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Abstract

This paper develops a model with high-skilled and low-skilled workers to show the expansionary effects of government spending despite large training costs for new hires. The main idea is that a fiscal stimulus induces changes in the composition of the labor force conditional on the extent of aggregate demand pressure. A period of high aggregate demand pressure is followed by a high value of forgone output as training activity causes production disruption. In this period firms decide to hire more low-skilled workers, who constitute a cheaper part of the labor force. When aggregate demand pressure is diminished, firms switch to hiring more high-skilled workers. However, the current literature considers only high-skilled workers, who tend to increase saving in government bonds to protect against poor employment prospects. In this case, the combination of weak employment prospects and the crowding-out effects of higher lump-sum taxes and government debt on private consumption and capital investment gives rise to recessionary effects. In contrast, this paper provides a model with a more realistic labor and financial market structure and suggests that countercyclical government spending in the form of government consumption and especially government investment can be used to deal with recessions.

JEL Classification: E22, E24, E32, E62.

Keywords: Government spending, training cost, search and matching frictions, financial friction.

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1. Introduction

The U.S. economy experienced its largest contraction since the 1930s during the 2008 Great Recession. To spur aggregate demand and job creation, the U.S. fiscal authorities responded by implementing a large-scale fiscal stimulus in the form of government spending. According to Hagedorn et al. (2019), increased government spending follows almost every recession, but there is still plenty of room to improve our understanding of its effectiveness and propagation. Indeed, there is a lack of consensus about the estimated size of the fiscal multiplier (see e.g., Ramey, 2011 and Parker, 2011) and the sign of the fiscal multiplier (see e.g., Alesina et al., 2002 and Ilzetzki et al., 2013). Recent papers by Picco (2020) and Faccini and Yashiv (2022) indicate that large training costs of new hires in a representative agent framework are a crucial reason for the counterintuitive recessionary effects of expansionary policies. Our paper contributes to the literature by studying the role of training costs within the heterogeneous agent framework. Our main finding is that a rise in government spending induces an economic expansion despite large training costs.

When a hiring process includes training activities for new hires, production is disrupted. Specifically, a firm's ability to produce is lowered due to a temporal reallocation of some experienced workers from production to training activities. The output costs associated with production disruption can be large for a high value of output. This perfectly corresponds to the case of expansionary government spending, which under sticky prices generates excess aggregate demand pressure. In the representative agent New Keynesian (RANK) model à la Picco (2020) and Faccini and Yashiv (2022), a higher value of output, coupled with large training costs, leads to a larger rise in the marginal cost than the marginal benefit of hiring. Consequently, firms decide to postpone hiring (or hire to a small extent). The presence of job separation and high savings induced by poor employment prospects translates into output contraction. This finding casts doubt on using countercyclical government spending as a general policymakers' tool to fight recessions.

By contrast, this paper considers a model economy populated with two types of workers, so that firms have a choice during the hiring process. Differently skilled workers typically face asymmetric labor and financial market frictions, whose effects are reflected in the wage bargaining process and the job creation condition. When the value of output is high, the hiring of low-skilled workers is more attractive for firms due to their lower training (non-wage labor) costs. The output expansion occurs as the economy experiences an extensive hiring activity for low-skilled workers. When financial friction is added to the setting with training costs, low-skilled workers as liquidity-constrained households could become an even cheaper labor force. This is because financial friction makes low-skilled workers willing to accept lower wage payments because the hiring allows them to improve their lifetime utility. Hence, the stimulative effects of increased government spending are more pronounced when the interaction of labor and financial market frictions is considered.

To isolate the impact of asymmetric training costs on the transmission of increased government spending to the real economy, we build a two-agent New Keynesian model with a representative household (TANKrep). Differently skilled workers, who live together in one big family, face different levels of training costs. Additionally, we build a two-agent New Keynesian (TANK) model to examine the importance of the interaction between asymmetric training costs and financial friction, where the latter is characterized by no risk-sharing between high- and low-skilled workers. This model assumes that differently skilled workers live separately in two big families due to their different access to financial markets, as in Galí et al. (2007)[1]. The presence of search and matching (SAM) frictions, which include matching efficiency, separation rates, and bargaining power, as in Dolado et al. (2021), are common to both the TANKrep and TANK models.

This paper contributes to the analytical heterogeneous agent New Keynesian (HANK) literature by developing a TANK framework with segmented labor and financial markets. Although a recently growing quantitative HANK literature is characterized by richer households' heterogeneity on the basis of idiosyncratic risk and incomplete markets, Debortoli and Galí (2018) find that a TANK model captures the implications of aggregate shocks in a full-scale HANK model reasonably well. In addition, the analytical and quantitative HANK literature abstracts from firm-specific hiring frictions, which are essentially the hallmark of the literature with a representative agent framework.

Our paper provides a bridge between the two strands of literature. In the first strand, hiring frictions are traditionally modeled as pecuniary costs (vacancy posting costs) within the heterogeneous agent setting (see, e.g., Gornemann et al., 2021 and Ravn and Sterk, 2021). The second strand emphasizes the non-pecuniary nature of hiring (training) costs in the representative agent framework (see, e.g., Picco, 2020, Faccini and Yashiv, 2022 and Faccini and Melosi, 2022). With respect to the first strand, hiring costs are expressed as asymmetric non-pecuniary costs and SAM frictions are asymmetric across skills. With respect to the second strand, segmented labor and financial markets are introduced to study the heterogeneous responses of households to higher government spending. Note that this paper follows Dolado et al. (2021) in modelling a segmented labor market, and additionally considers non-pecuniary training costs and a segmented financial market.

In our quantitative results, both government consumption and government investment generate an economic expansion, with the latter having a much larger fiscal multiplier. The impulse response analysis of government consumption is divided into three parts. The first part focuses on the output responses to an expansionary government consumption shock in models with flexible wages. In the

[1] High-skilled workers have access to financial markets and provide high-skilled labor services, while low-skilled workers do not have access to financial markets and supply low-skilled labor services. Two well-established premises provide the justification for considering these two groups of workers. First, the existence of employment and earnings polarization by skill level (Goos and Manning, 2007 and Autor and Dorn, 2013). Second, the difference in financial literacy (Lusardi and Mitchell, 2007) and participation costs (Vissing-Jorgensen, 2002), which underlies the unequal access of workers to financial markets.

RANK model, the output expansion is recorded for vacancy posting costs in non-pecuniary terms, but the fiscal multiplier is small on impact (0.039) and stays positive for another twenty quarters. By contrast, the RANK model with internal training costs characterizes persistent recessionary effects, with a multiplier of -0.101 after forty quarters. If vacancy posting costs are expressed in pecuniary terms, a rise in output is more pronounced, leading to a multiplier of 0.095 on impact. The rationale for the opposing output responses is that non-pecuniary hiring costs are related to production disruption, while pecuniary hiring costs imply payments for hiring services to an external labor agency. In contrast, TANKrep and TANK models with flexible wages report the expansionary output effects despite modelling hiring costs as internal training costs. The TANKrep model[2] generates a multiplier of 0.055 on impact, which gradually declines. With the exception of a small, initially negative multiplier of -0.015, the TANK model[3] also shows expansionary output effects with a peak multiplier of 0.236 after forty quarters.

The second part of the analysis examines the output responses under rigid wages. In the TANKrep model, rigid wages initially amplify the multiplier to 0.092. However, a large increase in demand for labor in the first two quarters, with a training costs specification, implies a more expensive hiring of new workers in the next five quarters. Consequently, output drops and the multiplier becomes lower. Later, the low value of output induced by lower aggregate demand pressure stimulates firms to hire more (productive) high-skilled workers so that output starts to rise. To determine the influence of financial friction in the TANK model, we assume that workers face symmetric labor market frictions. In this case, firms would still have lower wage labor costs by hiring low-skilled workers. As a result of increased hiring and associated investment activity, the initial drop in output is significantly limited in the TANK model relative to the RANK model. Moreover, the TANK model shows that output returns to its pre-crisis average level after nine quarters, while output in the RANK model does not complete its recovery even after forty quarters. In addition, adding asymmetric SAM frictions to the TANKrep and TANK models only slightly amplifies the effects of asymmetric training costs and financial friction through an improved labor market position of high-skilled workers. After forty quarters, the size of the cumulative multiplier is 0.158.

The third part of the analysis investigates the responses of several real economic variables in addition to output. In the RANK model, higher government spending leads to a rise in aggregate demand pressures, which under large training costs increase the marginal cost of hiring more than the marginal benefit and accordingly discourage firms from hiring new workers. As this fiscal stimulus is followed by increasing taxes, there are standard negative wealth effects that induce wealthy households to decrease consumption and to increase labor supply. However, they face a problem of finding a job due to reduced hiring incentives for firms. In addition to poor employment prospects for wealthy households,

[2] The asymmetric training costs in this TANKrep model are specified with asymmetric hiring cost scaling parameters, $e_w = 5.07$ and $e_p = e_w/5.25$.

[3] This TANK model includes workers who are differently skilled due to their different skill intensity in production, have different access to financial markets but face symmetric SAM frictions and symmetric training costs.

the real interest rate rises as a government reacts to increased aggregate demand pressures, which in turn has crowding-out effects on capital investment. The combination of increasing taxes, low employment and decreasing capital investment puts downward pressure on output. By contrast, in the TANKrep and TANK models there is greater hiring activity, particularly of low-skilled workers, which has stimulative effects on investment and production activities.

In addition to our analysis of government consumption, we investigate output responses to expansionary government investment. There are two important observations regarding the effects of government investment when real wages are rigid. First, government investment generates stronger expansionary effects than government consumption because of a higher marginal productivity of labor inputs, which stimulates firms' labor demand. Thus, from the perspective of policy makers, government investment is a more efficient tool in dealing with recessions than government consumption. Second, the expansionary effects of government investment in the TANK model are larger and more persistent than in the RANK model. The size of the fiscal multiplier is 0.128 and 0.055 on impact in the TANK and the RANK models, respectively. After forty quarters, the cumulative fiscal multiplier is 0.755 in the TANK model, while it is 0.317 in the RANK model.

The rest of the paper is organized as follows. Section 3 presents the model economy. Section 4 accounts for the transmission mechanism. Section 5 is dedicated to the calibration, while Section 6 shows the impulse response analysis regarding expansionary government spending. Section 7 concludes.

3. Model economy

The model economy denoted as TANK has household, production, and government sectors[4]. The household sector includes a continuum of wealthy w and poor households p on the unit interval. These households are different in terms of the frictions they face in the financial and labor markets. In this regard, households have differential access to financial markets, in the spirit of Galí et al. (2007) [5]. In addition, households may have different productivity levels, reflected in skill intensity in production, and face asymmetric SAM frictions (matching efficiency, separation rates, and bargaining power), as in Dolado et al. (2021), as well as asymmetric training costs internal to intermediate goods firms. Taking financial and labor market segmentation together, a constant share $s_w \in [0, 1]$ of the household population[6] participates in financial/capital markets and provide high-skilled labor services, while the remaining fraction $s_p = 1 - s_w$ are non-participants in financial/capital markets and provides low-skilled labor services. In the production sector, there exists a distinction between intermediate and

[4] The description of the TANKrep model is provided in Appendix A.5.

[5] In this model economy, household members can be perfectly insured against unemployment risk (induced by SAM frictions and hiring costs) within a particular skill group, but not between them. As in Merz (1995), the head of each household provides perfect risk sharing within a given household type by pooling the income of all its members and then allocating it to consumption, so that all members consume the same amount of consumption goods regardless of their employment state.

[6] When $s_w = 1$, our two-agent model collapses to a standard representative agent model with only wealthy households.

final goods firms to avoid difficulties arising from having the hiring and pricing decisions within the same firm. Perfectly competitive intermediate producers hire labour and rent capital from households to produce a homogeneous intermediate good, which is then differentiated by final goods firms that face price-setting rigidities. The final output is used for private consumption, investment, and public consumption. In the government sector, the monetary authority sets the short-term nominal interest rate following a standard Taylor rule, while the fiscal authority conducts government spending that is financed with lump-sum taxes and issuing short-term bonds.

3.1. Labor market

There is a large number of households, which are classified into two groups by the skill level of their members: high- and low-skilled workers. Workers can only participate in the labor market they belong to the basis of their skill level; high- or low-skill labor markets. In addition, we assume that workers cannot change their skill level over time, which makes their respective population share constant.

Following Galí (2010), in each period household members can be in one of three different employment states: employed, unemployed but actively looking for a job, and unemployed but inactive. The sum of those members who are employed $N_{k,t}$ and those who are unemployed but actively looking for a job $U_{k,t}$ constitutes a pool of people who participate in the labor market or the total workforce

$$L_{k,t} = N_{k,t} + U_{k,t}, \quad k \in \{w, p\} \tag{1}$$

The labor market as a place of interaction between (intermediate goods) firms and workers is characterized by labor market frictions in the form of SAM frictions and training costs. To find new workers, firms post job vacancies $\upsilon_{k,t}$ for which $U^k_{0,t}$ apply. The variable $U^k_{0,t}$ is the notation for the pool of unemployed people at the beginning of period t who are actively searching for a job. Only the beginning-of-period job seekers from the unemployment pool can be hired, while employed workers cannot search for jobs. The matching technology for new gross hires $H_{k,t}$ takes the standard Cobb-Douglas form:

$$H_{k,t}(\upsilon_{k,t}, U^k_{0,t}) = \psi_k \left(\upsilon_{k,t}\right)^{\varsigma} \left(U^k_{0,t}\right)^{1-\varsigma}, \quad k \in \{w, p\} \tag{2}$$

where $\psi_k > 0$ captures the matching efficiency and $\varsigma \in (0, 1)$ is the elasticity of the new hires to the beginning-of-period job seekers.

