



MASTER THESIS

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Fiscal Policy in the Context of an Aging Population: The Case of the Czech Republic

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Abstract: The trend of aging population presents a significant challenge, affecting sustainable financing of pension systems in most developed countries, including the Czech Republic. To gain better understanding of the impact of projected aging and alternative policies, I calibrated already established macroeconomic model for policy analysis under demographic change with focus on the pension system (McGrattan & Prescott, 2017) for the Czech Republic and used it to quantify the impact of ageing coupled with increase in pension age. The thesis shows the stand-alone impact of aged demography and combined impact of aged demography and raised pension age. Based on the model, aged demography has expected negative welfare impact of -10,5%, which can be offset by increase in pension age by 5 years, however the change in policy doesn't nullify the impact of ageing across the whole income distribution. In transition path, faster increase in pension age has positive welfare impact for all households younger than 45 years and large negative impact for households near pension age, high income households benefit almost irrespective of age, with only small welfare decrease for cohorts deep in pension.

Keywords: Fiscal policy, OLG model, Ageing Population, Pension system, PAYG

I acknowledge the use of Google's Gemini model in drafting the initial outline, final editing, translating ideas into code and debugging Latex. The resulting output was critically evaluated and adapted to fit the academic scope of this thesis.

Název práce: Fiskální politika v kontextu stárnoucí populace: případ České republiky

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Abstrakt: Trend stárnutí populace představuje významnou výzvu ovlivňující udržitelné financování penzijních systémů, včetně penzijního systému České republiky. Pro získání lepšího pochopení vlivu projektovaného stárnutí populace a alternativních politik jsem kalibroval již zavedený makroekonomický model pro analýzu veřejných politik při změně demografie a s důrazem na penzijní systém (McGrattan & Prescott, 2017) pro Českou republiku a využil jsem ho pro kvantifikaci vlivu stárnoucí populace a zvýšení věku odchodu do důchodu. Práce ukazuje izolovaný vliv stárnutí populace a vliv kombinace tohoto stárnutí a zvýšeného věku odchodu do důchodu. Na základě modelu má stárnutí populace negativní vliv na blahobyt (-10,5 %), který může být vyvážen zvýšením věku odchodu do důchodu o 5 let, ovšem taková změna ve veřejné politice nevyváží vliv zestárlé populace pro všechny příjmové skupiny. V tranzici má rychlejší zvýšení věku odchodu do důchodu pozitivní vliv na blahobyt pro osoby mladší 45 let a významný negativní vliv na domácnosti těsně před odchodem do důchodu. Vysokopříjmové domácnosti benefitují bez ohledu na věk s pouze malým snížením blahobytu v pokročilé fázi penze.

Klíčová slova: Fiskální politika, OLG model, Stárnoucí populace, Penzijní systém, PAYG

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Introduction

Population aging presents one of the most impactful and irreversible macroeconomic challenges for developed economies over the next half-century. Driven by combination of persistently low fertility rates and sustained increases in life expectancy, the population shape of developed nations is rapidly transforming from pyramids into pillars. Such demographic shift fundamentally alters the dependency ratios, placing systemic pressures on public finances, labor markets, and possibly the trajectory of long-term economic growth. The transition from a period of a “demographic dividend”, where the working-age population was growing compared to dependents, to a period of shift towards demographic deficit may require a fundamental restructuring of the social contract and adjustment of the fiscal architecture.

The Czech Republic serves as a prime example of this structural shift. Total Fertility decreased from 2.4 in 1975 to its minimum of 1.1 in 1999 then recovering slightly to around 1.4 in 2025, amounting to 41% drop in the last 50 year period. During the same period life expectancy increased by 14% from 70 years to 80 years. (Czech Statistical Office, 2024b). Consequently, the old-age dependency ratio is projected to rise from 36% in 2024 to 60% by 2060.(Czech Statistical Office, 2024a) In practical terms, this shifts the demographic balance from nearly three working-age adults (aged 20–64) for every senior (aged 65+) today, to just under two. According to the projections for long - long-term sustainability of the Czech Fiscal Council (2025), the pension system is expected to move from a nearly - balanced position today to a structural yearly deficit in the range of 1–4% of GDP by 2050–2060, driven primarily by demographic ageing. Given the scale and persistence of these structural deficits, financing them solely through sustained debt accumulation is unrealistic, forcing policymakers to confront stark and unavoidable political tradeoffs in the near future.

In this thesis, I evaluate the extent of the macroeconomic and fiscal consequences with welfare analysis for multiple income groups across policy scenarios. Assessing the long-term sustainability of its public pension system and broader fiscal health requires advanced macroeconomic frameworks capable of capturing complex, multi-generational interactions. For this purpose I am using OLG model based on the work of McGrattan and Prescott (2017, 2018), which is able to capture demographic ageing with heterogeneous population and the PAYG pension system. Welfare impact of alternative policies is quantified using the equivalent consumption variation (CEV).

Ultimately, this thesis provides a quantitative assessment of the macroeconomic and welfare impacts of demographic aging and alternative reforms, mapping how distinct policies distribute the fiscal burden across heterogeneous cohorts while including major general equilibrium feedback channels.

1 Model

The following chapter presents the setup of the model including demography and outline of the solution methods.

1.1 Demography

The economy is populated by an overlapping generation of agents with finite lives and lifespan uncertainty. Agents enter the economy at age $j = 0$ with zero assets ($a = 0$). The conditional probability of survival from age j to age $j + 1$ at time t is denoted as $\sigma_{j,t}$. Agents live for a maximum of J periods with $\sigma_{J,t} = 0$ and $a_{J+1,t} = 0$ in all periods. Redistribution of leftover assets from deceased cohort members is redistributed to survivors through perfect annuity markets with gross return on investments $R_{j,t}$, reflecting the conditional probability of survival.

$$R_{j,t} = \frac{1 + r_t}{\sigma_{j-1,t-1}}$$

Agents are heterogeneous with respect to their labor efficiency $e_{j,k}$ which depends on age j and permanent type assigned at birth $k \in \{Low, Medium, High\}$, with each type representing one third of the income distribution with population weight $\mu_k = 1/3$.

Agents actively supply labor during their working life, corresponding to model ages $j < j_{ret}$ (where $j = 0$ corresponds to real-world age 20). At the statutory retirement age j_{ret} , households face mandatory retirement, which restricts their labor supply to zero. During retirement ($j \geq j_{ret}$), a household of productivity type $k \in \{Low, Med, High\}$ receives a type-specific public pension benefit $tr_{pension,k}$ calibrated to gross wage replacement rates.

In demographic steady state the amount of agents entering the economy to age $j = 0$ marked $N_{0,t}$ is given by the population growth g_n and by the number of preceding new entrants $N_{0,t-1}$.

$$N_{0,t} = (1 + g_n)N_{0,t-1}$$

The steady-state relationship of amount of agent between subsequent generations is:

$$N_{j,t} = \frac{1 + g_n}{\sigma_{j,t}} N_{j+1,t}$$

From size of generation $N_{j,t}$ the number of agents with specific type $N_{j,k,t}$ can be recovered by using the constant share of type in population $\mu_k = 1/3$ for all types.

$$N_{j,k,t} = \mu_k \cdot N_{j,t}$$

From cohort amounts population shares can be computed:

$$n_{j,t} = \frac{N_{j,t}}{\sum_j N_{j,t}} \quad n_{j,k,t} = \frac{N_{j,k,t}}{\sum_j N_{j,t}}$$

The state of the economy at any time t is fully characterized by the cross-sectional measure of households, denoted by $\lambda_{j,k,t}(a)$. This distribution tracks the

absolute mass of households of age j and productivity type k holding assets a . The asset space is discretized into a finite grid $\mathcal{A} = \{a_1 = 0, a_2, \dots, a_M\}$.

The mass of incoming households is distributed across productivity types k according to exogenous weights $\mu_k = 1/3$. The initial distribution over assets for newborns is therefore:

$$\lambda_{0,k,t}(a) = \begin{cases} N_{0,k,t} & \text{if } a = 0 \\ 0 & \text{if } a > 0 \end{cases}$$

Summing over the asset grid yields the total absolute mass of households in that specific age and type cohort:

$$N_{j,k,t} = \sum_{a \in \mathcal{A}} \lambda_{j,k,t}(a)$$

To map households from the current asset space to the future asset space, an indicator function $I_{j,k,t}(a, a')$ is used, which equals 1 if a household optimally chooses $a' \in \mathcal{A}$ given their current state at time t , and 0 otherwise:

$$I_{j,k,t}(a, a') = \begin{cases} 1 & \text{if } a'_{j,k,t}(a) = a' \\ 0 & \text{otherwise} \end{cases}$$

The absolute mass of households at time $t + 1$ for age $j + 1$ over the asset space is determined by summing over the optimal choices made by the surviving mass of households from the previous period:

$$\lambda_{j+1,k,t+1}(a') = \sigma_{j,t} \sum_{a \in \mathcal{A}} \lambda_{j,k,t}(a) \cdot I_{j,k,t}(a, a')$$

1.2 The Household Optimization Problem

1.2.1 Problem in Levels

Household of productivity type k born in year t_0 at age $j = 0$. The household lives for a maximum of J periods, corresponding to ages $j \in \{0, 1, \dots, J\}$. The household chooses a lifetime sequence of consumption $\{C_{j,k,t_0+j}\}_{j=0}^J$, labor supply $\{l_{j,k,t_0+j}\}_{j=0}^J$, and asset holdings $\{A_{j+1,k,t_0+j+1}\}_{j=0}^J$ to maximize expected discounted lifetime utility:

$$\max_{\{C_{j,k,t}, l_{j,k,t}, A_{j+1,k,t+1}\}} \sum_{j=0}^J \beta^j \Sigma_{j,t_0} \left[\frac{C_{j,k,t_0+j}^{1-\gamma}}{1-\gamma} - \phi Z_{t_0+j}^{1-\gamma} \frac{l_{j,k,t_0+j}^{1+1/\nu}}{1+1/\nu} \right]$$

where β is the discount factor, γ is the coefficient of relative risk aversion, ν is the Frisch elasticity of labor supply, ϕ is the disutility weight of labor, and Z_t is the level of labor-augmenting technology growing at a constant rate g ($Z_{t+1} = Z_t(1+g)$). The cumulative survival probability to age j is defined as:

$$\Sigma_{j,t_0} = \prod_{s=0}^{j-1} \sigma_{s,t_0+s} \quad \text{with} \quad \Sigma_{0,t_0} = 1$$

where $\sigma_{s,t}$ is the probability of surviving from age s to $s+1$ in year t . The household faces the sequential budget constraint for each age $j = 0, \dots, J$ in year $t = t_0 + j$:

$$C_{j,k,t}(1 + \tau_{c,t}) + A_{j+1,k,t+1} = \frac{1 + r_t}{\sigma_{j-1,t-1}} A_{j,k,t} + W_t \epsilon_{j,k} l_{j,k,t} (1 - \tau_{ss,t}) + Tr_{j,k,t}$$

subject to:

$$\begin{aligned} A_{0,k,t_0} &= 0 \\ A_{J+1,k,t_0+J+1} &= 0 \\ A_{j+1,k,t+1} &\geq 0 \quad \forall j = 0, \dots, J \\ 0 &\leq l_{j,k,t} \leq \bar{l} \quad \forall j < j_{ret,t} \\ l_{j,k,t} &= 0 \quad \forall j \geq j_{ret,t} \end{aligned}$$

where W_t is the level wage, $\epsilon_{j,k}$ is the lifecycle efficiency profile, $\tau_{c,t}$ is the consumption tax, $\tau_{ss,t}$ is the payroll tax, $Tr_{j,k,t}$ is government transfers (pensions), and $j_{ret,t}$ is the mandatory retirement age index.

1.2.2 Normalized Problem

To stationarise the model, I detrend the variables by the technology level Z_t :

$$c_{j,k,t} \equiv \frac{C_{j,k,t}}{Z_t}, \quad a_{j,k,t} \equiv \frac{A_{j,k,t}}{Z_t}, \quad w_t \equiv \frac{W_t}{Z_t}, \quad tr_{j,k,t} \equiv \frac{Tr_{j,k,t}}{Z_t}$$

The normalized sequential maximization problem is:

$$\max_{\{c_{j,k,t}, l_{j,k,t}, a_{j+1,k,t+1}\}} \sum_{j=0}^J \tilde{\beta}^j \Sigma_{j,t_0} \left[\frac{c_{j,k,t}^{1-\gamma}}{1-\gamma} - \phi \frac{l_{j,k,t}^{1+1/\nu}}{1+1/\nu} \right]$$

subject to:

$$c_{j,k,t}(1 + \tau_{c,t}) + a_{j+1,k,t+1}(1 + g) = \frac{1 + r_t}{\sigma_{j-1,t-1}} a_{j,k,t} + w_t \epsilon_{j,k} l_{j,k,t}(1 - \tau_{ss,t}) + tr_{j,k,t}$$

where the effective discount factor is adjusted to:

$$\tilde{\beta} \equiv \beta(1 + g)^{1-\gamma}$$

1.2.3 First-Order Conditions

By formulating the Lagrangian of the normalized problem, I obtain the optimality conditions for consumption, labor, and asset accumulation.

Intratemporal

For any working age $j < j_{ret,t}$, the intratemporal first-order condition equates the marginal rate of substitution between consumption and labor to the net real wage:

$$\phi \frac{l_{j,k,t}^{1/\nu}}{c_{j,k,t}^{-\gamma}} = \frac{w_t \epsilon_{j,k}(1 - \tau_{ss,t})}{1 + \tau_{c,t}}$$

Rearranging for the optimal labor supply hours:

$$l_{j,k,t} = \left[\frac{c_{j,k,t}^{-\gamma}}{\phi} \frac{w_t \epsilon_{j,k}(1 - \tau_{ss,t})}{1 + \tau_{c,t}} \right]^\nu$$

Intertemporal

For $j = 0, \dots, J - 1$, the first-order condition with respect to $a_{j+1,k,t+1}$ yields the Euler equation (expressed as a Kuhn-Tucker condition to account for the borrowing constraint):

$$c_{j,k,t}^{-\gamma} \geq \tilde{\beta} c_{j+1,k,t+1}^{-\gamma} (1 + r_{t+1}) \left(\frac{1 + \tau_{c,t}}{1 + \tau_{c,t+1}} \right)$$

which holds with strict equality whenever $a_{j+1,k,t+1} > 0$, however agents in this model in the beginning of their life are generally bounded by the restriction. The age-specific survival probability $\sigma_{j,t}$ cancels out from both sides of the Euler equation. This is because the perfect annuity market pays a higher return on assets, scaling the gross return to $\frac{1+r_{t+1}}{\sigma_{j,t}}$, which exactly compensates the surviving households for the risk of dying. The technology growth factor $(1 + g)^{-\gamma}$ appears in the normalized Euler equation because tomorrow's consumption is deflated by a higher technology level, $Z_{t+1} = Z_t(1 + g)$. The ratio $\left(\frac{1+\tau_{c,t}}{1+\tau_{c,t+1}} \right)$ captures the intertemporal substitution effect induced by changing tax rates. If consumption taxes are rising along the transition path ($\tau_{c,t+1} > \tau_{c,t}$), consumption today becomes relatively cheaper than consumption tomorrow, encouraging households to consume earlier.

