

Public Debt Maturity: The Role of Liquidity Provision*

Dallal Bendjellal[†]

Job Market Paper

[Click here for the latest version](#)

November 9, 2022

Abstract

What debt maturities should governments issue when sovereign bonds serve as collateral? Standard Debt Management frameworks advocate for the issuance of long-term debt because of its hedging benefits for the government's budget. These frameworks are premised on the assumption that public debt is solely used to finance fiscal deficits. In practice, government bonds play a central role in financial markets as they are used as collateral to borrow liquidity. This paper introduces the collateral role of public debt into a standard Debt Management model to analyze its impact on the optimal structure of debt maturity. My main finding is that optimal maturity management involves an additional objective which is the provision of collateral and thus liquidity. This raises a policy trade-off for the government. While long-term debt allows to reduce the debt borrowing costs, short-term debt proves more effective to enhance liquidity provision. I show that the optimal maturity structure depends on the extent to which private liquidity relies on collateral. For a plausible calibration, the government finds it optimal to issue short-term bonds to accommodate liquidity provision.

Keywords: Debt Management, Maturity Structure, Collateral, Financial Frictions

JEL classifications: E62, H63, H21, G18, D53.

*I would like to thank my advisors Aurélien Eyquem and Céline Poilly for their guidance and advice. This paper also benefited from conversations with Davide Debortoli, Albert Marcet, Alberto Martin, Fabrice Collard, Alain Venditti and Paul Hubert. This project has received funding from the french government under the "France 2030" investment plan managed by the French National Research Agency (reference: ANR-17-EURE-0020) and from Excellence Initiative of Aix-Marseille University - A*MIDEX.

[†]Aix Marseille Univ, CNRS, AMSE, Marseille, France. Email: dallal.bendjellal@univ-amu.fr.

1 Introduction

Over the last two decades, a succession of financial and economic crises has challenged the ability of governments to sustain their public finances. For many economies, recurring and large increases in government debt have been inevitable. This made debt managers pay particular attention to how the maturity structure affects the costs and risks related to a high stock of debt. A substantial literature in public finance (see *inter alia* [Angeletos, 2002](#) and [Barro, 2003](#))¹ studies optimal maturity management and concludes that governments should issue long-term debt because of its hedging benefit for the fiscal budget. As long prices fall in bad times, long bonds lower the total debt burden and prevent from costly rollovers of debt.

The optimal Debt Management literature is premised on the assumption that the unique function of public debt in the economy is to finance fiscal deficits. In practice, however, public debt plays a key role in facilitating the flow of liquidity within the financial system, since government bonds are typically used by market participants as collateral to borrow liquidity. In the Euro Area, the repo market—where more than 80% of funding is guaranteed by sovereign bonds—represents the largest segment of the money market, accounting for more than two thirds of the total market turnover ([ECB, 2020](#)). This implies that government bond prices have important implications for market liquidity and therefore the allocation of funding into the real economy².

Because the maturity of bonds is a key determinant of their market price, and thus their collateral value, this paper studies how the collateral role of public debt impacts the optimal maturity structure that the government should choose for its debt. To this end, I develop a dynamic fiscal policy model with multiple debt maturities as in the seminal work of [Angeletos \(2002\)](#). I introduce an additional role for government debt which is the provision of collateral: firms need borrowing to finance their wage bill and produce, and access to liquidity requires collateral.

My main finding is that optimal debt management involves an additional objective which is the provision of collateral, and thus liquidity. This raises a policy trade-off for the government. Long-term debt is a good hedge for the fiscal budget because it prevents from high rollover costs and reduces the actual value of the debt burden when shocks hit the economy since long prices deteriorate. However, because the market price of long bonds declines in bad times, long-term debt is costly

¹Other contributions to this literature include [Buera and Nicolini \(2004\)](#), [Nosbusch \(2008\)](#), [Lustig, Sleet, and Yeltekin \(2008\)](#), and [Fraglia, Marcet, and Scott \(2010\)](#).

²See [Brunnermeier and Pedersen \(2009\)](#) and [Huh and Infante \(2021\)](#) for the link between the value of collateral and market liquidity in financial markets.

for private liquidity. When shocks hit, the fall in the collateral value of long bonds reduces private agents' ability to borrow liquidity. The resulting decline in economic activity leads to lower tax revenues and significant welfare losses. I show that the optimal maturity of public debt hinges on the extent to which private liquidity relies on collateral. For a plausible calibration, I find that the government finds it optimal to issue short-term debt to boost private liquidity during periods of stressed public finances, because of its benefits on welfare and tax revenues.

My core framework is based on the classic model of [Lucas and Stokey \(1983\)](#). In this economy, markets are competitive and labor is the only factor of production. The government faces public spending shocks; it collects taxes on labor income and issues public debt to finance its expenditures. My model features two frictions. First, as in [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#), I assume that debt is noncontingent, and that the government is able to issue all debt maturities. Second, I introduce a financial friction into the production sector that gives rise to a private liquidity role for public debt, in the spirit of [Holmström and Tirole \(1998\)](#). In a similar way as [Niemann and Pichler \(2017\)](#), I assume that firms need advanced financing to pay their workers because production is subject to moral hazard. Borrowing requires collateral, and public debt holdings are pledged as collateral. The private agents' ability to borrow and to produce becomes tightly connected to the market value of their public debt holdings.

The work of [Lucas and Stokey \(1983\)](#) shows that, in the presence of fiscal shocks, the government is able to achieve full insurance and sustain a constant tax rate by issuing state-contingent debt. That is, bonds that pay badly when public spending is high, and well when public spending is low. In practice, however, such securities tend to not exist, which makes the structure of the public debt portfolio important to manage debt payments when shocks occur. [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#) show that, even if only noncontingent bonds are available, the government can achieve full insurance by choosing the right maturity structure for its debt. The optimal maturity is the one that generates a decline in the market value of public debt when the present value of the government's primary surpluses are low. Since short-term interest rates rise when public spending is low, the issuance of long-term debt and the purchase of short-term assets is then optimal.

When public debt serves as collateral, such a maturity structure inflicts a non-trivial cost on the government at a time when expenditures are high. First, a fall in the market value of collateral is welfare costly as it dries up liquidity for the private sector. Second, the associated fall in economic activity lowers the government's tax revenues when they are most needed. This exacerbates the

decline in the present value of the primary surpluses, and further tightens the government's budget constraint. In this sense, the lower the issuance of long-term debt—the price of which declines when public spending is high—the lower the liquidity shortage and thus the associated cost inflicted on the social planner. A switch of the maturity structure—that is the sell of short-term debt and the purchase of long-term assets—raises private liquidity in bad times, which improves welfare and increases tax revenues. However, a switched maturity comes at the cost of higher debt payments for the government. The resulting trade-off then gives rise to an inefficiency and makes full insurance against fiscal shocks no longer guaranteed.

In order to analyze how this trade-off impacts the optimal maturity, I study the economy's Ramsey equilibrium in a three-period model with endogenous public spending as in [Debortoli, Nunes, and Yared \(2017\)](#). The income tax rate being constant, any deviation from full insurance then translates into the government's choice of public expenditures. In a first exercise, I consider a special case that allows for an analytical solution of the optimal policy. I assume that labor is fixed and that the government is not benevolent—that is, private consumption is not valuable for the social planner. Although this shuts down the welfare effect of the collateral friction on private agents, it sheds light on the key channels driving the government's optimal policy.

I find that easing the financial friction becomes an additional policy objective for the government; and public debt serves as the instrument. In the absence of shocks, the social planner deviates from a smooth policy to provide collateral when private agents are financially constrained. This happens because the lack of liquidity impacts the government's tax revenues and therefore the financing of valuable public expenditures. In the presence of fiscal shocks, I show that the optimal maturity choice in the reduced form is to issue short-term debt and to purchase long-term assets. I characterize the existence of the optimal solution and show that it requires the constant tax rate to be greater than the severity of the collateral friction. As a consequence, the tax gain that the government realizes on the value of collateral is greater than the capital loss on the value of outstanding debt. The government then finds it optimal to raise the market value of public debt in bad times, and therefore to switch its debt maturity as compared to an economy without collateral friction. Finally, the analysis of the analytical solution also shows that the debt positions required for hedging are sensitive to the severity of the collateral constraint.

In the second exercise, I conduct numerical simulations to derive more precise quantitative implications for optimal policy. Results show that the conclusions of the reduced-form representation hold

approximately when the government is benevolent and labor supply is endogenous. I first analyze the optimal policy in a deterministic economy to understand the impact of the friction when fiscal insurance is not an issue. In comparison to a frictionless economy, I find that an economy with collateral constraint experiences an important liquidity shortage when public policy is not optimized. This highlights the critical role played by the government's optimal policy when public debt becomes a source of liquidity. Moreover, since liquidity boosts both economic activity and tax revenues, I find that the collateral service of debt allows the government to enhance social welfare. The welfare gain, however, depends on the severity of the friction; a collateral constraint that is too severe may lead to welfare losses because the provision of liquidity comes at the cost of a high debt burden.

In the presence of fiscal shocks, I find that the government's optimal maturity hinges on the tightness of the collateral friction. For reasonable values of the friction's severity, the social planner chooses to issue short-term bonds and to buy long-term assets so that the market value of public debt increases when public spending is high. However, when the collateral friction becomes too severe, the decline in the collateral value of public debt makes the government switch its debt maturity: the issuance of long-term debt becomes optimal. This happens because a variation in the value of public debt when the friction is too constraining has little impact on private liquidity, which makes the reduction in debt payments a better strategy for the government to finance its expenditures.

Overall, regardless of the optimal maturity choice of the government, full insurance in the presence of the collateral constraint is no longer possible as any variation in the value of outstanding debt carries a cost: it either raises debt payments or hurts private liquidity. As full hedging requires very large debt positions ([Angeletos, 2002](#)), the lack of insurance in the presence of a collateral constraint implies that the optimal debt positions are substantially lower, and thus more plausible, for most values of the friction's severity.

Related Literature. My work connects to the public finance literature on debt maturity which goes back to [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#). Contributions to this literature explore, in different environments, debt maturity as a fiscal insurance device. [Nosbusch \(2008\)](#) and [Lustig et al. \(2008\)](#), for instance, study the optimal maturity when inflation uncertainty is an issue and find that long-term debt remains optimal. [Faraglia, Marcet, and Scott \(2010\)](#) introduce habits and capital, and show that it is always optimal to issue long-term debt, but the debt positions are large and volatile. [Debortoli, Nunes, and Yared \(2017\)](#) show that introducing lack of commitment over fiscal policy makes the government choose a flat maturity to reduce the funding costs of debt.

Faraglia, Marcet, Oikonomou, and Scott (2019) focus on the case in which the government has only access to long-term bonds and find that issuing long-term debt may lead to higher tax volatility, which reduces its ability to provide insurance. Bigio, Nuño, and Passadore (2019) analyze how liquidity frictions affect optimal maturity, but in a setting in which the financial constraint applies on foreign debt holders. This paper is the first to provide an analysis of the optimal debt maturity taking into account the collateral function of government bonds, and therefore their impact on private liquidity.

The liquidity role of public debt that I introduce in this paper appears in various contributions, such as Woodford (1990), Aiyagari and McGrattan (1998), and Holmström and Tirole (1998). Their work explores how public debt can be used as an instrument to relax financial constraints and improve welfare. Considering a similar friction, Angeletos, Collard, and Dellas (2016) find that the provision of liquidity also raises the debt borrowing costs and makes the government deviate from tax smoothing. They show that even when the government has access to state-contingent debt, the financial friction prevents full insurance against shocks. As in this work, I find that, in the presence of the friction, the optimal debt maturity does not allow the government to completely insulate its budget even though it replicates the payments of state-contingent debt. Niemann and Pichler (2017) also study the optimal provision of public liquidity to domestic agents, but with the possibility of government default.

More generally, this paper relates to the literature that studies the Ramsey fiscal policy when the government has access only to noncontingent debt. Prominent examples include Barro (1979), Aiyagari et al. (2002), Bhandari et al. (2017). The article is also close to the literature that explores optimal debt maturity when the government has the option to default on its debt. In such settings, optimal maturity impacts the government's repayment incentives and rollover risk, which in turn influence debt prices (see, for instance, Aguiar et al. (2019), Arellano and Ramanarayanan (2012), Bocola and DAVIS (2019), Hatchondo et al. (2016)).

The rest of the article is organized as follows. Section 2 lays out the general model economy and defines the problem of the social planner. Section 3 provides an analytical analysis of the special case with fixed labor, and section 4 presents the results of the numerical exercise.

2 The Model

2.1 General Environment

I consider an economy similar to [Lucas and Stokey \(1983\)](#) populated by households, firms and a government. The government taxes labor income and issues debt in order to finance public spending. Time is discrete $t = \{1, \dots, \infty\}$, and stochastic disturbances in period t are defined by a stochastic state $s_t \in S$ which follows a first order Markov process (s_0 is given). Let $s^t = \{s_0, \dots, s_t\} \in S^t$ represent a history at time t , and let $\pi(s^{t+k} | s^t)$ represent the probability of s^{t+k} conditional on s^t for $t+k \geq t$.

2.1.1 Government

In every period t , the government issues debt B_t^{t+k} at a price q_t^{t+k} , with a promise to pay one unit of consumption at $t+k > t$. The government policies $\{\tau_t, g_t, \{B_t^{t+k}\}_{k=1}^\infty\}$ must satisfy the following budget constraint

$$\sum_{k=1}^{\infty} q_t^{t+k} (B_t^{t+k} - B_{t-1}^{t+k}) + \tau_t w_t n_t = B_{t-1}^t + g_t \quad (1)$$

where τ_t is the labor tax, w_t is the labor wage, n_t is labor and g_t is public spending. The initial level of government debt $\{B_{-1}^{k-1}\}_{k=1}^\infty$ is exogenous. As in [Angeletos \(2002\)](#), I assume that the government buys back all outstanding long-term debt in every period, and then reissues fresh debt at all maturities. In order to prevent Ponzi schemes, the stock of debt is constrained by an upper and lower debt limit $B_t^{t+k} \in [\underline{B}, \bar{B}]$. I assume that \underline{B} is sufficiently low and \bar{B} is sufficiently high so that the debt limit constraint is not binding.

In this environment, government debt is non-contingent as the value of outstanding debt $\{B_{t-1}^{t+k}\}_{k=0}^\infty$ is independent of the realization of s_{t+k} . If debt were state-contingent, each bond of the debt portfolio $\{\{B_{t-1}^{t+k} | s^{t+k}\}_{s^{t+k} \in S^{t+k}}\}_{k=0}^\infty$ maturing in $t+k$ would depend on the realization of a history of shocks $s^{t+k} \in S^{t+k}$.

