i \geq \hat{\eta}) = \theta_i$*

for $i = \hat{\eta} + \hat{\omega} + 1, ..., n$. *Then, for the set of truthful histories which contain* $\acute{\eta} \in [0, \hat{\eta} - 1]$ *waitings (and any* $\hat{\omega} \in [0, n-p]$ *withdrawals), we have* $s_{\acute{\eta}+\hat{\omega}+1}(\theta_{\acute{\eta}+\hat{\omega}+1}, .) = \theta_{\acute{\eta}+\hat{\omega}+1}$.

Proof. In the previous lemma we have shown the case when $\acute{\eta} = \eta - 1$. What happens if a patient depositor observes a truthful history with $\acute{\eta} = \eta - 2$ and $\hat{\omega} \in [0, n-p]$ withdrawals? By waiting, the resulting history will be a truthful one with $\hat{\eta} - 1$ waitings and $\hat{\omega} \in [0, n-p]$ withdrawals. By our common knowledge assumption all subsequent depositors will know that up to depositor $\hat{\eta} + \hat{\omega} - 1$ all actions have been truthful, so by the previous lemma all subsequent actions will be truthful as well. The resulting first best yields the highest possible payoff to any patient depositor, so there is no unilateral profitable deviation. Hence, given the belief embodied in the history the unique perfect Bayesian equilibrium strategy is to be truthful. The same argument can be applied to any truthful history with $\acute{\eta} \in [0, \hat{\eta} - 1]$ waitings. ■

Consider a patient depositor who observes a history which contains $p-1$ waitings and $\omega^i \in [0, n-p]$ withdrawals, so each patient depositor other than the one who observes the history has waited. Any such history is a truthful one, and the only equilibrium strategy is $s_i(pat, h^{tr}(p-1)) = wa$ because it leads to the first best which yields the highest obtainable payoff. Therefore, we may apply the corollary to this set of truthful histories.

Lemma 2 *For the set of truthful histories which contain* $\acute{\eta} \in [0, p-1]$ *waitings (and any* $\hat{\omega} \in [0, n-p]$ *withdrawals), we have* $s_{\acute{\eta}+\hat{\omega}+1}(\theta_{\acute{\eta}+\hat{\omega}+1}, h^{tr}(\acute{\eta})) = \theta_{\acute{\eta}+\hat{\omega}+1}$.

Proof. Apply corollary to $H^{tr}(p-1)$. ■

The previous lemma determines the best responses for any history that may come up in the game. Moreover, all these histories will be truthful! This is the case, because - for instance - any history starting with waitings is truthful, and since our strategy prescribes truthful action to these histories the resulting histories must be truthful as well. Any withdrawal after histories starting with waiting(s) must be due to impatient depositors. If a history begins with a withdrawal, then it is truthful, because a patient depositor would have waited according to the best responses we have found, so histories starting with withdrawals will be truthful as well.

Proposition 7 *The strategy* $\mathbf{s}_i(\boldsymbol{\theta}_i, \theta_1^{i-1}) = \theta_i$ *and the belief* $\mu(\theta_{i+1}^n \mid \theta_1^i)$ *for all* i *is the unique perfect Bayesian equilibrium of the game.*

Proof. Consider the history consisting of $\acute{\eta}$ waitings and no withdrawal, where $\acute{\eta} \in [0, p-1]$. The unique compatible belief is that it is a truthful history, so by the previous lemma $\mathbf{s}_{\acute{\eta}+1}(\theta_{\acute{\eta}+1}, \theta_1^{\acute{\eta}}) = \theta_{\acute{\eta}+1}$. As a consequence, the history starting with $\acute{\eta}$ waitings and followed by a withdrawal reveals that the last depositor must have been an impatient depositor. Therefore, a patient depositor observing this history knows that it is a truthful one, so $\mathbf{s}_{\acute{\eta}+2}(\theta_{\acute{\eta}+2}, \theta_1^{\acute{\eta}+1}) = \theta_{\acute{\eta}+2}$. This argument shows that any history starting with $\acute{\eta} \in [0, p-1]$ waitings must be a truthful one, so the previous lemma applies to them. Now consider how the game unfolds. If the first depositor is patient, then her belief is $\mu(\theta_2^n \mid \emptyset, pat) = \mu(\theta_2^n \mid pat)$ which corresponds to our definition of a truthful history. The previous lemma ensures that her optimal action is to wait. Thus, the second depositor can be sure to observe a truthful history, so her optimal action is to act truthfully as is the case for each later-coming depositor. depositors at any position can be sure to observe a truthful history to which the unique equilibrium strategy is to be truthful. ■

As a consequence of the proposition we have the following corollary.