1.2.4 Recursive Formulation

The household's lifetime optimization problem can be formulated recursively using dynamic programming. In each calendar year t , the state variables for a household of age $j \in \{0, 1, \dots, J\}$ and productivity type $k \in \{1, \dots, N_e\}$ are their starting asset holdings A (or a in normalized terms). The problem is solved backwards starting from the last period of life J to the entry age 0.

Normalizing the starting asset state $a = A/Z_t$, the choices $c = C/Z_t$ and $a' = A'/Z_{t+1}$, and the wages and transfers. The value function $V_{j,k,t}(a) \equiv V_{j,k,t}^{\text{level}}(A)Z_t^{\gamma-1}$ satisfies the following recursive equations:

Working-Age Households ($j < j_{\text{ret},t}$)

$$V_{j,k,t}(a) = \max_{c,l,a'} \left\{ \frac{c^{1-\gamma}}{1-\gamma} - \phi \frac{l^{1+1/\nu}}{1+1/\nu} + \tilde{\beta} \sigma_{j,t} V_{j+1,k,t+1}(a') \right\}$$

subject to:

$$\begin{aligned} c(1 + \tau_{c,t}) + a'(1 + g) &= R_{\text{eff},j,t}a + w_t \epsilon_{j,k} l (1 - \tau_{ss,t}) + tr_{j,t} \\ a' &\geq 0, \quad 0 \leq l \leq 1 \end{aligned}$$

Retired Households ($j \geq j_{\text{ret},t}$)

$$V_{j,k,t}(a) = \max_{c,a'} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \tilde{\beta} \sigma_{j,t} V_{j+1,k,t+1}(a') \right\}$$

subject to:

$$\begin{aligned} c(1 + \tau_{c,t}) + a'(1 + g) &= R_{\text{eff},j,t}a + tr_{j,t} \\ a' &\geq 0 \end{aligned}$$

where the effective discount factor is:

$$\tilde{\beta} \equiv \beta(1 + g)^{1-\gamma}$$

and the effective gross interest rate on the annuity market is:

$$R_{\text{eff},j,t} = \frac{1 + r_t}{\sigma_{j-1,t-1}}$$

Terminal Condition

$$V_{J+1,k,t+1}(a') = 0$$

1.3 Production

The production side of the economy consists of two sectors: the Corporate Sector (Sector 1) and the Non-Corporate Sector (Sector 2). Both sectors operate under perfect competition and utilize constant returns to scale (CRS) Cobb-Douglas technologies with two types of capital—tangible and intangible—alongside labor inputs.

1.3.1 Sectoral Production Function

Output in the Corporate sector ($Y_{1,t}$) and Non-Corporate sector ($Y_{2,t}$) in calendar year t is produced according to:

$$\begin{aligned} Y_{1,t} &= K_{1T,t}^{\theta_1} K_{1I,t}^{\theta_2} (Z_t L_{1,t})^{1-\theta_1-\theta_2} \\ Y_{2,t} &= K_{2T,t}^{\theta_3} K_{2I,t}^{\theta_4} (Z_t L_{2,t})^{1-\theta_3-\theta_4} \end{aligned}$$

where:

- $K_{iT,t}$ and $K_{iI,t}$ denote the tangible (T) and intangible (I) capital stocks in sector $i \in \{1, 2\}$.
- $L_{i,t}$ is the total labor hours hired by sector i .
- Z_t is the level of labor-augmenting technological progress, which grows at the constant exogenous rate g :

$$Z_{t+1} = Z_t(1 + g)$$

- $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$ represents the capital input shares in production, satisfying $\theta_1 + \theta_2 < 1$ and $\theta_3 + \theta_4 < 1$ where $\theta_1, \theta_2, \theta_3, \theta_4 > 0$, ensuring a positive labor share in both sectors.

1.3.2 CES Aggregation and Final Good Production

The outputs of the two sectors are combined by a competitive final-good representative firm using a constant elasticity of substitution (CES) aggregator to produce aggregate GDP (Y_t):

$$Y_t = \left(\omega Y_{1,t}^\rho + (1 - \omega) Y_{2,t}^\rho \right)^{1/\rho}$$

where $\omega \in (0, 1)$ is the sector weight of the corporate sector and $\rho < 1$ governs the elasticity of substitution between corporate and non-corporate goods, given by $\sigma = \frac{1}{1-\rho}$.

The final-good firm maximizes profit:

$$\max_{Y_{1,t}, Y_{2,t}} P_t Y_t - P_{1,t} Y_{1,t} - P_{2,t} Y_{2,t}$$

where P_t is the aggregate price index (normalized to $P_t = 1$ as the numeraire), and $P_{1,t}, P_{2,t}$ are the relative sector prices. The first-order conditions yield the relative demand for sectoral outputs:

$$\frac{Y_{1,t}}{Y_{2,t}} = \left(\frac{P_{1,t}}{P_{2,t}} \frac{1 - \omega}{\omega} \right)^{\frac{1}{\rho-1}}$$

And the aggregate price index zero-profit condition yields:

$$P_t = \left(\omega^\sigma P_{1,t}^{1-\sigma} + (1-\omega)^\sigma P_{2,t}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} = 1$$

1.3.3 Rental Rates of Capital

Firms rent capital from households in competitive markets. Because corporate income and distributions are taxed differently, the cost of renting capital includes sector-specific tax wedges:

- **Sector 1 (Corporate):** Corporate profits are taxed at rate $\tau_{\pi,t}$, and dividends distributed to households are taxed at rate $\tau_{d1,t}$. The net returns to households are adjusted by the corporate tax wedge $(1 - \tau_{d1,t})(1 - \tau_{\pi,t})$. The rental rates (user costs) of corporate tangible and intangible capital are:

$$R_{1T,t} = \frac{r_t}{(1 - \tau_{d1,t})(1 - \tau_{\pi,t})} + \delta_{1T}$$

$$R_{1I,t} = \frac{r_t}{(1 - \tau_{d1,t})(1 - \tau_{\pi,t})} + \delta_{1I}$$

- **Sector 2 (Non-Corporate):** Non-corporate income distributed to households is taxed at rate $\tau_{d2,t}$. The rental rates are:

$$R_{2T,t} = \frac{r_t}{1 - \tau_{d2,t}} + \delta_{2T}$$

$$R_{2I,t} = \frac{r_t}{1 - \tau_{d2,t}} + \delta_{2I}$$

where r_t is the risk-free net interest rate in the economy and $\delta_{i,j}$ represents the depreciation rate of capital type j in sector i .

1.3.4 Profit Maximization

Firms in each sector choose capital and labor inputs each period to maximize static profits. For Sector 1 (Corporate):

$$\max_{K_{1T,t}, K_{1I,t}, L_{1,t}} P_{1,t} Y_{1,t} - R_{1T,t} K_{1T,t} - R_{1I,t} K_{1I,t} - W_t L_{1,t}$$

The first-order conditions equate the marginal product of each input to its cost:

$$\theta_1 P_{1,t} \frac{Y_{1,t}}{K_{1T,t}} = R_{1T,t}$$

$$\theta_2 P_{1,t} \frac{Y_{1,t}}{K_{1I,t}} = R_{1I,t}$$

$$(1 - \theta_1 - \theta_2) P_{1,t} \frac{Y_{1,t}}{L_{1,t}} = W_t$$

Similar FOCs hold for Sector 2 (Non-Corporate):

$$\begin{aligned}\theta_3 P_{2,t} \frac{Y_{2,t}}{K_{2T,t}} &= R_{2T,t} \\ \theta_4 P_{2,t} \frac{Y_{2,t}}{K_{2I,t}} &= R_{2I,t} \\ (1 - \theta_3 - \theta_4) P_{2,t} \frac{Y_{2,t}}{L_{2,t}} &= W_t\end{aligned}$$

1.3.5 Normalization of the Firm Problem

To stationarise the firm's problem, I normalise all level variables by dividing them by the product of technology and population: $Z_t N_t$, where N_t is the total population size growing at rate $g_{n,t}$ ($N_{t+1} = N_t(1 + g_{n,t+1})$).

We define normalized per-capita variables:

$$\tilde{y}_{i,t} \equiv \frac{Y_{i,t}}{Z_t N_t}, \quad \tilde{k}_{iT,t} \equiv \frac{K_{iT,t}}{Z_t N_t}, \quad \tilde{k}_{iI,t} \equiv \frac{K_{iI,t}}{Z_t N_t}, \quad l_{i,t} \equiv \frac{L_{i,t}}{N_t}$$

Substituting these definitions into the Corporate sector production function:

$$\tilde{y}_{1,t} Z_t N_t = \left(\tilde{k}_{1T,t} Z_t N_t \right)^{\theta_1} \left(\tilde{k}_{1I,t} Z_t N_t \right)^{\theta_2} (Z_t l_{1,t} N_t)^{1-\theta_1-\theta_2}$$

Factoring out the growth terms ($Z_t N_t$) on the right-hand side:

$$\tilde{y}_{1,t} Z_t N_t = \tilde{k}_{1T,t}^{\theta_1} \tilde{k}_{1I,t}^{\theta_2} l_{1,t}^{1-\theta_1-\theta_2} (Z_t N_t)^{\theta_1+\theta_2+1-\theta_1-\theta_2}$$

Since the exponents sum to 1 (Constant Returns to Scale):

$$\tilde{y}_{1,t} Z_t N_t = \tilde{k}_{1T,t}^{\theta_1} \tilde{k}_{1I,t}^{\theta_2} l_{1,t}^{1-\theta_1-\theta_2} (Z_t N_t)$$

Dividing both sides by $Z_t N_t$ yields the stationary production function:

$$\tilde{y}_{1,t} = \tilde{k}_{1T,t}^{\theta_1} \tilde{k}_{1I,t}^{\theta_2} l_{1,t}^{1-\theta_1-\theta_2}$$

Applying the same detrending procedure to the FOCs and prices, and noting that the numeraire requires $P_t = 1$: Normalized Wage: $w_t = W_t/Z_t$ is stationary. The level wage grows at the technology growth rate:

$$\frac{W_{t+1}}{W_t} = 1 + g$$

Normalized FOCs:

$$\begin{aligned}\theta_1 P_{1,t} \frac{\tilde{y}_{1,t}}{\tilde{k}_{1T,t}} &= R_{1T,t} \\ \theta_2 P_{1,t} \frac{\tilde{y}_{1,t}}{\tilde{k}_{1I,t}} &= R_{1I,t} \\ (1 - \theta_1 - \theta_2) P_{1,t} \frac{\tilde{y}_{1,t}}{l_{1,t}} &= w_t\end{aligned}$$

Under perfect competition, price equals marginal cost. The relative sector prices are given by their normalized unit cost functions:

$$P_{1,t} = \left(\frac{R_{1T,t}}{\theta_1} \right)^{\theta_1} \left(\frac{R_{1I,t}}{\theta_2} \right)^{\theta_2} \left(\frac{w_t}{1 - \theta_1 - \theta_2} \right)^{1 - \theta_1 - \theta_2}$$

$$P_{2,t} = \left(\frac{R_{2T,t}}{\theta_3} \right)^{\theta_3} \left(\frac{R_{2I,t}}{\theta_4} \right)^{\theta_4} \left(\frac{w_t}{1 - \theta_3 - \theta_4} \right)^{1 - \theta_3 - \theta_4}$$

1.4 The Government Sector

The government in this overlapping generations model manages fiscal policy by raising tax revenues, spending on public consumption, distributing welfare transfers and pensions, and managing public debt. The government's actions are subject to a sequential budget constraint.

1.4.1 Sources of Government Revenue in Levels

The government collects revenue from three tax bases: labor income, private consumption, and capital income.

Labor Income Tax

Labor income is subject to a payroll tax (social security tax) at rate $\tau_{ss,t}$. For a household of age $j < j_{ret,t}$ and type k in calendar year t , labor earnings are $Y_{j,k,t}^{labor} = W_t \epsilon_{j,k} l_{j,k,t}$. There are no tax exemption so the taxable labor income is:

$$Y_{j,k,t}^{taxable} = Y_{j,k,t}^{labor}$$

The total revenue from the labor income tax is aggregated over the working-age population distribution:

$$Rev_{labor,t} = \tau_{ss,t} \sum_{j=0}^{j_{ret,t}-1} \sum_k \sum_a \lambda_{j,k,t}(a) Y_{j,k,t}^{taxable}$$

where $\lambda_{j,k,t}(a)$ is the cohort mass at asset level a .

Consumption Tax

The government levies a flat tax at rate $\tau_{c,t}$ on aggregate consumption:

$$Rev_{consumption,t} = \tau_{c,t} C_t^{agg} = \tau_{c,t} \sum_{j=0}^J \sum_k \sum_a \lambda_{j,k,t}(a) C_{j,k,t}(a)$$

Capital Income Taxes

Capital tax revenue is collected from both the corporate and non-corporate sectors, reflecting precise sectoral tax rules:

- **Corporate Income Tax:** Corporate profits are taxed at rate $\tau_{\pi,t}$. Taxable profits are defined as corporate revenue minus wage payments and tangible/intangible depreciation:

$$\Pi_{1,t}^{corp} = P_{1,t} Y_{1,t} - W_t L_{1,t} - \delta_{1T} K_{1T,t} - \delta_{1I} K_{1I,t}$$

The corporate income tax revenue is:

$$Rev_{corp_inc,t} = \tau_{\pi,t} \Pi_{1,t}^{corp}$$

- **Dividend Tax:** After-tax corporate profits are distributed as dividends, $Div_{1,t} = (1 - \tau_{\pi,t})\Pi_{1,t}^{\text{corp}}$, which are taxed at rate $\tau_{d1,t}$:

$$Rev_{\text{dividend},t} = \tau_{d1,t}(1 - \tau_{\pi,t})\Pi_{1,t}^{\text{corp}}$$

- **Non-Corporate Distribution Tax:** Non-corporate profits are defined as revenue minus wages and depreciation:

$$\Pi_{2,t}^{\text{noncorp}} = P_{2,t}Y_{2,t} - W_tL_{2,t} - \delta_{2T}K_{2T,t} - \delta_{2I}K_{2I,t}$$

Non-corporate profits distributed to owners are taxed at rate $\tau_{d2,t}$:

$$Rev_{\text{noncorp},t} = \tau_{d2,t}\Pi_{2,t}^{\text{noncorp}}$$

Total capital tax revenue is the sum of these three components:

$$Rev_{\text{capital},t} = Rev_{\text{corp_inc},t} + Rev_{\text{dividend},t} + Rev_{\text{noncorp},t}$$

Total government revenue is:

$$Rev_t = Rev_{\text{labor},t} + Rev_{\text{consumption},t} + Rev_{\text{capital},t}$$

1.4.2 Government Expenditures in Levels

Government expenditures consist of public consumption, welfare transfers, pensions, and debt servicing costs.