2.1.2 Households

There is a continuum of measure one of identical, infinitely lived households that derive the following utility

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t [u(c_t, n_t) + \theta_t(s_t) v(g_t)] \quad (2)$$

where $\beta \in (0, 1)$ is a time discount factor. The utility function $u(\cdot)$ is strictly increasing in consumption and strictly decreasing in labor, globally concave, additively separable in c_t and n_t , and

continuously differentiable. $v(\cdot)$ is strictly increasing, concave and continuously differentiable; and $\theta(s_t)$ is high (low) when public spending is more (less) valuable.

Each household is composed of two types of members: workers and bankers. Workers supply labor to competitive firms, and bankers act as intermediaries between creditors and firms. Households enter the period with an initial stock of debt held by bankers.

2.1.3 Firms

Firms are perfectly competitive and have access to a production technology that transforms labor into consumption goods at a unitary rate $y_t = n_t$. Production is subject to a moral hazard problem that prevents firms from pledging funds to workers or outside creditors. Therefore, firms must borrow a working capital loan l_t from bankers in order to pay the wage bill $w_t n_t$ at the beginning of the period; hence, $l_t = w_t n_t$. Firms' profit at the end of the period writes

$$P_t^f = n_t - w_t n_t - (r_t - 1)w_t n_t \quad (3)$$

where r_t is the gross interest rate on working capital loans.

2.1.4 Bankers

Bankers act as intermediaries between outside creditors and firms by monitoring the production process. They provide intra-period loans l_t to firms using outside creditors' deposits d_t . Even though banks have a greater capacity to pledge funds to depositors, they are also subject to a moral hazard problem. The collateral that bankers are able to pledge must cover at least a fraction $\lambda \in (0, 1)$ of the deposits they receive. Government bonds represent the only source of collateral available to bankers. Hence, the collateral constraint faced by a representative banker from household j writes³

$$\lambda d_t^j \leq \sum_{k=0}^{\infty} q_t^{t+k} B_{t-1}^{j,t+k} \quad (4)$$

As deposits are used by the banker to provide loans to firms; it follows that $\lambda l_t^j \leq \sum_{k=0}^{\infty} q_t^{t+k} B_{t-1}^{j,t+k}$. Since bankers are competitive, the gross interest rate on working capital loans is greater than one only if the collateral that bankers can pledge constraints the supply of loans (i.e. the collateral constraint is binding).

³Outside creditors are assumed to be workers from households other than the banker's. This rules out the possibility of personal market interactions allowing the banker to avoid the financial friction.

2.1.5 Aggregation

At the end of the production process, workers and bankers transfer their earnings back to their respective households. Consumption-savings decisions are then made at the household level, hence there is perfect consumption insurance within households. Households use their earnings to consume and purchase government debt. The aggregate dynamic budget constraint for households is given by

$$c_t + \sum_{k=1}^{\infty} q_t^{t+k} (B_t^{t+k} - B_{t-1}^{t+k}) = (1 - \tau_t)w_t n_t + (r_t - 1)l_t + B_{t-1}^t \quad (5)$$

where $(r_t - 1)l_t$ is the bankers' profit. Aggregation across bankers implies the following collateral constraint

$$\lambda l_t \leq \sum_{k=0}^{\infty} q_t^{t+k} B_{t-1}^{t+k} \quad (6)$$

The resource constraint of the economy writes

$$c_t + g_t = n_t \quad (7)$$

2.2 Equilibrium

2.2.1 Competitive equilibrium

Let $\{c_t, n_t\}_{t=0}^{\infty}$ represent an *allocation*, $\{\tau_t, g_t, \{B_t^{t+k}\}_{k=1}^{\infty}\}_{t=0}^{\infty}$ represents a *government policy* and $\{w_t, \{q_t^{t+k}\}_{k=1}^{\infty}\}_{t=0}^{\infty}$ denote a *price system*.

Definition 1. A *competitive equilibrium* is an allocation, a government policy and a price system such that

1. given the price system and the government policy, the allocation maximizes the firm's profits (3) and the household's objective function (2) subject to the sequence of household's budget constraints (5) and collateral constraints (6); and
2. given the allocation and the price system, the government policy satisfies the sequence of government budget constraints (1);

From the above definition, Walras law implies that a stochastic sequence $\{c_t, n_t, g_t\}_{t=0}^{\infty}$ generated by a competitive equilibrium necessarily satisfies the resource constraint of the economy (7)⁴.

⁴See the Online Appendix for proof.

Households and firms take prices $(w_t, \{q_t^{t+k}\}_{k=1}^\infty)$ and government policies $(\tau_t, g_t, \{B_t^{t+k}\}_{k=1}^\infty)$ as given. Households choose consumption, labor supply, loans supply and savings to maximize their objective function (2), subject to the budget constraint and the collateral constraint

$$\begin{aligned}
V_t = \max_{c_t, n_t, l_t, \{B_t^{t+k}\}_{k=1}^\infty} & [u(c_t, n_t) + \theta_t(s_t)v(g_t)] + \beta \mathbb{E}_t V_{t+1} \\
& - \mu_{1,t} \left[c_t + \sum_{k=1}^{\infty} q_t^{t+k} (B_t^{t+k} - B_{t-1}^{t+k}) - (1 - \tau_t)w_t n_t - (r_t - 1)l_t - B_{t-1}^t \right] \\
& - \mu_{2,t} \left[l_t - \frac{1}{\lambda} \sum_{k=0}^{\infty} q_t^{t+k} B_{t-1}^{t+k} \right]
\end{aligned}$$

The first-order conditions for the household's problem imply the following intratemporal and intertemporal conditions

$$(1 - \tau_t)w_t = -\frac{u_{n,t}}{u_{c,t}} \quad (8)$$

$$q_t^{t+k} = \mathbb{E}_t \left\{ \beta \frac{u_{c,t+1}}{u_{c,t}} q_{t+1}^{t+k} \left[1 + \underbrace{\frac{1}{\lambda} (r_{t+1} - 1)}_{\text{liquidity premium}} \right] \right\} \quad \forall k > 0 \quad (9)$$

The Euler equation (9) highlights the dual role played by public debt in the model:

(i) it allows households to shift consumption over the maturity of the bond; and

(ii) it is a source of collateral for bankers. The liquidity premium depends positively on the interest rate on loans as bankers increase their demand for collateral when the return on lending is higher, and negatively on λ as a higher severity of the friction increases the collateral required for the same amount of loans, making the liquidity service of each unit of debt lower.

Firms choose labor demand to maximize their profits (3). The first-order condition of their problem implies

$$w_t = \frac{1}{r_t} \quad (10)$$

A competitive equilibrium can thus be summarized by the following equations

$$(1 - \tau_t)w_t = -\frac{u_{n,t}}{u_{c,t}} \quad (11)$$

$$q_t^{t+k} = \mathbb{E}_t \left\{ \beta \frac{u_{c,t+1}}{u_{c,t}} q_{t+1}^{t+k} \left[1 + \frac{1}{\lambda} \left(\frac{1}{w_{t+1}} - 1 \right) \right] \right\} \quad (12)$$

$$\sum_{k=1}^{\infty} q_t^{t+k} (B_t^{t+k} - B_{t-1}^{t+k}) + \tau_t w_t n_t = B_{t-1}^t + g_t \quad (13)$$

$$\lambda w_t n_t = \sum_{k=0}^{\infty} q_t^{t+k} B_{t-1}^{t+k} \quad (14)$$

$$c_t + g_t = n_t \quad (15)$$

When the collateral constraint is not binding, my model reduces to the one in Angeletos (2002). Since the production function is linear, the frictionless labor demand is given by

$$w_t = 1 \quad (16)$$

The gross interest rate on loans being equal to one, the liquidity premium on public debt becomes zero which implies the following Euler equation⁵

$$q_t^{t+k} = \mathbb{E}_t \left\{ \beta \frac{u_{c,t+1}}{u_{c,t}} q_{t+1}^{t+k} \right\} \quad (17)$$

Comparisons between the Ramsey outcomes with and without collateral constraint are presented in the results.

2.2.2 The Ramsey problem

The government faces an optimal policy problem. I consider a Ramsey problem in which the government is benevolent and shares the same preferences as the households in equation (2). In addition, the government has access to a commitment technology that allows it to commit to a particular sequence of policies from period $t = 0$.

Definition 2. A *Ramsey equilibrium* is an allocation, a price system and a government policy such that

1. the allocation and the price system generate a competitive equilibrium for every given government policy; and
2. given an initial level of debt $\{B_{-1}^{k-1}\}_{k=1}^{\infty}$, the government policy maximizes the household's objective function (2) subject to the competitive equilibrium constraints (11 to 15).

⁵Derivations of the frictionless competitive equilibria are available in the Online Appendix.

2.3 Three-Period Economy

I conduct my analysis in a three-period economy. I first consider a special case in which labor is fixed and the government is not benevolent. Such a framework allows to explicitly characterize the Ramsey equilibrium, and therefore highlight the role that the collateral friction has on optimal debt maturity. I then conduct a numerical exercise in which I relax those limiting assumptions in order to study the quantitative implications of the optimal policy.

In an economy *à la* [Lucas and Stokey \(1983\)](#), the government has access to linear labor taxes and to state-contingent debt to finance public expenditures. When the government faces public spending shocks, it finds it optimal to sustain a constant tax rate on labor income and to issue state-contingent debt to finance the stochastic spending needs. [Angeletos \(2002\)](#) shows that when state-contingent debt is not available, the government is able to completely insulate itself against any shock by choosing an appropriate maturity structure for the non-contingent debt portfolio. The optimal maturity structure allows the government to replicate the state-contingent payments and therefore to implement an invariant tax rate. I explicitly characterize this outcome in a three-period model without collateral constraint, as in [Debortoli, Nunes, and Yared \(2017\)](#)⁶. I then solve the Ramsey problem when private agents face a binding collateral constraint, that makes the economy's activity dependent on the market value of public debt, and therefore its maturity.

Let $t = 0, 1, 2$ and define θ^H and θ^L with $\theta^L = 1 - \delta$ and $\theta^H = 1 + \delta$ for $\delta \in [0, 1)$. Suppose that $\theta_1 = \theta^H$ with probability $\frac{1}{2}$ and $\theta_1 = \theta^L$ with probability $\frac{1}{2}$. In period $t = 2$, define θ_2 as follows

$$\theta_2 = \begin{cases} \alpha\theta^H + (1 - \alpha)\theta^L, & \text{if } \theta_1 = \theta^H \\ \alpha\theta^L + (1 - \alpha)\theta^H, & \text{if } \theta_1 = \theta^L \end{cases}$$

where $\alpha \in [0.5, 1)$. Hence, uncertainty is only present in period $t = 1$, with δ capturing the volatility of the shock. Parameter α captures the persistence of the shock between dates $t = 1$ and $t = 2$.

As in [Lucas and Stokey \(1983\)](#), I characterize competitive equilibria using the primal approach which allows to abstract from prices so that the government can be thought of as directly choosing a feasible allocation that maximizes welfare. The government's welfare can be represented by

$$\mathbb{E} \sum_{t=0,1,2} \beta^t \left\{ (1 - \psi) \left[\log(c_t) - \frac{1}{\phi + 1} n_t^{\phi+1} \right] + \psi \theta_t(g_t) \right\} \quad (18)$$

⁶The case in which the collateral constraint is not binding is identical to the one in [Debortoli, Nunes, and Yared \(2017\)](#) with full commitment.

where ϕ is the inverse of the Frisch elasticity and $\psi \in [0, 1]$. I also consider that all debt is paid in the end of period $t = 2$ and that there is zero initial debt $\{B_{-1}^k\}_{k=0}^2 = 0$. Since public expenditures are endogenous and can therefore be chosen by the government, I assume that the government is committed to a fixed tax rate τ , as in [Debortoli, Nunes, and Yared \(2017\)](#). Any deviation from full insurance against shocks would then translate into the government's choice of public spending.

3 A special case with fixed labor

I first consider a special case that allows for an analytical characterization of the optimal policy. I assume that labor is exogeneously fixed to some n that households supply at the labor wage set by firms. The government's welfare can then be represented by

$$\mathbb{E} \sum_{t=0,1,2} \beta^t \{(1 - \psi) \log(c_t) + \psi \theta_t(g_t)\} \quad (19)$$

I assume that $\beta = 1$ and $\psi \rightarrow 1$. The latter implies that the government is no longer benevolent since its welfare is almost insensitive to the variations in private consumption. This simplification is dropped in the next section.

3.1 Without Collateral Constraint

In order to understand the optimal maturity structure when debt serves as collateral, it is useful to start the analysis from a frictionless economy. In such an environment, the absence of state-contingent debt is the only departure from the dynamic fiscal policy model of [Lucas and Stokey \(1983\)](#). The government can only issue non-contingent debt of all maturities, as in [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#).

In a frictionless economy, the government faces the following implementability conditions

$$t = 0 : \quad 0 = \tau w_0 n - n + c_0 + \mathbb{E}\{q_0^1 B_0^1\} + \mathbb{E}\{q_0^2 B_0^2\} \quad (20)$$

$$t = 1 : \quad B_0^1 + q_1^2 B_0^2 = \tau w_1 n - n + c_1 + q_1^2 B_1^2 \quad (21)$$

$$t = 2 : \quad B_1^2 = \tau w_2 n - n + c_2 \quad (22)$$

where the labor wage $\{w_t\}_{t=0}^2$ and the bond price $\{q_t^{t+k}\}_{k=0}^2\}_{t=0}^2$ are defined by equations (16) and (17), respectively. Substituting for prices and debt levels in equations (20), (21) and (22) gives the following constraint

$$\mathbb{E} \sum_{t=0,1,2} \left(1 - \frac{1-\tau}{c_t} n_t \right) = 0 \quad (23)$$

Equation (23) is the intertemporal implementability condition in period $t = 0$. Lucas and Stokey (1983) show that under complete markets, the only relevant constraint on the planner is the intertemporal implementability condition at date $t = 0$ (equation (23)). The reason is that in the presence of state-contingent debt, an allocation that satisfies the intertemporal implementability condition in date $t = 0$ and $s^t = s^0$ necessarily satisfies the intertemporal implementability constraints for all other histories s^t , since the government can freely choose the state-contingent payments to satisfy the constraint at all future histories s^t . This implies that under complete markets, the government maximizes welfare (19) under the constraint in equation (23). The resulting optimality condition writes

$$c_t = \frac{1}{\theta_t^{\frac{1}{2}}} \frac{n(1-\tau)}{3} \mathbb{E} \sum_{i=0,1,2} \theta_i^{\frac{1}{2}} \quad \forall t. \quad (24)$$

The optimal allocation in equation (24) implies that public expenditures and private consumption have two possible realizations in $t = 1$ and $t = 2$. When the shock is high ($\theta_1 = \theta^H$), private consumption in $t = 1$ and $t = 2$ is low, and therefore, public spending (*i.e.* $g_t = n - c_t$) is high. It follows that the government's primary surpluses in period $t = 1$ are low (high) when the shock is high (low). The government would then like to reduce (increase) the market value of outstanding debt—that is, the due payments to the private sector—when the shock is high (low). In the presence of state-contingent debt, the government's due payments in $t = 1$ are contingent to the realization of θ_1 . This allows the government to lower its payments to the private sector when public expenditures are high. As a consequence, the government is perfectly insured against the θ_1 shock. The maturity of debt is therefore irrelevant.