Labor market tightness $\theta_{k,t}$, vacancy filling probabilities $\nu_{k,t}$ and hiring probabilities $\mu_{k,t}$ differ by the skill type of workers $k \in \{w, p\}$:

$$\theta_{k,t} = \frac{\upsilon_{k,t}}{U^k_{0,t}} \tag{3}$$

$$\nu_{k,t} = \frac{H_{k,t}}{\upsilon_{k,t}} \tag{4}$$

$$\mu_{k,t} = \frac{H_{k,t}}{U^k_{0,t}} \tag{5}$$

Aggregate employment in the wholesale sector evolves according to the following law of motion:

$$N_{k,t} = (1-\sigma_k)N_{k,t-1} + H_{k,t}, \quad k \in \{w,p\} \tag{6}$$

where $\sigma_k \in (0,1)$ is a constant exogenous separation rate, which indicates a share of employed workers who leave the firm and consequently become unemployed until the next period. Note that equation (6) indicates that newly hired workers become productive (or start working) immediately in the same period in which they are hired. This is in line with Blanchard and Galí (2010) timing specification, where employment is a choice variable that can contemporaneously respond to shocks in the economy.

In addition to SAM frictions, intermediate goods firms face hiring costs. Faccini and Yashiv (2022) provide micro-evidence that around 80% of hiring costs are post-match and expressed in intermediate goods or as foregone output. Accordingly, in our benchmark model specification, hiring costs are treated as internal training costs. These costs occur after the establishment of a job relationship, and assume the discrepancy between newly hired workers and experienced workers regarding the level of productivity. To close the gap between them, the new hires pass through the training process. If the training activity is not delegated to some third-party labor agency, firms resort to internal training. With this internal training activity, production disruption takes place as some experienced workers are diverted from production to training the new hires.

3.2 Households

3.2.1 Ricardian high-skilled households (the wealthy)

Wealthy households maximize their expected lifetime utility, which is a separably additive function of consumption $c_{w,t}$ and labor supply $\ell_{w,t}$:

$$\max_{c_{w,t},i_t,k_t,b_t,\ell_{w,t},n_{w,t},u_{w,t}} \quad \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Big\{ \frac{1}{1-\sigma_c}(c_{w,\tau} - hC_{w,\tau-1})^{1-\sigma_c} - \varphi_{n,w}\frac{(\ell_{w,\tau})^{1+\eta}}{1+\eta} \Big\}$$

where $\mathbb{E}_t$ is the conditional expectations operator in period t, $\beta \in (0,1)$ is the subjective discount factor, $c_{w,t}(C_{w,t})$ is the time-t individual (aggregate) level of consumption of the final good, $\sigma_c \geq 0$ is the inverse of the intertemporal elasticity of substitution, $h < 1$ measures the degree of external consumption habits, $\eta > 0$ is the inverse Frisch elasticity of labour supply, and $\varphi_{n,w} > 0$ specifies the weight on the disutility of labor market activities $\ell_{w,t}$ [7].

The real budget constraint of a wealthy household in every period t is:

$$c_{w,t} + t_{w,t} + i_t + b_t \leq w_{w,t}n_{w,t} + r_t^k k_{t-1} + \frac{R_{t-1}b_{t-1}}{\pi_t} + \frac{\Pi_t^{int}}{s_w} + \frac{\Pi_t^r}{s_w}$$

[7] Similarly to Galí (2010), we focus on the extensive margin (the changes in the number of workers), and abstract from the intensive margin (the changes in the working time). Moreover, Dossche et al. (2019) indicate that firms in the US adjust their labor input mainly along the extensive margin as only 6% of variation in aggregate hours is attributed to the variation in hours per worker, which is much less than the 48% in the Euro Area.

and the employment law of motion:

$$n_{w,t} = (1-\sigma_w)n_{w,t-1} + \frac{\mu_{w,t}}{1-\mu_{w,t}}u_{w,t}(= h_{w,t})$$

and the law of motion of physical capital:

$$i_t = k_t - (1-\delta_k)k_{t-1} + \frac{\phi_k}{2}\Big(\frac{k_t}{k_{t-1}} - 1\Big)^2 k_{t-1}$$

Note that the nominal variables are transformed in real terms by being divided with the price of the final composite good P_t, and $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

Wealthy households receive real labor income $w_{w,t}n_{w,t}$ when employed, income from renting capital stock $r_t^k k_{t-1}$, real return on government bonds $\frac{R_{t-1}b_{t-1}}{\pi_t}$ (where R_t is the nominal interest rate set by the central bank), and real profits in the form of dividends $\Pi_t^{int} + \Pi_t^r$ from ownership of intermediate and final goods firms. The household chooses to save these total resources in the form of risk-free government bonds b_t and physical capital i_t, and to spend them by purchasing consumption goods $c_{w,t}$ and paying real lump-sum taxes $t_{w,t}$ to the government.

The Lagrangian function associated with the maximization problem of wealthy household is:

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\Bigg\{ \frac{1}{1-\sigma_c}(c_{w,\tau} - hC_{w,\tau-1})^{1-\sigma_c} - \varphi_{n,w}\frac{(\ell_{w,\tau})^{1+\eta}}{1+\eta} - \lambda_{w,\tau}^c \Bigg(c_{w,t} + t_{w,t} + i_t + b_t -$$

$$- w_{w,t}n_{w,t} - r_t^k k_{t-1} - \frac{R_{t-1}b_{t-1}}{\pi_t} - \frac{\Pi_t^{int}}{s_w} - \frac{\Pi_t^r}{s_w} + \lambda_{w,\tau}^n \Big(n_{w,t} - (1-\sigma_w)n_{w,t-1} - \frac{\mu_{w,t}}{1-\mu_{w,t}}u_{w,t}\Big)\Bigg) +$$

$$+ \lambda_{w,\tau}^l (n_{w,t} + u_{w,t} - \ell_{w,t})\Bigg\}$$

Let $\lambda_{w,t}^c$, $\lambda_{w,t}^n$, $\lambda_{w,t}^l$ be the Lagrangian multipliers corresponding to the budget constraint, the employment law of motion and the labor force participation, respectively. The first-order conditions for the intertemporal problem of wealthy households are

$$[c_{w,t}]: \quad \lambda_{w,t}^c = \frac{1}{(c_{w,t} - hC_{w,t-1})^{\sigma_c}} \tag{7}$$

$$[n_{w,t}]: \quad \lambda_{w,t}^n = \frac{\lambda_{w,t}^l}{\lambda_{w,t}^c} + w_{w,t} + \mathbb{E}_t\, \beta \frac{\lambda_{w,t+1}^c}{\lambda_{w,t}^c}\lambda_{w,t+1}^n(1-\sigma_w) \tag{8}$$

$$[\ell_{w,t}]: \quad \lambda_{w,t}^l = -\varphi_{n,w} \cdot \ell_{w,t}^{\eta} \tag{9}$$

$$[u_{w,t}]: \quad \lambda_{w,t}^l = -\lambda_{w,t}^c \cdot \lambda_{w,t}^n \cdot \frac{\mu_{w,t}}{1-\mu_{w,t}} \tag{10}$$

$$[k_t]: \quad \lambda_{w,t}^c \left(1 + \phi_k\Big(\frac{k_t}{k_{t-1}} - 1\Big)\right) = \mathbb{E}_t\, \beta\lambda_{w,t+1}^c \left((1-\delta_k) + r_{t+1}^k + \frac{\phi_k}{2}\Big(\Big(\frac{k_{t+1}}{k_t}\Big)^2 - 1\Big)\right) \tag{11}$$

$$[b_t]: \quad \lambda^c_{w,t} = \mathbb{E}_t\, \beta \lambda^c_{w,t+1} \frac{R_t}{\pi_{t+1}} \tag{12}$$

The first optimality condition states that the Lagrange multiplier $\lambda^c_{w,t}$ must equal the marginal utility of private consumption. The next three conditions determine the real marginal values of being employed and participating in the labor market. The last two conditions are arbitrage conditions related to the returns on capital and bonds.

The real marginal value of a job for a skilled worker $\lambda^n_{w,t}$ is a function of the disutility of labor market participation (forgone leisure), the real wage and the continuation value of a job (or the expected discounted value of staying employed in the next period). Note that the disutility from labor supply is divided by the marginal utility of consumption to transform utils into consumption goods. In the absence of labor market frictions, there is no surplus for the household of having one more employed member $\lambda^n_{w,t} = 0$. In this case, equation (8) reduces to the standard labor supply condition, where the marginal rate of substitution between consumption and leisure $\frac{\lambda^l_{w,t}}{\lambda^c_{w,t}}$ equals the real wage.

3.2.2 Non-Ricardian low-skilled households (the poor)

A continuum of infinitely-lived poor households maximizes their expected lifetime utility:

$$\max_{c_{p,t}, l_{p,t}, n_{p,t}, u_{p,t}} \quad \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Big\{ \frac{1}{1-\sigma_c} (c_{p,\tau} - hC_{p,\tau-1})^{1-\sigma_c} - \varphi_{n,p} \frac{(\ell_{p,\tau})^{1+\eta}}{1+\eta} \Big\}$$

subject to the real budget constraint in every period t:

$$c_{p,t} + t_{p,t} \leq w_{p,t} n_{p,t}$$

and subject to the constraint on employment flows:

$$n_{p,t} = (1-\sigma_p) n_{p,t-1} + \frac{\mu_{p,t}}{1-\mu_{p,t}} u_{p,t}$$

The Lagrangian function associated with the maximization problem of a poor household is:

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Bigg\{ \frac{1}{1-\sigma_c} (c_{p,\tau} - hC_{p,\tau-1})^{1-\sigma_c} - \varphi_{n,p} \frac{(\ell_{p,\tau})^{1+\eta}}{1+\eta} - \lambda^c_{p,\tau} \Bigg(c_{p,t} + t_{p,t} - w_{p,t} n_{p,t} +$$

$$+ \lambda^n_{p,\tau} (n_{p,t} - (1-\sigma_p) n_{p,t-1} - \frac{\mu_{p,t}}{1-\mu_{p,t}} u_{p,t}) \Bigg) + \lambda^l_{p,\tau} (n_{p,t} + u_{p,t} - \ell_{p,t}) \Bigg\}$$

The optimization with respect to the choice variables of the poor gives the following optimality conditions:

$$[c_{p,t}]: \quad \lambda^c_{p,t} = \frac{1}{(c_{p,t} - hC_{p,t-1})^{\sigma_c}} \tag{13}$$

$$[n_{p,t}]: \quad \lambda^n_{p,t} = \frac{\lambda^l_{p,t}}{\lambda^c_{p,t}} + w_{p,t} + \mathbb{E}_t\, \beta \frac{\lambda^c_{p,t+1}}{\lambda^c_{p,t}} \lambda^n_{p,t+1} (1-\sigma_p) \tag{14}$$

$$[\ell_{p,t}]: \quad \lambda^l_{p,t} = -\varphi_{n,p} \cdot \ell^{\eta}_{p,t} \tag{15}$$

$$[u_{p,t}]: \quad \lambda^l_{p,t} = -\lambda^c_{p,t} \cdot \lambda^n_{p,t} \cdot \frac{\mu_{p,t}}{1-\mu_{p,t}} \tag{16}$$

The Lagrangian multipliers associated with the budget constraint, the employment law of motion and the labor force participation have the same interpretation as in the optimality problem of wealthy households.

The poor can only participate in the labor market as they are excluded from financial/capital markets. For supplying low-skilled labor services to intermediate goods firms, employed poor households receive real labor income $w_{p,t}n_{p,t}$. This total disposable income is used for purchases of consumption goods $c_{p,t}$ and the payment of real lump-sum taxes $t_{p,t}$ to the government. Given that poor households spend all their net disposable income each period in a hand to-mouth manner as in Galí et al. (2007), they are expected to have a larger marginal propensity to consume than wealthy households, and thus be more sensitive to transitory labor income changes. Differently to Galí et al. (2007), hand-to-mouth workers do not have pure myopic behavior due to the dynamic nature of the employment law of motion. They consider the benefits of being employed today. If they get a job today, they are likely to stay employed in the future due to a relatively low separation rate. They will enjoy labor income from employment, which will be used for consumption tomorrow and according improvement of their lifetime utility.

3.3 Producers

3.3.1 Intermediate (wholesale) goods producers

There is a unit continuum of perfectly competitive firms that produce a homogeneous good $f_{int,t}$ and sell it to retail firms at price $P_{int,t}$ in a competitive market. In the production process, wholesale firms rent the aggregate stock of capital K_t, and hire aggregate skilled labor $N_{w,t}$, and aggregate unskilled labor $N_{p,t}$. The production function takes a standard Cobb-Douglas form with a nested CES composite of two labor inputs:

$$f_{int,t} = F(K_t, N_{w,t}, N_{p,t}) = AK_t^{\iota}\Big[m(N_{w,t})^{\sigma} + (1-m)(N_{p,t})^{\sigma}\Big]^{\frac{1-\iota}{\sigma}} \tag{17}$$

where $A > 0$ stands for the level of aggregate productivity, the parameter ι indicates the income share of physical capital, the parameter m determines the skill intensity (or the productivity level) of labor input, and the parameter σ governs the elasticity of substitution between skilled and unskilled labor in the production process.

When making hiring decisions, intermediate goods firms face labor adjustment costs, which are modelled as training costs and expressed in non-pecuniary terms. Differently to Faccini and Yashiv (2022), the hiring cost function is specified to be asymmetric for differently skilled workers:

$$\tilde{g}^k_{int,t} = \frac{e_k}{2}\left(\frac{H_{k,t}}{N_{k,t}}\right)^2, \quad k \in \{w, p\}$$

where e_k measures the degree of curvature of hiring cost, and $H_{k,t}/N_{k,t}$ is the hiring rate.

The net output of an intermediate goods firm is:

$$Y_{int,t} = f_{int,t}\Big(1 - \sum_{k\in\{w,p\}} \tilde{g}^k_{int,t}\Big) = f_{int,t} - g_{int,t} \tag{18}$$

Intermediate goods producers seek to maximize their nominal profits subject to the employment law of motion (6) and the production function net of hiring costs (18):

$$P_t\Pi^{int}_t = P_{int,t}Y_{int,t} - W_{w,t}N_{w,t} - W_{p,t}N_{p,t} - R^k_t K_t$$

The real profit of the intermediate goods firms is expressed as follows:

$$\Pi^{int}_t = \frac{Y_{int,t}}{x_t} - w_{w,t}N_{w,t} - w_{p,t}N_{p,t} - r^k_t K_t,$$

where $x_t = \frac{P_t}{P_{int,t}}$ is the retail-price markup defined as a ratio of the price of the final good P_t and the price of the intermediate good $P_{int,t}$. The inverse of retail-price markup $\frac{1}{x_t}$ is the real marginal cost for retail firms.