- **Public Consumption (G_t):** Public spending is set as a constant fraction g_{share} of aggregate output:

$$G_t = g_{\text{share}}Y_t$$

- **Pension Benefits ($Tr_{\text{pension},t}$):** A type-specific pension transfer (scaled by technology) is paid exclusively to retired households ($j \geq j_{\text{ret},t}$) based on their productivity type $k \in \{1, 2, 3\}$:

$$Tr_{\text{pension},t} = tr_{\text{base}}Z_t \sum_{j=j_{\text{ret},t}}^J \sum_{k=1}^3 \phi_k^{\text{pension}} \sum_a \lambda_{j,k,t}(a)$$

where tr_{base} represents the calibrated baseline pension benefit, ϕ_k^{pension} is the type-specific pension multiplier for low, medium, and high productivity types, respectively), and $\lambda_{j,k,t}(a)$ represents the distribution of households over age j , productivity type k , and asset holdings a in period t .

1.4.3 Government Budget Constraint and Normalization

The government budget constraint in levels is:

$$B_{t+1} = (1 + r_t)B_t + G_t + Tr_{\text{welfare},t} + Tr_{\text{pension},t} - Rev_t$$

where B_t is the total nominal stock of government debt.

To normalize the budget constraint, we divide both sides by the technology-population trend $Z_t N_t$:

$$\frac{B_{t+1}}{Z_t N_t} = (1 + r_t) \frac{B_t}{Z_t N_t} + \frac{G_t}{Z_t N_t} + \frac{Tr_{welfare,t}}{Z_t N_t} + \frac{Tr_{pension,t}}{Z_t N_t} - \frac{Rev_t}{Z_t N_t}$$

Let $\tilde{b}_t \equiv \frac{B_t}{Z_t N_t}$, $\tilde{g}_t \equiv \frac{G_t}{Z_t N_t}$, $\tilde{tr}_t \equiv \frac{Tr_{welfare,t} + Tr_{pension,t}}{Z_t N_t}$, and $r\tilde{ev}_t \equiv \frac{Rev_t}{Z_t N_t}$ be the normalized variables. Using the relationships $Z_{t+1} = Z_t(1 + g)$ and $N_{t+1} = N_t(1 + g_{n,t+1})$, we rewrite the left-hand side:

$$\frac{B_{t+1}}{Z_t N_t} = \frac{B_{t+1}}{Z_{t+1} N_{t+1}} \frac{Z_{t+1} N_{t+1}}{Z_t N_t} = \tilde{b}_{t+1}(1 + g)(1 + g_{n,t+1})$$

Substituting this into the normalized constraint yields the stationary law of motion for government debt:

$$\tilde{b}_{t+1}(1 + g)(1 + g_{n,t+1}) = (1 + r_t)\tilde{b}_t + \tilde{g}_t + \tilde{tr}_t - r\tilde{ev}_t$$

Rearranging to express required tax revenue yields:

$$r\tilde{ev}_t = \tilde{g}_t + \tilde{tr}_t + \underbrace{(1 + r_t)\tilde{b}_t - (1 + g)(1 + g_{n,t+1})\tilde{b}_{t+1}}_{\text{Net Debt Service}_t}$$

1.4.4 Derivation of the Net Debt Service

The term $\text{Net Debt Service}_t$ represents the net interest payments the government must make, accounting for the issuance of new debt. If the government maintains a target debt-to-GDP ratio along a BGP (or transition path) where $\tilde{b}_{t+1} \approx \tilde{b}_t = \tilde{b}$, it's possible to expand the Net Debt Service:

$$\begin{aligned} \text{Net Debt Service}_t &= (1 + r_t)\tilde{b} - (1 + g)(1 + g_{n,t+1})\tilde{b} \\ &= [1 + r_t - (1 + g + g_{n,t+1} + g \cdot g_{n,t+1})]\tilde{b} \\ &= [r_t - g - g_{n,t+1} - g \cdot g_{n,t+1}]\tilde{b} \end{aligned}$$

1.4.5 The Public Pension System

The economy features a pay-as-you-go (PAYG) public pension system that provides type-specific retirement benefits to retired cohorts. The structure of the pension system is formalized as follows:

Retirement Decision and Pension Benefits

Households actively supply labor during their working life, corresponding to model ages $j < j_{ret}$ (where $j = 0$ corresponds to real-world age 20). At the statutory retirement age j_{ret} , households face mandatory retirement, which restricts their labor supply to zero:

$$l_{j,k,t} = 0 \quad \forall j \geq j_{ret} \quad (1.1)$$

During retirement ($j \geq j_{ret}$), a household of productivity type $k \in \{Low, Med, High\}$ receives a type-specific public pension benefit $tr_{pension,j,k,t}$. The transfer income $tr_{j,k,t}$ received by retired household is given by:

$$tr_{j,k,t} = tr_{pension,j,k,t} \quad \forall j \geq j_{ret} \quad (1.2)$$

To capture progressivity or earnings-dependence, the type-specific pension benefit is determined as the product of a base pension benefit and a type-specific multiplier:

$$tr_{pension,j,k,t} = \bar{tr}_{pension} \times \phi_k^{pension} \quad (1.3)$$

where $\bar{tr}_{pension}$ represents the base pension benefit, and $\phi_k^{pension}$ is the pension multiplier for productivity type k .

1.5 Competitive Equilibrium

Given a demographic profile of survival probabilities $\sigma_{j,t}$ and population growth rate, technology growth rate g , and a fiscal policy of tax rates, government consumption share, public debt share, and transfer policies

$\{\tau_{c,t}, \tau_{ss,t}, \tau_{exempt}, g_{share}, \tilde{b}_t, tr_{welfare}, tr_{pension}\}$, a competitive equilibrium is defined as:

- a sequence of household value functions $\{V_{j,k,t}(a)\}_{j=0}^J$ and optimal policy rules for consumption, labor supply, and next-period assets $\{c_{j,k,t}(a), l_{j,k,t}(a), a'_{j,k,t}(a)\}_{j=0}^J$,
- a sequence of sectoral production allocations and capital demands $\{\tilde{y}_{i,t}, \tilde{k}_{iT,t}, \tilde{k}_{iI,t}, l_{i,t}\}$ for sectors $i \in \{1, 2\}$,
- a sequence of relative sectoral prices $\{P_{1,t}, P_{2,t}\}$ and relative factor prices $\{r_t, w_t\}$,
- sequence of labor income tax rates $\{\tau_{ss,t}\}$,
- sequence of cross-sectional household distributions $\{\lambda_{j,k,t}(a)\}$

such that in every calendar period t :

1. **Household Optimality:** Given relative prices and fiscal policies, the household policy functions $\{c_{j,k,t}(a), l_{j,k,t}(a), a'_{j,k,t}(a)\}$ and value functions $\{V_{j,k,t}(a)\}$ solve the recursive Bellman equations:

$$V_{j,k,t}(a) = \max_{c,l,a'} \left\{ u(c, l) + \beta \sigma_{j,t} V_{j+1,k,t+1}(a') \right\}$$

subject to the normalized budget constraint:

$$c(1 + \tau_{c,t}) + a'(1 + g) = R_{eff,j,t}a + w_t \epsilon_{j,k} l(1 - \tau_{ss,t}) + tr_{j,k,t}$$

where $\tilde{\beta} \equiv \beta(1 + g)^{1-\gamma}$ and $a' \geq 0$ for all $j \in \{0, 1, \dots, J\}$.

2. **Firm Optimality:** Given relative factor prices $\{r_t, w_t\}$, relative sector prices $\{P_{1,t}, P_{2,t}\}$, and corporate taxes, sectoral firms choose capital and labor inputs to satisfy their static marginal product conditions:

$$\begin{aligned} \theta_1 P_{1,t} \frac{\tilde{y}_{1,t}}{\tilde{k}_{1T,t}} &= R_{1T,t}, & \theta_2 P_{1,t} \frac{\tilde{y}_{1,t}}{\tilde{k}_{1I,t}} &= R_{1I,t} \\ \theta_3 P_{2,t} \frac{\tilde{y}_{2,t}}{\tilde{k}_{2T,t}} &= R_{2T,t}, & \theta_4 P_{2,t} \frac{\tilde{y}_{2,t}}{\tilde{k}_{2I,t}} &= R_{2I,t} \\ (1 - \theta_1 - \theta_2) P_{1,t} \frac{\tilde{y}_{1,t}}{l_{1,t}} &= w_t, & (1 - \theta_3 - \theta_4) P_{2,t} \frac{\tilde{y}_{2,t}}{l_{2,t}} &= w_t \end{aligned}$$

3. **Final Good Aggregator Choice:** The competitive aggregate final-good firm maximizes profits, satisfying the relative demand and aggregate price zero-profit condition:

$$P_t = \left(\omega^\sigma P_{1,t}^{1-\sigma} + (1-\omega)^\sigma P_{2,t}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} = 1$$

4. **Government Budget Balance:** The government budget constraint holds, determining the labor tax rate $\tau_{ss,t}$ required to support public spending and the public debt path $B_t^{\text{agg}} = N_t \tilde{b}_t$:

$$r\tilde{v}_t = \tilde{g}_t + \tilde{t}r_t + (r_t - g - g_{n,t+1} - g \cdot g_{n,t+1}) \tilde{b}_t$$

5. **Demographic and Distributional Dynamics:**

- The total population size N_t is the sum of the absolute cohort masses:

$$N_t = \sum_{j=0}^J \sum_k \sum_a \lambda_{j,k,t}(a)$$

- The population growth rate $g_{n,t}$ is:

$$g_{n,t} = \frac{N_t}{N_{t-1}} - 1$$

6. **Market Clearing:** To close the model, we require that the asset, labor, and goods markets clear in every calendar period t in total aggregate detrended terms. The cross-sectional distribution $\lambda_{j,k,t}(a)$ represents the absolute mass of households in the economy, which integrates to the total population size N_t :

$$\sum_{j=0}^J \sum_k \sum_a \lambda_{j,k,t}(a) = N_t$$

- **Asset Market Clearing:** The total aggregate detrended supply of assets by households must equal the total aggregate detrended demand for capital by firms in both sectors plus the total aggregate stock of government debt:

$$A_t^{\text{supply}} = K_{1T,t}^{\text{agg}} + K_{1I,t}^{\text{agg}} + K_{2T,t}^{\text{agg}} + K_{2I,t}^{\text{agg}} + B_t^{\text{agg}}$$

where total aggregate detrended asset supply is:

$$A_t^{\text{supply}} = \sum_{j=0}^J \sum_k \sum_a \lambda_{j,k,t}(a) a$$

and the aggregate demands for capital and public debt are scaled by the population size N_t :

$$\begin{aligned} K_{ij,t}^{\text{agg}} &= N_t \tilde{k}_{ij,t} \quad \forall i \in \{1, 2\}, j \in \{T, I\} \\ B_t^{\text{agg}} &= N_t \tilde{b}_t \end{aligned}$$

Therefore, the asset market clearing condition can be rewritten as:

$$\sum_{j=0}^J \sum_k \sum_a \lambda_{j,k,t}(a) a = N_t \left(\tilde{k}_{1T,t} + \tilde{k}_{1I,t} + \tilde{k}_{2T,t} + \tilde{k}_{2I,t} + \tilde{b}_t \right)$$

- Labor Market Clearing: The total aggregate labor hours hired by the corporate and non-corporate sectors must equal the aggregate effective labor hours supplied by households:

$$L_{1,t} + L_{2,t} = L_t^{\text{supply}}$$

where the aggregate effective labor supply is:

$$L_t^{\text{supply}} = \sum_{j=0}^{j_{\text{ret},t}-1} \sum_k \sum_a \lambda_{j,k,t}(a) \epsilon_{j,k} l_{j,k,t}(a)$$

- Goods Market Clearing (Feasibility Constraint): The total aggregate detrended output must equal the sum of aggregate private consumption, aggregate public consumption, and aggregate investment:

$$Y_t^{\text{agg}} = C_t^{\text{agg}} + G_t^{\text{agg}} + I_t^{\text{agg}}$$

where the aggregate components are defined as:

$$Y_t^{\text{agg}} = N_t Z_t \tilde{y}_t$$

$$G_t^{\text{agg}} = N_t Z_t \tilde{g}_t$$

$$C_t^{\text{agg}} = Z_t \sum_{j=0}^J \sum_k \sum_a \lambda_{j,k,t}(a) c_{j,k,t}(a)$$

The aggregate investment I_t^{agg} is the sum of investment across all capital types:

$$I_t^{\text{agg}} = \sum_{i \in \{1,2\}} \sum_{j \in \{T,I\}} \left[K_{ij,t+1}^{\text{agg}} - (1 - \delta_{ij}) K_{ij,t}^{\text{agg}} \right]$$

Substituting $K_{ij,t}^{\text{agg}} = Z_t N_t \tilde{k}_{ij,t}$ and $K_{ij,t+1}^{\text{agg}} = Z_t (1+g) N_t (1+g_{n,t+1}) \tilde{k}_{ij,t+1}$ recovers the investment dynamics:

$$I_t^{\text{agg}} = Z_t N_t \sum_{i \in \{1,2\}} \sum_{j \in \{T,I\}} \left[\tilde{k}_{ij,t+1} (1+g)(1+g_{n,t+1}) - (1 - \delta_{ij}) \tilde{k}_{ij,t} \right]$$

1.6 Welfare Decomposition and CEV Calculation

To compute the Consumption Equivalent Variation (CEV) for a cohort, lifetime value function $V_{j,k,t}(a)$ is decomposed into a consumption utility component $V_{j,k,t}^c(a)$ and a labor disutility component $V_{j,k,t}^l(a)$. Additionally, a value-of-life constant u_{bar} is added to prevent negative utility levels from distorting policy evaluations.

1.6.1 Recursive Utility Decomposition

Let the period utility be decomposed as:

$$u^c(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

$$u^l(l) = -\phi \frac{l^{1+1/\nu}}{1+1/\nu}$$

The value function is decomposed as:

$$V_{j,k,t}(a) = V_{j,k,t}^c(a) + V_{j,k,t}^l(a)$$

Working backwards from the terminal period J , the components are defined recursively as:

$$V_{j,k,t}^c(a) = u^c(c_{j,k,t}^*(a)) + \tilde{\beta}\sigma_{j,t}V_{j+1,k,t+1}^c(a'_{j,k,t}(a))$$

$$V_{j,k,t}^l(a) = u^l(l_{j,k,t}^*(a)) + \tilde{\beta}\sigma_{j,t}V_{j+1,k,t+1}^l(a'_{j,k,t}(a))$$

with the terminal conditions $V_{J,k,t+1}^c(a') = 0$ and $V_{J,k,t+1}^l(a') = 0$, where $c_{j,k,t}^*(a)$ and $l_{j,k,t}^*(a)$ are the optimal policy functions.