In the absence of state-contingent debt, full insurance is not guaranteed due to the fact that the government needs to fulfill its non-contingent payments whatever is the realization of the primary surpluses. In this case, [Aiyagari, Marcet, Sargent, and Seppälä \(2002\)](#) show that the optimal allocation must satisfy the intertemporal implementability condition at date $t = 0$, as well as additional ones that impose that the government issue only non-contingent debt for all future histories s^t . In this economy, this implies that the optimal allocation must also satisfy the intertemporal implementability in period $t = 1$, which is the only period in which there is uncertainty⁷. Accordingly, to show that the full insurance allocation (24) is sustainable when state-contingent debt is not avail-

⁷Note that once the shock θ_1 is realized in period $t = 1$, the government has no uncertainty surrounding the realization of θ_2 , and therefore public expenditures g_2 . As a result, the government chooses the level of debt B_1^2 in $t = 1$ such that the latter is equal to the *certain* primary surplus in $t = 2$.

able, one must show that there exists B_0^1 and B_0^2 for which the allocation in equation (24) satisfies the following intertemporal implementability constraint in period $t = 1$ in the high state as well as the low state of nature⁸

$$\underbrace{B_0^1 + \frac{c_1}{c_2} B_0^2}_{\text{market value of outstanding debt}} = \underbrace{c_1 - n(1 - \tau) + \frac{c_1}{c_2} (c_2 - n(1 - \tau))}_{\text{net present value of primary surpluses}} \quad (25)$$

Hence, the government must choose B_0^1 and B_0^2 in period $t = 0$ such that the net present value of primary surpluses in period $t = 1$ —which depends on the realization of θ_1 —is equal to the debt payments that the government has to make in this period, that is the market value of outstanding non-contingent debt. The optimal debt maturity is the one that satisfies equation (25) for all the realizations of θ_1 (*i.e.* θ^H and θ^L) while allowing to sustain the full insurance allocation. As B_0^1 and B_0^2 are chosen in $t = 0$ and they are not contingent on s^1 —that is, they do not depend on θ_1 in date $t = 1$ —, the two debt positions can be derived using equation (25) when $\theta_1 = \theta^H$ and $\theta_1 = \theta^L$. Accordingly, the levels of B_0^1 and B_0^2 are given by

$$B_0^1 = \frac{2(c_2^H - c_2^L) - n(1 - \tau) \left(\frac{c_2^H}{c_1^H} - \frac{c_2^L}{c_1^L} \right)}{\frac{c_2^H}{c_1^H} - \frac{c_2^L}{c_1^L}} \quad (26)$$

$$B_0^2 = \frac{2(c_1^H - c_1^L) - n(1 - \tau) \left(\frac{c_1^H}{c_2^H} - \frac{c_1^L}{c_2^L} \right)}{\frac{c_1^H}{c_2^H} - \frac{c_1^L}{c_2^L}} \quad (27)$$

with $c_1 = c_1^H$ when $\theta_1 = \theta^H$, and $c_1 = c_1^L$ when $\theta_1 = \theta^L$. Using the optimal allocation in equation (24), it can be shown that⁹

$$B_0^1 < 0 \quad \text{and} \quad B_0^2 > 0$$

Hence, issuing long-term debt ($B_0^2 > 0$) and purchasing short-term assets ($B_0^1 < 0$) allows the government to sustain full insurance with non-contingent debt. Indeed, when the full insurance allocation is implemented, the net present value of primary surpluses in period $t = 1$ is low when the high shock $\theta_1 = \theta^H$ is realized (see equation (24)). If the government has access to state-contingent debt, it can choose to absorb the resulting deficit using a state-contingent payment from households. In the absence of state-contingent bonds, the government can replicate the state-contingent payment with a capital gain on the portfolio of outstanding debt, and thereby sustaining the full

⁸Notice that if the government has only access to one-period debt, the intertemporal implementability constraint in period $t = 1$ would lead to an indetermination since the non-contingent level of debt B_0^1 has to satisfy the equation in the high state *and* the low state. The full insurance allocation in equation (24) is therefore not sustainable in the absence of debt maturity choice.

⁹See the Online Appendix for proof.

insurance allocation. This is possible because, since the shock is mean-reverting, the one-period bond price in $t = 1$, $q_1^2 = c_1/c_2$, is lower when the shock is high. Therefore, issuing long-term debt in period $t = 0$ ($B_0^2 > 0$) allows to reduce the market value of outstanding debt during the high shock. The government purchases short-term assets in period $t = 0$ so that the payments on these assets cover a part of the buyback of outstanding long-term debt when the shock is high and therefore the government's resources are lower.

3.2 With Collateral Constraint

In this section, I show that the conclusion on public debt optimal maturity in [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#) is altered in the presence of a collateral friction. In this environment, the government faces one additional constraint in the Ramsey problem, which is the collateral constraint (equation (14)). Since the initial level of debt is zero, the working capital constraint is not binding in period $t = 0$. For simplicity, I also consider that the collateral constraint does not bind in period $t = 2$. The government's implementability conditions in period $t = 0$, $t = 1$ and $t = 2$ are given by

$$t = 0 : \quad 0 = \tau w_0 n - n + c_0 + \mathbb{E}\{q_0^1 B_0^1\} + \mathbb{E}\{q_0^2 B_0^2\} \quad (28)$$

$$t = 1 : \quad B_0^1 + q_1^2 B_0^2 = \tau w_1 n - n + c_1 + q_1^2 B_1^2 \quad (29)$$

$$B_0^1 + q_1^2 B_0^2 = \lambda w_1 n \quad (30)$$

$$t = 2 : \quad B_1^2 = \tau w_2 n - n + c_2 \quad (31)$$

where the labor wages w_0 and w_2 are defined by equation (16), and the bond prices $\{\{q_t^{t+k}\}_{k=0}^2\}_{t=0}^2$ are defined by equation (12). In the presence of the collateral constraint in $t = 1$, the labor wage set by firms is no longer equal to 1. Since a fraction λ of working capital loans ($w_1 n$) has to be covered by collateral, the labor wage is increasing in the supply of loans, and therefore the amount of collateral available in the economy in $t = 1$. After substituting for prices and debt levels, the intertemporal implementability constraint in period $t = 0$ is given by

$$1 - \frac{1 - \tau}{c_0} n + \frac{1 - \lambda}{\tau - \lambda} \mathbb{E} \left\{ 2 - \frac{1 - \tau}{(1 - \lambda)c_1} n - \frac{1 - \tau}{c_2} n \right\} = 0 \quad (32)$$

The government's problem is to maximize welfare (19) subject to equation (32). Note that when $\tau - \lambda < 0$, there is no allocation $\{c_0, c_1, c_2\}$ that satisfy the implementability constraint in equation (32). When $\tau - \lambda > 0$, the optimal allocation is given by

$$c_0 = \frac{1}{\theta_0^{\frac{1}{2}}} \frac{n(1-\tau)(\tau-\lambda)^{\frac{1}{2}}}{2(1-\lambda) + (\tau-\lambda)} \mathbb{E} \left[(\tau-\lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + \theta_1^{\frac{1}{2}} + (1-\lambda)^{\frac{1}{2}} \theta_2^{\frac{1}{2}} \right] \quad (33)$$

$$c_1 = \frac{1}{\theta_1^{\frac{1}{2}}} \frac{n(1-\tau)}{2(1-\lambda) + (\tau-\lambda)} \mathbb{E} \left[(\tau-\lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + \theta_1^{\frac{1}{2}} + (1-\lambda)^{\frac{1}{2}} \theta_2^{\frac{1}{2}} \right] \quad (34)$$

$$c_2 = \frac{1}{\theta_2^{\frac{1}{2}}} \frac{n(1-\tau)(1-\lambda)^{\frac{1}{2}}}{2(1-\lambda) + (\tau-\lambda)} \mathbb{E} \left[(\tau-\lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + \theta_1^{\frac{1}{2}} + (1-\lambda)^{\frac{1}{2}} \theta_2^{\frac{1}{2}} \right] \quad (35)$$

Deterministic Economy In the absence of shocks, the optimal allocation in dates $t = 1$ and $t = 2$ in a frictionless economy is given by

$$c_1 = c_2 = \frac{n(1-\tau)}{3} \left(\theta_0^{\frac{1}{2}} + 2 \right) \quad (36)$$

As the labor tax is fixed to τ , the government chooses the same amount of public expenditures in periods $t = 1$ and $t = 2$. As a result, private consumption is equal in dates $t = 1$ and $t = 2$ (*i.e.* $c_1/c_2 = 1$). When the collateral constraint is binding in $t = 1$, the allocation in $t = 1$ and $t = 2$ is given by

$$c_1 = \frac{n(1-\tau)}{2(1-\lambda) + (\tau-\lambda)} \mathbb{E} \left[(\tau-\lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + (1-\lambda)^{\frac{1}{2}} + 1 \right] \quad (37)$$

$$c_2 = \frac{n(1-\tau)(1-\lambda)^{\frac{1}{2}}}{2(1-\lambda) + (\tau-\lambda)} \mathbb{E} \left[(\tau-\lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + (1-\lambda)^{\frac{1}{2}} + 1 \right] \quad (38)$$

With a binding collateral constraint in $t = 1$, the government deviates from a smooth policy in the absence of shocks to provide aggregate collateral to the private sector. This results in

$$c_1/c_2 = 1/(1-\lambda)^{\frac{1}{2}} > 1 \quad (39)$$

Why does the government deviate from a smooth policy (*i.e.* $c_1/c_2 = 1$)? To understand this, it is first useful to understand the impact of the collateral constraint on the economy. In this economy, the production of firms in $t = 1$ is tightly connected to the supply of loans, and therefore to the aggregate collateral available in the economy (equation (30)). Provided that the labor tax is fixed, this implies that the tax revenue that the government receives in period $t = 1$ ($\tau w_1 n$) is tightly connected to the supply of collateral, that is the holdings of public debt by the private sector. As a result, the government's issuance of debt in period $t = 0$ does not only affect the payments of debt in period $t = 1$ (as in a frictionless economy), but also the government's tax base through the supply of collateral.

When the government issues one extra unit of debt in $t = 0$ (short- or long-term debt), it increases

its debt payments in $t = 1$ by one unit, but it also raises the tax revenue in the same period by $\frac{\tau}{\lambda}$. This is because one extra unit of total debt translates into private collateral in $t = 1$ and raises the supply of working capital loans by $\frac{1}{\lambda}$ (see equation 30). The resulting increase in the wage bill $w_1 n$ by $\frac{1}{\lambda}$ induces a rise in tax revenues equal to $\frac{\tau}{\lambda}$.

The government uses the collateral service of public debt to provide liquidity to the private sector and therefore raise its tax revenues in $t = 1$. To do so, the social planner increases the supply of debt in period $t = 0$ as compared to a frictionless economy in order to provide the bankers with more collateral in the next period $t = 1$. The higher supply of debt in $t = 0$ requires the government to increase the present value of its primary surpluses in $t = 1$ to meet the debt payments to the private sector. This goes through a decline in public spending g_1 as compared to g_2 , and therefore a rise in private consumption ($c_1/c_2 > 1$) as long as production is fixed to n . A direct implication of the increase in the bankers' collateral is a rise in the supply of loans to firms. Since labor is fixed in this economy, more liquidity translates into a rise in the labor wage w_1 (see equation (30)), which raises the government's tax base and thus tax revenue in $t = 1$.

The deviation of the optimal policy from a smooth policy depends on the severity of the financial friction λ (see equation (39)). A more severe collateral friction makes public debt less valuable as collateral. For the same debt issuance, the government's provision of liquidity is lower, as well as the associated tax revenue. This is a direct consequence of the fact that one extra unit of public debt induces a tax gain equal to $\frac{\tau}{\lambda}$. Therefore, the government needs to issue more debt when the friction becomes more constraining. This translates into a larger decline in g_1 relatively to g_2 , c_1/c_2 is then higher.

In the absence of uncertainty, the government has no hedging motive to use the maturity of public debt. Since there is a unique state of nature in $t = 1$, any combination between long-term debt (B_0^2) and short-term debt (B_0^1) that satisfies the implementability constraint in $t = 1$ (*i.e.* $B_0^1 + \frac{c_1}{c_2} B_0^2 = \bar{B}$) is optimal. The maturity structure is therefore indeterminate in the absence of shocks, whether the collateral constraint is binding or not. However, under a collateral friction, the government's set of optimal maturity combinations in date $t = 1$ is altered given that the optimal allocation induces a price of long-term debt B_0^2 that is higher than one (*i.e.* $c_1/c_2 > 1$). While the set of optimal maturities in a frictionless economy satisfy $B_0^1 + B_0^2 = \bar{B}$ (with $c_1/c_2 = 1$), the set of optimal

maturities in the presence of a financial friction satisfy the following equation

$$B_0^1 + \frac{1}{(1-\lambda)^{\frac{1}{2}}} B_0^2 = \bar{B} \quad (40)$$

The market value of long-term debt being higher compared to a frictionless economy, the government issues a lower level of B_0^2 and/or B_0^1 for the same levels of total debt. The higher price of B_0^2 , though, does not shift the government's preferences towards long- or short-term debt as long as the market value of total outstanding debt \bar{B} is unchanged over the set of combinations between B_0^1 and B_0^2 satisfying equation (40). This comes back to the fact that the debt maturity choice is irrelevant in the absence of uncertainty, whether the financial constraint is binding or not.

Stochastic Economy and Optimal Maturity The optimal allocation in equations (33) to (35) implies that, as in a frictionless economy, the government decreases private consumption in $t = 1$ and $t = 2$ when a high shock is realized in $t = 1$. This translates into a rise in public expenditures ($g_t = n - c_t$) when their value is higher in the utility function (*i.e.* θ_1 and θ_2 are high).