The present discounted value of real profits of intermediate goods firms is:

$$\max_{K_t,N_{w,t},N_{p,t},H_{w,t},H_{p,t}} \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda^c_{t,\tau}\Pi^{int}_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\frac{\lambda^c_{w,\tau}}{\lambda^c_{w,t}}\Bigg\{\frac{Y_{int,t}}{x_t} - w_{w,t}N_{w,t} - w_{p,t}N_{p,t} - r^k_t K_t -$$
$$- \sum_{k\in\{w,p\}} Q^N_{k,t}\Big(N_{k,t} - (1-\sigma_k)N_{k,t-1} - H_{k,t}\Big)\Bigg\}$$

where $\Lambda^c_{t,t+1} = \beta\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}}$ is the real stochastic discount factor of wealthy households who only own the intermediate goods firms, and $Q^N_{k,t}$ is the Lagrange multiplier on the employment constraint (6).

The first order conditions of the real profit function with respect to the firm's choice variables are

$$[K_t] : r^k_t = \frac{1}{x_t}(f_{K,t} - g_{K,t}) \tag{19}$$

$$[N_{w,t}] : Q^N_{w,t} = \frac{1}{x_t}(f_{N_{w,t}} - g_{N_{w,t}}) - w_{w,t} + (1-\sigma_w)\,\mathbb{E}_t\,\beta\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}}Q^N_{w,t+1} \tag{20}$$

$$[N_{p,t}] : Q^N_{p,t} = \frac{1}{x_t}(f_{N_{p,t}} - g_{N_{p,t}}) - w_{p,t} + (1-\sigma_p)\,\mathbb{E}_t\,\beta\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}}Q^N_{p,t+1} \tag{21}$$

$$[H_{w,t}] : Q^N_{w,t} = \frac{1}{x_t}g_{H_{w,t}} \tag{22}$$

$$[H_{p,t}] : Q^N_{p,t} = \frac{1}{x_t} g_{H_{p,t}} \tag{23}$$

The derivatives of the production function and the hiring cost function are given by

$$f_{K,t} = A\iota K_t^{\iota-1} \Big[m(N_{w,t})^\sigma + (1-m)(N_{p,t})^\sigma\Big]^{\frac{1-\iota}{\sigma}}$$

$$f_{N_{w,t}} = AK_t^\iota(1-\iota)\Big[m(N_{w,t})^\sigma + (1-m)(N_{p,t})^\sigma\Big]^{\frac{1-\iota}{\sigma}-1} mN_{w,t}^{\sigma-1}$$

$$f_{N_{p,t}} = AK_t^\iota(1-\iota)\Big[m(N_{w,t})^\sigma + (1-m)(N_{p,t})^\sigma\Big]^{\frac{1-\iota}{\sigma}-1} (1-m)N_{p,t}^{\sigma-1}$$

$$g_{N_{w,t}} = -e_w\left(\frac{H_{w,t}}{N_{w,t}}\right)^2 \frac{1}{N_{w,t}} f_{int,t} + f_{N_{w,t}} \left(\frac{e_w}{2}\left(\frac{H_{w,t}}{N_{w,t}}\right)^2 + \frac{e_p}{2}\left(\frac{H_{p,t}}{N_{p,t}}\right)^2\right)$$

$$g_{N_{p,t}} = -e_p\left(\frac{H_{p,t}}{N_{p,t}}\right)^2 \frac{1}{N_{p,t}} f_{int,t} + f_{N_{p,t}} \left(\frac{e_w}{2}\left(\frac{H_{w,t}}{N_{w,t}}\right)^2 + \frac{e_p}{2}\left(\frac{H_{p,t}}{N_{p,t}}\right)^2\right)$$

$$g_{H_{w,t}} = e_w\left(\frac{H_{w,t}}{N_{w,t}}\right) \frac{1}{N_{w,t}} f_{int,t}$$

$$g_{H_{p,t}} = e_p\left(\frac{H_{p,t}}{N_{p,t}}\right) \frac{1}{N_{p,t}} f_{int,t}$$

$$g_{K,t} = \frac{e_w}{2}\left(\frac{H_{w,t}}{N_{w,t}}\right)^2 f_{K,t} + \frac{e_p}{2}\left(\frac{H_{p,t}}{N_{p,t}}\right)^2 f_{K,t}$$

The first optimality condition is related to the demand for capital, which equates the rental rate of capital with the marginal revenue from using an additional unit of capital. The latter term is the net marginal product of capital multiplied by the real marginal costs. The next two conditions specify the labor demand for two types of workers. In equations (20) and (21), the real value of a marginal job for a firm is the sum of current real profits from an additional worker and the expected continuation value. Note that the current profits consist of the marginal revenue from employing an additional worker less the real wage payment, while the continuation value presents the expected discounted future profits provided that the worker remains employed. The last two optimality conditions define the firm's hiring decision, which relates the real marginal value of employment for a firm to the real marginal cost of hiring. Accordingly, a wholesale firm tends to hire a new worker until the benefit of hiring equals the cost of hiring that worker.

3.3.2 Final (retail) goods producers

A continuum of retail firms indexed by $j \in [0, 1]$ operate in a monopolistically competitive market. Each firm purchases the quantity $Y_t(j)$ of the homogeneous intermediate good $Y_t = Y_{int,t}$, which is then used as an input in the production of the final differentiated good $Y_t^f(j)$. The transformation technology is linear, $Y_t^f(j) = Y_t(j)$, so that aggregate final output is given by:

$$Y_t^f = \Big[\int_0^1 Y_t^f(j)^{\frac{\epsilon}{\epsilon-1}} dj\Big]^{\frac{\epsilon}{\epsilon-1}} = \Big[\int_0^1 Y_t(j)^{\frac{\epsilon}{\epsilon-1}} dj\Big]^{\frac{\epsilon}{\epsilon-1}} = Y_t$$

where $\epsilon > 1$ is the elasticity of substitution across varieties. It can be shown that the final consumption bundle Y_t^f gives the aggregate price index P_t by solving the standard cost-minimization problem of the firm.

Final goods firms buy intermediate goods at wholesale price $P_{int,t}$, costlessly differentiate them, and then sell a variety of final goods at price $P_t(j)$. When changing their prices, retailers have to pay quadratic price adjustment costs in terms of the final good as in the Rotemberg (1982) model specification. Specifically, the cost is present whenever the ratio between the current price and the price set in the previous period, $P_t(j)/P_{t-1}(j)$, deviates from the steady state inflation rate π:

$$\frac{\phi_p}{2}\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)^2 Y_t \tag{24}$$

where $\phi_p \geq 0$ is a parameter that measures the extent of price adjustment costs.

Each final goods firm chooses its own price $P_t(j)$ to maximize real profits

$$\max_{P_t(j)} \ \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau}^c \Pi_t^r(j) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\frac{\lambda_{w,\tau}^c}{\lambda_{w,t}^c}\left(\left(\frac{P_\tau(j)}{P_\tau} - \frac{P_{int,\tau}}{P_\tau}\right) Y_t(j) - \frac{\phi_p}{2}\left(\frac{P_\tau(j)}{\pi P_{\tau-1}(j)} - 1\right)^2 Y_\tau\right)$$

subject to (24) and the demand of households for final goods variety

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$$

Taking the derivative with respect to the price $P_t(j)$ gives

$$FOC[P_t(j)]: \ (1-\epsilon)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_t} - (-\epsilon)\left(\frac{P_{int,t}}{P_t}\right)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon-1}\frac{Y_t}{P_t} - \phi_p\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)\frac{Y_t}{\pi P_{t-1}(j)}$$
$$+ \mathbb{E}_t\,\beta\frac{\lambda_{w,t+1}^c}{\lambda_{w,t}^c}\phi_p\left(\frac{P_{t+1}(j)}{\pi P_t(j)} - 1\right)\frac{P_{t+1}(j)Y_{t+1}}{\pi P_t(j)^2} = 0$$

Since all retailers produce the same quantity of output in equilibrium, they all set the same price. Given this statement and the aggregate price level in the economy $P_t = (\int_0^1 P_t(j)^{1-\epsilon}dj)^{\frac{1}{1-\epsilon}}$, it follows that $P_t^*(j) = P_t^*$. Accordingly, the optimal pricing condition for retailers is given by

$$(1-\epsilon) + \epsilon\frac{P_{int,t}}{P_t} - \phi_p\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)\frac{P_t}{\pi P_{t-1}(j)} + \mathbb{E}_t\,\Lambda_{t,t+1}^c\phi_p\left(\frac{P_{t+1}(j)}{\pi P_t(j)} - 1\right)\frac{P_{t+1}(j)}{\pi P_t(j)}\frac{Y_{t+1}}{Y_t} = 0$$

$$\Leftrightarrow \quad (1-\epsilon) + \epsilon\, mc_t^r - \phi_p\left(\frac{\pi_t}{\pi} - 1\right)\frac{\pi_t}{\pi} + \mathbb{E}_t\,\beta\frac{\lambda_{w,t+1}^c}{\lambda_{w,t}^c}\phi_p\left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{\pi_{t+1}}{\pi}\frac{Y_{t+1}}{Y_t} = 0$$

When log-linearized around a zero-inflation steady state, the above pricing equation becomes the log-linearized New Keynesian Phillips Curve

$$\widetilde{\pi}_t = \frac{(\epsilon - 1)}{\phi_p}\widetilde{mc}_t^r + \beta\,\mathbb{E}_t\,\widetilde{\pi}_{t+1}$$

In symmetric equilibrium where all retailers are identical, the value of aggregate real profits distributed to all wealthy households is defined as follows

$$\Pi_t^r = \int_0^1 \Pi_t^r(j)dj = \int_0^1 \left(\frac{P_t(j)}{P_t} Y_t(j) - mc_t^r \cdot Y_t(j) - \frac{\phi_p}{2}\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right)^2 Y_t \right) dj$$

$$\Leftrightarrow \quad \Pi_t^r = \left(1 - mc_t^r - \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi} - 1 \right)^2 \right) Y_t$$

3.4 Monetary and fiscal policies

The monetary authority implements monetary policy through a standard Taylor rule that takes the following form:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\theta_r} \left[\left(\frac{\pi_t}{\pi} \right)^{\theta_\pi} \left(\frac{Y_t}{Y} \right)^{\theta_y} \right]^{1-\theta_r} \tag{25}$$

where R is the steady-state short term (gross) nominal interest rate, $0 \leq \theta_r \leq 1$ is a parameter associated with interest rate smoothing, $\theta_\pi > 0$ and $\theta_y > 0$ capture the interest rate response to deviations of inflation and output from their respective steady states.

The fiscal authority collects lump-sum taxes from households and issues one-period bonds to finance (unproductive) government purchases and interest payments on its outstanding debt.

The intertemporal government budget constraint expressed in aggregate real terms is:

$$T_t + B_t = \frac{R_{t-1}B_{t-1}}{\pi_t} + G_t \tag{26}$$

The real government spending G_t evolves exogenously over time and follows an AR(1) process

$$G_t = G^{1-\phi_g}(G_{t-1})^{\phi_g} exp(\epsilon_t^g), \quad \epsilon_t^g \sim \mathcal{N}(0, \sigma_g)$$

where G is the steady state fraction of government spending, $\phi_g \in (0, 1)$ is the persistence parameter, and ϵ_t^g is the government spending shock.

Lump-sum taxes $T_t = \sum_k s_k t_{k,t}$ follow the passive fiscal policy rule specified as:

$$\frac{T_t}{Y} - \frac{T}{Y} = \phi_B\Big(\frac{B_{t-1}}{Y} - \frac{B}{Y} \Big) + \phi_{BG}\Big(\frac{G_t}{Y} - \frac{G}{Y} \Big)$$

where $\phi_B > 0$ and $\phi_{BG} > 0$ stand for the tax-feedback parameters related to the government debt and spending, respectively. Lump-sum taxes are assumed to be the same for both types of workers.

3.5 Wage bargaining

Following a successful job match between a wholesale firm and a worker, real wages are determined by a standard Nash bargaining process. The negotiation of real wages takes place separately for the two distinct labor markets $k \in \{w, p\}$. The Nash wages maximise the joint match surplus of a worker and a firm weighted by the parameter $\vartheta^k \in [0,1]$, which refers to the bargaining power of a worker:

$$\max_{w_{k,t}} \left(\frac{\lambda^l_{k,t}}{\lambda^c_{k,t}} + w_{k,t} + \mathbb{E}_t\,\beta \frac{\lambda^c_{k,t+1}}{\lambda^c_{k,t}} \lambda^n_{k,t+1}(1-\sigma_k)\right)^{\vartheta^k} \left(\frac{1}{x_t}(f_{N_{k,t}} - g_{N_{k,t}}) - w_{k,t} + (1-\sigma_k)\,\mathbb{E}_t\,\beta \frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}} Q^N_{k,t+1}\right)^{1-\vartheta^k}$$

The optimality condition to this problem characterizes the surplus sharing rule:

$$\vartheta^k Q^{*,N}_{k,t} = (1-\vartheta^k)\lambda^{*,n}_{k,t}$$

The real value of a marginal job for a firm $Q^{*,N}_{k,t}$ and for a household $\lambda^{*,n}_{k,t}$ are specified as follows:

$$Q^{*,N}_{k,t} = \frac{1}{x_t}(f_{N_{k,t}} - g_{N_{k,t}}) - w^*_{k,t} + (1-\sigma_k)\,\mathbb{E}_t\,\beta \frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}} Q^{*,N}_{k,t+1}$$

$$\lambda^{*,n}_{k,t} = \frac{\lambda^l_{k,t}}{\lambda^c_{k,t}} + w^*_{k,t} + (1-\sigma_k)\,\mathbb{E}_t\,\beta \frac{\lambda^c_{k,t+1}}{\lambda^c_{k,t}} \lambda^{*,n}_{k,t+1}$$

The substitution of $Q^{*,N}_{k,t}$ and $\lambda^{*,n}_{k,t}$ in the bargaining solution leads to the real wage for $k \in \{w,p\}$:

$$\begin{aligned} w^*_{k,t} \equiv w^{NASH}_{k,t} = \vartheta^k \frac{1}{x_t}(f_{N_{k,t}} - g_{N_{k,t}}) - (1-\vartheta^k)\frac{\lambda^l_{k,t}}{\lambda^c_{k,t}} + \\ + \vartheta^k(1-\sigma_k)\,\mathbb{E}_t\,\beta \frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}} Q^{*,N}_{k,t+1} - (1-\vartheta^k)(1-\sigma_k)\,\mathbb{E}_t\,\beta \frac{\lambda^c_{k,t+1}}{\lambda^c_{k,t}} \lambda^{*,n}_{k,t+1} \end{aligned}$$

The Nash bargained wage includes two terms that are common to both types of workers. The first term is a fraction ϑ^k of the marginal revenue product of a worker. The second term is a fraction $1-\vartheta^k$ of the worker's reservation wage (or the outside option), which is the MRS between consumption and leisure. There is also another term that constitutes a part of the reservation wage distinctive only to the low-skilled. Boscá et al. (2011) call this third term an inequality term in utility. Different access of the two types of workers to financial markets induces this third term. As risk-sharing exists within the household type, but not between them, a difference in the intertemporal MRS is present, $\frac{\lambda^c_{w,t+1}}{\lambda^c_{w,t}} - \frac{\lambda^c_{p,t+1}}{\lambda^c_{p,t}} \gtrless 0$. Note that the inequality term in utility disappears in the steady state.