1.6.2 Incorporating the Value-of-Life Constant (u_{bar})

Under CRRA preferences with $\gamma > 1$, utility is negative everywhere. To avoid welfare measures being distorted by negative numbers, I define a cumulative discounted sum of the value-of-life parameter u_{bar} along the transition path:

$$U_{j,t}^{\text{bar}} = u_{\text{bar}} + \tilde{\beta}\sigma_{j,t}U_{j+1,t+1}^{\text{bar}}$$

with the terminal condition $U_{J+1,t+1}^{\text{bar}} = 0$.

The total welfare of a cohort of age j at time t under a policy transition is:

$$W_{j,k,t}^{\text{trans}}(a) = V_{j,k,t}^{\text{trans}}(a) + U_{j,t}^{\text{bar}}$$

1.6.3 The CEV Formulation

The CEV (Δ) measures the uniform percentage scaling of consumption in the status quo steady state (SS_0) required to yield the same welfare as the transition path. It is defined implicitly by:

$$V_{j,k,t}^{\text{trans}}(a) + U_{j,t}^{\text{bar,trans}} = V_{j,k,t}^{\text{ss0}}(a; \Delta) + U_{j,t}^{\text{bar,ss0}}$$

Since consumption utility is homogeneous of degree $1 - \gamma$:

$$V_{j,k,t}^{\text{ss0}}(a; \Delta) = (1 + \Delta)^{1-\gamma} V_{j,k,t}^{c, \text{ss0}}(a) + V_{j,k,t}^{l, \text{ss0}}(a)$$

Substituting this into the equivalence equation and solving for Δ yields:

$$\Delta = \left(\frac{V_{j,k,t}^{\text{trans}}(a) + U_{j,t}^{\text{bar,trans}} - V_{j,k,t}^{l, \text{ss0}}(a) - U_{j,t}^{\text{bar,ss0}}}{V_{j,k,t}^{c, \text{ss0}}(a)} \right)^{\frac{1}{1-\gamma}} - 1$$

1.6.4 Ex-Ante Aggregate Welfare

To evaluate the aggregate societal impact of the policy reform, I compute the expected welfare gain for a newborn cohort entering the economy in calendar year t_0 . This is evaluated under a “veil of ignorance,” before the newborn draws their permanent productivity type k and starts with zero initial assets ($a_0 = 0$).

The expected utility of a newborn born in year t_0 under the policy transition path is:

$$\mathbb{E} [V_{0,t_0}^{\text{trans}}(0)] = \sum_k \mu_k V_{0,k,t_0}^{\text{trans}}(0)$$

where μ_k represents the exogenous entry probability weight for productivity type k , satisfying $\sum_k \mu_k = 1$ (with $\mu_k = 1/3$ for all k).

Correspondingly, if the newborn were to live in the initial status quo steady state (SS_0) forever, the ex-ante expected utility is:

$$\mathbb{E} [V_0^{\text{ss0}}(0)] = \sum_k \mu_k V_{0,k}^{\text{ss0}}(0)$$

By splitting the initial steady state value function into consumption and labor components, we define the ex-ante baseline aggregates as:

$$\begin{aligned} \mathbb{E} [V_{0,\text{ss0}}^c(0)] &= \sum_k \mu_k V_{0,k,\text{ss0}}^c(0) \\ \mathbb{E} [V_{0,\text{ss0}}^l(0)] &= \sum_k \mu_k V_{0,k,\text{ss0}}^l(0) \end{aligned}$$

The ex-ante Consumption Equivalent Variation, Δ_{ex,t_0} , is defined as the percentage consumption scaling in the status quo steady state required to make the household indifferent ex-ante (before their productivity type is revealed) to the policy transition:

$$\sum_k \mu_k V_{0,k,t_0}^{\text{trans}}(0) + U_{0,t_0}^{\text{bar,trans}} = \sum_k \mu_k V_{0,k}^{\text{ss0}}(0; \Delta_{\text{ex},t_0}) + U_0^{\text{bar,ss0}}$$

$U_0^{\text{bar,ss0}}$ drops the t_0 subscript since survival probabilities are constant in the steady state)

Substituting the homogeneity of the consumption value function, this equivalence is written as:

$$\sum_k \mu_k V_{0,k,t_0}^{\text{trans}}(0) + U_{0,t_0}^{\text{bar,trans}} = (1 + \Delta_{\text{ex},t_0})^{1-\gamma} \mathbb{E} [V_{0,\text{ss0}}^c(0)] + \mathbb{E} [V_{0,\text{ss0}}^l(0)] + U_0^{\text{bar,ss0}}$$

Solving for the ex-ante welfare gain Δ_{ex,t_0} yields the final closed-form equation:

$$\Delta_{\text{ex},t_0} = \left(\frac{\sum_k \mu_k V_{0,k,t_0}^{\text{trans}}(0) + U_{0,t_0}^{\text{bar,trans}} - \mathbb{E} [V_{0,\text{ss0}}^l(0)] - U_0^{\text{bar,ss0}}}{\mathbb{E} [V_{0,\text{ss0}}^c(0)]} \right)^{\frac{1}{1-\gamma}} - 1$$

This ex-ante welfare metric provides a single aggregate measure of generational welfare for each cohort born along the transition path, combining the gains and losses across different productivity types into a unified expectation.

1.7 Gini Coefficient

To evaluate the distributional impacts of population aging and alternative fiscal policies, I compute Gini inequality coefficients for assets, consumption, and disposable income. This section outlines the theoretical definition and the discrete numerical computation algorithm implemented in the model.

1.7.1 Definition

The Gini coefficient is a standard measure of statistical dispersion representing inequality. Geometrically, it is defined via the Lorenz curve, which plots the cumulative share of total resources (y-axis) against the cumulative share of the population (x-axis), sorted in ascending order from poorest to richest.

In a perfectly equal economy, the Lorenz curve corresponds to the 45° diagonal line (the line of perfect equality). Let A denote the area between the line of perfect equality and the Lorenz curve, and let B denote the area under the Lorenz curve. The Gini coefficient G is defined as:

$$G = \frac{A}{A + B} \tag{1.4}$$

Since the total area under the 45° line is $A + B = 0.5$, the formula simplifies to:

$$G = \frac{A}{0.5} = 2A = 1 - 2B \tag{1.5}$$

The Gini coefficient is bounded in the interval $[0, 1]$, where $G = 0$ corresponds to perfect equality (all individuals hold identical resource shares) and $G = 1$ corresponds to perfect inequality (a single individual holds the entirety of the resources).

1.8 Calibration

1.8.1 Households

Households (HHs) enter the economy at age $j = 0$, which corresponds to a real-world age of 20, with zero assets ($a = 0$). They live for a maximum of $J = 75$ periods corresponding to a real-world age of 95 and face lifespan uncertainty, with the probability of survival to the next age dictated by the parameter $\sigma_{j,t}$. The model age span covers more than 99.5% of the population aged 20+ for the period from 2023 to 2100 in the Czech Republic (Czech Statistical Office, 2024a).

Parameter	Value	Description
β	0.985	Subjective discount factor
γ	1.3	Coefficient of relative risk aversion
ϕ	120	Disutility of labor weight
ν	0.4	Frisch elasticity of labor supply

Table 1.1 Household Preference Parameters

The preference parameters are calibrated to match macroeconomic empirical targets and established literature. The subjective discount factor (β) is calibrated internally to target the aggregate capital-to-output ratio ($\tilde{k}/\tilde{y}$) of 3.6 which is 33% higher than the reported 2.7 for the 2024 Czech Republic (European Commission, 2024) this is caused by broader definition of capital, in the original paper McGrattan and Prescott (2017) the US capital to output ratio is 5.8 times GNP compared to BEA reported 2.8 times GNP due to inclusion of consumer durables, inventories, land, and business intangible capital, however in this thesis the increase is not calibrated thus rather conservative. The coefficient of relative risk aversion (γ) is set to 1.3 to match empirical estimates specific to the Czech Republic (Opatrný and Ščasný, 2025). The disutility of labor weight (ϕ) is calibrated such that households optimally choose to work on average 30.5% of their available time, matching the reported 1779 hours worked per year (OECD, 2024a). Finally, the Frisch elasticity of labor supply (ν) is set to 0.4, which is strictly in line with standard macroeconomic estimates Congressional Budget Office (2012).

1.8.2 Production

Level of technology evolves according to:

$$Z_t = Z_{t-1} \cdot (1 + g)$$

with $g = 1.6\%$ based on long term macro prediction from The Czech Fiscal Council (2019). The elasticity parameters are set close to the original baseline model of McGrattan and Prescott (2017). Corporate sector share of output

has been set to 0.53 to match the National Accounts (Czech Statistical Office, 2026a). Depreciation rates have been increased compared to the original to better match capital output ratio.

Parameter	Description	Value
ρ	CES substitution parameter (final good aggregator)	0.5
ω	CES sector weight (corporate sector share)	0.53
θ_{1T}	Tangible capital output elasticity (Corporate)	0.182
θ_{1I}	Intangible capital output elasticity (Corporate)	0.190
θ_{2T}	Tangible capital output elasticity (Non-corporate)	0.502
θ_{2I}	Intangible capital output elasticity (Non-corporate)	0.095
δ_{1T}	Tangible capital depreciation rate (Corporate)	0.06
δ_{1I}	Intangible capital depreciation rate (Corporate)	0.08
δ_{2T}	Tangible capital depreciation rate (Non-corporate)	0.04
δ_{2I}	Intangible capital depreciation rate (Non-corporate)	0.08

Table 1.2 Firm Production and Capital Parameters

1.8.3 The Government

Parameter	Description	Value
j_{ret}	Mandatory retirement age	65
τ_c	Consumption tax rate	0.126
$tr_{pension}$	Pension benefit transfer	0.10
g_{share}	Public consumption share of output	0.19
$\tilde{b}$	Target government debt	0.43
τ_π	Corporate income tax rate	0.23
τ_{d1}	Dividend tax rate (corporate sector)	0.15
τ_{d2}	Distribution tax rate (non-corporate sector)	0.15

Table 1.3 Policy and Fiscal Parameters

Mandatory retirement age is set to 65. Consumption tax rate is set to 0.126, as the effective consumption tax rate based on the data from OECD (2024b) consumption tax trends data. The share of public consumption is set to 0.19 based on data on final consumption of the general government from Eurostat (2024). Output share of government debt is set to 0.43 as to match the data from Czech Statistical Office (2026b). Corporate income tax rate was set based on average effective corporate tax rate from the European Commission (2026). Dividend and Distribution tax rate was calibrated using PwC (2026) tax summary.

1.8.4 Demography

Demography in the model is used to assign weights to households and thus codetermines aggregate outcomes.

Households are heterogeneous with respect to their labor efficiency type k and age j . The distribution of households over type is given by the entry weights of HH types within the new cohort and population growth.

The probability of survival is set so that the overall demographic shape fits the calibration targets, while maintaining consistency with the data, thus keeping the probability of survival close to 1 with a decline beginning near the pension age. Over history, the probability of survival is starting to decline later in life, this trend is expected to continue and is reflected in forecasts both from Czech Statistical Office and EUROSTAT.

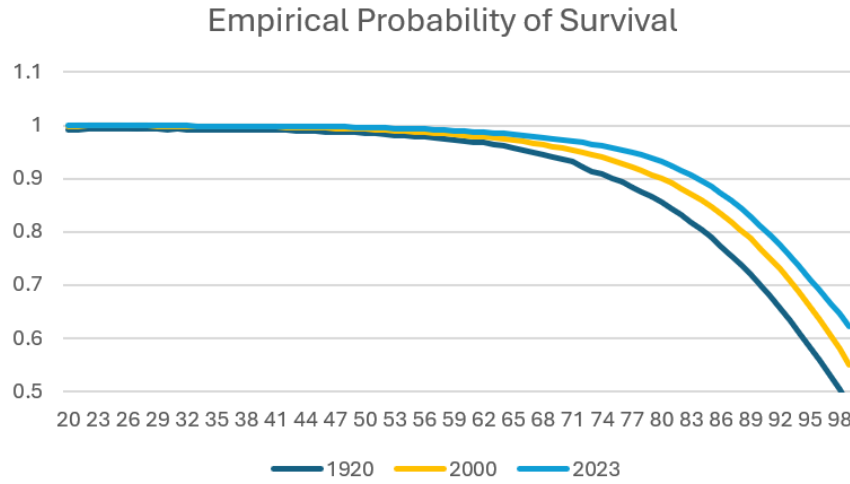


Figure 1.1 Empirical Probability of Survival
Source: Czech Statistical Office, 2025a

Main calibration target for the demography is the old age dependency ratio (OADR) defined as population aged 65+ divided by population aged 20-64. This metric is expected to rise from 36% in 2024 to around 60% in 2060 according to the medium variant demographic projection from Czech Statistical Office (2024a), with the total population roughly stable over the same period, there is only around 3% decline when comparing 2024 population estimate with 2060 forecast, which is less than one tenth of one percent per year.

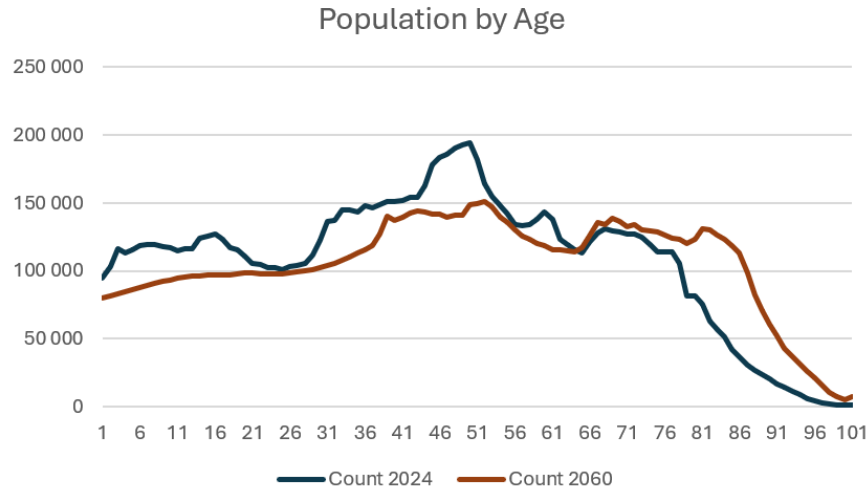


Figure 1.2 Population by Age
Source: Czech Statistical Office, 2025a

The empirical population density by age used for calibration of the model is transformed so that the density of all households with ages below 20 is set to 0, and the sum of all densities of the remaining ages is normalized to 1, as to be consistent with the model.