When the government has access to state-contingent debt, it is able to implement the optimal allocation in period $t = 0$ under the uncertainty surrounding the realization of θ_1 . When the government does not have access to state-contingent debt, the optimal allocation must satisfy the implementability constraint in period $t = 1$ in which the shock is realized. The constraint requires that the government's due payments of debt in $t = 1$ be equal to the present value of its *realized* surpluses after the occurrence of the shock at the beginning of the period. The optimal debt maturity is then the one that sustains the optimal allocation in equations (33) to (35) such that the following implementability constraint in date $t = 1$ is satisfied for $\theta_1 = \theta^H$ and for $\theta_1 = \theta^L$

$$B_0^1 + \frac{c_1}{c_2} B_0^2 = -\frac{\lambda}{\tau - \lambda} \left[c_1 - n + \frac{c_1}{c_2} (c_2 - n(1 - \tau)) \right] \quad (41)$$

B_0^1 and B_0^2 can be derived using equation (41) when $\theta_1 = \theta^H$ and $\theta_1 = \theta^L$. Accordingly, the debt levels are defined by

$$B_0^1 = -\frac{\lambda}{\tau - \lambda} \frac{2(c_2^H - c_2^L) - n \left(\frac{c_2^H}{c_1^H} - \frac{c_2^L}{c_1^L} \right)}{\frac{c_2^H}{c_1^H} - \frac{c_2^L}{c_1^L}} \quad (42)$$

$$B_0^2 = -\frac{\lambda}{\tau - \lambda} \frac{2(c_1^H - c_1^L) - n(1 - \tau) \left(\frac{c_1^H}{c_2^H} - \frac{c_1^L}{c_2^L} \right)}{\frac{c_1^H}{c_2^H} - \frac{c_1^L}{c_2^L}} \quad (43)$$

Using the optimal allocation in equations (33) to (35), it can be shown that¹⁰

$$B_0^1 > 0 \quad \text{and} \quad B_0^2 < 0$$

This implies that the government finds it optimal to issue short-term debt and to purchase long-term assets to sustain the optimal allocation when debt serves as collateral in period $t = 1$. Therefore, under the optimal allocation in equations (33) to (35), the government switches the maturity of its debt when public debt serves as collateral for private agents.

Why does the maturity of debt switch? When a high shock is realized in period $t = 1$, the government increases public expenditures in $t = 1$ and $t = 2$ (see equations (33) to (35)). This induces a decline in the government's surpluses in $t = 1$ and $t = 2$. In order to finance this decline, the government would want to replicate the state-contingent payment by manipulating the fall in the one-period price of debt $q_1^2 = c_1/c_2$. In a frictionless economy, the government issues long-term debt and purchases short-term assets in date $t = 0$ so that the market value of outstanding debt in $t = 1$ decreases with the decline the primary surpluses. The resulting capital gain then serves to finance the increase in public expenditures. In the presence of the collateral constraint, the government adopts the opposite policy because any variation in the value of public debt does not only impact the debt payments, but also the government's tax revenues through collateral. A one unit fall in the market value of total debt induces a one unit fall in debt payment, but also a $\frac{\tau}{\lambda}$ unit fall in tax revenue. When $\tau > \lambda$ —which is the condition under which the optimal allocation exists—, the government realizes a tax loss (since $\frac{\tau}{\lambda} > 1$) if it reduces the value of total debt when public spending is high, because of the high collateral value of government debt. As a result, the government would want to *raise* the market value of outstanding debt after a high shock when debt serves as collateral. This goes through the issuance of short-term debt and the purchase of long term assets. In doing so, the government increases the market value of the private collateral, which boosts economic activity and brings its tax base up. The resulting tax gain serves to finance public expenditures.

$$\underbrace{B_0^1 + \frac{c_1}{c_2} B_0^2}_{\text{debt payments in } t=1} = \frac{\tau}{\lambda} \overbrace{\left(B_0^1 + \frac{c_1}{c_2} B_0^2 \right)}^{\substack{\text{aggregate collateral} \\ (= \lambda w_1 n)}} - g_1 + \frac{c_1}{c_2} (\tau n - g_2) \quad (44)$$

tax revenue in $t=1$

Therefore, when the shock is high ($\theta_1 = \theta^H$) the government realizes a capital loss on its long-term

¹⁰See Appendix A for proof.

assets ($B_0^2 < 0$) because of the decline in the price of these assets $q_1^2 = c_1/c_2$. As a consequence, the total value of outstanding debt goes up in $t = 1$. This raises the debt payments, but it also increases the private collateral and thus the working capital loans ($l_1 = w_1n$). Provided that labor is fixed in this economy, this translates into a rise in the labor wage w_1 . The resulting increase in the government's tax revenue finances the loss on debt payments as well as the increase in public expenditures. When the shock is low ($\theta_1 = \theta^L$), the rise in the price of debt induces a capital gain on the government's long-term assets. This decreases not only the debt payments, but also the aggregate collateral and therefore the firms' wage bill. As a consequence, the tax revenue net of debt payments falls with the decline in public spending. In this way, the maturity policy allows the government to replicate the state-contingent payments after the realization of the shock.

Debt Positions. The debt positions B_0^1 and B_0^2 required to replicate the state-contingent payments depend on two factors (see equation (44)). The first factor is the variation in the government's surpluses in $t = 1$ which determines the government's need for hedging. The second factor is the variation in the short-term interest rate c_1/c_2 (*i.e.* the numerator in equations (42) and (43)); it governs the variation in the market value of total debt and therefore the collateral. Indeed, when the variation in the government's surpluses is high, the hedging position required in period $t = 1$ shrinks in the variation of the short-term interest rate. The higher is the variation in the price of debt, the smaller is the debt position needed for insurance.

The variations in the primary surpluses and the short-term interest rate are governed by the characteristics of the public spending shock (*i.e.* the volatility δ and the persistence α), whether the collateral constraint is binding or not. The presence of the financial constraint, though, adds an additional channel that has an impact on these variations dependent on the severity of the friction. The friction first influences the variation in the value of total debt—which is equal to the primary surpluses—through the tax gain (loss) realized on collateral. The latter is determined by the wedge between the tax rate and the severity of the financial constraint, since a one-unit rise in the market value of debt induces a tax gain equal to $\frac{\tau}{\lambda} - 1$. This implies that the higher the wedge between τ and λ , the larger is the tax gain (loss) on each unit of debt, and therefore the smaller is the variation in the primary surpluses. As a result, the debt positions required for insurance are low. The reasoning behind is that, for a given tax rate τ , a less severe financial friction makes the collateral value of each unit of debt higher. In this sense, when the market value of debt increases, it induces a rise in private liquidity that is larger. The associated tax revenue is therefore larger as well.

The presence of the collateral friction also alters the variations in the primary surpluses and the short-term interest rate by distorting the optimal allocation. To better understand this impact, I follow [Debortoli, Nunes, and Yared \(2017\)](#) and I determine the optimal short- and long-term debt positions as the volatility of shock δ goes to zero (*i.e.* deterministic limit); and as the persistence of the shock α goes to one (*i.e.* full persistence limit). I then compare the debt positions to the ones in a frictionless economy.

As the volatility of the shock δ goes to zero, the optimal levels of B_0^1 and B_0^2 when the collateral constraint is binding in $t = 1$ are given by¹¹

$$\lim_{\delta \rightarrow 0} B_0^1 = n \frac{\lambda}{\tau - \lambda} \left\{ \frac{\frac{(2\alpha-1)(1-\tau)}{2(1-\lambda)+(\tau-\lambda)} \left[(\tau - \lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + 1 + (1 - \lambda)^{\frac{1}{2}} \right]}{1 - \alpha} + 1 \right\} > 0 \quad (47)$$

$$\lim_{\delta \rightarrow 0} B_0^2 = -n(1 - \tau) \frac{\lambda}{\tau - \lambda} \left\{ \frac{\frac{(1-\lambda)^{1/2}}{2(1-\lambda)+(\tau-\lambda)} \left[(\tau - \lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + 1 + (1 - \lambda)^{\frac{1}{2}} \right]}{1 - \alpha} - 1 \right\} < 0 \quad (48)$$

The government maintains its optimal maturity policy, that is, issuing short-term debt and purchasing long-term assets, even when the volatility of the shock tends to zero. This result is consistent with the one in a frictionless economy explained in [Debortoli, Nunes, and Yared \(2017\)](#). Indeed, as the volatility of the shock goes to zero, the government's need for hedging (*i.e.* the variation in the surplus) and the volatility in short-term interest rates tends to zero as well. However, as long as the need for hedging is not zero—as apposed to a deterministic economy—the government needs yet to maintain its debt maturity positions such that, with the low volatility in the interest rates, the variation in the market value of debt allows for insurance.

How is the size of the debt positions impacted in the presence of the financial distortion? Using the

¹¹Proofs on both limits are provided in Appendix A. For comparison, the levels of B_0^1 and B_0^2 when the collateral constraint does not bind in $t = 1$ are as follow

$$\lim_{\delta \rightarrow 0} B_0^1 = -n(1 - \tau) \left\{ \frac{(2\alpha - 1)(\theta_0^{1/2} + 2)/3}{1 - \alpha} + 1 \right\} < 0 \quad (45)$$

$$\lim_{\delta \rightarrow 0} B_0^2 = n(1 - \tau) \left\{ \frac{(\theta_0^{1/2} + 2)/3}{1 - \alpha} - 1 \right\} > 0 \quad (46)$$

expressions in equations (47) and (48), it can be shown that¹²

$$|B_0^1| > n \frac{\lambda}{\tau - \lambda} \quad \text{and} \quad |B_0^2| < n(1 - \tau) \frac{\lambda}{\tau - \lambda} \quad (49)$$

while in a frictionless economy the debt positions in absolute value are both higher than the households' disposable income (*i.e.* $|B_0^1| > n(1 - \tau)$ and $|B_0^2| > n(1 - \tau)$). The collateral constraint then changes the size of the debt positions in two ways. First, the debt positions are now determined by the fraction of tax gain (loss) realized on every unit of debt measured by $\frac{\lambda}{\tau - \lambda}$. This implies that the debt positions can be larger than the ones in a frictionless economy—which are already large—when $\frac{\lambda}{\tau - \lambda} > 1$, that is, when the tax gain (loss) on a unit of debt is lower than one. Second, the effect that the collateral constraint has on the optimal allocation makes the short-term debt position strictly higher than the long-term asset position, $|B_0^1| > |B_0^2|$. This happens because the government needs to maintain positive position of total debt for any variation in its market value, so that the private sector has access to collateral.

When the persistence of shock goes to one, the government's debt positions are explosive

$$\lim_{\alpha \rightarrow 1} B_0^1 \rightarrow \infty \quad \text{and} \quad \lim_{\alpha \rightarrow 1} B_0^2 \rightarrow -\infty \quad (50)$$

This result is similar to the one in a frictionless economy¹³. As the shock persistence tends to one, the variation in the short-term interest c_2/c_1 goes to zero. However, the government's need for hedging does not go to zero since public expenditures vary in $t = 2$ almost as much as they vary in $t = 1$. The government therefore requires infinite levels of debt positions in order to preserve insurance¹⁴.

4 A Numerical Exercise with time-varying labor

In this section, I conduct a numerical exercise in which I move away from the limiting case discussed in the previous section. First, I now consider that labor is endogenously determined, and therefore time-varying. Second, I consider a government that is benevolent (that is, ψ is no longer arbitrarily small). This implies that private consumption and leisure are now valuable for the social planner. This allows to explore how the optimal policy is sensitive to these assumptions and to study the quantitative implications of my results in a three-period economy.

¹²See proof in Appendix A.

¹³In a frictionless economy, the government adopts the opposite infinite positions, $B_0^1 \rightarrow -\infty$ and $B_0^2 \rightarrow \infty$.

¹⁴This result is explained in Debortoli, Nunes and Yared (2017)

In a frictionless economy, the government's implementability conditions are given by equations (20) to (22) where n becomes n_t . Since labor demand pins down the wage $\{w_t\}_{t=0}^2$ to one (see equation (16)), the variations in labor n_t —and therefore production—are solely driven by the variations in private consumption, *i.e.* the income effect (see equation 8). Hence, labor is determined by the labor supply equation and it is equal to $n_t = \left(\frac{1-\tau}{c_t}\right)^{\frac{1}{\phi}}$ for $t = \{0, 1, 2\}$. The government's problem is as follow

$$\begin{aligned} \max_{c_t} \mathbb{E} \sum_{t=0,1,2} \beta^t \left\{ (1-\psi) \left[\log(c_t) - \frac{1}{\phi+1} n_t^{\phi+1} \right] + \psi \theta_t g_t \right\} \\ \text{s.t.} \quad \mathbb{E} \sum_{t=0,1,2} \beta^t \left(1 - \frac{1-\tau}{c_t} n_t \right) = 0 \end{aligned}$$

where $g_t = n_t - c_t$ and $n_t = \left(\frac{1-\tau}{c_t}\right)^{\frac{1}{\phi}}$ for $t = \{0, 1, 2\}$.

In the presence of a collateral constraint in $t = 1$, the implementability conditions are given by equations (20) to (22) where n becomes n_t . In this economy, the labor wage in period $t = 1$ is no longer equal to one. Labor demand is determined by the supply of working capital loans, and therefore the aggregate collateral available in the economy. As a consequence, labor n_1 and the wage rate w_1 are defined by the labor supply equation (8) as well as the collateral constraint (10). The government's problem in this economy is given by

$$\begin{aligned} \max_{c_t, n_1} \mathbb{E} \sum_{t=0,1,2} \beta^t \left\{ (1-\psi) \left[\log(c_t) - \frac{1}{\phi+1} n_t^{\phi+1} \right] + \psi \theta_t g_t \right\} \\ \text{s.t.} \quad 1 - \frac{1-\tau}{c_0} n_0 + \mathbb{E} \left\{ \left[1 + \frac{1}{\lambda} \left(\frac{1}{w_1} - 1 \right) \right] \left[\beta \left(1 - \frac{1-\tau w_1}{c_1} n_1 \right) + \beta^2 \left(1 - \frac{1-\tau}{c_2} n_2 \right) \right] \right\} = 0 \end{aligned} \quad (51)$$

$$\lambda w_1 n_1 = c_1 - (1 - \tau w_1) n_1 + \beta \frac{c_1}{c_2} [c_2 - (1 - \tau) n_2] \quad (52)$$

where $g_t = n_t - c_t$ for $t = \{0, 1, 2\}$, $n_t = \left(\frac{1-\tau}{c_t}\right)^{\frac{1}{\phi}}$ for $t = \{0, 2\}$, and $w_1 = \frac{n_1^\phi c_1}{1-\tau}$. Equation (51) is the intertemporal implementability constraint, and equation (52) is the collateral constraint. The latter requires that a fraction λ of the working capital loans ($w_1 n_1$) be equal to the present value of the primary surpluses in $t = 1$, which are in turn equal to the outstanding level of debt.

The relative weight of public spending in the utility function is calibrated such that the marginal utility of private consumption equals that of public spending in the deterministic economy ($u_c = u_g$),

which gives $\psi = 0.53$. I calibrate the labor tax rate τ to 28% and I set the inverse of the Frisch elasticity to $\phi = 1$. The discount factor β is set such that the risk-free rate is equal to 4.1 percent annually, which implies $\beta = 0.99$. I first study the optimal policy when θ_t is deterministic to highlight the effect of the collateral constraint on the optimal allocation without uncertainty. I then analyze the effects of a shock in period $t = 1$ in the presence of the collateral constraint. I generate a 3% shock on θ_1 by calibrating the shock volatility δ to 0.03. The persistence of the shock is set to $\alpha = 0.85$.