Although Boscá et al. (2011) specify different reservation wages for Ricardian and non-Ricardian workers, both types of workers receive the same wage and have the same employment level. The reason is the assumption of the same skill level for those two types of workers. Accordingly, the union structure can pool together both types of workers in the labor market and then bargain with firms about the wage and employment. Our model, however, assumes a segmented labor market, so differently skilled workers have different Nash bargained wages in addition to different reservation wages.

We also introduce real wage rigidity, as in Hall (2005), such that the actual real wage is a weighted average between the actual real wage from the previous period and the Nash wage:

$$w_{k,t} = \rho_w^k w_{k,t-1} + (1 - \rho_w^k) w_{k,t}^*$$

where ρ_w^k controls the degree of real wage rigidity and refers to the fraction of wages not adjusted each period. The importance of sticky wages is to make a search and matching model better in terms of matching empirically observed high volatility of unemployment and low volatility of wages (Shimer, 2005).

3.6 Aggregate variables and market clearing

In equilibrium, the market clearing conditions for skilled and unskilled labour, physical capital, bonds, and goods markets are respectively

$$N_{w,t} = s_w n_{w,t}$$

$$N_{p,t} = s_p n_{p,t}$$

$$K_t = s_w k_{t-1}$$

$$B_t = s_w b_t$$

$$Y_t = C_t + I_t + G_t + \frac{\phi_p}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 Y_t$$

In the aggregate resource constraint, the aggregate consumption is defined as $C_t = s_w c_{w,t} + s_p c_{p,t}$, and aggregate investment as $I_t = s_w i_t$. The goods market clearing condition[8] stems from the combination of the budget constraints of the two types of households, the government budget constraint, and the definition of firms' profits. It requires the net aggregate output to be equal to aggregate demand plus the resources allocated to the cost of price adjustment.

4. Transmission Mechanism

This section highlights that hiring costs are essential for transmitting the effects of increased government spending on the real economy. How hiring costs are modelled and the composition of hiring costs largely determine whether the government spending induces expansionary or recessionary effects. Accordingly, the following question arises: What are the effects of government spending in the presence of large hiring costs?

In general, there are two strands of literature that take different approaches to modelling the hiring costs. One strand of literature (see e.g. Gertler et al., 2008 and Mayer et al., 2010) generates the expansionary effects of government spending using the standard NK model with DMP framework. The reason behind this result lies in treating hiring costs as *vacancy posting costs*. To see this, let us analyse

[8] Derivation of the goods market clearing is provided in Appendix A.1.

the (log-linearized) labor demand equation that relates the marginal revenue product of labor to the real wage:

$$(\widetilde{mc}_t^r + \widetilde{f}_{N_t}) - \widetilde{w}_t = 0$$

When the fiscal authorities increase government consumption, the demand for intermediate goods increases. Under the sticky price setting, the difference between intermediate and final good prices increases. The higher aggregate demand pressures, which are reflected in higher real marginal costs for retailers, stimulate intermediate goods firms to hire more workers. As a result of higher employment in the economy, the amount of goods produced becomes larger. In addition to this direct influence of employment on production, there is an indirect stimulus. Specifically, higher labor demand is followed by larger vacancy posting costs, which form a part of aggregate demand as they are pecuniary third-party payments for the provision of hiring services.

The other strand of literature generates the recessionary effects of government spending. The reason lies in modelling hiring costs as *training costs*. To provide the rationale for this result, we can use the job creation condition that equates the marginal benefit of hiring to the marginal cost of hiring:

$$mc^r f_N(\widetilde{mc}_t^r + \widetilde{f}_{N_t}) - mc^r g_N(\widetilde{mc}_t^r + \widetilde{g}_{N_t}) - w\widetilde{w}_t + \mathbb{E}_t\,\beta(1-\sigma)Q^N(\widetilde{\lambda}_{t+1}^c - \widetilde{\lambda}_t^c + \widetilde{Q}_{t+1}^N) = mc^r g_H(\widetilde{mc}_t^r + \widetilde{g}_{H_t})$$

In the above expression, the derivative of the hiring cost function with respect to the new hires in the steady state is

$$g_H = e \cdot \frac{H}{N} \cdot \frac{f_{int}}{N}$$

The higher level of government spending leads to a rise in real marginal costs, which appear on both sides of the job creation condition. Thus, the net output effect is ambiguous. Two studies stand out by assessing the response of the economy to aggregate demand shocks when hiring costs are larger. First, Faccini and Yashiv (2022) perform a sensitivity analysis on the scaling parameter e in the hiring cost function. They find that a higher value of parameter e makes the marginal benefit of hiring lower that the marginal cost of hiring. This reduces the incentives of firms for hiring, which under job separation may translate into a lower level of employment and then output. Second, Picco (2020) examines what happens if the economy starts with a higher steady state level of hiring rate H/N. She also finds recessionary effects of government spending.

Despite the presence of large training costs, the model of this paper may generate expansionary effects. The reason lies in the coexistence of two different types of workers in the production process. The population division is a result of asymmetric training costs in the TANKrep model or the interaction between financial and labor market frictions in the TANK model. Firms do not need to postpone their hiring decision for the times when $\widetilde{mc}_t^r$ are lower, which is the case with the second strand of literature. Instead, at the times of higher $\widetilde{mc}_t^r$ firms may choose to hire low-skilled workers whose hiring (training) costs are lower. We can identify the intuition by analysing the optimal hiring condition for the skill level

$k \in \{w, p\}$:

$$mc^r f_{N_k}(\widetilde{mc}^r_t + \widetilde{f}_{N_{k,t}}) - mc^r g_{N_k}(\widetilde{mc}^r_t + \widetilde{g}_{N_{k,t}}) - w_k \widetilde{w}_{k,t} +$$

$$+ \mathbb{E}_t\, \beta(1-\sigma_k) Q^N_k (\widetilde{\lambda}^c_{k,t+1} - \widetilde{\lambda}^c_{k,t} + \widetilde{Q}^N_{k,t+1}) = mc^r g_{H_k}(\widetilde{mc}^r_t + \widetilde{g}_{H_{k,t}})$$

This job creation condition stems from the combination of firm's optimality conditions for employment and hiring.

What is noticeable in the second strand of literature is that it only considers the effects coming from differences in the steady state values of marginal hiring costs $g_H = e \cdot \frac{H}{N} \cdot \frac{f_{int}}{N}$ by changing the values of parameter e and the hiring rate H/N. Similarly, our paper considers how different steady state values of marginal hiring costs for high-skilled workers g_{H_w} and low-skilled workers g_{H_p} affect the job creation condition of respective workers.

Our paper also highlights the importance of dynamic responses of variables. Introducing financial friction in the TANK model causes population decomposition. Poor households who have restricted access to financial/capital markets exhibit high MPC, which increases the sensitivity of real marginal costs depending on their net disposable income. Moreover, the members of poor households are low-skilled workers with no pure myopic behaviour. They perceive the chance of improving their lifetime utility by being hired, which lowers their reservation wage and then their market wage. With higher real marginal costs and lower wage payments, the hiring of low-skilled workers becomes more attractive even in the presence of training costs. The marginal benefit is higher than the marginal cost of hiring low-skilled workers, incentivizing intermediate goods firms to hire them more. Increased hiring activity is also closely followed by higher investment in capital due to the complementarity between inputs in the CD production function. As a result, an economic expansion may arise.

A lower reservation wage of low-skilled workers is associated with a decreasing inequality term in utility. This claim is supported by two important reasons. First, the discrepancy between today's and tomorrow's level of consumption is much more pronounced for low-skilled workers, $\widetilde{\lambda}^c_{w,t+1} - \widetilde{\lambda}^c_{w,t} < \widetilde{\lambda}^c_{p,t+1} - \widetilde{\lambda}^c_{p,t}$. The rationale is that low-skilled workers cannot use wealth to smooth their consumption over time. Second, low-skilled workers enjoy higher consumption today than tomorrow due to a larger net disposable income today, $\widetilde{\lambda}^c_{p,t+1} - \widetilde{\lambda}^c_{p,t} > 0$. As increased government expenditure is financed by an increasing level of taxes, net disposable income becomes relatively higher today than tomorrow. Hence, poor household tends to have as many (low-skilled) workers employed as possible in order to collect more labor income sources tomorrow, which will then be used for consumption. Taking both reasons together, the gap in the intertemporal MRS can be interpreted as a readiness of low-skilled workers to accept a lower wage payment.

5. Calibration

Table 1 shows the calibrated values of structural parameters for the US economy at a quarterly frequency. The values of these parameters are determined internally by solving a non-stochastic steady state corresponding to the long run pre-crisis average values of targets, and externally in accordance with the estimates from the existing literature.

There are two baseline model economies, TANKrep and TANK, which are populated with high- and low-skilled workers. These workers either live together in one big family (TANKrep) or separately in two big families, due to their different access to financial markets (TANK). Population shares in both models are set to $s_w = 0.5$ and $s_p = 0.5$, which implies that 50 percent of the total population provides high-skilled labor services to intermediate goods firms. This is in line with Wolcott (2021) who indicates that 56 percent of the US population in 2007 can be regarded as high-skilled as they have at least one year of college education and accordingly search for high-skilled jobs. In addition, the TANK model is characterized by high-skilled workers who only have access to financial markets, and thus allows us to examine the role of financial friction in driving the output responses to government spending. The same population share for high-skilled workers who are treated as 'Ricardian' and low-skilled workers who are 'hand-to-mouth' can be found in Bhattarai et al. (2022).

In calibrating the parameters related to SAM frictions in the labor market, this paper closely follows Dolado et al. (2021). Accordingly, the parameters $\varphi_{n,k}$ and ϑ^k are jointly determined by matching the pre-crisis average values of participation and unemployment rates for the two types of workers $k \in \{w, p\}$

$$partic\_k = \frac{N_k + U_k}{s_k} \quad \text{and} \quad unemp\_k = \frac{U_k}{N_k + U_k}$$

The parameters $\varphi_{n,k}$ and ϑ^k stand for the weight on the disutility of labor market activities and the bargaining power of workers, respectively.

The baseline calibration in Table 1 specifies symmetry in participation and unemployment rates for the two types of workers. As in Dolado et al. (2021), this implies a participation rate of 0.675 and an unemployment rate of 0.053. There is also a symmetry in the training costs scaling parameters $e_w = e_p$ and the two parameters associated with SAM frictions: the job separation rate $\sigma_w = \sigma_p$ and the matching efficiency $\psi_w = \psi_p$. In the matching function, the matching elasticity $\varsigma = 0.5$ is assumed to be the same for both types of workers.

We also consider two additional models, Model 3 and Model 4 in Table 2, which examine asymmetric training costs and the interaction of financial friction with asymmetric training costs and SAM frictions, respectively. Model 3 relies on the calibrated values of parameters from the TANKrep and includes asymmetric training costs. Model 4 uses the parameter values from the TANK and incorporates both asymmetric training costs and asymmetric SAM frictions. In these two additional models, keeping the same values of parameters from the TANKrep and the TANK implies that the skill premium, participation rate and unemployment rate become non-targeted. Table 2 reports three results that suggest the good performance of the models. First, the model-induced values of the stated variables are close to their real-data counterparts. Second, the model non-targeted steady state ratios θ_w/θ_p, μ_w/μ_p, and ν_w/ν_p match well the estimates of Wolcott (2021). Third, training costs are close to one percent of aggregate output, which is comparable to the aggregate hiring costs in Blanchard and Galí (2010) due to the small vacancy costs in the data.

In addition to the internally calibrated parameters $\varphi_{n,w}$ and $\varphi_{n,p}$, the other parameters in the utility function specification include β, σ_c, h, η. The subjective discount factor, $\beta = 0.9945$, is calibrated to match a quarterly gross interest rate of around 1 percent ($R = 1 + \frac{2.21}{4 \cdot 100} = 1.0055$). The inverse of the intertemporal elasticity of substitution σ_c is set to 1, giving the log form of the utility function in consumption. The degree of external habit formation h takes the conventional value of 0.75. The inverse Frisch elasticity of labor supply η on the extensive margin is set to 1. Chang et al. (2019) indicate that the value of 1 for η is quite a reasonable value.

In the production process of intermediate goods firms, the steady-state value of the technological process A is normalized to 1. To match an investment rate of 2.5%, the quarterly depreciation rate of physical capital δ_k is set to 0.025, which corresponds to 10% in annual terms. As is standard in the literature, the income share of capital is $\iota = 0.35$. This choice implies the elasticity of substitution between capital and high-skilled/low-skilled labor of $1/(1 - \iota) = 1.538$. The parameter governing the income share of high-skilled labor input $m = 0.6241$ is calibrated to match a skill (wage) premium of 1.55, the value provided by Bhattarai et al. (2022). Following Katz and Murphy (1992), the parameter σ is set to 0.2908, which implies the elasticity of substitution between high- and low-skilled labor of $1/(1 - \sigma) = 1.41$. The degree of real wage rigidity for both types of workers is set to 0.8 to be consistent with Dolado et al. (2021).