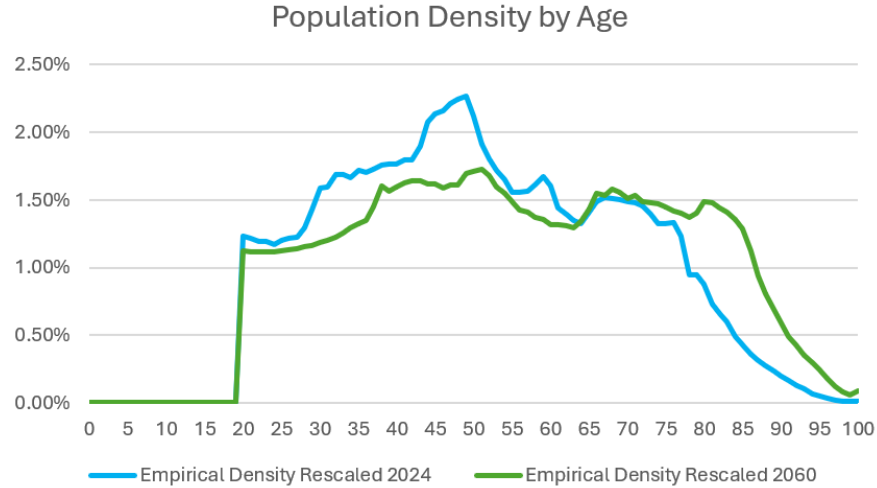


Figure 1.3 Population Density by Age
Source: Czech Statistical Office, 2024a

The model demography used for the initial steady state reflecting 2024 values, is graphed below matching exactly the 2024 OADR of 36%, however with slight differences in shape compared to empirical density as steady state model density does not allow for shock like increase in density with age.

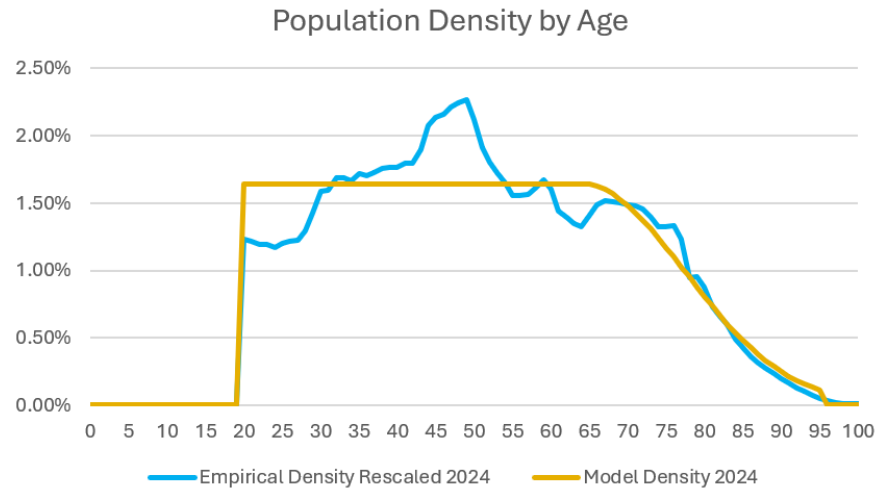


Figure 1.4 Population Density by Age 2024
Source: Czech Statistical Office, 2025a

The model demography used for the final steady state reflecting 2060 values, is graphed below matching exactly the 2060 OADR of 60%, again with slight differences in shape compared to empirical density as steady state model density does not allow for shock like increase in density with respect to age. The change in between these two is the delay of the point from which probability of survival starts declining and thus people live longer and the mass of people aged 65+ is larger.

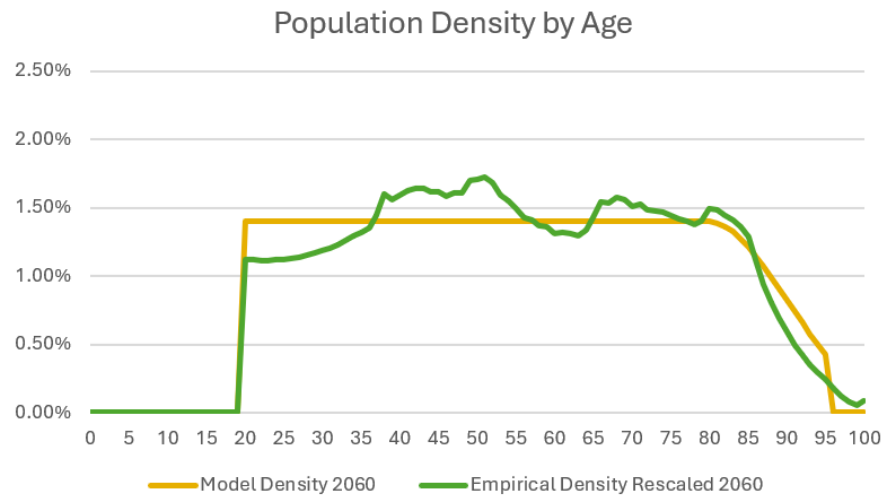


Figure 1.5 Population Density by Age 2060
Source: Czech Statistical Office, 2024a

Labor Income and Productivity Profile

Households differ with respect to their labor productivity. The pre-tax labor income of a household of age j and productivity type k in calendar year t , denoted in normalized terms, is determined by their chosen labor hours $l_{j,k,t}$, their idiosyncratic labor efficiency $\epsilon_{j,k}$, and the aggregate normalized wage rate w_t :

$$y_{j,k,t}^{\text{labor}} = w_t \epsilon_{j,k} l_{j,k,t}$$

The lifecycle efficiency profile $\epsilon_{j,k}$ is deterministically governed by two components: a baseline age-dependent productivity profile that is common across all households, and a fixed level multiplier specific to the household's permanent productivity type k :

$$\epsilon_{j,k} = \bar{\epsilon}_j \cdot \alpha_k$$

where α_k is the productivity multiplier for type k , and $\bar{\epsilon}_j$ is the base efficiency at age j .

The base efficiency evolves over the lifecycle according to a quadratic polynomial in age:

$$\bar{\epsilon}_j = a_0 - a_1(j - j_{\text{peak}})^2 + a_2j$$

where j_{peak} represents the age of peak productivity, and a_0 , a_1 , and a_2 are structural parameters calibrated to match empirical lifecycle earnings profiles.

Parameter	Value
a_0	1.0000
a_1	0.0005
j_{peak}	45.00
a_2	0.0075

Table 1.4 Calibrated Parameters for Lifecycle Efficiency Profiles

Earnings by Age Group

Based on the data from CSO earnings profile by age group is increasing in age with slight decline around 45 years of age and continuation of decline towards retirement. Earnings of workers close to retirement are lower than of workers in peak-earner age group, but still above new entrants. Both facts are consistent for average and median wages, with median wages declining faster and earlier than average (Czech Statistical Office, 2025b). Earnings are normalised based on entrant age group earnings and the shape of earnings is used to calibrate the shape of age dependent efficiency parameter of workers, such that the earning profile fits with the cross-sectional data.

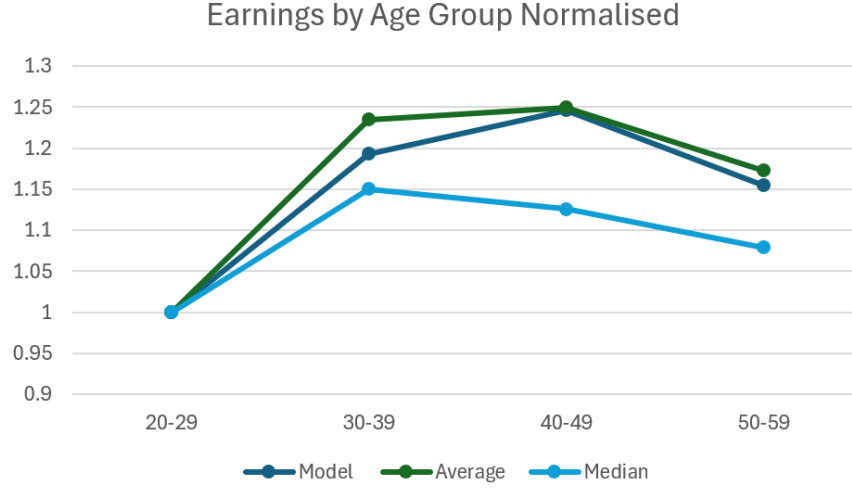


Figure 1.6 Earnings by Age Group
Source: Czech Statistical Office, 2025b

To match the relative level of earnings of household types $k \in \{Low, Medium, High\}$, 2024 data on the distribution of earnings from Czech Statistical Office (2025b) are used. The population is divided into 3 groups each representing 1/3 of the population and for each group the average of earnings is constructed. The average earnings of the highest earner group is constructed such that the average of average income of groups matches the official report for the average wage. The average wage of given group is translated into productivity type multiplier by normalisation by the average wage of the middle group.

Type	Average Wage	α_k
Low	27.7	0.65
Middle	42.6	1.00
High	79.5	1.87

Table 1.5 Average Wage and Type multiplier by Type

1.8.5 Inequality

Table 1.6 presents the Gini coefficients for assets, consumption, and disposable income, comparing the 2024 baseline (SS_{2024}) with the empirical gini.

Variable	Model	Empirical	Diff
Assets Gini ($Gini_A$)	0.42	0.67	-0.25
Consumption Gini ($Gini_C$)	0.34	x	x
Disposable Income Gini ($Gini_Y$)	0.28	0.24	0.04

Table 1.6 Gini Coefficients: Baseline Model (2024, Retire 65) vs. Empirical

1.8.6 Pension Calibration

Parameter	Description	Value
$\bar{tr}_{pension}$	Base Pension Benefit	0.1958
$\phi^{pension}$	Pension Multipliers [<i>Low, Med, High</i>]	[0.441, 0.512, 0.667]

Table 1.7 Pension System and Fiscal Calibration Parameters

The pension system parameters are calibrated to match the gross wage replacement rate in the czech economy based on OECD (2025) Pension at Glance data. The replacement rate(RR) is reported for multiple of mean wage of men. RR for 0.5 of mean is 71%, for mean is 44.2% and for double of mean is 30.6%. To get target RR for types of HH, I use the mean wage of types of households and get the mean wage multiple. Using the multiple with linear interpolation I get the following target replacement rates.

HH type	Multiple	Data RR(%)	Model RR(%)
Low	0.565	67.8	68.3
Med	0.869	51.3	53.84
High	1.624	35.7	36.91

Table 1.8 Gross pension replacement rates

The baseline 2024 model economy has replacement rates of 68.3% for Low, 53.84% for Med and 36.91% for High. The parameter $\bar{tr}_{pension}$ reflect the wage level in the model economy and avg. share of time worked. The parameter $\phi^{pension}$ reflect the target RR and type specific productivity.

2 Results

2.1 Demographic Aging: Macroeconomic Aggregates

The general equilibrium adjustment of the economy to the demographic aging shock under a constant retirement age of 65 is summarized in Table 2.1. The analysis compares the baseline steady state (SS_{2024}), representing the demographic structure of 2024, with the aged steady state (SS_{2060}), representing the demographics of 2060.

Variable	2024	2060	% Change
<i>Demography</i>			
Old Age Dependency Ratio	0.3544	0.5906	+66.65%
<i>1. Prices and Fiscal Policy</i>			
Labor Tax (τ_{ss})	33.28%	43.28%	+10.00 pp
Interest Rate (r)	6.87%	6.33%	-53.08 bp
Wage (w)	0.5077	0.5205	+2.53%
<i>2. Macroeconomic Variables</i>			
Output ($\tilde{y}$)	0.3251	0.2946	-9.39%
Capital ($\tilde{k}$)	1.1710	1.1192	-4.42%
Labor ($\tilde{l}$)	0.3237	0.2856	-11.75%
Avg. Labor hours	30.41%	31.37%	+0.97 pp
<i>3. Key Macro Ratios</i>			
Capital-to-Output ($\tilde{k}/\tilde{y}$)	3.6021	3.7995	+5.48%
Capital-to-Labor ($\tilde{k}/\tilde{l}$)	3.6181	3.9185	+8.30%
<i>4. Government Budget & Fiscal Structure</i>			
Public Goods ($\tilde{g}$)	0.0634	0.0574	-9.39%
Total Transfers ($\tilde{tr}$)	0.0287	0.0403	+40.44%
Labor Tax Revenue	0.0546	0.0643	+17.68%
Capital Tax Revenue	0.0218	0.0191	-12.21%
Cons. Tax Revenue	0.0233	0.0206	-11.77%
Transfers as % of GDP	8.83%	13.68%	+4.85 pp

Table 2.1 Baseline vs. Demographic Aging (Constant Retirement Age 65)

2.1.1 Macroeconomic Aggregates and Price Dynamics

Output falls by 9.39 percent, dropping from 0.3251 to 0.2946. This contraction is primarily driven by a substantial 11.75 percent decline in the aggregate labor supply, which falls from 0.3237 to 0.2856, as the workforce shrinks and the retiree population expands. Aggregate capital also falls, but its contraction is much milder, dropping by only 4.42 percent from 1.1710 to

1.1192. Because the aggregate labor supply contracted much more than the capital stock, the economy undergoes significant capital deepening, which is reflected in an 8.30 percent increase in the capital-to-labor ratio.

This capital deepening alters the relative scarcity of production inputs, triggering adjustments in factor prices. The increased abundance of capital relative to labor drives down the marginal product of capital, causing the real interest rate to fall by 53 basis points, from 6.87 percent in the baseline to 6.33 percent in the counterfactual steady state. Conversely, labor becomes relatively scarce, which drives up its marginal productivity and results in a 2.53 percent increase in the real gross wage, which rises from 0.5077 to 0.5205.

This gross wage increase, however, does not translate into higher purchasing power for households. Because the aging of the population is accompanied by a massive increase in the pension dependency ratio, the pay-as-you-go pension system faces severe deficit pressures. Total pension expenditures increase by 41.88 percent in absolute terms, rising from 0.0277 to 0.0393, which corresponds to an expansion in the pension share of output from 8.52 percent to 13.34 percent. To balance the government budget under a constant retirement age of 65, the statutory payroll tax rate must be raised by 10.00 percentage points, increasing from 33.28 percent to 43.28 percent. This massive tax hike completely absorbs the gross wage gains, leading to a substantial drop in net take-home wages and driving an 11.77 percent contraction in aggregate consumption, which falls from 0.1852 to 0.1634.

2.1.2 Individual Savings and Labor Supply Behavior

At the microeconomic level, individual household decisions over their life-cycles respond directly to the biological and economic changes brought by longevity. Because households face a higher probability of survival into old age, they adjust their saving behavior to prepare for a longer retirement. Households maintain higher saving rates during their active working years to build up retirement portfolios, which provides the microeconomic foundation for the aggregate capital deepening observed.

Due to capital deepening and higher wages individual active households increase their average labor supply hours by 0.97 percentage points of their time endowment, rising from 30.41 percent to 31.37 percent.