Table 1: Parameter values.

β	0.99	Discount factor
ψ	0.53	Weight of public spending
ϕ	1	Inverse of the Frisch elasticity
τ	0.28	Tax rate on labor income
δ	0.03	Shock volatility
α	0.85	Shock persistence

4.1 Deterministic Economy

In this section, I study the optimal policy of the government without uncertainty (*i.e.* $\delta = 0$). Abstracting from the need for fiscal insurance allows to better understand the impact of the financial friction in the economy, and therefore the role that the debt instrument plays in the optimal policy. Figure (1) shows the evolution of consumption, labor, public spending and the government's outstanding debt in the optimal equilibrium with and without collateral constraint.

In the absence of a collateral constraint in $t = 1$, the government chooses to smooth private consumption and public expenditures over the three periods. The government then balances its budget each period by setting public expenditures g_t to the constant tax revenue τn_t . As a consequence, there is no public debt issuance over the three periods.

When the collateral constraint is binding in period $t = 1$, the social planner internalizes the private agents' need for collateral in that period to have access to liquidity for production. This makes the zero debt issuance policy no longer optimal. One additional objective of the social planner in such an economy is to provide collateral, and therefore liquidity, to the private sector. Because the government is benevolent, the provision of liquidity becomes valuable in two ways: (i) it increases the

government's tax revenues, and (ii) it raises the households' income and thus private consumption.

To provide the private sector with collateral in period $t = 1$, the government issues public debt in period $t = 0$. The supply of public debt opens the door for bank lending, which increases firms' production (n_1) and wages as compared to a frictionless economy. The associated increase in households' income raises private consumption. The stock of public debt issued in $t = 0$ needs to be paid back by the government in $t = 1$. To meet those payments, the government issues debt and reduces public expenditures in date $t = 1$. The latter induces a fall in aggregate demand that dampens production and thus hours worked n_1 .

The government's deviation from a smooth policy increases with the severity of the financial friction. Indeed, as the collateral value of public debt decreases, the private sector is required to hold a larger stock of collateral for the same amount of working capital loans. To maintain the provision of liquidity in the economy, the government then issues a larger stock of debt in $t = 1$ when the λ is high. The fall in public expenditures needed to meet the debt payments in $t = 1$ is consequently stronger. This has a positive wealth effect on households as it makes them work less and consume more (Baxter and King (1993)). The rise in debt issuance has the opposite impact on public expenditures in period $t = 0$ as it allows to raise the government's revenue in that period.

The optimal maturity of debt issued in period $t = 0$ is indeterminate in the absence of shocks since there is no hedging motive for the government. As explained in the previous section, many debt maturities allow the government to satisfy the implementability constraint in $t = 1$ as there is a unique state of nature. This is true whether the economy is frictionless or not. However, regardless of the maturity structure of debt, Figure (1) shows that the presence of the financial friction requires that the short- and/or long-term debt issuance in $t = 0$ be strictly positive such that the private sector can have access to collateral in $t = 1$. The government's optimal policy is therefore aligned with the provision of public debt to alleviate the financial friction.

How important is the provision of public debt? To analyze the implications of a lack of public debt in this economy, it is useful to compare the planner's optimal policy to a competitive equilibrium in which debt issuances are solely driven by the government's budget imbalances. I characterize the competitive equilibrium by fixing public expenditures to $g = 0.14$. As the income tax rate τ is fixed, the levels of debt are determined such that the budget constraint of the government (equation (1)) is satisfied in each period. Figure (2) depicts the evolution of consumption, labor, public spend-

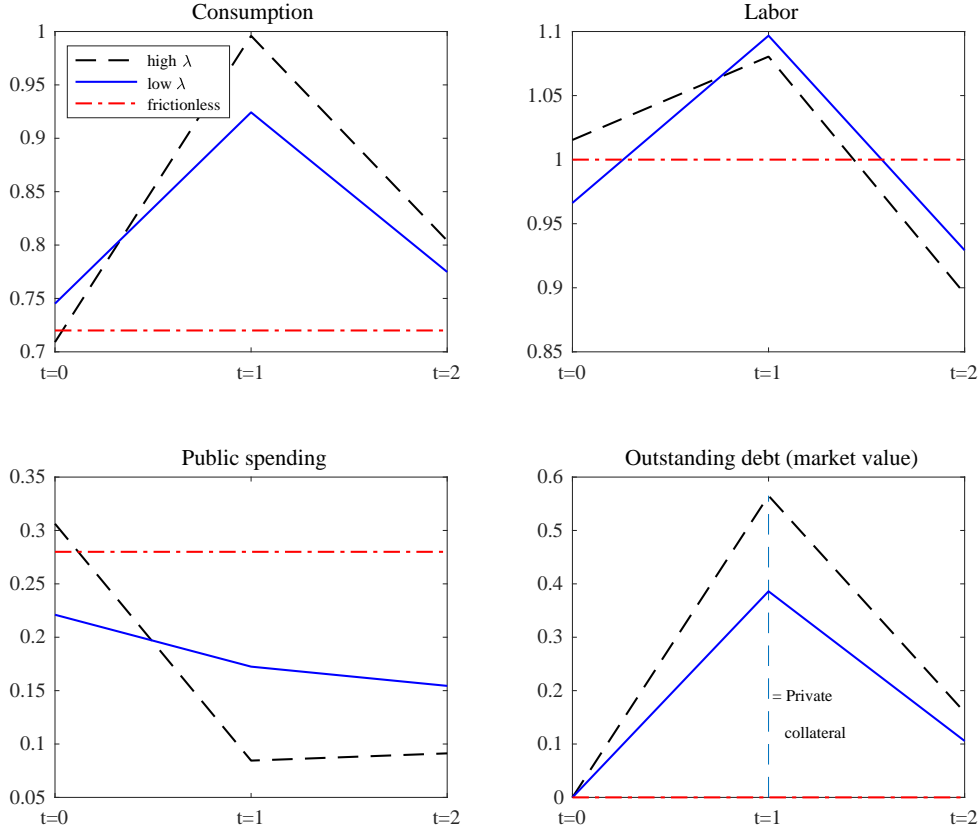


Figure 1: Optimal policy with and without collateral friction.

ing and outstanding debt in the Ramsey optimal equilibrium, as well as the competitive equilibrium.

In a decentralized economy, private agents realize in period $t = 0$ that their liquidity in $t = 1$ will be constrained by their collateral holdings. However, the government has little incentives to issue public debt in $t = 0$ as long as public expenditures are fixed and there is no debt payment (*i.e.* outstanding debt is zero). As a result, the lack of collateral in $t = 1$ leads to a shortage of liquidity in the private sector, which in turn induces a decline in production, labor wages and subsequently private consumption. The fall in economic activity cuts the government's tax revenues. This constraints the government to issue more debt in $t = 1$ to finance its fixed public spending. In contrast to a decentralized economy, the social planner in a centralized economy internalizes the social value of the liquidity service of public debt. The optimal policy is then designed to satiate the private agents' demand for public debt in $t = 0$ in order to use it as collateral against working capital loans in $t = 1$. This ultimately prevents the liquidity shortage that takes place in the competitive equilibrium. In addition, the increased taxes allow for higher public expenditures. This result highlights

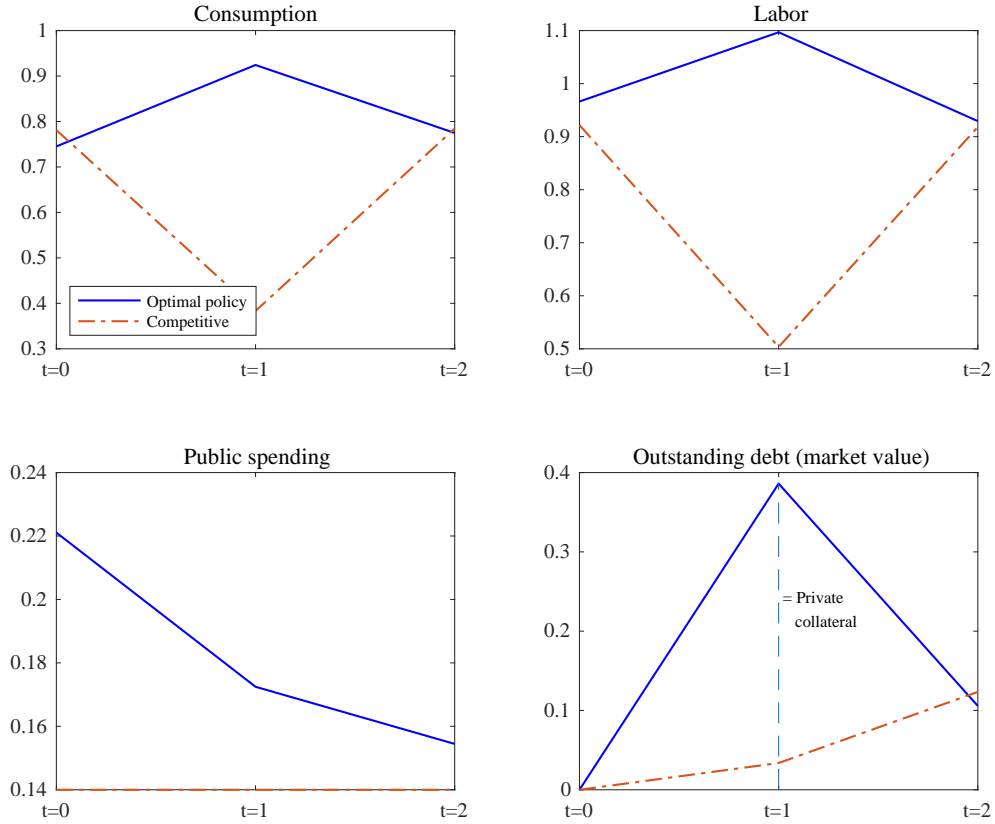


Figure 2: Optimal policy and competitive equilibrium.

the critical role that the government's provision of debt plays when private agents are financially constrained.

The Welfare Impact of The Collateral Friction To provide a better understanding of the implications of the financial distortion on optimal policy in a deterministic economy, I compare welfare in the baseline economy to the one in a frictionless economy. I compute the welfare gain (cost) of the optimal policy in the baseline as the Hicksian consumption equivalent ϵ solving the following equation

$$\sum_{t=0,1,2} \beta^t \{u(c_t, n_t, g_t) - u(c_{f,t}(1 + \epsilon/100), g_{f,t}, n_{f,t})\} = 0$$

ϵ measures the percentage of consumption gain (loss) associated with the optimal policy in the baseline $\{c_t, n_t, g_t\}$ compared to the frictionless equilibrium $\{c_{v,t}, n_{v,t}, g_{v,t}\}$. Figure (3) plots the evolution of the welfare effect of the collateral friction (*i.e.* the value of ϵ) as a function of the

tightness of the friction measured by λ . The asterisks capture the welfare measure with the baseline parametrization in which the labor tax rate is fixed to $\tau = 0.28$; and the circles capture the welfare measure with a lower tax rate equal to $\tau = 0.25$. Figure (3) also reports the average value of λ and the peak of the curves, which reflects the maximum welfare gain. The average value of λ is equal to 0.21, which corresponds to a debt-to-equity ratio of 4.55. The latter matches the relevant statistic for financial corporations in OECD countries in 2020 (OECD, 2022).

As Figure (3) illustrates, the presence of the collateral friction allows the government to improve social welfare up to 1% of consumption in a frictionless economy. In this economy, the collateral constraint on private agents turns the government's debt instrument into a liquidity vehicle. The social planner then takes advantage of the collateral service of public debt to raise the liquidity of the private sector and therefore the households' income. The welfare gain, though, depends on the severity of the collateral friction. Because the supply of liquidity is tightly connected to the public debt burden, the friction has two conflicting effects on social welfare. On the one hand, a high stock of debt gives access to more liquidity, which raises private consumption and tax revenues. On the other hand, a high debt burden requires a large decline in public expenditures for the government to meet its debt payments. When the collateral friction is too constraining (*i.e.* λ is high), access to liquidity requires large government debt holdings. The resulting loss in public expenditures dampens the welfare gain, and for very high values of λ , it gives rise to welfare loss. When $\lambda = 0.45$, the government decreases public expenditures to zero in order to payback the stock of debt needed for collateral¹⁵.

Comparing the two curves, one can notice that for a lower labor tax rate, (i) the welfare gain is lower, and (ii) the maximum welfare gain is reached with a smaller value of λ . The reason is that the government's tax revenue determines the decline in public expenditures needed to raise public debt. The lower is the tax rate, the higher is the fall in public expenditures required to raise the aggregate collateral. This reduces the utility gain associated with the rise in private liquidity. Hence, for the average value of λ , the welfare gain that the government realizes when debt serves as collateral is lower when the tax rate is lower. In addition, because the severity of friction is more costly in terms of public spending, the maximum welfare gain is reached with a lower tightness of the friction ($\lambda = 0.18$, against $\lambda = 0.2$ for the baseline tax rate, $\tau = 0.28$).

¹⁵Note that the starting value of λ in Figure (3) is $\lambda = 0.01$. The welfare effect of the friction when $\lambda = 0$ is by construction zero since the economy becomes frictionless.

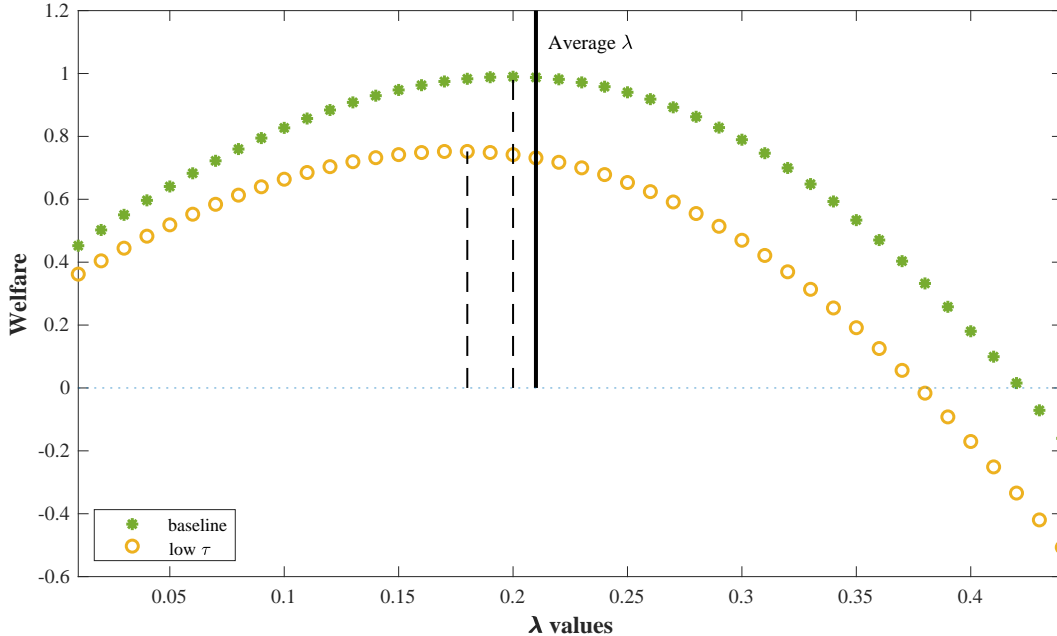


Figure 3: Welfare effect of the collateral friction as a function of λ .