Silva and Toledo (2009) report that average training costs are 55% of quarterly wages in the US, while only around 5% of quarterly wages goes to average vacancy costs. According to Faccini and Melosi (2022), the corresponding value of the scaling parameter for training costs is $e = 5.0417$. Given that Faccini and Yashiv (2022) take this value as an approximation of high training costs, we assume that $e_w = e$. For the case of symmetric SAM frictions and symmetric training costs, the ratio of the scaling parameters is $e_w/e_p = 1$. If the case of asymmetric training costs is considered, this ratio is determined from the ratio of average hiring (training) costs in terms of wages.

There are two observations that the ratio of average hiring costs in terms of wages equals one. First, Blatter et al. (2012) compare the construction sector with the industrial (and service) sectors in terms of hiring costs in weeks of wage payments. They find that hiring costs in the construction sector are around 1/1.55 of those in the industrial (and service) sectors. Second, the construction sector is known to be characterized by lower skill requirements. Accordingly, the ratio of average hiring costs in terms of wages is given by:

$$\frac{(g^w_{int} \cdot mc^r / H_w)/w_w}{(g^p_{int} \cdot mc^r / H_p)/w_p} = 1, \quad \text{where} \quad g^k_{int} = \frac{e_k}{2}\left(\frac{H_k}{N_k}\right)^2 f_{int}, \quad \text{for} \quad k \in \{w, p\}$$

From the above equation, we can express a ratio of scaling parameters:

$$\frac{e_w}{e_p} = \frac{w_w}{w_p}\frac{\sigma_p}{\sigma_w}\frac{N_w}{N_p}$$

For the case of symmetric SAM frictions and asymmetric training costs, this ratio is $e_w/e_p = 1.55$. We will also consider the alternative values for the ratio e_w/e_p as Belo et al. (2017) indicate that the ratio of the labor adjustment costs parameters in the high- and low-skill industries is 10.5. A ratio of similar value can be found in Blatter et al. (2012) when comparing average hiring costs of occupations with the highest labor skills (an automation technician) and the lowest labor skills (a medical assistant).

The steady state gross inflation rate is normalized to 1. The elasticity of substitution across varieties ϵ is set to 11, which refers to a final good price mark-up of 10% over the intermediate good ($\mu^p = \frac{\epsilon}{\epsilon-1} = 1.1$). The Rotemberg quadratic adjustment cost parameter is set to 118.0521 to be consistent with the Calvo (1983) price stickiness model, where prices change on average once every fourth quarter. If the share of retailers that can adjust their prices is given by $1 - \theta$, then the value for parameter ϕ_p is

$$\phi_p = \frac{(\epsilon - 1)\theta}{(1-\theta)(1-\beta\theta)} = \frac{11 \cdot 0.75}{(1-0.75)(1-0.75 \cdot 0.9945)} = 118.0521$$

The capital adjustment costs parameter ϕ_k is set to 4 as in Dolado et al. (2021), which together with $\delta_k = 0.025$ implies that the elasticity of the investment-to-capital ratio with respect to Tobin's q is 10. The detailed derivation of this elasticity is given in Appendix A.2.

The steady state share of government expenditure in output is set to 20%, while a ratio of government debt to output is set to 2.8 or to 70% in annual terms. As for the fiscal and monetary policy parameters, they take common values in the literature. Specifically, the tax-feedback parameters related to government debt and spending are set to 0.33 and 0.1. In addition, the interest rate responsiveness to the inflation and output gaps are set to 1.5 and 0.5/4, while the interest rate smoothing parameter is 0.75.

Table 1: Parameter values for Model 1 (TANKrep: SAM+TC) and Model 2 (TANK: SAM+TC+FF)

Notation	Description	Model1 (Model2)	Source
Households			
β	Subjective discount factor	0.9945	quarterly R of 1%
η	Elasticity of labor supply	1	Convention
σ_c	Coefficient of relative risk aversion	1	Convention
$\varphi_{n,w}$	Relative weight on ℓ_w	23.1420 (2.9992)	Target is $partic\_w = 0.675$
$\varphi_{n,p}$	Relative weight on ℓ_p	14.9304 (6.5372)	Target is $partic\_p = 0.675$
s_w	Population share of the wealthy	0.5	Bhattarai et al. (2022)
Inter goods firms			
A	Production scale parameter	1	Convention
δ_k	Capital depreciation rate	0.025	quarterly I/K of 2.5%
ι	Income share of capital	0.35	Convention
m	Income share of high-skilled labor	0.6075	Target is $w_w/w_p = 1.55$
σ	Measure of elas of subs b/w N_w and N_p	0.2908	Katz and Murphy (1992)
ϕ_k	Capital adjustment cost	4	Dolado et al. (2021)
Final goods firms			
ϕ_p	Price adjustment cost	118.05	reset prices every 4 quarters
ϵ	Elas of subs between retail goods	11	mark-up of 10%
Labor market			
σ_w	Separation rate-wealthy	0.0404	Dolado et al. (2021)
σ_p	Separation rate-poor	0.0404	Dolado et al. (2021)
ρ_w^k	Wage stickeness	0.8	Dolado et al. (2021)
e_w	Hiring friction parameter-wealthy	5.079	Average hiring cost/wage
e_p	Hiring friction parameter-poor	5.079	Average hiring cost/wage
ϑ^w	Bargaining power-wealthy	0.7131	Target is $unemp\_w = 0.053$
ϑ^p	Bargaining power-poor	0.6159	Target is $unemp\_p = 0.053$
ς	Matching elasticity	0.5	Dolado et al. (2021)
ψ_w	Matching efficiency-wealthy	0.5875	Dolado et al. (2021)
ψ_p	Matching efficiency-poor	0.5875	Dolado et al. (2021)
Fis and mon policy			
ϕ_B	Tax response to debt	0.33	Convention
ϕ_{BG}	Tax response to gov spending	0.1	Convention
θ_π	Monetary policy response to inflation	1.5	Convention
θ_y	Monetary policy response to output	0.125	Convention
θ_r	Monetary policy inertia	0.75	Convention
ϕ_g	Gov spending persistence	0.9	Faccini and Melosi (2022)
σ_g	Volatility of gov spending shock	0.01	Faccini and Melosi (2022)

Notes: TANKrep - two types of workers live together in one representative household, TANK - two types of workers live separately in their own representative household, SAM - symmetric search and matching frictions, TC - symmetric training costs, and FF - financial friction.

Table 2: Selected Steady-state values of variables

Notation	Description	Model 1	Model 2	Model 3	Model 4	Data
Targeted						
G/Y	Government consumption to GDP ratio	0.2	0.2	0.2	0.2	0.2
$B/4Y$	Debt to GDP ratio (annualised)	0.7	0.7	0.7	0.7	0.7
ci_share	Capital income share	0.35	0.35	0.35	0.35	0.35
w_w/w_p	Skill premium	1.55	1.55			1.55
$partic\_w$	Participation rate - wealthy	0.675	0.675			0.675
$partic\_p$	Participation rate - poor	0.675	0.675			0.675
$unemp\_w$	Unemployment rate - wealthy	0.053	0.053			0.053
$unemp\_p$	Unemployment rate - poor	0.053	0.053			0.053
Non-targeted						
μ_w/μ_p	Ratio of job finding rates			0.78	1.15	1.15
θ_w/θ_p	Ratio of labor market tightness			0.61	0.53	0.65
ν_w/ν_p	Vacancy filling probabilities			1.28	2.17	1.74
w_w/w_p	Skill premium			1.58	1.50	1.55
$partic\_w$	Participation rate - wealthy			0.674	0.677	0.69
$partic\_p$	Participation rate - poor			0.678	0.677	0.66
$unemp\_w$	Unemployment rate - wealthy			0.053	0.019	0.028
$unemp\_p$	Unemployment rate - poor			0.034	0.067	0.078
$\frac{(g^w_{int} \cdot mc^r/H_w)/w_w}{(g^p_{int} \cdot mc^r/H_p)/w_p}$	Ratio of average hiring cost to wage	0.645	0.645	1	0.429	1
g_{int}/Y	Hiring cost to GDP ratio	0.008	0.008	0.007	0.007	0.01

Notes: Model 1 is TANKrep: SAM+TC. Model 2 is TANK: SAM+TC+FF. Model 3 is TANKrep: SAM+ATC, which has the same parameter values from Model 1 and $e_w = 5.079 > e_p = 3.277$. Model 4 is TANK: ASAM+ATC+FF, which has the same parameter values from Model 2 and $\psi_w = 0.720 > \psi_p = 0.455$, $\sigma_w = 0.025 < \sigma_p = 0.056$ and $e_w = 5.079 > e_p = 3.277$.

6. Results

This section is divided into three parts. The first part reports the responses of the real economy in the TANKrep and TANK models and how the change in the form of hiring costs[9] (vacancy costs or training costs) affects the propagation of a government spending shock to the real economy. The second part examines whether the symmetric or asymmetric forms of search and matching frictions interacted with training costs play a dominant role in driving the responses of the real output. In this part of the analysis, the focus is on training costs rather than vacancy costs as they may generate counterintuitive recessionary effects of expansionary policies (see, for instance, Picco, 2020 and Faccini and Yashiv, 2022). The third part provides a more general picture of the dynamic responses of many real economic variables of interest. The analysis in all three parts is conducted in both the representative and heterogeneous agent frameworks. The consideration of the latter setting is the novel contribution of this paper to the literature.

Figure 1 shows impulse responses of output, hiring, employment and the value of output in the TANKrep model to an expansionary fiscal policy shock, which corresponds to a rise in government spending of one percent of steady-state output. As opposed to the RANK model, which shows recessionary effects, the TANKrep model reports economic expansion. The rise in output in the TANKrep model is dependent on the extent of training costs of the new hires. With smaller training costs for low-skilled workers, the output records a stronger expansion. When the value of output is high, firms decide to hire low-skilled workers to a large extent, while the hiring of high-skilled workers is negative or small. When the value of output is low, firms choose to hire more high-skilled workers whose marginal productivity is higher.

In Figure 2, the expansionary effects of government spending can be observed for the RANK models with hiring costs modelled as vacancy posting costs, and the TANK model with training costs. By contrast, the persistent recessionary effects are distinctive to the model with one type of workers where hiring frictions are expressed as training costs or in terms of foregone output. These results are in line with the section dedicated to the transmission mechanism. Additionally, the RANK model with vacancy costs in pecuniary terms or in units of final good generates larger expansionary effects than the RANK model with vacancy costs in non-pecuniary terms or in units of intermediate good. The reason is that non-pecuniary hiring costs cause disruption in production, while pecuniary hiring costs are characterized by third-party payments for the provision of hiring services.

The impulse response analysis in Figures 1 and 2 emphasizes that expansionary effects are still present despite modelling hiring costs in terms of foregone output, and even surpass the effects of vacancy costs. Note that these results are generated under the assumption of flexible wages. In the next part of the analysis, we examine how the responses of the real economy change when real wage rigidity is

[9] A formal presentation of different forms of hiring costs is provided in Appendix A.3.

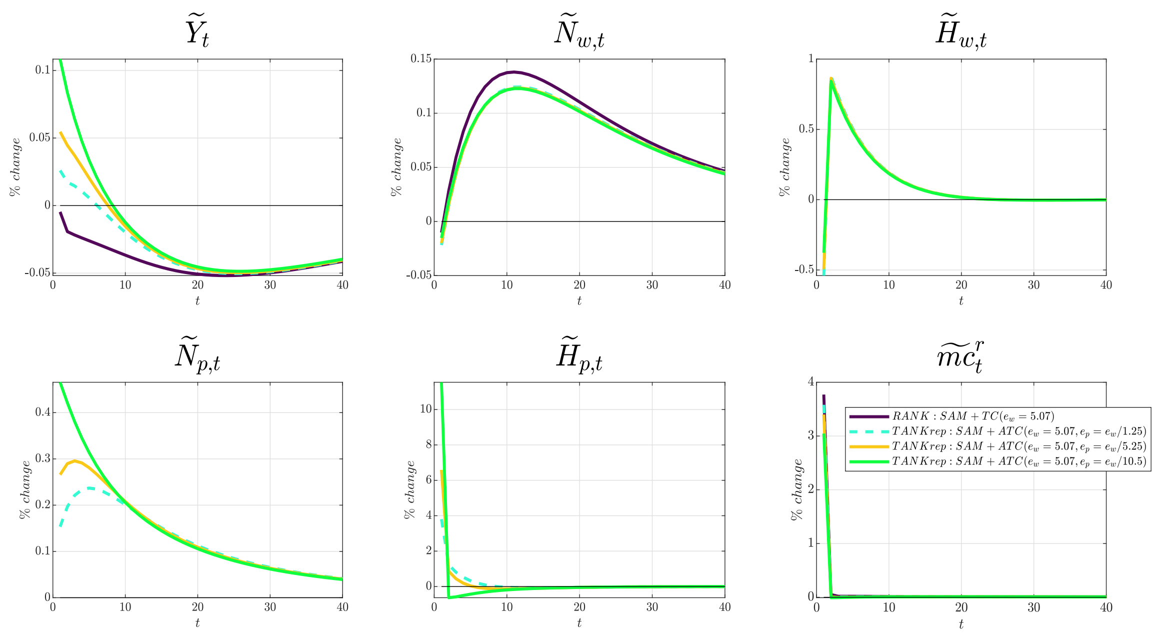

Figure 1: Impulse responses to a fiscal expansion when real wages are flexible

Notes: RANK: SAM + TC refers to the representative agent New Keynesian model with search and matching frictions SAM and training cost TC. TANKrep: SAM + ATC is the two agent New Keynesian model with a representative household. SAM frictions include matching efficiency ψ_k, separation rate σ_k, and bargaining power ϑ^k. The results of these models are generated for the case of absent real wage rigidity, $\rho_w^k = 0$.

introduced.