Corollary 4 *The first best is strongly implementable.*

8.4 Appendix D

Consider the leading example which has that $u(c_2(\mu = 3)) > u(c_1^*) > u(c_2(\mu \leq 2)$ and $3c_1^* < 4$, so patient depositors at position 1,2 and 3 would only want to wait if all the other patient depositors wait. The optimal decision for a patient depositor in the last position is easily defined. When she observes a history with 2 withdrawals she withdraws, otherwise she waits. We know also that if a patient depositor observes any history containing 2 waitings, then her best response is to wait.

Thus, in any equilibrium we should have

$$
\begin{gathered}
s_3(wa, wa) = wa, \\
s_4(wa, wa, wi) = s_4(wa, wi, wa) = s_4(wi, wa, wa) = s_4(wi, wi, wi) = wa, \\
s_4(wa, wi, wi) = s_4(wi, wi, wa) = s_4(wi, wa, wi) = wi,
\end{gathered}
$$

where the subscript in the strategy denotes position and we have the history in brackets.

The previous strategies also imply that we must have

$$s_2(wa) = wa,$$

because otherwise a patient depositor would deviate unilaterally and receive $u(c_2^*)$, the highest possible payment.

Run happens if at least one of the patient depositors withdraws. When patient depositors at the beginning of the line wait, then later-coming patient depositors will wait as well, so to generate a run we should have the first patient depositors withdraw. Hence, we propose the following strategies:

$$
\begin{aligned}
s_1(\emptyset) &= wi, \\
s_2(wi) &= wi, \\
s_3(wa, wi) &= s_3(wi, wa) = s_3(wi, wi) = wi.
\end{aligned}
$$

THIS IS NOT OK. GIVEN $s_2(wa) = wa$ AGENT CAN UPDATE HER BELIEFS WHEN OBSERVING $(WA, WI) \rightarrow \mu_3(\theta_2 = pat \mid (wa, wi)) = 0$, SO $s_3(wa, wi) = WI$ IS NOT OPTIMAL

If these strategies really form an equilibrium, then depositors at any position should observe histories consisting only of withdrawals. It is straightforward to compute the beliefs on the proposed equilibrium path.

Whether we have a perfect Bayesian equilibrium with run boils down to the question if a deviation at the first position is profitable or not. If a patient depositor at the first position deviates, then subsequent depositors will observe histories which *ex ante* have zero probability. The only way that the deviation is not profitable could occur if the second depositor is an impatient one (who consequently withdraws), and the third depositor believes that the withdrawal has been due to a patient depositor, so her best response is to withdraw.[17] But is that belief possible? Given $s_2(wa) = wa$, only an impatient depositor would withdraw, so a patient depositor observing (wa, wi) cannot believe that the second depositor has been untruthful. Consequently, a patient depositor at the first position would deviate, so the proposed strategies do not form a run equilibrium.

8.5 Appendix E

We use the same definition of truthful history as before. Remember that we focus on patient depositors.

Lemma 3 *If waiting is the unique rationalizable strategy to any truthful history containing at least $\hat{\eta}$ waitings, then it is the unique rationalizable strategy to any truthful history with $\hat{\eta} - 1$ waitings.*

Proof. If a patient depositor observing any truthful history with $\hat{\eta} - 1$ waitings waits, then she knows that all later-coming patient depositors will observe a truthful history containing at least $\hat{\eta}$ waitings, so they will wait. This means that in the end all patient depositors wait which yields the highest possible payoff for patient depositors. Thus, waiting is the unique rationalizable strategy. ■

Corollary 5 *If waiting is the unique rationalizable strategy to any truthful history containing at least $\hat{\eta}$ waitings, then it is the unique rationalizable strategy to any truthful history with $[\hat{\eta} - 1, 0]$ waitings.*

Proof. Apply the previous lemma repeatedly. ■

Apply the corollary to any history containing $p - 1$ waitings (a necessarily truthful history) to obtain the proposition.

[17] Notice that a deviation at the first position results in a gain in two of the three possible alignments. Nevertheless, it is possible to find parameters such that the losses of the third alignment outweigh the gains of the other two.

8.6 Appendix F

To illustrate the possibility of a run, consider the leading example when only withdrawals are observed. The possible histories are: observing $0, 1, 2$ or 3 withdrawals. In the last case, a patient depositor knows to be in the last position and her best response is to wait to earn the interest rate. A patient depositor observing a different number of withdrawals may believe to be in the following positions:

wi	possible position
0	1,2,3
1	2,3,4
2	3,4

We cannot restrict in any way these conjectures, so for example a patient depositor observing 2 withdrawals may attach a high probability of being in position 3. As a consequence, she should withdraw. Similarly, a patient depositor observing a withdrawal may believe to be at the second position and she may believe that later-coming patient depositor will withdraw, so her best response to this conjecture is to withdraw. These conjectures are possible because waitings are not observable. We do not claim that run is the unique outcome, we say that it is a possible outcome.