4.2 Stochastic Economy and Optimal Maturity

In what follows, I study the optimal policy of the government in response to a shock on θ_1 . The goal is to first examine how the presence of the financial friction distorts the optimal allocation under uncertainty, and second to analyze the implications of the financial distortion on the optimal maturity of debt. To that end, I first analyse the optimal allocation in a frictionless economy in which the collateral constraint does not bind. I then analyze the optimal allocation in the baseline model. Finally, I compare the debt maturity structure with and without financial distortion.

The optimal allocation in the frictionless economy is depicted in Figure (4). In the absence of a collateral friction, a high shock on θ_1 (*i.e.* $\theta_1 = \theta^H$) increases the value of public spending in welfare in period $t = 1$ and $t = 2$. In response to that, the government increases public expenditures over periods $t = 1$ and $t = 2$. This raises the economy's production (*i.e.* n_1 and n_2) and decreases the households' private consumption. The associated fall in the primary surpluses requires the government to lower its debt payments. The government adopts the opposite policy when the shock is low.

Figure (5) plots the optimal policy when the collateral constraint is binding in $t = 1$. The tightness of the friction is set to $\lambda = 0.25$. As in a the frictionless economy, the government increases its expenditures in $t = 1$ when a high shock is realized. This rise, though, does not have the same

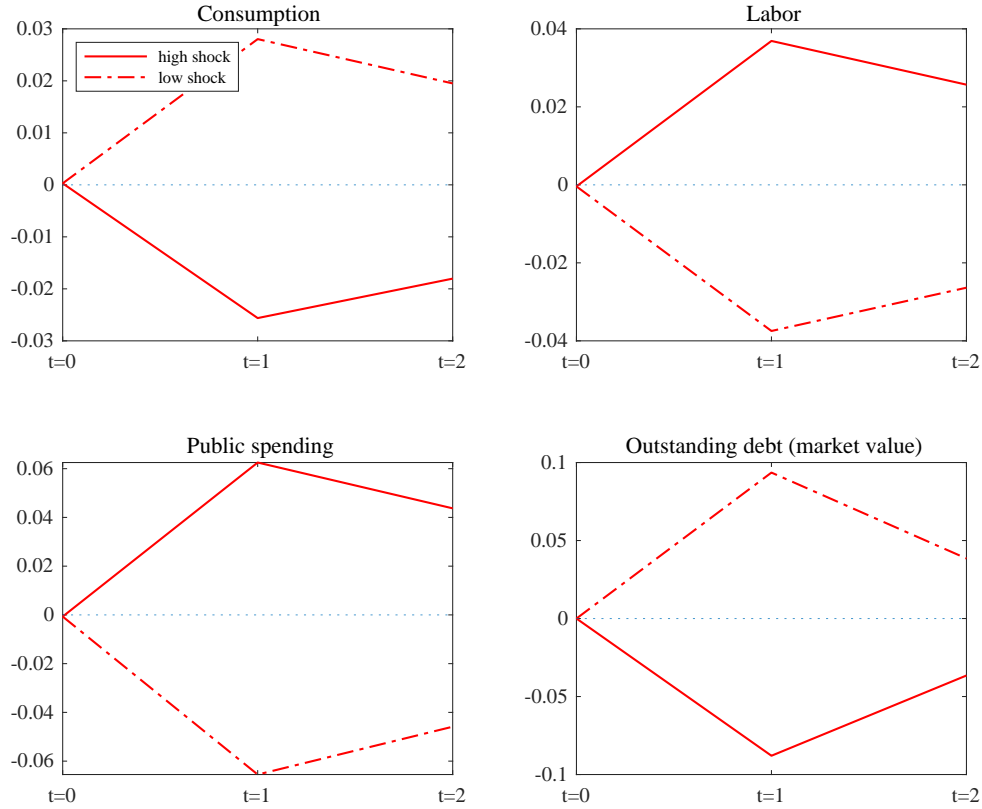


Figure 4: Optimal policy in a frictionless stochastic economy (deviations from the deterministic state).

impact on the economy when the collateral constraint is binding. In this economy, any variation in the market value of the outstanding public debt in $t = 1$ does not only impact the amount of payments that the government makes to households, but also the amount of collateral at the private sector's disposal. The latter determines the economy's liquidity, production and therefore tax revenues. In this sense, the government decides on the variation of the market value of outstanding debt depending on the tightness of the collateral friction—measured by λ —with respect to the labor tax rate τ .

With a collateral friction for which $\lambda < 0.34$, the social planner finances the rise in public spending in $t = 1$ through an *increase* in the market value of outstanding debt (*i.e.* $B_0^1 + \frac{c_1}{c_2} B_0^2$) and a rise in debt issuance in the same period (*i.e.* B_1^2). By raising the market value of outstanding debt, the government raises the value of the collateral available for the private sector, which increases economic activity and thus the tax base. This translates into a rise in production n_1 and private consumption c_1 in Figure (5). Because the collateral value of public debt is relatively high, the

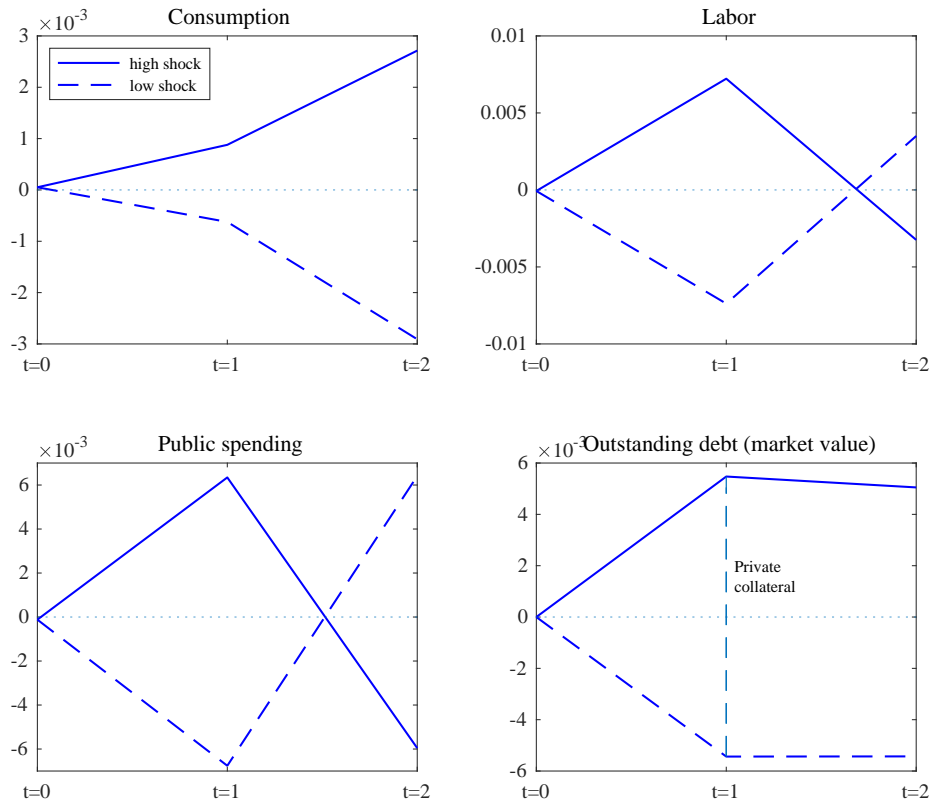


Figure 5: Optimal policy with collateral friction (deviations from the deterministic state).

rise in tax revenues—induced by the increased liquidity—outweighs the increase in debt payments. This gives rise to a tax gain which, along with a debt issuance in $t = 1$, allows to finance the rise in public expenditures g_1 . As opposed to a frictionless economy, the government does not raise public spending in $t = 2$ in response to a high shock in $t = 1$ even though its social value increases. This comes back to the fact that the government’s freedom to manipulate the market value of outstanding debt is constrained by the effect of the latter on economic activity and tax revenues. By increasing the market value of outstanding debt, the tax gain realized on the collateral in $t = 1$ is not high enough to finance the rise in public expenditures in period $t = 1$ as well as in period $t = 2$ (*i.e.* a decline in debt issuance in $t = 1$). This makes the government reduce g_2 to meet its debt payments in the same period.

When the financial friction is too severe ($\lambda > 0.34$), the government *reduces* the market value of outstanding debt in response to a high shock on θ_1 (see Figure (8) in Appendix B). Indeed, since the collateral value of public debt is low, the debt payments associated with increasing the value

outstanding debt are higher than tax revenues induced by the creation of liquidity. The social planner then finds it optimal to reduce the market value of outstanding debt when the primary surplus falls down in $t = 1$. This gives rise to a collateral shortage in the economy that induces a fall in liquidity, economic activity and tax revenues. The decline in tax revenues makes the government issue debt in $t = 1$, which translates into a fall in public expenditures in $t = 2$.

In comparison to a frictionless economy, one can notice that the presence of the collateral friction leads to a lower variation in public expenditures in response to a shock in $t = 1$, whatever is the value of the friction's severity λ (see Figures (5) and (8)). This implies a lower variation in private consumption, labor and the market value of outstanding debt. The lower response of public expenditures to a shock on their social value in this economy reflects the lack of insurance induced by the financial friction. When private liquidity is connected to the value of public debt, the high variations in the value of debt required to achieve full insurance against the shock are costly. The associated cost is either the increase in debt payments (when the market value of debt rises with a high shock) or the squeeze of private liquidity (when the value of debt falls down with a high shock). Because the state-contingent payments that the optimal policy provides are limited, the government lowers the variation of its expenditures.

The Optimal Debt Maturity When the payments of debt are contingent on the realization of uncertainty, the government is able to implement the optimal allocations in Figures (4) and (5). That is, the government varies its debt payments to households depending on the realization of the shock in period $t = 1$, and therefore the realized primary surpluses. When the debt payments are non-contingent, however, they only depend on the risk-free interest rate. The optimal allocation must then satisfy the government's implementability constraint when the shock is realized (*i.e.* in period $t = 1$). The latter requires that the government's debt payments—that is, the market value of outstanding debt—be equal to the realized present value of the primary surpluses, so that the government can pay back its debt. The optimal allocation of public debt that allows to replicate the state-contingent payments—and thus to sustain the optimal allocations in Figures (4) and (5)—is depicted in Figure (6). The figure reports the levels of short- and long-term debt issued in period $t = 0$ (*i.e.* B_0^1 and B_0^2 , respectively) in the frictionless economy as well as the baseline economy.

When the collateral constraint is not binding, the government finds it optimal to issue long-term debt ($B_0^2 > 0$) and to invest in short-term assets ($B_0^1 < 0$) to achieve full insurance. As Figure (4) shows, the bond price of debt in $t = 1$ (*i.e.* c_1/c_2) goes down after the rise in public expenditures.

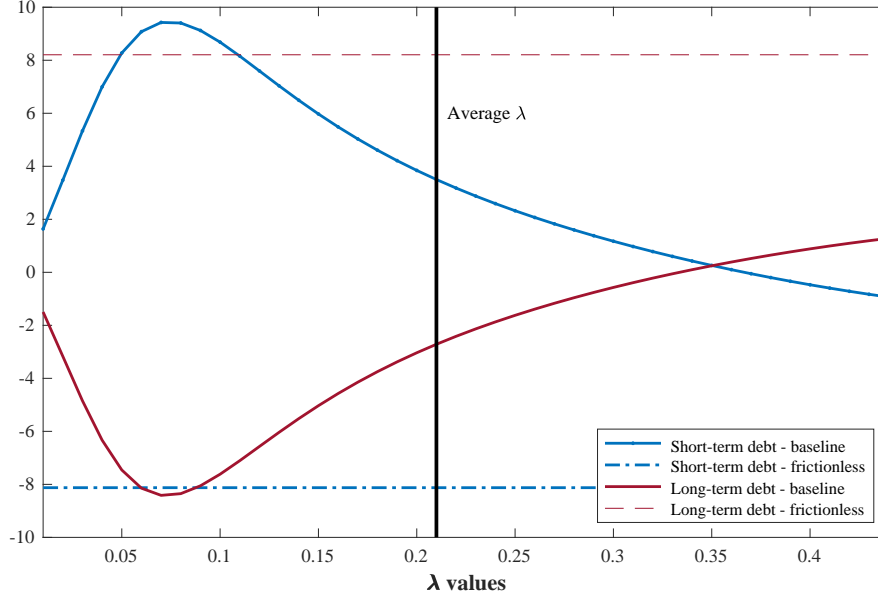


Figure 6: The optimal debt maturity with (without) collateral friction as a function of λ .

By issuing long-term debt, the government replicates the fall in the state-contingent payments, required after the decline in the primary surpluses, through the capital gain realized on the fall in the value of outstanding debt. When public expenditures decrease after a low shock, the government realizes a capital loss on the value of outstanding debt when the primary surpluses are high.

The optimal maturity of public debt in the presence of a collateral friction hinges on the tightness of the friction. This implication follows from the fact that the government's optimal allocation, as explained above, depends on the severity of the collateral friction. With a collateral friction for which $\lambda < 0.34$, the government would want to *increase* (decrease) the value of outstanding debt when public spending is high (low) (see Figure (5)), since the high collateral value of public debt allows for a tax gain (loss). To do so, the government then issues short-term debt and purchases long-term assets to insure itself against the uncertainty surrounding the social value of public spending. When $\lambda > 0.34$, the low collateral value of public debt makes the government *reduce* (raise) the market value of outstanding debt when public expenditures are high (low) (see Figure (8)). This is because the variations in debt payments are higher than the variations the in tax revenues induced by collateral.