Figure 3 displays the output responses to an increase in government spending assuming real wage stickiness and the interaction of SAM frictions with training costs. The left panel differs from the right panel in that it considers only the economies characterized by symmetry in SAM frictions and training costs. If we focus on the left panel, despite the symmetry in labor market frictions, the response of output is markedly different. Initially, the economies with two types of workers (blue and red solid lines) experience a drop in output, which is then followed by an expansion. However, this output reduction is much less persistent and pronounced relative to the economy with only one type of workers (purple solid line). In addition, models with two types of workers document output recovery to its pre-crisis average level after around two years, while output in a model with one type of workers does not complete its recovery even after 10 years.

The firm's hiring decision lies at the core of the output responses. The models with one type and two types of workers both report a rise in output in the first period, which is an indication of a greater marginal benefit than the marginal cost of hiring. However, this output expansion is small, as hiring is associated with training costs that swallow up output. In the next period, the assumption of sticky wages and a training costs specification imply a more expensive hiring of new workers. Specifically, a higher aggregate demand pressure in the first period leads to a rise in labor demand and wages, which

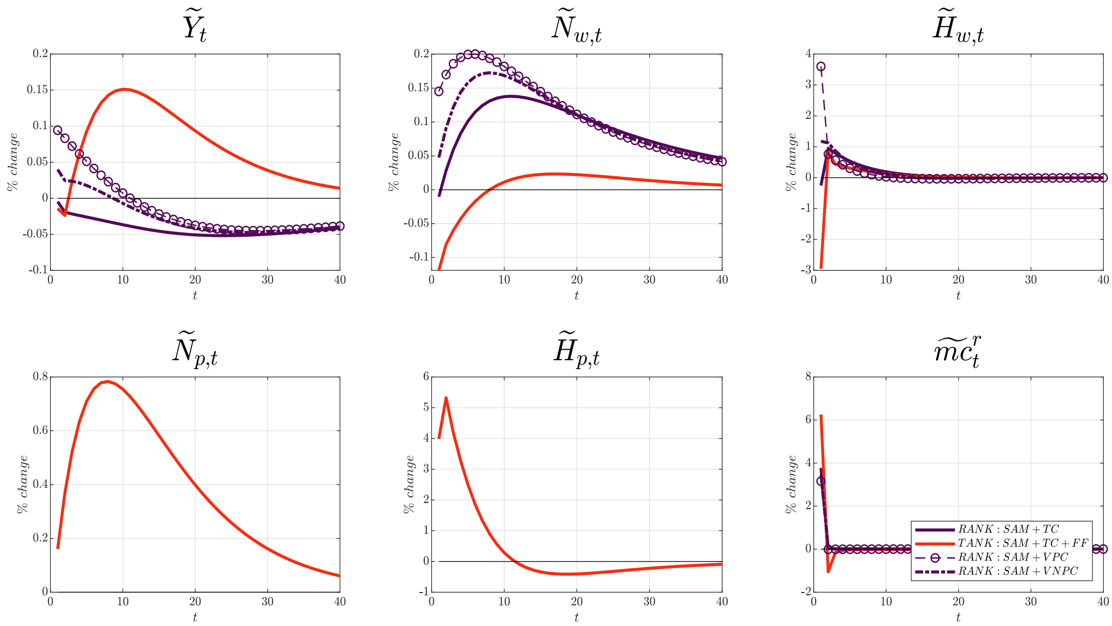

Figure 2: Impulse responses to a fiscal expansion when real wages are flexible

Notes: RANK: SAM + TC refers to the representative agent New Keynesian model with search and matching frictions SAM and training cost TC. TANK: SAM + TC + FF is the two agent New Keynesian model where households face asymmetric financial friction. SAM frictions include matching efficiency ψ_k, separation rate σ_k, and bargaining power ϑ^k. VPC is vacancy posting costs in pecuniary terms while VNPC is vacancy posting costs in non-pecuniary terms. The results of the stated models are generated for the case of absent real wage rigidity, $\rho_w^k = 0$.

are largely transmitted to wages in the next period due to wage rigidity. In addition, training activity causes production disruption so that firms would rather choose to postpone hiring and focus on sales that are more profitable at the time of a high value of output. With relatively high hiring costs in the RANK model as in Faccini and Yashiv (2022) and Picco (2020), weak employment and output occur. However, in the economy populated with two types of workers, firms have a choice in a hiring process. When aggregate demand pressure is large, a cheaper labor force such as low-skilled workers can be used to sustain production until the period of relatively low value of output.

In the left panel of Figure 3, a blue solid line represents the contribution of adding financial friction (FF) to the RANK model. To measure the influence of FF, workers need to be equally productive, alongside the symmetry in SAM frictions and training costs. This is achieved by adding symmetry in their skill intensity $m = 0.5$ and perfect substitutability in the production function $\sigma = 1$. With no skill mismatch in production, the fall in output and subsequent expansion are mitigated compared to the red solid line. Note that higher skill intensity and higher complementarity of labor inputs are incorporated in the model in line with observed labor market dynamics in the US economy[10]. When

[10] As stated in calibration, skill premium and imperfect substitutability between high-skilled and low-skilled workers underlie the labor market in the US.

$m > 0.5$, high-skilled workers are more present in production. However, initial periods feature a higher value of output, which under higher marginal costs of hiring of high-skilled workers leads to a larger output contraction[11]. In later periods, when the value of output is lower, firms are incentivized to hire more for two reasons: the lower value of foregone output and larger production capacity, as higher skill intensity is associated with higher marginal productivity of high-skilled workers. Firms also hire more low-skilled workers due to higher complementarity between the two types of workers, $\sigma < 1$.

The right panel of Figure 3 focuses on the interaction of the symmetric and asymmetric forms of SAM frictions and training costs. In comparison with the RANK model, all models presented with two groups of workers characterize the expansionary effects of government spending. Asymmetric SAM frictions go in favour of high-skilled workers, but generate a slightly stronger economic expansion (yellow solid line) than symmetric SAM frictions (red solid line). A stronger economic expansion is recorded for asymmetric training costs that favor low-skilled workers (compare the green and red solid lines).

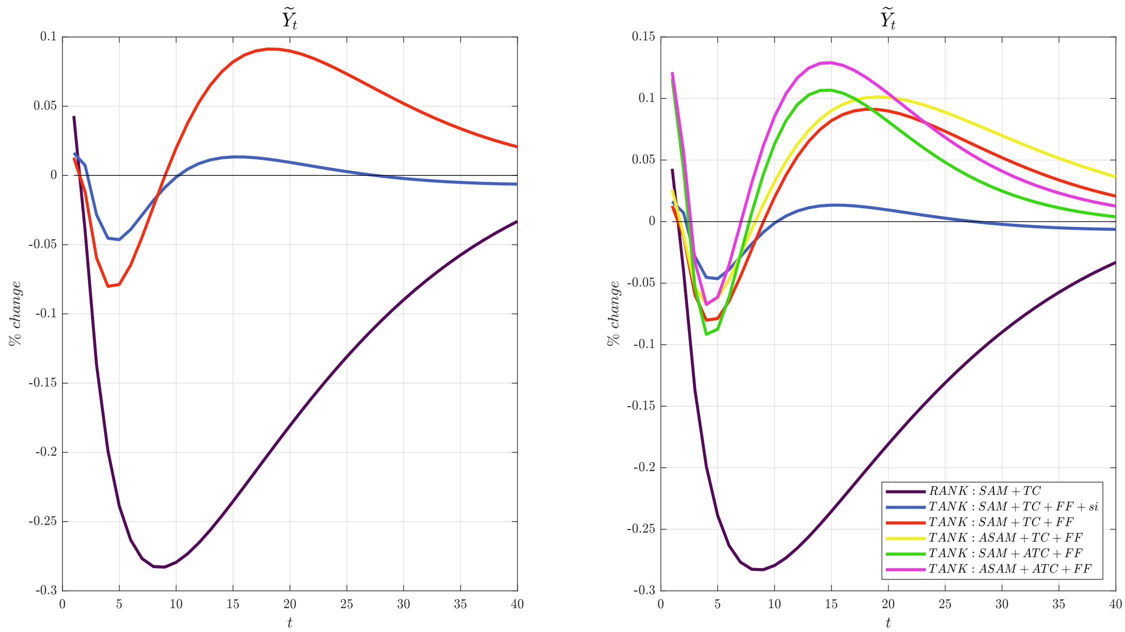

Figure 3: Impulse responses of the economy to a fiscal expansion when real wages are rigid

Notes: SAM - search and matching frictions, TC - training costs, ASAM - asymmetric search and matching frictions in all three parameters ($\psi_w \neq \psi_p$, $\vartheta^w \neq \vartheta^p$, $\sigma_w \neq \sigma_p$), ATC - asymmetric training costs ($e_p = e_w/5.25$), si - symmetric skill intensity, and FF - financial friction. The results of the stated models are generated for the case of real wage rigidity, $\rho_w^k = 0.8$.

The third part of this section provides the impulse response analysis of several real economic variables

[11] A larger skill intensity in production leaves less space for low-skilled workers, who are a cheaper labor force, due to financial friction. The role of this type of workers is especially important for sustaining production in the period of a high value of output.

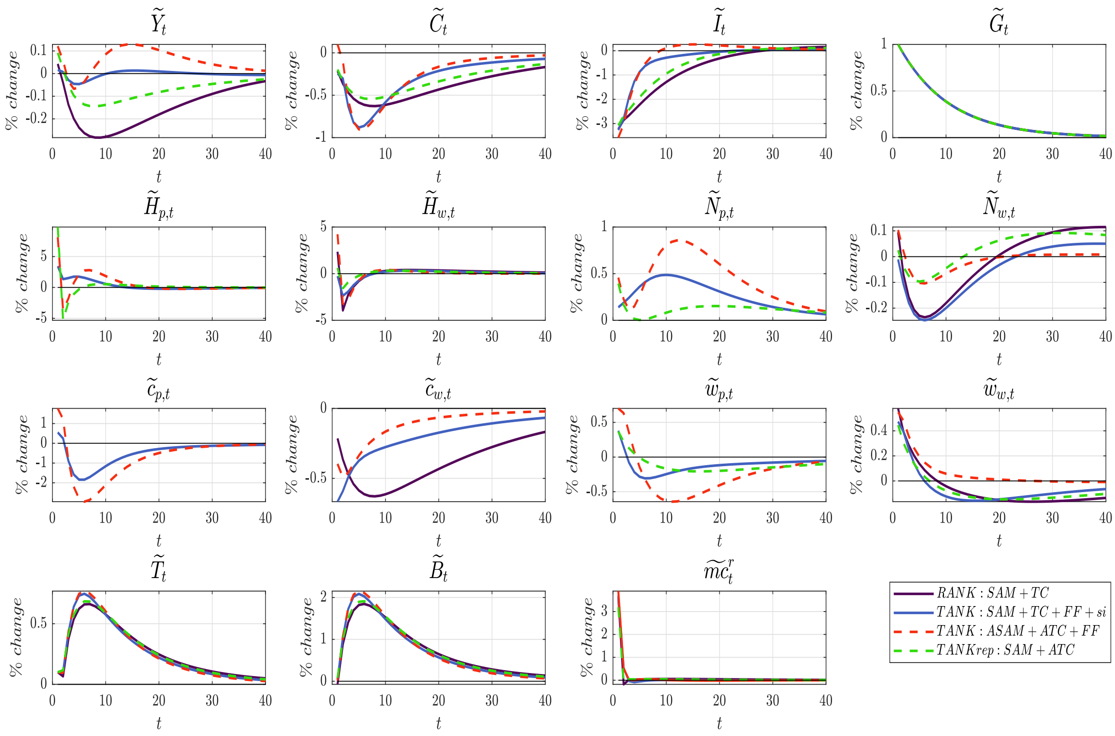

Figure 4: Impulse responses of the economy to a fiscal expansion when real wages are rigid

Notes: SAM - search and matching frictions, TC - training costs, ASAM - asymmetric search and matching frictions in all three parameters ($\psi_w \neq \psi_p$, $\vartheta^w \neq \vartheta^p$, $\sigma_w \neq \sigma_p$), ATC - asymmetric training costs ($e_p = e_w/5.25$), *si* - symmetric skill intensity, and FF - financial friction. The results of the stated models are generated for the case of real wage rigidity, $\rho_w^k = 0.8$.

of interest besides the real output. A government spending shock, which can be interpreted as an aggregate demand shock, gives rise to elevated aggregate demand pressures in the economy (see the dynamics of $\widetilde{mc}_t^r$ in Figure 4). To keep the budget balanced over time, the government finances an increased demand for goods by raising lump-sum taxes and issuing debt. The negative wealth effect of government spending comes into play as agents in the economy perceive that the fiscal stimulus goes hand-in-hand with higher tax payments. In response to a lower disposable income, workers participate more actively in the labor market. In addition to consuming less leisure, high-skilled workers decide to consume less consumption goods and save more. More precisely, they save more in the form of government bonds at the expense of capital investment due to a greater demand of the government for bonds and a greater real interest rate.

In Figure 4, the four model specifications display qualitatively similar results regarding the crowding-out of private consumption and capital investment. However, the quantitative effects are different, especially those related to the response of investment in capital. A faster recovery of investment is observed for the TANKrep and TANK models. These models underlie a higher level of hiring and associated employment, which limits a drop in investment. Firms accumulate more capital to ensure that an increasing number of workers in production is equipped with a sufficient level of capital. Better investment opportunities exert a stimulative impact on production activities.