8.7 Appendix G

Proof. We assumed that at least two patient depositors should wait to make waiting a better choice. Nevertheless, the strategy ensures that at most one depositor will wait. This happens if there are funds in the bank for the last depositor and this last depositor is patient. Unilateral deviations are not profitable, because waitings are not observable. No patient depositor can induce later-coming patient depositors to deviate from the run strategy. ■

Consider the leading example as an illustration. The proposed strategy for patient depositors would be

$$s_i = wi \text{ for } i = 0, 1, 2,$$
$$s_3 = wa,$$

where a patient depositor who knows to be the last would wait and earn the return. Given these strategies on the equilibrium path depositors know their position. The implied beliefs on the equilibrium path are the following

$$\mu(\theta_4^4 \quad | \quad 2wi, pat) = \begin{cases} imp \text{ with prob. } \frac{1}{3}, \\ pat \text{ with prob. } \frac{2}{3}. \end{cases}$$

$$\mu(\theta_3^4 \quad | \quad 1wi, pat) = \begin{cases} pat, imp \text{ with prob. } \frac{1}{3}, \\ imp, pat \text{ with prob. } \frac{1}{3}, \\ pat, pat \text{ with prob. } \frac{1}{3}. \end{cases}$$

$$\mu(\theta_2^4 \quad | \quad \varnothing, pat) = \begin{cases} pat, pat, imp \text{ with prob. } \frac{1}{3}, \\ pat, imp, pat \text{ with prob. } \frac{1}{3}, \\ imp, pat, pat \text{ with prob. } \frac{1}{3}. \end{cases}$$

The interesting thing about this setup is that information sets off the equilibrium path are not observed. Thus, if a patient depositor deviates later-coming patient depositors will not know about it. There is no evidence on deviations, nor is there any clue which would suggest that a previous patient depositor did so. Having the previous beliefs and knowing that a deviation will not be detected results in that deviations are not profitable. Therefore, the best any depositor can do is to follow her prescribed strategy.

8.8 Appendix H

We use the same definition of truthful history as before and again we focus on patient depositors. Denote an element of the truthful histories with $\hat{\rho}$ reports by $h^{tr}(\hat{\rho}) \in H^{tr}(\hat{\rho})$.

Lemma 4 *If waiting is the unique rationalizable strategy to any truthful history containing at least $\hat{\rho}$ reports, then it is the unique rationalizable strategy to any truthful history with $\hat{\rho} - 1$ reports.*

Proof. For a patient depositor observing any truthful history with $\hat{\rho}-1$ reports reporting dominates withdrawal, because by reporting she induces all later-coming patient depositors to wait and she gets $u(c_2^*)-k$, whereas by withdrawing she only would receive $u(c_1^*)$. Thus, withdrawal is not a rationalizable strategy for a patient depositor observing any $h^{tr}(\hat{\rho} - 1) \in H^{tr}(\hat{\rho} - 1)$. Given this, a withdrawal after such a truthful history must be due to an impatient depositor, so the new history is also a truthful history with $\hat{\rho}-1$ reports. Hence, reporting dominates withdrawal again. Applying the same argument, any withdrawal after any $h^{tr}(\hat{\rho}-1) \in H^{tr}(\hat{\rho}-1)$ must be truthful, so these histories will be truthful as well. Hence, for patient depositors observing these histories, reporting will dominate withdrawal. Knowing that no patient depositor will withdraw once any $h^{tr}(\hat{\rho}-1) \in H^{tr}(\hat{\rho}-1)$ is reached, means that a patient depositor does not need to report to avoid that later-coming depositors withdraw. Therefore, the best response to any $h^{tr}(\hat{\rho} - 1) \in H^{tr}(\hat{\rho} - 1)$ is to wait. ■

Corollary 6 *If waiting is the unique rationalizable strategy to any truthful history containing at least $\hat{\rho}$ reports, then it is the unique rationalizable strategy to any truthful history with $[\hat{\rho} - 1, 0]$ reports.*

Proof. Apply the previous lemma repeatedly. ■

A patient depositor observing any history containing $p-1$ reports knows that the history is truthful. Thus, we can apply the corollary. As a consequence, a patient depositor observing nothing knows that the history is truthful, so her unique rationalizable strategy is to wait. Given this, a patient depositor observing a withdrawal knows that it must have been an impatient depositor, so the history is truthful. She will wait, as any patient depositor after observing at most $n-p$ withdrawals. Hence, the first best obtains, and no patient depositor reports.