Debt Positions. As explained in the previous section, the collateral friction affects the government's positions of short- and long-term debt (*i.e.* B_0^1 and B_0^2 , respectively). This comes from the fact that

the friction affects the variations in the primary surpluses (*i.e.* the value of outstanding public debt) and the short-term interest rate. When the severity of the constraint is very low (λ ranging roughly between 0.01 and 0.07), the debt positions increase in the tightness of the financial constraint. This happens because the tax gain that the government realizes on collateral decreases when the liquidity value of debt declines. This raises the government's need for hedging. As a consequence, the social planner needs higher variations in the market value of debt to achieve insurance (see Figure (9) in Appendix B). Given that the variation in the short-term interest rate (*i.e.* c_2/c_1) does not increase with the severity of the constraint, larger debt positions are required to attain those variations.

As the financial constraint starts being more severe, the government's debt positions decrease in the tightness of the friction (see Figure (6) for values of $\lambda > 0.07$). When the collateral value of public debt starts becoming low, the variations in the market value of debt have a lower impact on private liquidity. As a consequence, the crowding-out effect of public spending on private consumption becomes dominant in date $t = 1$. This ultimately induces a large variation in the short-term interest rates c_2/c_1 (see Figure (9) in Appendix B). Therefore, even though the variation in the market value of debt—and thus the primary surpluses—increase with λ , the debt positions required for insurance become smaller because the variation in the short-term interest rate increase as well.

Figure (7) reports the government's debt positions for low persistence and volatility of the shock. When the persistence of the shock is lower, the variation in the primary surpluses is smaller whereas the interest rate volatility is higher. This leads to lower debt positions for all values of λ . Notice that the evolution of the debt positions is flatter when the severity of the friction is low, as compared to the baseline shock persistence. This is mainly driven by the higher interest rate volatility as the crowding-out effect of public spending on private consumption becomes more dominant than the liquidity effect. A lower shock volatility has little impact on the government's debt positions. Indeed, while the government's need for hedging is smaller when uncertainty is low, the interest rate volatility goes down as well. High debt positions are therefore necessary for the government to maintain insurance. For comparison, Figure (10) in Appendix B plots the sensitivity of the debt positions in a frictionless to the shock persistence and volatility.

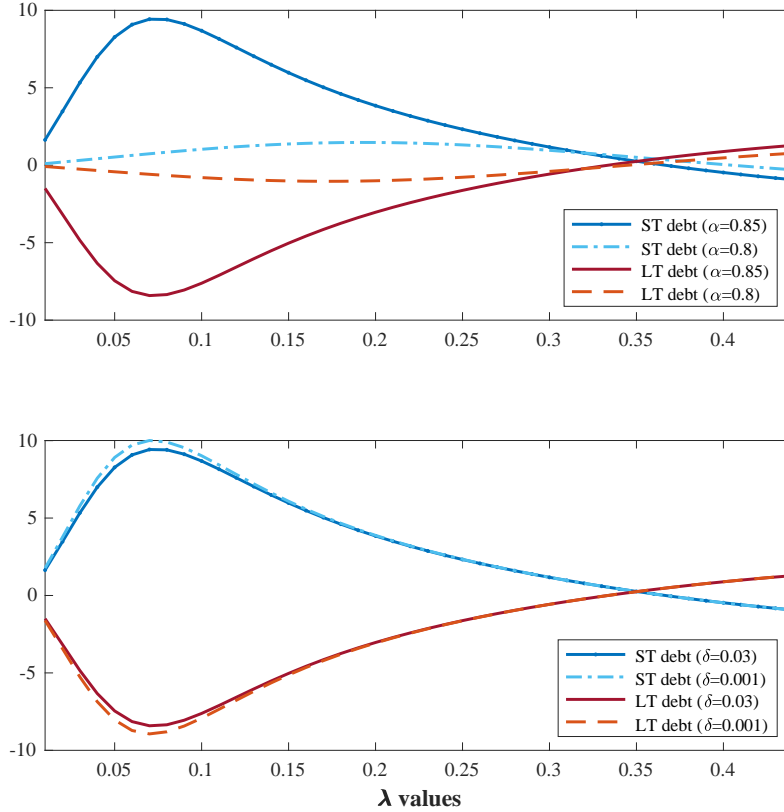


Figure 7: Sensitivity of the debt positions to the shock characteristics.

5 Conclusion

This paper presents a model on optimal debt maturity, in which the government’s policy has implications on the liquidity of the private sector. The standard literature on optimal debt maturity argue that the issuance of long-term bonds and the purchase of short-term assets is optimal and allows the government to achieve full insurance against shocks. I show in this article that the introduction of a collateral role for public debt alters the optimal maturity of the government. In the presence of the collateral friction, public debt becomes a key policy instrument which allows to relax the financial constraint on private agents and therefore raise their liquidity. Because of this additional policy channel, long-term borrowing becomes costly as it squeezes private liquidity during periods of public budget stress. Thus, the government’s maturity choice involves a trade-off between the benefit of reducing debt payments and the cost of squeezing liquidity. This trade-off prevents the government from achieving full insurance. In this economy, the optimal debt maturity hinges on the severity of the collateral constraint. For plausible values of the friction’s severity, issuing short-term debt and buying long-term assets is optimal, as it loosens the liquidity constraint which improves welfare and raises tax revenues.

References

- Aguiar, M., M. Amador, H. Hopenhayn, and I. Werning (2019). Take the short route: Equilibrium default and debt maturity. *Econometrica* 87(2), 423–462.
- Aiyagari, S. R., A. Marcet, T. J. Sargent, and J. Seppälä (2002). Optimal taxation without state-contingent debt. *Journal of Political Economy* 110(6), 1220–1254.
- Aiyagari, S. R. and E. R. McGrattan (1998). The optimum quantity of debt. *Journal of Monetary Economics* 42(3), 447–469.
- Angeletos, G.-M. (2002). Fiscal policy with noncontingent debt and the optimal maturity structure. *The Quarterly Journal of Economics* 117(3), 1105–1131.
- Angeletos, G.-M., F. Collard, and H. Dellas (2016). Public debt as private liquidity: Optimal policy. Technical report, National Bureau of Economic Research.
- Arellano, C. and A. Ramanarayanan (2012). Default and the maturity structure in sovereign bonds. *Journal of Political Economy* 120(2), 187–232.
- Barro, R. J. (1979). On the determination of the public debt. *Journal of political Economy* 87(5, Part 1), 940–971.
- Barro, R. J. (2003). Optimal management of indexed and nominal debt. *Annals of Economics and Finance* (4), 1–15.
- Baxter, M. and R. G. King (1993). Fiscal policy in general equilibrium. *The American Economic Review*, 315–334.
- Bhandari, A., D. Evans, M. Golosov, and T. J. Sargent (2017). Fiscal policy and debt management with incomplete markets. *The Quarterly Journal of Economics* 132(2), 617–663.
- Bigio, S., G. Nuño, and J. Passadore (2019). A framework for debt-maturity management.
- Bocola, L. and A. Dovis (2019). Self-fulfilling debt crises: A quantitative analysis. *American Economic Review* 109(12), 4343–77.
- Brunnermeier, M. K. and L. H. Pedersen (2009). Market liquidity and funding liquidity. *The review of financial studies* 22(6), 2201–2238.
- Buera, F. and J. P. Nicolini (2004). Optimal maturity of government debt without state contingent bonds. *Journal of Monetary Economics* 51(3), 531–554.

- Debortoli, D., R. Nunes, and P. Yared (2017). Optimal time-consistent government debt maturity. *The Quarterly Journal of Economics* 132(1), 55–102.
- ECB (2020). Euro money market study 2020.
- Faraglia, E., A. Marcet, R. Oikonomou, and A. Scott (2019). Long term government bonds.
- Faraglia, E., A. Marcet, and A. Scott (2010). In search of a theory of debt management. *Journal of Monetary Economics* 57(7), 821–836.
- Hatchondo, J. C., L. Martinez, and C. Sosa-Padilla (2016). Debt dilution and sovereign default risk. *Journal of Political Economy* 124(5), 1383–1422.
- Holmström, B. and J. Tirole (1998). Private and public supply of liquidity. *Journal of Political Economy* 106(1), 1–40.
- Huh, Y. and S. Infante (2021). Bond market intermediation and the role of repo. *Journal of Banking & Finance* 122, 105999.
- Lucas, R. E. and N. L. Stokey (1983). Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics* 12(1), 55–93.
- Lustig, H., C. Sleet, and Ş. Yeltekin (2008). Fiscal hedging with nominal assets. *Journal of Monetary Economics* 55(4), 710–727.
- Niemann, S. and P. Pichler (2017). Collateral, liquidity and debt sustainability. *The Economic Journal* 127(604), 2093–2126.
- Nosbusch, Y. (2008). Interest costs and the optimal maturity structure of government debt. *The Economic Journal* 118(527), 477–498.
- Woodford, M. (1990). Public debt as private liquidity. *The American Economic Review* 80(2), 382–388.

A Mathematical Derivations

A.1 General Environment

The resource constraint of the economy is driven by the combination of the households' aggregate resource constraint (equation (5)) and the government's budget constraint (equation (1))

$$c_t + g_t = w_t n_t + (r_t - 1)l_t$$

The working capital loans are equal to the wage bill ($l_t = w_t n_t$) and labor demand implies that $r_t = 1/w_t$ (equation (10)), this leads to the following resource constraint

$$c_t + g_t = n_t \tag{53}$$

A.2 Three-Period Model

A.2.1 Special case with fixed labor

After substitution for prices, the government's implementability conditions with a binding collateral constraint are given by

$$t = 0 : \quad 0 = \frac{c_0 - n(1 - \tau)}{c_0} + \mathbb{E} \left\{ \frac{c_0}{c_1} \left[n + \left(1 - \frac{1}{\lambda} \right) \left(B_0^1 + \frac{c_1}{c_2} B_0^2 \right) \right] \right\} \tag{54}$$

$$t = 1 : \quad B_0^1 + q_1^2 B_0^2 = \frac{\lambda}{\lambda - \tau} \left[c_1 - n + \frac{c_1}{c_2} B_1^2 \right] \tag{55}$$

$$t = 2 : \quad B_1^2 = c_2 - n(1 - \tau) \tag{56}$$

After substitution of the debt levels, the government's optimal policy problem writes

$$\begin{aligned} & \max_{c_t} \mathbb{E} \sum_{t=0,1,2} \beta^t [(1 - \psi) \log(c_t) + \psi \theta_t (n - c_t)] \\ & + \gamma \left\{ \frac{c_0 - n(1 - \tau)}{c_0} + \frac{1}{\tau - \lambda} \mathbb{E} \left[\frac{(1 - \lambda)c_1 - n(1 - \tau)}{c_1} + (1 - \lambda) \frac{c_2 - n(1 - \tau)}{c_2} \right] \right\} \end{aligned}$$

the associated optimality conditions are given by equations (33) to (35).

To study the sign of B_0^1 and B_0^2 in equations (42) and (43), respectively, I first substitute for $c_1^{H,L}$ and $c_2^{H,L}$ using equations (34) and (35), respectively. This yields the following expressions for B_0^1 and B_0^2

$$B_0^1 = -n \frac{\lambda}{\tau - \lambda} \frac{\left\{ \frac{2(1-\tau)}{2(1-\lambda)+(\tau-\lambda)} \mathbb{E} \left[(\tau - \lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + \theta_1^{\frac{1}{2}} + (1 - \lambda)^{\frac{1}{2}} \theta_2^{\frac{1}{2}} \right] - (\theta^H)^{\frac{1}{2}} \right\} (\theta_2^L)^{\frac{1}{2}} - \left\{ \frac{2(1-\tau)}{2(1-\lambda)+(\tau-\lambda)} \mathbb{E} \left[(\tau - \lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + \theta_1^{\frac{1}{2}} + (1 - \lambda)^{\frac{1}{2}} \theta_2^{\frac{1}{2}} \right] - (\theta^L)^{\frac{1}{2}} \right\} (\theta_2^H)^{\frac{1}{2}}}{(\theta^H \theta_2^L)^{\frac{1}{2}} - (\theta^L \theta_2^H)^{\frac{1}{2}}} \quad (57)$$

$$B_0^2 = -n \frac{\lambda(1-\tau)}{\tau - \lambda} \frac{\left\{ \frac{2(1-\lambda)^{\frac{1}{2}}}{2(1-\lambda)+(\tau-\lambda)} \mathbb{E} \left[(\tau - \lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + \theta_1^{\frac{1}{2}} + (1 - \lambda)^{\frac{1}{2}} \theta_2^{\frac{1}{2}} \right] - (\theta_2^H)^{\frac{1}{2}} \right\} (\theta^L)^{\frac{1}{2}} - \left\{ \frac{2(1-\lambda)^{\frac{1}{2}}}{2(1-\lambda)+(\tau-\lambda)} \mathbb{E} \left[(\tau - \lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + \theta_1^{\frac{1}{2}} + (1 - \lambda)^{\frac{1}{2}} \theta_2^{\frac{1}{2}} \right] - (\theta_2^L)^{\frac{1}{2}} \right\} (\theta^H)^{\frac{1}{2}}}{(\theta^L \theta_2^H)^{\frac{1}{2}} - (\theta^H \theta_2^L)^{\frac{1}{2}}} \quad (58)$$

where $\theta_2^H = \alpha \theta^H + (1 - \alpha) \theta^L$ and $\theta_2^L = \alpha \theta^L + (1 - \alpha) \theta^H$. I have appealed to the fact that $\theta^H > \theta^L$ and $\frac{2(1-\lambda)^{\frac{1}{2}}}{2(1-\lambda)+(\tau-\lambda)} \mathbb{E} \left[(\tau - \lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + \theta_1^{\frac{1}{2}} + (1 - \lambda)^{\frac{1}{2}} \theta_2^{\frac{1}{2}} \right] > \theta^H$. To prove that $B_0^1 > 0$, one can show that

$$(\theta^H \theta_2^L)^{\frac{1}{2}} > (\theta^L \theta_2^H)^{\frac{1}{2}}, \text{ and}$$

$$\begin{aligned} & \left\{ \frac{2(1-\lambda)^{\frac{1}{2}}}{2(1-\lambda)+(\tau-\lambda)} \mathbb{E} \left[(\tau - \lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + \theta_1^{\frac{1}{2}} + (1 - \lambda)^{\frac{1}{2}} \theta_2^{\frac{1}{2}} \right] - (\theta_2^H)^{\frac{1}{2}} \right\} (\theta^L)^{\frac{1}{2}} \\ & < \left\{ \frac{2(1-\lambda)^{\frac{1}{2}}}{2(1-\lambda)+(\tau-\lambda)} \mathbb{E} \left[(\tau - \lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + \theta_1^{\frac{1}{2}} + (1 - \lambda)^{\frac{1}{2}} \theta_2^{\frac{1}{2}} \right] - (\theta_2^L)^{\frac{1}{2}} \right\} (\theta^H)^{\frac{1}{2}} \end{aligned}$$