6.1. Productivity-enhancing government spending and the fiscal multiplier

Given a close relationship between capital investment and job creation in the economy, it is useful to perform a counterfactual analysis on whether that relationship can be improved. Specifically, this analysis compares the effectiveness unproductive government spending considered so far with productive spending. Productivity-enhancing government spending assumes that government capital enters the aggregate production function of intermediate goods firms

$$f_{int,t} = AK_t^{\iota}\Big[m(N_{w,t})^{\sigma} + (1-m)(N_{p,t})^{\sigma}\Big]^{\frac{1-\iota}{\sigma}} K_{g,t}^{\zeta}$$

where $K_{g,t}$ is productive government capital and ζ is a parameter that determines the productivity of government capital. Moreover, we specify the law of motion of government investment

$$G_{I,t} = K_{g,t} - (1-\delta_{k_g})K_{g,t-1}$$

and an AR(1) process of government investment

$$G_{I,t} = G_I^{1-\phi_{g_I}}(G_{I,t-1})^{\phi_{g_I}} exp(\epsilon_t^{g_I}), \quad \epsilon_t^{g_I} \sim \mathcal{N}(0, \sigma_{g_I})$$

We follow Sims and Wolff (2018) in setting a value of parameter $\zeta = 0.05$, the depreciation rate on government capital $\delta_{k_g} = 0.025$, a value of parameter $\phi_{g_I} = 0.9338$, and the value of the steady-state capital ratio $\frac{K_{g,t}}{K_t} = 0.165$. Note that a value of parameter $\zeta = 0$ returns the benchmark specification with only unproductive government spending. There are two useful observations in Figure 5 regarding the effects of government investment. First, productivity-enhancing government spending generates larger expansionary effects in both models (compare dashed lines with solid lines). This is because government investment leads to a higher marginal productivity of labor inputs, which is then transmitted to increased labor demand of firms. Hence, from a policy maker's perspective, it is better to use government investment than government consumption to deal with recessions. Second, the expansionary effects of government investment in the TANK model are stronger and more persistent than in the RANK model.

The next part of the analysis focuses on the cumulative fiscal multiplier, which is defined as a ratio of the cumulative sum of the discounted percentage output changes and that of government spending changes for a given horizon k

$$fm = \frac{\sum_{k=0}^{\infty}\beta^k dY_k}{\sum_{k=0}^{\infty}\beta^k dG_k}$$

For $k = 0$, the above expression refers to the impact fiscal multiplier.

Table 3 shows that the RANK model has negative fiscal multipliers of government consumption for both types of wage specifications and over all horizons (the exception is the first period in the model with

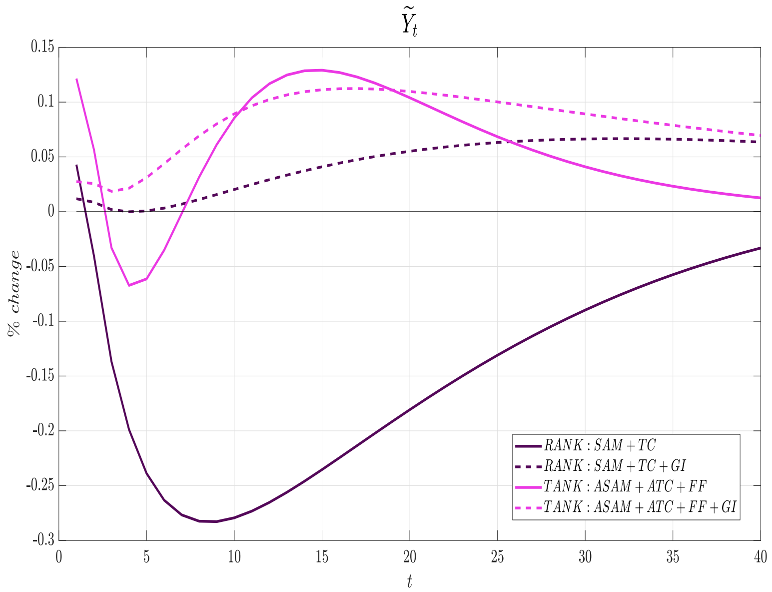

Figure 5: Impulse responses of the economy to a fiscal expansion when real wages are rigid

Notes: SAM - search and matching frictions, TC - training costs, ASAM - asymmetric search and matching frictions in all three parameters ($\psi_w \neq \psi_p$, $\vartheta^w \neq \vartheta^p$, $\sigma_w \neq \sigma_p$), ATC - asymmetric training costs ($e_p = e_w/5.25$), and FF - financial friction. GI is government investment. The results of the stated models are generated for the case of real wage rigidity, $\rho_w^k = 0.8$. Solid lines indicate impulse responses of output to government consumption shocks, while dashed lines indicate impulse responses of output to government investment shocks.

rigid wages). The reason lies in strong crowding-out of aggregate demand components such as private consumption and capital investment. When flexible wages are considered, the TANK model shows positive (cumulative) fiscal multipliers of government consumption. As for the rigid wages specification, positive fiscal multipliers are documented for the TANK model with asymmetric search and matching frictions and asymmetric training costs. In addition, the TANK model with government investment has positive fiscal multipliers over all horizons. Compared to the RANK model with government investment, the fiscal multiplier in the TANK model is more than twice as large.

Table 3: Fiscal multipliers across different models and different horizons

Model	Horizon k 1Q	1Y	5Y	10Y	250Y
Flexible wages					
$RANK : SAM + TC$	-0.005	-0.017	-0.055	-0.101	-0.245
$TANKrep : SAM + ATC$	0.055	0.050	0.009	-0.042	-0.186
$TANK : SAM + TC + FF$	-0.015	0.001	0.167	0.236	0.288
$RANK : SAM + VPC$	0.095	0.092	0.058	0.010	-0.130
$RANK : SAM + VNPC$	0.039	0.033	0.008	-0.038	-0.179
Rigid wages					
$RANK : SAM + TC$	0.042	-0.061	-0.363	-0.493	-0.561
$TANKrep : SAM + ATC$	0.092	0.015	-0.161	-0.225	-0.302
$TANK : SAM + TC + FF + si$	0.016	-0.005	-0.019	-0.012	-0.028
$TANK : SAM + TC + FF$	0.012	-0.026	-0.009	0.069	0.145
$TANK : ASAM + TC + FF$	0.026	-0.017	0.012	0.102	0.216
$TANK : SAM + ATC + FF$	0.116	0.032	0.042	0.103	0.136
$TANK : ASAM + ATC + FF$	0.122	0.048	0.080	0.158	0.216
$RANK : SAM + TC + GI$	0.055	0.035	0.113	0.317	1.055
$TANK : ASAM + ATC + FF + GI$	0.128	0.121	0.437	0.755	1.550

Notes: SAM - search and matching frictions, TC - training costs, ASAM - asymmetric search and matching frictions in all three parameters ($\psi_w \neq \psi_p$, $\vartheta^w \neq \vartheta^p$, $\sigma_w \neq \sigma_p$), ATC - asymmetric training costs ($e_w = 5.07, e_p = e_w/5.25$) and FF - financial friction, VPC and VNPC are vacancy posting costs in pecuniary and non-pecuniary terms, si - symmetric skill intensity. GI is government investment. Real wage rigidity is introduced with $\rho_w^k = 0.8$.

7. Conclusion

This study examines the effects of increased government spending on the real economy in the presence of large training costs of newly hired workers. For that purpose, we build a TANKrep model with asymmetric training costs and a TANK model that includes the interaction between asymmetric training costs and financial friction. The heterogeneous market structure of these models shows the expansionary effects of the fiscal stimulus, in contrast to the recessionary effects indicated by the literature that relies on the representative agent setting. A different hiring decision of firms plays an essential role in shaping a different response of the real economy to the fiscal stimulus. The firms' investment in training activity for new hires causes production disruption, as some experienced workers are diverted from production to training the new hires. Training costs are a common feature of both the representative and heterogeneous agent frameworks. In the period of high aggregate demand pressure, the value of forgone output is large, which under large training costs reduces the incentives of firms to hire. What makes two frameworks different is the firms' chance to choose the cheaper type of workers at a time of high marginal costs of hiring driven by high aggregate demand pressure. This is the case with the heterogeneous agent framework, where firms choose low-skilled workers and postpone hiring of high-skilled workers. Lower training costs for low-skilled workers stimulates their hiring, and the addition of financial friction further amplifies this hiring. Financial friction constrains the access of low-skilled workers to financial markets, which leads to their lower reservation wage and then market wage. There are two broad types of government spending that fiscal authorities can implement: government consumption and government investment. Given that government investment generates more expansionary effects in terms of the output multiplier, fiscal authorities may use it as a more efficient tool to deal with recessions.

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Appendix A: Model Derivation

A.1 Derivation of the aggregate resource constraint

If $n-1$ market clearing conditions are satisfied in equilibrium, then by Walras's Law, the n^{th} (goods) market clears in equilibrium too.

To derive the aggregate resource constraint, we combine the following equations:

1. The real budget constraint of wealthy households:

$$s_w\left(c_{w,t}+t_{w,t}+i_t+b_t=w_{w,t}n_{w,t}+r_t^k k_{t-1}+\frac{R_{t-1}b_{t-1}}{\pi_t}+\frac{\Pi_t^{int}}{s_w}+\frac{\Pi_t^r}{s_w}\right)$$

2. The real budget constraint of poor households:

$$s_p\Big(c_{p,t}+t_{p,t}=w_{p,t}n_{p,t}\Big)$$

3. The definition of real profits and output:

$$\Pi_t^{int}=\frac{P_{int,t}}{P_t}Y_{int,t}-w_{w,t}N_{w,t}-w_{p,t}N_{p,t}-r_t^k K_t,$$

$$\Pi_t^r=\left(1-\frac{1}{x_t}-\frac{\phi_p}{2}(\frac{\pi_t}{\pi}-1)^2\right)Y_t,$$

$$Y_t=Y_{int,t}$$

4. The real government budget constraint:

$$T_t+B_t=\frac{R_{t-1}B_{t-1}}{\pi_t}+G_t$$

The distribution of lump-sum taxes can be expressed from the government budget constraint:

$$T_t\equiv s_w t_{w,t}+s_p t_{p,t}=\frac{R_{t-1}B_{t-1}}{\pi_t}+G_t-B_t$$

Substitution of lump-sum taxes paid by households into the government budget constraint yields:

$$s_w w_{w,t}n_{w,t}+s_w r_t^k k_{t-1}+s_w\frac{R_{t-1}b_{t-1}}{\pi_t}+\Pi_t^{int}+\Pi_t^r-s_w(c_{w,t}+i_t+b_t)+$$

$$+s_p w_{p,t}n_{p,t}-s_p c_{p,t}=\frac{R_{t-1}B_{t-1}}{\pi_t}+G_t-B_t$$

Aggregating terms in the previous expression

$$w_{w,t}N_{w,t}+r_t^k K_t+\frac{R_{t-1}B_{t-1}}{\pi_t}-C_{w,t}-I_t-B_t$$

$$+\left[\frac{Y_{int,t}}{x_t}-w_{w,t}N_{w,t}-w_{p,t}N_{p,t}-r_t^k K_t\right]+\left[\left(1-\frac{1}{x_t}-\frac{\phi_p}{2}(\frac{\pi_t}{\pi}-1)^2\right)Y_t\right]+$$

$$+w_{p,t}N_{p,t}-C_{p,t}=\frac{R_{t-1}B_{t-1}}{\pi_t}+G_t-B_t$$

and given that market clearing conditions hold for labor, capital and bond markets, the aggregate resource constraint (or the goods market clearing condition) becomes:

$$Y_t = C_t + I_t + G_t + \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t$$

A.2 Derivation of the elasticity of the investment to capital ratio with respect to Tobin's q

The Lagrangean function for the optimization problem of wealthy households:

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Bigg\{ \frac{1}{1-\sigma_c}(c_{w,\tau} - hC_{w,\tau-1})^{1-\sigma_c} - \varphi_{n,w}\frac{(\ell_{w,\tau})^{1+\eta}}{1+\eta} - \lambda^c_{w,\tau}\Bigg(c_{w,t} + t_{w,t} + i_t + b_t -$$

$$- w_{w,t}n_{w,t} - r^k_t k_{t-1} - \frac{R_{t-1}b_{t-1}}{\pi_t} - \frac{\Pi^{int}_t}{s_w} - \frac{\Pi^r_t}{s_w} + \lambda^n_{w,\tau}\left(n_{w,t} - (1-\sigma_w)n_{w,t-1} - \frac{\mu_{w,t}}{1-\mu_{w,t}}u_{w,t}\right)\Bigg) +$$

$$+ \lambda^l_{w,\tau}(n_{w,t} + u_{w,t} - \ell_{w,t}) - Q_t\Big((k_t - (1-\delta_k)k_{t-1}) + \frac{\phi_k}{2}\left(\frac{i_t}{k_{t-1}} - \delta_k\right)^2 k_t - i_t\Big)\Bigg\}$$

where $q_t = Q_t/\lambda^c_{w,t}$ is the Tobin's q marginal ratio.