While individual active households work longer hours on average, the aggregate labor supply still contracts by 11.75 percent. The structural shift in the population pyramid—specifically, the rising proportion of retired individuals who supply zero labor hours—completely dominates the intensive-margin labor supply increase of the active population.

2.1.3 Pension System Replacement Rates

To evaluate how well the public pension system cushions the income drop at retirement across different productivity types under the demographic transi-

tion, I compute the lifetime working average replacement rate. Table 2.2 presents this measure for the baseline steady state (SS_{2024} , retirement at 65) and the aged steady state ($SS_{2060,CF}$, retirement at 65) under the calibrated pension multipliers.

Productivity Type	2024 (Ret 65)	2060 (Ret 65)	Change
<i>Lifetime Working Average Replacement Rate (%)</i>			
Low Productivity (Type 1)	68.30%	64.06%	−4.24 pp
Med Productivity (Type 2)	53.84%	50.80%	−3.04 pp
High Productivity (Type 3)	36.91%	34.66%	−2.25 pp

Table 2.2 Pension Replacement Rates by Productivity Type: Baseline vs. Demographic Aging

Table 2.2 shows that the lifetime working average replacement rate—which measures the pension benefit relative to the average gross labor income earned over the entire working life (from age 20 to 64)—declines across all types. The lifetime rate falls by 4.24 percentage points for the low-productivity type, by 3.04 percentage points for the medium-productivity type, and by 2.25 percentage points for the high-productivity type.

2.1.4 Inequality

The general equilibrium adjustment of the economy to demographic aging under a constant retirement age has divergent effects on different dimensions of inequality. Table 2.3 presents the Gini coefficients for aggregate assets, consumption, and disposable income, comparing the 2024 baseline (SS_{2024}) with the 2060 aged counterfactual ($SS_{2060,CF}$).

Variable	2024	2060	Change
Assets Gini ($Gini_A$)	0.4155	0.4636	+4.81 pp
Consumption Gini ($Gini_C$)	0.3341	0.3252	-0.89 pp
Disposable Income Gini ($Gini_Y$)	0.2815	0.2713	-1.02 pp

Table 2.3 Inequality Gini Coefficients: Baseline vs. Demographic Aging (Retire 65)

The demographic transition induces a significant divergence between wealth inequality and consumption or income inequality. Wealth (asset) inequality, as measured by the assets Gini coefficient, increases by 4.81 percentage points, rising from 0.4155 in 2024 to 0.4636 in 2060. High productivity households, who enjoy higher wages and larger pension benefits, accumulate significantly larger lifecycle savings to fund their extended retirement years. Low productivity households, by contrast, face low lifetime earnings, which prevents them from accumulating substantial assets during their active lives. When longevity increases, the gap in asset accumulation between the wealth

holding types and the hand-to-mouth types widens, leading to the observed concentration of wealth.

Conversely, disposable income inequality declines, with the income Gini coefficient falling by 1.02 percentage points, from 0.2815 to 0.2713. This compression of the disposable income distribution is driven by two main factors. First, the real interest rate falls due to capital deepening. Because high-productivity households hold the vast majority of capital, this decline in the return to capital disproportionately reduces the asset income of wealthy households, compressing the top of the income distribution. Second, the large labor tax hike required to balance the pension budget reduces net labor income for all active workers. Because retirees make up a much larger share of the population in 2060, and because their primary source of income, public pensions, is less unequal than the wage income of active workers, the expansion of the retiree population compresses the overall disposable income distribution.

In line with the compression of disposable income, consumption inequality also declines, with the consumption Gini coefficient falling by 0.89 percentage points, from 0.3341 in the baseline to 0.3252 in the 2060 steady state. Because consumption decisions are based on lifetime wealth and disposable income profiles, the general equilibrium changes that compress the disposable income distribution, the lower return on capital and the higher labor tax burden, force a corresponding convergence in household consumption levels across types.

2.1.5 Welfare Compensating Variation (CEV) Analysis

To quantify the aggregate and distributional welfare consequences of demographic aging, I compute the Consumption Equivalent Variation (CEV) across cohorts and productivity types. Table 2.4 summarizes the CEV gains and losses comparing the 2024 baseline (SS_{2024}) to the 2060 aged counterfactual ($SS_{2060,CF}$) under a constant retirement age of 65.

Cohort Age	Aggregate	Low	Med	High
Age 20	-10.54%	-13.56%	-11.34%	-6.72%
Age 30	-7.88%	-13.25%	-8.49%	-1.91%
Age 40	-1.66%	-8.57%	-3.65%	+7.24%
Age 50	+8.91%	-1.86%	+4.40%	+24.19%
Age 65	+61.41%	+12.83%	+40.02%	+131.37%
Age 80	+51.94%	+14.58%	+37.48%	+103.75%
Expected Newborn CEV	-10.94%			

Table 2.4 Welfare CEV (%) by Cohort and Productivity Type ($u_{\text{bar}} = 7.0$)

Intergenerational Welfare

The comparison of the 2024 baseline to the 2060 aged steady state reveals a large intergenerational difference. Younger cohorts, particularly newborns (Age 20) and young active workers (Age 30), experience substantial lifetime welfare losses. A newborn faces an aggregate CEV loss of 10.54 percent. This occurs because younger cohorts must pay the higher payroll tax rate ($\tau_{ss} = 43.28\%$ compared to 33.28%) over their entire working lives to fund the expanded retiree cohorts. Because retirement is far in the future, the discounted utility gain from living longer is heavily discounted, leaving them with a net welfare loss due to the immediate and long-lasting tax squeeze.

Conversely, older cohorts and retirees experience massive welfare gains. A retiree at age 65 enjoys an aggregate CEV gain of 61.41 percent. These older cohorts are already retired and pay zero payroll taxes. Instead, they benefit directly from living longer in the 2060 demographics. Because the utility flow of being alive is positive, surviving additional years adds substantial positive utility to their remaining years. The different demographic thus changes the welfare decreasing it for the young tax-paying workforce increasing it to the elderly retirees.

Welfare Distribution

Within any given cohort, the welfare gains and losses are highly differentiated. High-productivity households gain significantly more (or lose less) than low-productivity households. For newborns (Age 20), low-productivity types face a welfare loss of 13.56 percent, while high-productivity types lose only 6.72 percent. This distributional gap is even more pronounced at retirement: at age 65, high-productivity retirees enjoy an extraordinary CEV gain of 131.37 percent, while low-productivity retirees gain only 12.83 percent.

This regressivity is driven by being able to accumulate more assets and higher pensions. High-productivity households earn higher lifetime wages and receive larger pension transfers ($0.1958 \times 0.667 = 0.1306$ compared to $0.1958 \times 0.441 = 0.0863$). This high consumption level in retirement means that their quality of life is higher. Under the Value-of-Life framework, surviving additional years is exceptionally valuable to them because their period utility is highly positive. Low-productivity households, by contrast, have consumption levels closer to the borrowing constraints. The payroll tax hike hurts them severely during their working years, and their longevity gain is small because their retirement consumption is low.

Finally, this regressivity shifts the age at which a household becomes a net beneficiary of demographic aging:

- High-productivity types cross over to a net welfare gain by Age 40 (+7.24%).
- Medium-productivity types cross over by Age 50 (+4.40%).
- Low-productivity types only cross over after retirement at Age 65 (+12.83%).

Ex-Ante Welfare Gains and the Veil of Ignorance

To evaluate the aggregate societal impact of the demographic aging transition on future generations, I compute the ex-ante expected welfare gain for a newborn cohort entering the economy. This evaluation is conducted under a “veil of ignorance,” before the newborn draws their permanent productivity type and starts with zero initial assets ($a_0 = 0$). Table 2.5 presents the ex-ante CEV for each productivity type and the expected ex-ante welfare gain.

Demographic Group	CEV Gain (%)
Expected Gain (Newborn)	-10.94%
<i>Ex-Ante Gain by Productivity Type</i>	
Type 1 (Low Efficiency)	-13.56%
Type 2 (Medium Efficiency)	-11.34%
Type 3 (High Efficiency)	-6.72%

Table 2.5 Ex-Ante Welfare Gains for Newborns ($a_0 = 0$)

The ex-ante welfare analysis confirms that demographic aging is a net welfare-reducing event for future generations, yielding an expected ex-ante welfare loss of 10.94 percent in consumption-equivalent terms. This aggregate measure combines the welfare losses across all three productivity types before their specific earnings status is revealed.

The ex-ante losses by type highlight that even without initial wealth cushion, low productivity newborns face the most severe welfare reduction, losing 13.56 percent of lifetime consumption, while high-productivity newborns lose only 6.72 percent. The regressivity of the transition is thus preserved at the moment of birth: the prospect of facing a lifetime labor tax rate of 43.28 percent under a constant retirement age is much more damaging to lower productivity types, whose expected lifetime earnings and subsequent pension replacements are lower.

2.1.6 Welfare and Lifecycle Behavior by Productivity Type

To understand the microeconomic foundations of the transition to an aged demographic structure under a constant retirement age of 65, I examine the lifecycle profiles of consumption, labor supply, and asset accumulation. Figures 2.1 and 2.2 illustrate these profiles and the corresponding consumption equivalent variation (CEV) by productivity type, comparing the 2024 baseline (SS_{2024} , dashed lines) and the 2060 aged counterfactual ($SS_{2060,CF}$, solid lines) across the full life cycle from age 20 to 95. The shape of consumption profiles is in line with finding of Čahlíková (2010) for the Czech republic.

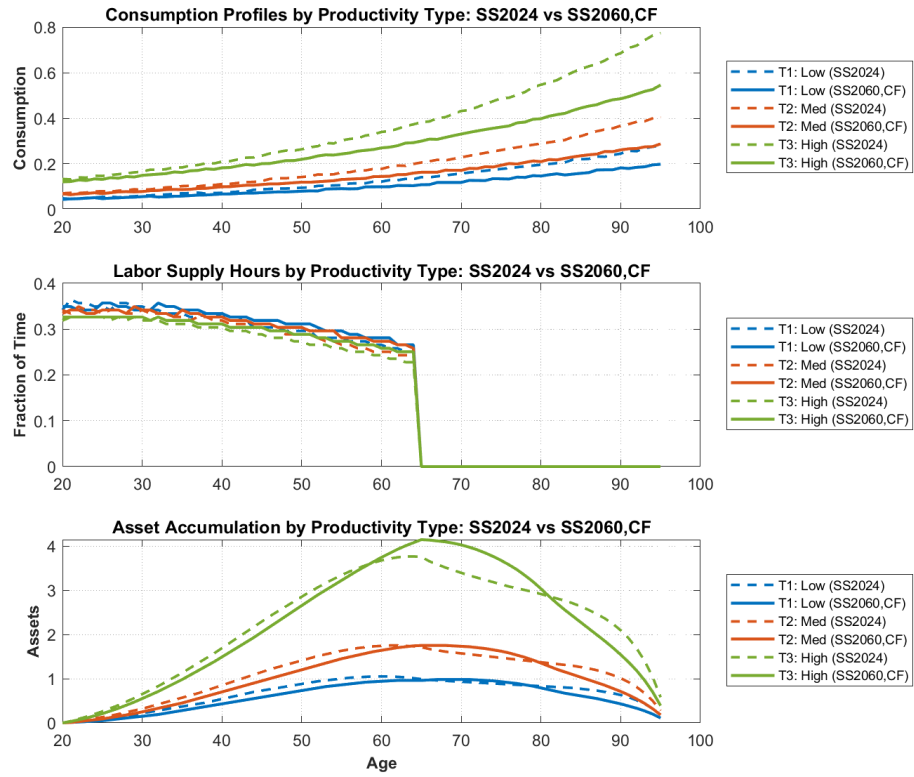


Figure 2.1 Lifecycle Profiles by Productivity Type: 2024(Ret. 65) vs. 2060(Ret. 65)

Welfare Distribution by Type

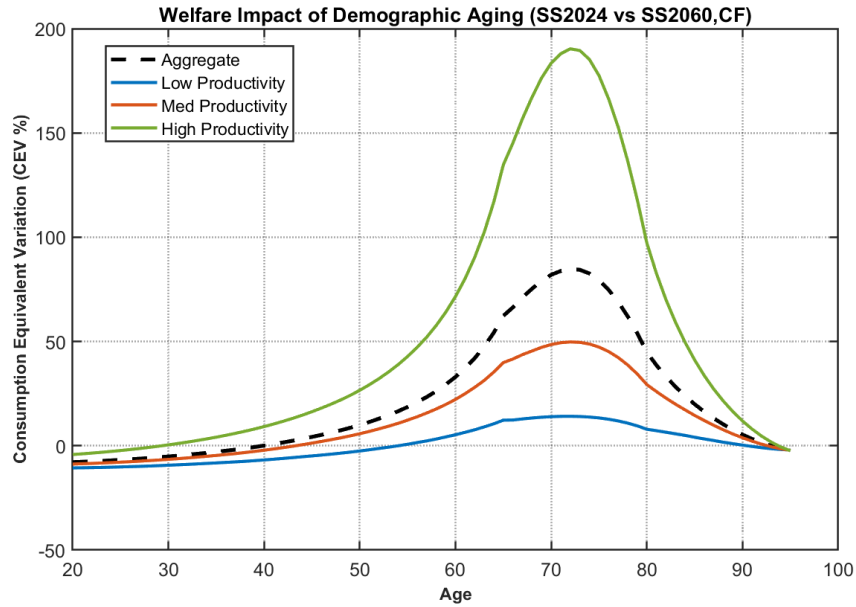


Figure 2.2 Welfare Impact (CEV %) of Demographic Aging by Type (Ages 20-95)

Figure 2.2 illustrates the welfare impact (CEV) across cohorts and productivity types. The welfare changes reveal a distributional divide. Newborns (Age 20) and young active workers face severe welfare losses (varying from -11.58 percent CEV for low-productivity to -4.71 percent for high-productivity newborns) because they must pay the higher labor tax over their entire careers.

Older cohorts close to or in retirement experience large welfare gains (with high-productivity retirees gaining +115.66 percent CEV, while low-productivity retirees gain +10.31 percent). These cohorts are already retired and pay zero payroll taxes, allowing them to collect the longevity dividend of living longer in the aged economy. The regressivity of these gains is clear: high-productivity households enjoy a high standard of living in retirement, making surviving additional years exceptionally valuable under the Value-of-Life framework, whereas low-productivity households have consumption levels close to borrowing constraints, resulting in much smaller welfare gains.

2.2 Policy Reform: Postponing Retirement to Age 70

This section evaluates the general equilibrium, distributional, and welfare effects of the primary policy reform: raising the statutory retirement age from 65 to 70 under the aged demographic structure of 2060. The analysis compares the 2060 counterfactual steady state with a retirement age of 65 ($SS_{2060,65}$), which serves as the status quo, with the reform steady state ($SS_{2060,70}$), where retirement is postponed to age 70. Table 2.6 summarizes the macroeconomic and fiscal adjustments.