Provided that $\lambda < \tau$ is a necessary condition, the above inequalities imply that $B_0^1 > 0$. It is then straightforward to show that $B_0^2 < 0$ since the numerator $\{(\theta^L \theta_2^H)^{\frac{1}{2}} - (\theta^H \theta_2^L)^{\frac{1}{2}}\}$ is negative, and it can be shown that

$$\begin{aligned} & \left\{ \frac{2(1-\lambda)^{\frac{1}{2}}}{2(1-\lambda)+(\tau-\lambda)} \mathbb{E} \left[(\tau - \lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + \theta_1^{\frac{1}{2}} + (1 - \lambda)^{\frac{1}{2}} \theta_2^{\frac{1}{2}} \right] - (\theta_2^H)^{\frac{1}{2}} \right\} (\theta^L)^{\frac{1}{2}} \\ & < \left\{ \frac{2(1-\lambda)^{\frac{1}{2}}}{2(1-\lambda)+(\tau-\lambda)} \mathbb{E} \left[(\tau - \lambda)^{\frac{1}{2}} \theta_0^{\frac{1}{2}} + \theta_1^{\frac{1}{2}} + (1 - \lambda)^{\frac{1}{2}} \theta_2^{\frac{1}{2}} \right] - (\theta_2^L)^{\frac{1}{2}} \right\} (\theta^H)^{\frac{1}{2}} \end{aligned}$$

A.2.2 Numerical Exercise

The social planner's first order conditions in a frictionless economy are given by

$$c_t : (1 - \psi) \left[\frac{1}{c_t} + \frac{1}{\phi} \frac{1}{c_t} \left(\frac{1-\tau}{c_t} \right)^{\frac{1}{\phi}+1} \right] + \psi \theta_t \left[-\frac{1}{\phi} \frac{1}{c_t} \left(\frac{1-\tau}{c_t} \right)^{\frac{1}{\phi}} - 1 \right] + \gamma \left(\frac{1}{\phi} + 1 \right) \frac{1}{c_t} \left(\frac{1-\tau}{c_t} \right)^{\frac{1}{\phi}+1} = 0$$

$$\gamma_f : \mathbb{E} \sum_{t=0,1,2} \beta^t \left\{ 1 - \left(\frac{1-\tau}{c_t} \right)^{\frac{1}{\phi}+1} \right\} = 0$$

for $t = \{0, 1, 2\}$. Parameter γ_f is the Lagrange multiplier on the intertemporal implementability constraint in period $t = 0$. Once the optimal allocation $\{c_0, c_1, c_2\}$ is determined, it is straightforward to deduce the substituted variables. Note that

$$g_t = n_t - c_t \quad \forall t \in \{0, 1, 2\}$$

$$n_t = \left(\frac{1-\tau}{c_t} \right)^{\frac{1}{\phi}} \quad \text{and} \quad w_t = 1 \quad \forall t \in \{0, 1, 2\}$$

The first order conditions of the social planner when the collateral constraint is binding in $t = 1$ are as follow

$$c_0 : (1-\psi) \left[\frac{1}{c_0} + \frac{1}{\phi} \frac{1}{c_0} \left(\frac{1-\tau}{c_0} \right)^{\frac{1}{\phi}+1} \right] + \psi \theta_0 \left[-\frac{1}{\phi} \frac{1}{c_0} \left(\frac{1-\tau}{c_0} \right)^{\frac{1}{\phi}} - 1 \right] + \gamma \left(\frac{1}{\phi} + 1 \right) \frac{1}{c_0} \left(\frac{1-\tau}{c_0} \right)^{\frac{1}{\phi}+1} = 0$$

$$c_1 : \beta(1-\psi) \frac{1}{c_1} - \beta \psi \theta_1 - \gamma_1 \beta \frac{n_1}{c_1^2} + \gamma_2 \frac{n_1}{c_1^2} = 0$$

$$n_1 : -\beta(1-\psi) n_1^\phi + \beta \psi \theta_1 + \gamma_1 \left[\beta \frac{1}{c_1} - (\phi+1) \frac{1-\lambda}{1-\tau} n_1^\phi \right] + \gamma_2 \left[(\phi+1) \frac{\tau-\lambda}{1-\tau} n_1^\phi - \frac{1}{c_1} \right] = 0$$

$$c_2 : \beta^2(1-\psi) \left[\frac{1}{c_2} + \frac{1}{\phi} \frac{1}{c_2} \left(\frac{1-\tau}{c_2} \right)^{\frac{1}{\phi}+1} \right] + \beta^2 \psi \theta_2 \left[-\frac{1}{\phi} \frac{1}{c_2} \left(\frac{1-\tau}{c_2} \right)^{\frac{1}{\phi}} - 1 \right] + \gamma_2 \beta \left(\frac{1}{\phi} + 1 \right) \frac{1}{c_2} \left(\frac{1-\tau}{c_2} \right)^{\frac{1}{\phi}+1} = 0$$

$$\gamma_1 : 1 - \left(\frac{1-\tau}{c_0} \right)^{\frac{1}{\phi}+1} + \beta \mathbb{E} \left\{ \frac{n_1}{c_1} - \frac{1-\lambda}{1-\tau} n_1^{\phi+1} \right\} = 0$$

$$\gamma_2 : \frac{\tau-\lambda}{1-\tau} n_1^{\phi+1} - \frac{n_1}{c_1} + 1 - \beta \left(\frac{1-\tau}{c_2} \right)^{\frac{1}{\phi}+1} + \beta = 0$$

where γ_1 and γ_2 are the Lagrange multipliers associated with the implementability conditions in periods $t = 0$ and $t = 1$, respectively. As for the substituted variables, one can deduce them using the following equations

$$g_t = n_t - c_t \quad \forall t \in \{0, 1, 2\}$$

$$n_t = \left(\frac{1-\tau}{c_t} \right)^{\frac{1}{\phi}} \quad \text{and} \quad w_t = 1 \quad \forall t \in \{0, 2\}$$

$$w_1 = \frac{n_1^\phi c_1}{1-\tau}$$

The Optimal Debt Levels. I derive the levels of short- and long-term debt in the frictionless economy ($B_{f,0}^1$ and $B_{f,0}^2$, respectively) using the intertemporal constraint of the government in date $t = 1$ when $\theta_1 = \theta^H$ and when $\theta_1 = \theta^L$. Accordingly, B_0^1 and B_0^2 are as follow

$$B_{f,0}^1 = c_1^H - \beta \frac{c_1^H}{c_2^H} n_2^H (1 - \tau) - n_1^H (1 - \tau) + \beta c_1^H - \beta \frac{c_1^H}{c_2^H} B_{f,0}^2 \quad (59)$$

$$B_{f,0}^2 = \frac{1}{\beta} \frac{c_1^L \left(1 - \beta \frac{1-\tau}{c_2^L} n_2^L + \beta\right) - c_1^H \left(1 - \beta \frac{1-\tau}{c_2^H} n_2^H + \beta\right) + (1 - \tau) n_1^H - (1 - \tau) n_1^L}{\frac{c_1^L}{c_2^L} - \frac{c_1^H}{c_2^H}} \quad (60)$$

In the baseline economy, one can derive the optimal debt levels B_0^1 and B_0^2 using either the intertemporal budget constraint in period $t = 1$ or the binding collateral friction in $t = 1$ when $\theta_1 = \theta^H$ and when $\theta_1 = \theta^H$. Using the collateral friction, the expressions of B_0^1 and B_0^2 write

$$B_0^1 = \frac{\lambda}{1 - \tau} c_1^H \frac{(n_1^H)^{\phi+1} \frac{c_1^L}{c_2^L} - (n_1^L)^{\phi+1} \frac{c_1^L}{c_2^H}}{\frac{c_1^L}{c_2^L} - \frac{c_1^H}{c_2^H}} \quad (61)$$

$$B_0^2 = \frac{1}{\beta} \frac{\lambda}{1 - \tau} \frac{(n_1^L)^{\phi+1} \frac{c_1^L}{c_2^L} - (n_1^H)^{\phi+1} \frac{c_1^H}{c_2^H}}{\frac{c_1^L}{c_2^L} - \frac{c_1^H}{c_2^H}} \quad (62)$$

B Additional Figures

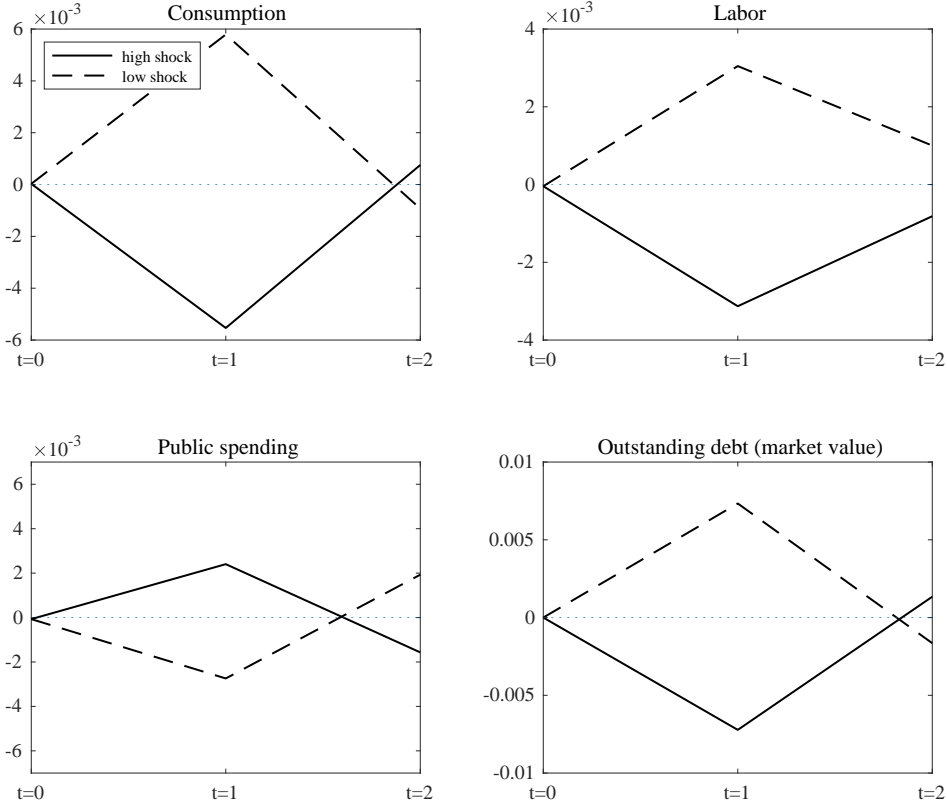


Figure 8: Optimal policy with $\lambda = 0.4$ (deviations from the deterministic state).

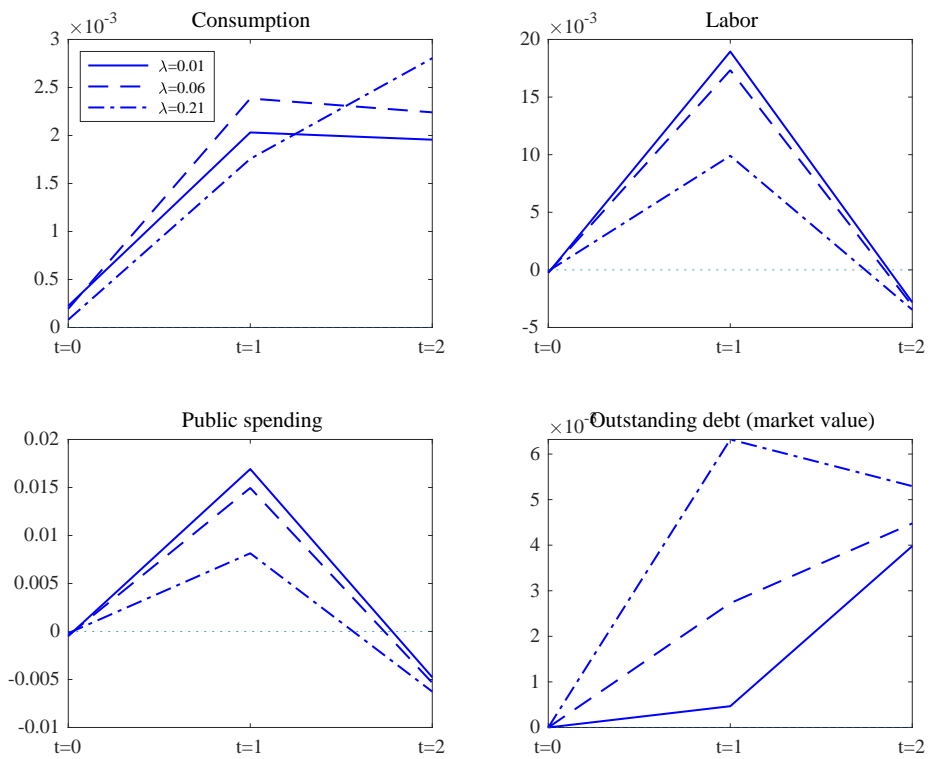


Figure 9: The sensitivity of the optimal allocation to the severity of the collateral friction (high shock).

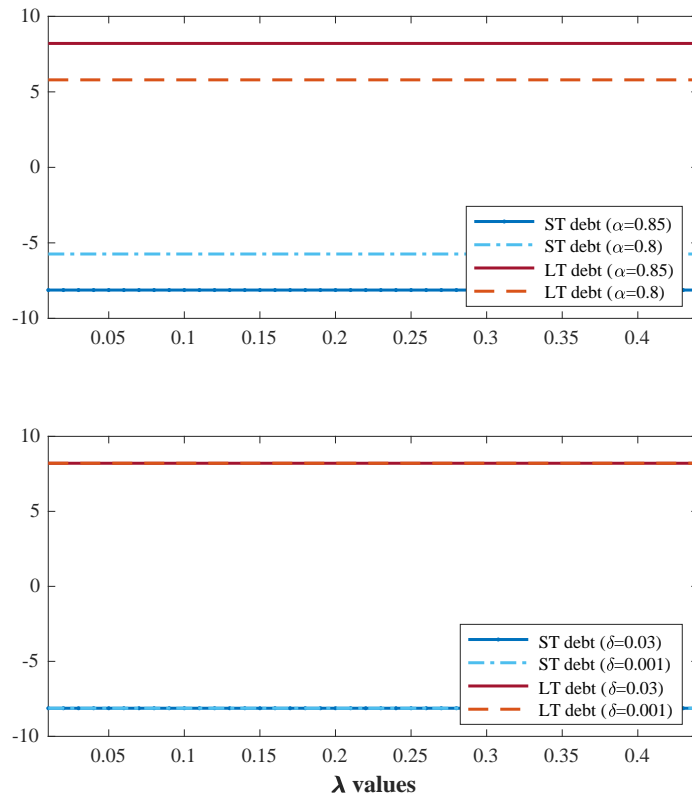


Figure 10: Sensitivity of the debt positions to the shock characteristics in a frictionless economy.