Variable	2060 (65)	2060 (70)	Change
<i>Demography</i>			
Old Age Dependency Ratio	0.5906	0.5906	
Pensioners to Working Ratio	0.5906	0.4316	-26.93%
<i>1. Prices and Fiscal Policy</i>			
Labor Tax (τ_{ss})	43.28%	36.31%	-6.97 pp
Interest Rate (r)	6.33%	6.29%	-4.79 bp
Aggregate Wage (w)	0.5205	0.5217	+0.23%
<i>2. Macroeconomic Variables</i>			
Output ($\tilde{y}$)	0.2946	0.3221	+9.34%
Capital ($\tilde{k}$)	1.1192	1.2298	+9.88%
Labor ($\tilde{l}$)	0.2856	0.3115	+9.07%
Avg. Labor hours	31.37%	30.62%	-0.75 pp
<i>3. Key Macro Ratios</i>			
Capital-to-Output ($\tilde{k}/\tilde{y}$)	3.7995	3.8184	+0.50%
Capital-to-Labor ($\tilde{k}/\tilde{l}$)	3.9185	3.9476	+0.74%
<i>4. Government Budget & Fiscal Structure</i>			
Public Goods ($\tilde{g}$)	0.0574	0.0628	+9.34%
Total Transfers ($\tilde{tr}$)	0.0403	0.0329	-18.35%
Labor Tax Revenue	0.0643	0.0590	-8.28%
Capital Tax Revenue	0.0191	0.0209	+9.00%
Cons. Tax Revenue	0.0206	0.0225	+9.22%
Transfers as % of GDP	13.68%	10.22%	-3.46 pp

Table 2.6 Pension Reform Comparative Statics: Retire 65 vs. Retire 70 (2060)

2.2.1 Macroeconomic and Fiscal Effects of the Reform

Postponing the statutory retirement age from 65 to 70 acts as a structural counterweight to demographic aging. By keeping workers aged 65 to 69 in the labor force, the reform directly alters the population pyramid's dependency ratios. The Pensioner to Worker ratio falls by 26.93 percent, dropping from 0.5906 in the status quo to 0.4316 in the reform steady state.

This contraction in the Pensioner to Worker ratio triggers a massive fiscal dividend: total pension transfers as a share of output drop from 13.68 percent to 10.22 percent (a reduction of 3.46 percentage points). To balance the pay-as-you-go government budget, the statutory payroll tax rate (τ_{ss}) falls by 6.97 percentage points, dropping from 43.28 percent to 36.31 percent.

This payroll tax relief stimulates aggregate supply. Aggregate labor supply ($\tilde{l}$) expands by 9.07 percent (from 0.2856 to 0.3115), while the aggregate capital stock ($\tilde{k}$) rises by 9.88 percent (from 1.1192 to 1.2298) as households continue saving for a slightly shorter but still extended retirement. The combination of these factors drives a 9.34 percent expansion in output ($\tilde{y}$), which rises from 0.2946 to 0.3221. In general equilibrium, because capital and labor expand at similar rates, the capital-to-labor ratio rises by only 0.74 percent, leaving factor prices largely unchanged: the real interest rate falls by a minor 4.79 basis points (to 6.29 percent), and the gross real wage rises by 0.23 percent (to 0.5217). Thus, the economic gains from the reform are primarily transmitted through tax relief and labor force expansion rather than factor price adjustments.

At the microeconomic level, the average active labor share drops by 0.75 percentage points, from 31.37 percent to 30.62 percent. This intensive-margin contraction represents a behavioral response to the tax cut: because net take-home wages are higher under the lower payroll tax, households experience a positive wealth effect that allows them to reduce their working hours slightly during their prime working years while still achieving higher consumption and savings.

2.2.2 Inequality Responses to Retirement Postponement

Unlike demographic aging, which increases wealth inequality, the policy reform compresses inequality across all three major dimensions. Table 2.7 presents the Gini coefficients for assets, consumption, and disposable income under the reform.

Variable	2060 (65)	2060 (70)	Change
Assets Gini (Gini_A)	0.4636	0.4400	-2.36 pp
Consumption Gini (Gini_C)	0.3252	0.3161	-0.91 pp
Disposable Income Gini (Gini_Y)	0.2713	0.2627	-0.86 pp

Table 2.7 Inequality Gini Coefficients: Retire 65 vs. Retire 70 (2060)

Wealth (asset) inequality declines by 2.36 percentage points, with the assets Gini dropping from 0.4636 to 0.4400. Consumption and disposable income inequality also decline, dropping by 0.91 percentage points and 0.86 percentage points, respectively.

2.2.3 Pension Replacement Rates under the Reform

Table 2.8 presents the lifetime working average replacement rate comparing the status quo to the reform.

Productivity Type	2060 (65)	2060 (70)	Change
Low Productivity (Type 1)	64.06%	65.07%	+1.01 pp
Med Productivity (Type 2)	50.80%	51.50%	+0.70 pp
High Productivity (Type 3)	34.66%	35.28%	+0.62 pp

Table 2.8 Pension Replacement Rates by Productivity Type: SS 2060 (Retire 65) vs. SS 2060 (Retire 70)

2.2.4 Welfare Compensating Variation (CEV) Analysis

To quantify the welfare impact of raising the retirement age, I compute the cohort-specific and ex-ante CEV. Because the survival probabilities are identical in both steady states (both under 2060 demographics), the expected lifetime length and the Value-of-Life constant cancel out mathematically from the welfare equations. Table 2.9 summarizes the CEV gains and losses.

Cohort Age	Aggregate	Low	Med	High
Age 20	+10.92%	+10.38%	+10.75%	+11.62%
Age 30	+9.54%	+9.71%	+8.91%	+9.98%
Age 40	+6.37%	+5.75%	+5.86%	+7.49%
Age 50	+2.91%	+0.66%	+2.87%	+5.21%
Age 65	-2.76%	-6.48%	-3.06%	+1.26%
Age 80	-0.19%	-0.15%	-0.19%	-0.22%
Expected Newborn CEV	+10.86%			

Table 2.9 Welfare CEV (%) of Raising the Retirement Age from 65 to 70

The welfare results reveal a strong intergenerational trade-off. Newborns (Age 20) and young active workers (Age 30, 40) experience massive welfare gains, with newborns gaining 10.92 percent in aggregate CEV terms. This indicates that the 6.97 percentage point payroll tax cut they enjoy over their entire working careers easily compensates them for having to work five additional years. Under the veil of ignorance, a newborn cohort achieves an expected ex-ante welfare gain of 10.86 percent (Table 2.9).

Conversely, older cohorts close to or at retirement experience welfare losses. New retirees at age 65 face an aggregate welfare loss of 2.76 percent. This loss is concentrated among low-productivity workers, who lose 6.48 percent. However, high-productivity retirees at age 65 experience a net welfare gain of 1.26 percent, as they benefit from the tax relief on their labor earnings.

2.2.5 Welfare and Lifecycle Behavior by Productivity Type

To understand the microeconomic foundations of the policy reform's welfare effects, I examine the lifecycle profiles of consumption, labor supply, and asset accumulation. Figures 2.3 and 2.4 illustrate these profiles and the corresponding consumption equivalent variation (CEV) by productivity type, comparing the Retire 65 status quo ($SS_{2060,65}$, dashed lines) and the Retire 70 reform ($SS_{2060,70}$, solid lines) across the full life cycle from age 20 to 95.

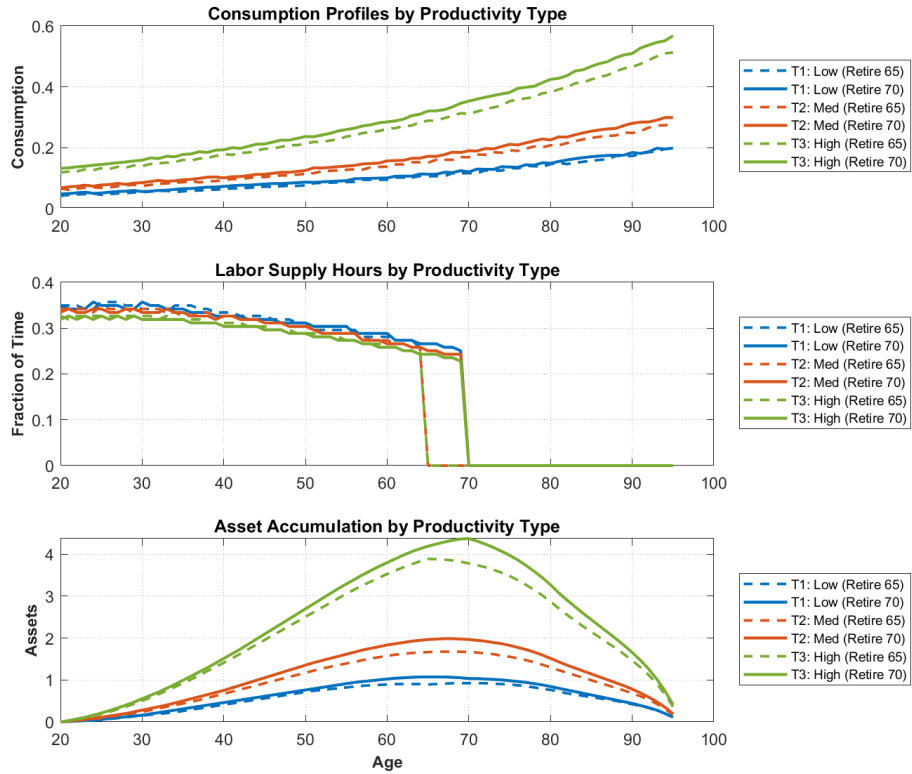


Figure 2.3 Lifecycle Profiles by Productivity Type: Retire 65 vs. Retire 70

Consumption and Savings Channels

The lower panel of Figure 2.3 shows that asset accumulation peaks significantly later and at a higher level under the Retire 70 reform. Under the Retire 65 status quo, assets peak around age 65 as households prepare to enter retirement. When retirement is postponed to age 70, the asset accumulation phase is extended by five years, peaking at age 70. Because households work longer and benefit from a lower payroll tax rate, their lifetime net earnings expand. This expansion enables households across all three productivity types to achieve higher consumption levels throughout their entire lifecycles, as shown in the top panel of Figure 2.3.

Labor Supply Hours response

The middle panel of Figure 2.3 highlights the behavioral adjustments in labor supply. Between ages 65 and 70, households under the status quo are retired and supply zero labor hours, while households under the reform continue to supply labor. However, during their prime working years (ages 20 to 64), workers of all types supply slightly fewer hours under the Retire 70 reform compared to the status quo.

Welfare Distribution by Type

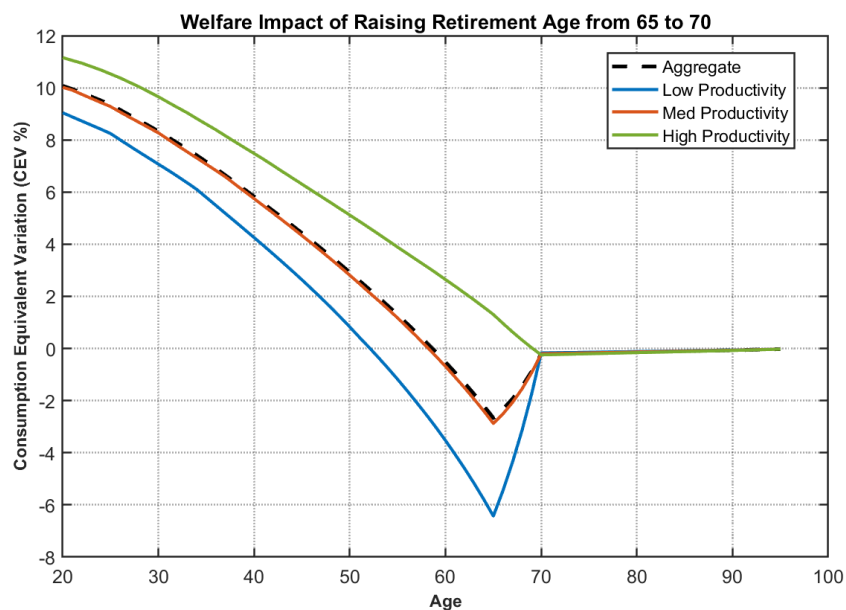


Figure 2.4 Welfare Impact (CEV %) of Raising the Retirement Age by Type

Figure 2.4 illustrates the welfare impact (CEV) across cohorts and productivity types. The welfare gains of the reform are heavily concentrated among younger cohorts, who benefit from the 6.73 percentage point payroll tax cut over their entire active careers. At age 20, high-productivity newborns achieve the largest welfare gain (+11.17 percent CEV), followed by medium-productivity (+10.29 percent) and low-productivity (+9.93 percent) newborns.

2.3 Combined Impact: Demographic Aging and Pension Reform

To evaluate the long-term net welfare impact of demographic aging combined with pension reform, Table 2.10 compares the initial baseline economy (SS_{2024} , retirement at 65) directly with the reformed aged economy (SS_{2060} , retirement at 70).

Cohort Age	Aggregate	Low	Med	High
Age 20	+0.19%	-3.85%	-0.93%	+5.34%
Age 30	+1.53%	-4.60%	+0.01%	+9.18%
Age 40	+5.14%	-4.03%	+2.58%	+16.86%
Age 50	+12.93%	-1.96%	+7.89%	+32.85%
Age 65	+57.66%	+0.67%	+32.33%	+139.99%
Age 80	+51.39%	+14.24%	+37.03%	+102.89%
Expected Newborn CEV	-0.37%			

Table 2.10 Welfare CEV (%) of Overall Transition: SS 2024 (Retire 65) vs. SS 2060 (Retire 70)

The combined results show that postponing the retirement age to 70 almost completely neutralizes the severe welfare losses induced by demographic aging for younger generations. Aggregate newborn welfare moves from -10.54% under status quo aging to $+0.19\%$ under the reform, while expected newborn welfare under the veil of ignorance rises from -10.94% to a nearly neutral -0.37% . However, the distributional differences remains evident: high-productivity newborns achieve a net welfare gain of $+5.34\%$ due to the substantial payroll tax relief (36.31% vs. 43.28%), whereas low-productivity newborns still experience a net loss of -3.85% . Older cohorts experience large net welfare gains ($+57.66\%$ aggregate CEV at age 65) as they have large benefit from longevity even without having to pay much higher payroll taxes during their working lives and thus have more assets.

2.4 Transition path

Welfare Comparison: Transitional Policy Reform vs. Baseline Transition

To isolate the transitional welfare impacts of the pension reform, I calculate the Consumption Equivalent Variation (CEV) of a policy transition path where the statutory retirement age is fixed at 70 from the outset relative to a baseline transition path where the statutory retirement age linearly increases from 65 to 70 over 36 years.

Because the exogenous demographic transition is identical in both scenarios, the cumulative value-of-life constant (U_{bar}) cancels out, yielding a clean measure of the economic and fiscal trade-offs of immediate policy implementation. Table 2.11 reports the CEV values by productivity type and ex-ante average for newborn cohorts born at different years of the transition, as well as select existing cohorts alive in year 1.

Cohort	Low	Medium	High	Average
Newborn Cohorts (t_0)				
Year 1 ($t_0 = 1$)	+3.00%	+3.17%	+3.38%	+3.17%
Year 10 ($t_0 = 10$)	+2.22%	+2.32%	+2.45%	+2.32%
Year 20 ($t_0 = 20$)	+1.43%	+1.46%	+1.54%	+1.47%
Year 35 ($t_0 = 35$)	+0.55%	+0.58%	+0.60%	+0.57%
Year 150 ($t_0 = 150$)	-0.00%	-0.00%	-0.00%	-0.00%
Existing Cohorts (at $t = 1$)				
Age 20	+3.00%	+3.17%	+3.38%	+3.17%
Age 30	+2.73%	+2.95%	+3.17%	+2.95%
Age 40	+1.72%	+2.29%	+3.01%	+2.34%
Age 50	-0.92%	+1.00%	+3.16%	+1.08%
Age 65	-9.67%	-3.63%	+3.61%	-3.23%
Age 80	-0.15%	-0.19%	-0.24%	-0.19%

Table 2.11 Welfare Gains/Losses of Cohorts (CEV % relative to Baseline Transition)

2.5 Summary

Isolated demographic aging under a constant statutory retirement age of 65 generates a severe macroeconomic contraction and fiscal pressure. As the old-age dependency ratio expands by 66.65 percent (rising from 0.3544 to 0.5906), the structural shrinkage of the working-age population contracts aggregate labor supply by 11.75 percent and aggregate output by 9.39 percent. Because capital contracts much less (-4.42 percent), the economy undergoes significant capital deepening, raising the capital-to-labor ratio by 8.30 percent. This capital deepening depresses the real interest rate by 53 basis points (to 6.33 percent) and elevates the gross real wage by 2.53 percent. However, the surge in public pension expenditure, which expands from 8.52 percent to 13.34 percent of output, forces a 10.00 percentage point increase in the labor tax rate (from 33.28 percent to 43.28 percent) to balance the fiscal budget. This tax hike completely neutralizes the gross wage gains, reducing net take-home pay and driving an 11.77 percent contraction in aggregate private consumption. Demographic aging generates a marked divergence in economic inequality. Wealth (asset) inequality increases by 4.81 percentage points (the assets Gini rises to 0.4636), driven by the longevity savings channel: high-productivity households accumulate substantial lifecycle assets to fund their extended retirement, whereas low-productivity households remain constrained by borrowing limits and low lifetime earnings. Conversely, disposable income and consumption inequality decline by 1.02 and 0.89 percentage points, respectively. This compression is driven by the falling return on capital—which disproportionately reduces the asset income of wealthy households—and the progressive compression resulting from high payroll tax rates and redistributive public transfers. Postponing the statutory retirement age to 70 provides a structural countermeasure that reverses much of the demographic macroeconomic contraction. By extending working lives by five years, the Pensioner-to-Worker ratio drops by 26.93 percent (from 0.5906 to 0.4316), reducing pension expenditure as a share of output by 3.46 percentage points. The resulting fiscal space allows the payroll tax rate to be cut by 6.97 percentage points (from 43.28 percent to 36.31 percent). This tax relief stimulates aggregate supply, expanding aggregate labor supply by 9.07 percent, aggregate capital by 9.88 percent, and output by 9.34 percent. Furthermore, the reform compresses inequality across all dimensions, reducing the assets Gini by 2.36 percentage points, consumption Gini by 0.91 percentage points, and disposable income Gini by 0.86 percentage points.

The welfare analysis, conducted using the Consumption Equivalent Variation (CEV) metric calibrated with a Value-of-Life constant ($u_{\text{bar}} = 7.0$), reveals significant intergenerational and intragenerational trade-offs across both policy scenarios. Under the demographic aging transition (holding the retirement age at 65), future generations face substantial lifetime welfare losses. Evaluated under the Rawlsian "veil of ignorance" before productivity status is revealed, a newborn entering the economy in 2060 faces an expected ex-ante welfare loss of 10.94 percent CEV. This loss is highly regressive: low-productivity newborns suffer a 13.56 percent welfare loss compared to a 6.72 percent loss for high-productivity newborns, reflecting the vulnerability

of low-wage earners to sustained labor tax hikes. In contrast, existing retirees at age 65 experience an aggregate welfare gain of 61.41 percent CEV because they collect the longevity dividend of living longer in the aged economy without facing the payroll tax burden. Raising the retirement age to 70 creates a sharp intergenerational trade-off. Future generations benefit substantially from the reform, with newborns achieving an expected ex-ante CEV gain of 10.86 percent. The 6.97 percentage point reduction in the labor tax rate easily compensates younger workers for five additional years of labor supply. Conversely, older cohorts near retirement bear the costs of the policy shift: new retirees at age 65 suffer an aggregate CEV loss of 2.76 percent. This loss is concentrated among low-productivity retirees (-6.48 percent CEV), who rely heavily on pension income and lose five years of leisure. High-productivity retirees at age 65 experience a minor net gain (+1.26 percent CEV) due to the tax relief on their remaining labor earnings. Finally, the transitional path analysis demonstrates that immediate policy implementation generates immediate fiscal relief for younger workers, but imposes transitional costs on middle-aged and older active cohorts who must delay retirement without having enjoyed the cumulative benefits of lower taxes over their earlier active careers.

Conclusion

This thesis provided general equilibrium quantitative evaluation of demographic aging in the Czech Republic and assessed postponing the statutory retirement age by 5 years as a structural policy counterweight.

The quantitative findings showed that unmitigated demographic aging between 2024 and 2060 imposes severe macroeconomic contraction and fiscal stress. As the old-age dependency ratio expands by 66.65 percent, aggregate labor supply contracts by 11.75 percent and output falls by 9.39 percent. Capital deepening (+8.30 percent k/l ratio) depresses interest rate by 53 basis points and pushes real gross wages up by 2.53 percent. However, to fund the expanding public pension expenditure (rising from 8.52 percent to 13.34 percent of output), the statutory payroll tax rate must be raised by 10.00 percentage points (from 33.28 percent to 43.28 percent). This tax spike completely neutralizes gross wage gains, causing aggregate consumption to fall by 11.77 percent and generating an ex-ante expected newborn welfare loss of 10.94 percent CEV. Postponing the statutory retirement age by 5 years acts as a powerful structural stabilizer. By reducing the Pensioner-to-Worker ratio by 26.93 percent, pension expenditures drop by 3.46 percentage points of GDP, allowing the payroll tax rate to fall by 6.97 percentage points (to 36.31 percent). This tax relief stimulates aggregate supply, expanding output by 9.34 percent and labor supply by 9.07 percent, while compressing inequality across asset (-2.36 pp Gini), consumption (-0.91 pp Gini), and income (-0.86 pp Gini) dimensions. The reform on average neutralizes the demographic welfare drag for future generations, lifting expected ex-ante newborn welfare from -10.94 percent to a nearly neutral -0.37 percent CEV. However with diverse effect across age and income distribution. In conclusion, while postponing the retirement age by 5 years restores long-term fiscal sustainability and aggregate growth, it generates a clear trade-off by imposing transitional welfare losses on low productivity cohorts nearing retirement. Policymakers may therefore pair retirement age extensions with targeted safety nets or flexible retirement windows to protect vulnerable households.

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A Attachments

A.1 Gini Numerical Computation

In the model, individuals are characterized by their age j , productivity type k , and asset holdings a . Each discrete state is associated with a resource level $x_{j,k}(a)$ (representing assets, consumption, or disposable income) and a probability mass (weight) $\lambda_{j,k}(a)$ from the cross-sectional distribution.

To compute the weighted Gini coefficient numerically, the discrete states are vectorized into an array of resource values $X = \{x_1, x_2, \dots, x_N\}$ and corresponding population weights $W = \{w_1, w_2, \dots, w_N\}$. The computation proceeds as follows:

- (a) **Non-Negativity Correction:** To prevent Gini values from exceeding unity due to household debt, resource values are truncated at zero:

$$x_i = \max(x_i, 0) \quad \forall i \in \{1, \dots, N\} \quad (\text{A.1})$$

- (b) **Sorting:** The observations are sorted in ascending order such that:

$$x_1 \leq x_2 \leq \dots \leq x_N \quad (\text{A.2})$$

The weight vector W is reordered to maintain alignment with the sorted resource values.

- (c) **Weight Normalization:** The population weights are normalized to sum to one:

$$\bar{w}_i = \frac{w_i}{\sum_{j=1}^N w_j} \quad (\text{A.3})$$

- (d) **Cumulative Resource Shares:** The cumulative share of total resources S_i held by the poorest i fraction of the population is calculated recursively:

$$S_i = \frac{\sum_{j=1}^i \bar{w}_j x_j}{\sum_{j=1}^N \bar{w}_j x_j}, \quad \text{with } S_0 = 0 \quad (\text{A.4})$$

- (e) **Trapezoidal Integration:** The area under the Lorenz curve B is approximated using the trapezoidal rule. For each interval i , the area of the trapezoid is given by the normalized width $\bar{w}_i$ multiplied by the average height of the cumulative resource share at the endpoints:

$$B = \sum_{i=1}^N \frac{1}{2} \bar{w}_i (S_i + S_{i-1}) \quad (\text{A.5})$$

- (f) **Gini Calculation:** Substituting B back into the Gini formula yields the final discrete weighted Gini coefficient:

$$G = 1 - \sum_{i=1}^N \bar{w}_i (S_i + S_{i-1}) \quad (\text{A.6})$$

A.2 Transition Path Calibration

Due to time constraint transition path experiment was run under slightly different calibration.

Parameter	Value	Description
β	1.0	Subjective discount factor
γ	1.3	Coefficient of relative risk aversion
ϕ	140	Disutility of labor weight
ν	0.4	Frisch elasticity of labor supply

Table A.1 Household Preference Parameters

Parameter	Description	Value
ρ	CES substitution parameter (final good aggregator)	0.5
ω	CES sector weight (corporate sector share)	0.6
θ_{1T}	Tangible capital output elasticity (Corporate)	0.182
θ_{1I}	Intangible capital output elasticity (Corporate)	0.190
θ_{2T}	Tangible capital output elasticity (Non-corporate)	0.502
θ_{2I}	Intangible capital output elasticity (Non-corporate)	0.095
δ_{1T}	Tangible capital depreciation rate (Corporate)	0.06
δ_{1I}	Intangible capital depreciation rate (Corporate)	0.08
δ_{2T}	Tangible capital depreciation rate (Non-corporate)	0.04
δ_{2I}	Intangible capital depreciation rate (Non-corporate)	0.08

Table A.2 Firm Production and Capital Parameters

Parameter	Description	Value
j_{ret}	Mandatory retirement age	65
τ_c	Consumption tax rate	0.21
$tr_{welfare}$	Universal welfare transfer (normalized)	0.01
$tr_{pension}$	Pension benefit transfer (normalized)	0.10
g_{share}	Public consumption share of output	0.27
$\tilde{b}$	Target government debt (normalized per-capita)	0.43
τ_π	Corporate income tax rate	0.35
τ_{d1}	Dividend tax rate (corporate sector)	0.15
τ_{d2}	Distribution tax rate (non-corporate sector)	0.30

Table A.3 Policy and Fiscal Parameters

To model transition path, Husaks children generation is artificially injected into the model as to better match the population transition dynamics of the Czech Republic. Although this might have slight distortionary effects in the beginning of transition path, it is reasonable to assume that it improved

the performance of the model during later years, which are of interest in the thesis. The transition demography matches better the dynamics in the official projection for the Czech Republic.

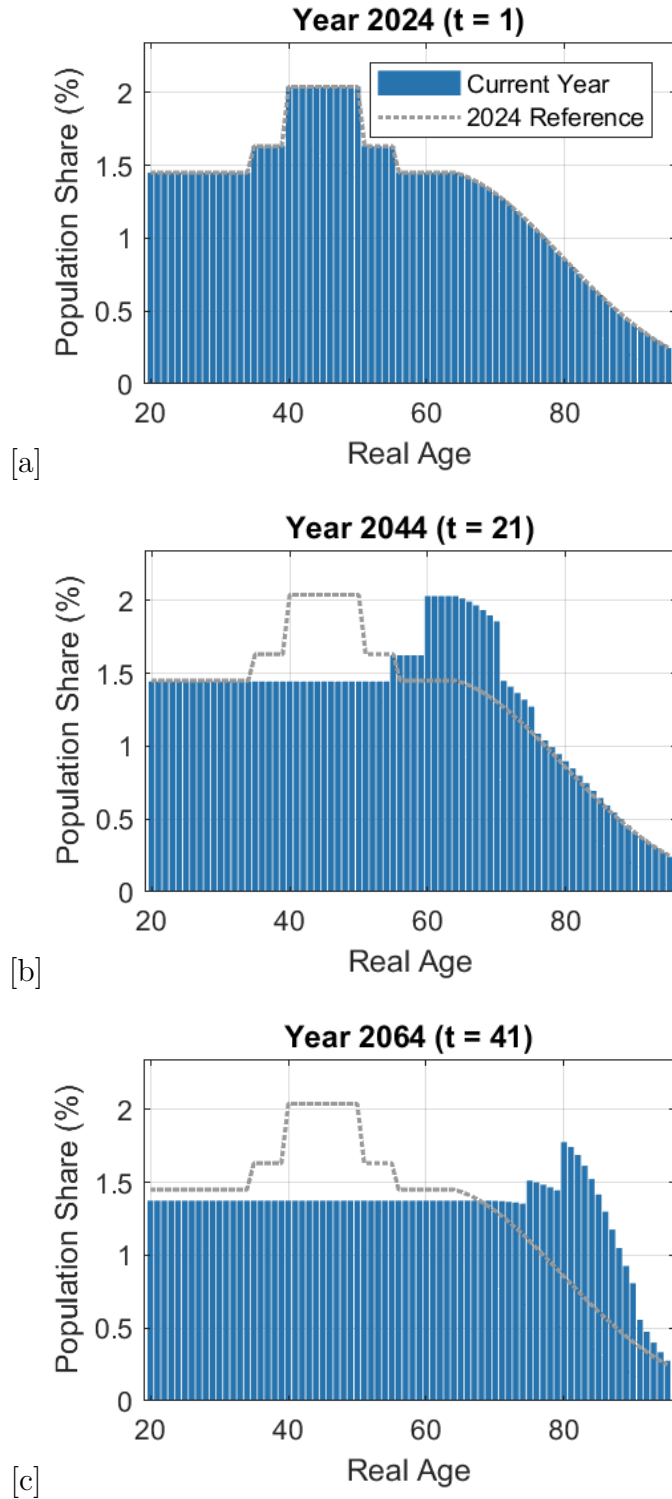


Figure A.4 Evolution of Transition Path Demography