The derivative of the optimization problem of wealthy households with respect to investment is:

$$-\lambda^c_{w,t} - Q_t\phi_k\left(\frac{i_t}{k_{t-1}} - \delta_k\right)\frac{k_t}{k_{t-1}} + Q_t = 0$$

$$\frac{\lambda_{w,t}}{Q_t} = 1 - \phi_k\left(\frac{i_t}{k_{t-1}} - \delta_k\right)\frac{k_t}{k_{t-1}}$$

$$\frac{1}{q_t} = 1 - \phi_k\left(\frac{i_t}{k_{t-1}} - \delta_k\right)\frac{k_t}{k_{t-1}}$$

$$\frac{i_t}{k_{t-1}} = \left(-\frac{1}{q_t} + 1\right)\frac{k_{t-1}}{\phi_k k_t} + \delta_k$$

$$log\left(\frac{i_t}{k_{t-1}}\right) = log\left(\left(-e^{-log(q_t)} + 1\right)\frac{k_{t-1}}{\phi_k k_t} + \delta_k\right)$$

The elasticity of the investment to capital ratio with respect to Tobin's q is

$$\frac{\partial log\left(\frac{i_t}{k_{t-1}}\right)}{\partial log(q_t)} = \frac{1}{\left(-e^{-log(q_t)} + 1\right)\frac{k_{t-1}}{\phi_k k_t} + \delta_k}\left(-\frac{k_{t-1}}{\phi_k k_t}e^{-log(q_t)}(-1)\right)$$

In steady state, the previous expression is evaluated as

$$\frac{\partial log\left(\frac{i}{k}\right)}{\partial log(q)} = \frac{1}{\delta_k}\frac{1}{\phi_k}$$

For $\delta_k = 0.025$ and $\phi_k = 4$, we have:

$$\varrho_k = \frac{1}{\delta_k \cdot \phi_k} = \frac{1}{0.025 \cdot 4} = 10$$

A.3 Different forms of hiring costs

The real profit of intermediate goods firms given different forms of hiring costs:

1. training costs

$$\Pi_t^{int} = \frac{Y_{int,t}}{x_t} - w_{w,t}N_{w,t} - w_{p,t}N_{p,t} - r_t^k K_t,$$

$$Y_{int,t} = f_{int,t}\Big(1 - \sum_{k\in\{w,p\}} \tilde{g}_{int,t}^k\Big) \quad \text{for} \quad \tilde{g}_{int,t}^k = \frac{e_k}{2}\left(\frac{H_{k,t}}{N_{k,t}}\right)^2$$

2. vacancy costs in pecuniary terms

$$\Pi_t^{int} = \frac{f_{int,t}}{x_t} - w_{w,t}N_{w,t} - w_{p,t}N_{p,t} - r_t^k K_t - \sum_{k\in\{w,p\}} \frac{e_k}{2}\left(\frac{\upsilon_{k,t}}{N_{k,t}}\right)^2 f_{int,t}$$

3. vacancy costs in non-pecuniary terms

$$\Pi_t^{int} = \frac{Y_{int,t}}{x_t} - w_{w,t}N_{w,t} - w_{p,t}N_{p,t} - r_t^k K_t,$$

$$Y_{int,t} = f_{int,t}\Big(1 - \sum_{k\in\{w,p\}} \tilde{g}_{int,t}^k\Big) \quad \text{for} \quad \tilde{g}_{int,t}^k = \frac{e_k}{2}\left(\frac{\upsilon_{k,t}}{N_{k,t}}\right)^2$$

A.4 The log-linearized system of equations

To study the dynamics of the model, this section specifies the log-linearized version of the model, where $\widetilde{x}_t$ indicates the log deviation of any variable x_t from its non-stochastic steady state x, i.e. $\widetilde{x}_t = log(x_t/x) \simeq (x_t - x)/x$. The exception holds for the fiscal variables (government spending, taxes and bonds) and profits, which are measured in percentage deviation relative to the non-stochastic steady-state level of output.

A.4.1 Labor market

1. Aggregate labor force participation:

$$L_k\widetilde{L}_{k,t} = N_k\widetilde{N}_{k,t} + U_k\widetilde{U}_{k,t}$$

2. Aggregate number of newly hired workers:

$$\widetilde{H}_{k,t} = \varsigma\widetilde{\upsilon}_{k,t} + (1-\varsigma)\widetilde{U}_{0,t}^k$$

3. Labor market tightness:

$$\widetilde{\theta}_{k,t} = \widetilde{\upsilon}_{k,t} - \widetilde{U}_{0,t}^k$$

4. Vacancy filling probabilities:

$$\widetilde{\nu}_{k,t} = \widetilde{H}_{k,t} - \widetilde{\upsilon}_{k,t}$$

5. Hiring probabilities:

$$\widetilde{\mu}_{k,t} = \widetilde{H}_{k,t} - \widetilde{U}_{0,t}^k$$

6. The aggregate job seekers at the beginning of period t:

$$U_0^k \widetilde{U}_{0,t}^k - \mu_k U_0^k(\widetilde{\mu}_{k,t} + \widetilde{U}_{0,t}^k) = U\widetilde{U}_t$$

7. Law of motion for employment:

$$\widetilde{N}_{k,t} = (1-\sigma_k)\widetilde{N}_{k,t-1} + \sigma_k \widetilde{H}_{k,t}$$

8. The Nash bargained wage for households $k \in \{w,p\}$:

$$w_k^* \widetilde{w}_{k,t}^* = \vartheta^k \left(\frac{f_{N_k}}{x}(\widetilde{f}_{N_{k,t}} - \widetilde{x}_t) - \frac{g_{N_k}}{x}(\widetilde{g}_{N_{k,t}} - \widetilde{x}_t) \right) - (1-\vartheta^k)\frac{\lambda_k^l}{\lambda_k^c}\left(\widetilde{\lambda}_{k,t}^l - \widetilde{\lambda}_{k,t}^c\right) +$$

$$+ \vartheta^k(1-\sigma_k)\beta Q_k^{*,N}(\widetilde{\lambda}_{w,t+1}^c - \widetilde{\lambda}_{w,t}^c + \widetilde{Q}_{k,t+1}^{*,N}) - (1-\vartheta^k)(1-\sigma_k)\beta\lambda_k^{*,n}(\widetilde{\lambda}_{k,t+1}^c - \widetilde{\lambda}_{k,t}^c + \widetilde{\lambda}_{k,t+1}^{*,n})$$

In Nash bargaining process, the surplus sharing rule is $\widetilde{Q}_{k,t}^{*,N} = \widetilde{\lambda}_{k,t}^{*,n}$.

9. Inertial real wage for households with skill level $k \in \{w,p\}$:

$$\widetilde{w}_{k,t} = \rho_w^k \widetilde{w}_{k,t-1} + (1-\rho_w^k)\widetilde{w}_{k,t}^*$$

A.4.2 Wealthy households

1. FOC with respect to consumption:

$$\widetilde{\lambda}_{w,t}^c = \frac{-\sigma_c(c_w\widetilde{c}_{w,t} - hC_w\widetilde{C}_{w,t-1})}{(c_w - hC_w)}$$

2. FOC with respect to employment:

$$\lambda_w^n \widetilde{\lambda}_{w,t}^n = \frac{\lambda_w^l}{\lambda_w^c}(\widetilde{\lambda}_{w,t}^l - \widetilde{\lambda}_{w,t}^c) + w_w\widetilde{w}_{w,t} + \mathbb{E}_t\,\beta(1-\sigma_w)\lambda_w^n(\widetilde{\lambda}_{w,t+1}^c - \widetilde{\lambda}_{w,t}^c + \widetilde{\lambda}_{w,t+1}^n)$$

3. FOC with respect to the participation in the labor market:

$$\widetilde{\lambda}_{w,t}^l = \eta\widetilde{\ell}_{w,t}$$

4. FOC with respect to unemployment:

$$\widetilde{\lambda}_{w,t}^l = \widetilde{\lambda}_{w,t}^c + \widetilde{\lambda}_{w,t}^n + \frac{1}{1-\mu_w}\widetilde{\mu}_{w,t}$$

5. FOC with respect to bonds:

$$\widetilde{\lambda}_{w,t}^c = \mathbb{E}_t\,\frac{\beta R}{\pi}(\widetilde{\lambda}_{w,t+1}^c + \widetilde{R}_t - \widetilde{\pi}_{t+1})$$

6. FOC with respect to physical capital:

$$\widetilde{\lambda}_{w,t}^c + \phi_k\widetilde{k}_t - \phi_k\widetilde{k}_{t-1} = \mathbb{E}_t\,\beta\Big((1-\delta_k)\widetilde{\lambda}_{w,t+1}^c + r^k(\widetilde{\lambda}_{w,t+1}^c + \widetilde{r}_{t+1}^k) + \phi_k\widetilde{k}_{t+1} - \phi_k\widetilde{k}_t\Big)$$

7. The budget constraint of wealthy households:

$$c_w\widetilde{c}_{w,t} + Y\widetilde{t}_{w,t} + k(\widetilde{k}_t - (1-\delta_k)\widetilde{k}_{t-1}) + Y\widetilde{b}_t$$

$$= w_w n_w(\widetilde{w}_{w,t} + \widetilde{n}_{w,t}) + kr^k(\widetilde{k}_{t-1} + \widetilde{r}^k_t) + \frac{Rb}{\pi}(\widetilde{R}_{t-1} + \frac{Y}{b}\widetilde{b}_{t-1} - \widetilde{\pi}_t) + \frac{Y}{s_w}\widetilde{\Pi}^{int}_t + \frac{Y}{s_w}\widetilde{\Pi}^r_t$$

8. The law of motion of capital:

$$\widetilde{i}^k_t = \frac{1}{\delta_k}(\widetilde{k}_t - (1-\delta_k)\widetilde{k}_{t-1})$$

A.4.3 Poor households

1. FOC with respect to consumption:

$$\widetilde{\lambda}^c_{p,t} = \frac{-\sigma_c(c_p\widetilde{c}_{p,t} - hC_p\widetilde{C}_{p,t-1})}{(c_p - hC_p)}$$

2. FOC with respect to employment:

$$\lambda^n_p\widetilde{\lambda}^n_{p,t} = \frac{\lambda^l_p}{\lambda^c_p}(\widetilde{\lambda}^l_{p,t} - \widetilde{\lambda}^c_{p,t}) + w_p\widetilde{w}_{p,t} + \mathbb{E}_t\,\beta(1-\sigma_p)\lambda^n_p(\widetilde{\lambda}^c_{p,t+1} - \widetilde{\lambda}^c_{p,t} + \widetilde{\lambda}^n_{p,t+1})$$

3. FOC with respect to the participation in the labor market:

$$\widetilde{\lambda}^l_{p,t} = \eta\widetilde{\ell}_{p,t}$$

4. FOC with respect to unemployment:

$$\widetilde{\lambda}^l_{p,t} = \widetilde{\lambda}^c_{p,t} + \widetilde{\lambda}^n_{p,t} + \frac{1}{1-\mu_p}\widetilde{\mu}_{p,t}$$

5. The budget constraint of poor households:

$$c_p\widetilde{c}_{p,t} + Y\widetilde{t}_{p,t} = w_p n_p(\widetilde{w}_{p,t} + \widetilde{n}_{p,t})$$

A.4.4 Intermediate goods firms

1. Production function:

$$\widetilde{f}_{int,t} = \iota\widetilde{K}_t + (1-\iota)\left(mN^\sigma_w + (1-m)N^\sigma_p\right)^{-1}(mN^\sigma_w\widetilde{N}_{w,t} + (1-m)N^\sigma_p\widetilde{N}_{p,t})$$

2. The net output of intermediate goods firm:

$$Y\widetilde{Y}_t = f_{int}\widetilde{f}_{int,t} - g_{int}\widetilde{g}_{int,t}$$

$$g_{int}\widetilde{g}_{int,t} = f_{int}\frac{e_w}{2}\left(\frac{H_w}{N_w}\right)^2(\widetilde{f}_{int,t} + 2\widetilde{H}_{w,t} - 2\widetilde{N}_{w,t}) + f_{int}\frac{e_p}{2}\left(\frac{H_p}{N_p}\right)^2(\widetilde{f}_{int,t} + 2\widetilde{H}_{p,t} - 2\widetilde{N}_{p,t})$$

3. FOC with respect to capital:

$$r^k x(\widetilde{r}^k_t + \widetilde{x}_t) = f_K\widetilde{f}_{K,t} - g_K\widetilde{g}_{K,t}$$

4. FOC with respect to skilled labor:

$$Q^N_w\widetilde{Q}^N_{w,t} = \frac{f_{N_w}}{x}(\widetilde{f}_{N_{w,t}} - \widetilde{x}_t) - \frac{g_{N_w}}{x}(\widetilde{g}_{N_{w,t}} - \widetilde{x}_t) - w_w\widetilde{w}_{w,t} + \mathbb{E}_t\,\beta(1-\sigma_w)Q^N_w(\widetilde{\lambda}^c_{w,t+1} - \widetilde{\lambda}^c_{w,t} + \widetilde{Q}^N_{w,t+1})$$

5. FOC with respect to unskilled labor:

$$Q_p^N \widetilde{Q}_{p,t}^N = \frac{f_{N_p}}{x}(\widetilde{f}_{N_{p,t}} - \widetilde{x}_t) - \frac{g_{N_p}}{x}(\widetilde{g}_{N_{p,t}} - \widetilde{x}_t) - w_p \widetilde{w}_{p,t} + \mathbb{E}_t\, \beta(1-\sigma_p) Q_p^N (\widetilde{\lambda}^c_{w,t+1} - \widetilde{\lambda}^c_{w,t} + \widetilde{Q}^N_{p,t+1})$$

6. FOC with respect to hiring of skilled labor:

$$\widetilde{Q}^N_{w,t} = \widetilde{g}_{H_{w,t}} - \widetilde{x}_t$$

7. FOC with respect to hiring of unskilled labor:

$$\widetilde{Q}^N_{p,t} = \widetilde{g}_{H_{p,t}} - \widetilde{x}_t$$

8. The derivatives of the production function and hiring cost function:

$$\widetilde{f}_{K_t} = (\iota - 1)\widetilde{K}_t + (1-\iota)(mN_w^\sigma + (1-m)N_p^\sigma)^{-1}\Big(mN_w^\sigma \widetilde{N}_{w,t} + (1-m)N_p^\sigma \widetilde{N}_{p,t}\Big)$$

$$\widetilde{f}_{N_{w,t}} = \iota\widetilde{K}_t + \Big(\frac{1-\iota}{\sigma} - 1\Big)(mN_w^\sigma + (1-m)N_p^\sigma)^{-1}\Big(m\sigma N_w^\sigma \widetilde{N}_{w,t} + (1-m)\sigma N_p^\sigma \widetilde{N}_{p,t}\Big) + (\sigma - 1)\widetilde{N}_{w,t}$$

$$\widetilde{f}_{N_{p,t}} = \iota\widetilde{K}_t + \Big(\frac{1-\iota}{\sigma} - 1\Big)(mN_w^\sigma + (1-m)N_p^\sigma)^{-1}\Big(m\sigma N_w^\sigma \widetilde{N}_{w,t} + (1-m)\sigma N_p^\sigma \widetilde{N}_{p,t}\Big) + (\sigma - 1)\widetilde{N}_{p,t}$$

$$\begin{aligned} g_{N_w}\widetilde{g}_{N_{w,t}} =& -e_w\Big(\frac{H_w}{N_w}\Big)^2 \frac{1}{N_w} f_{int}\Big(2\widetilde{H}_{w,t} - 2\widetilde{N}_{w,t} - \widetilde{N}_{w,t} + \widetilde{f}_{int,t}\Big) + \\ & f_{N_w}\frac{e_w}{2}\Big(\frac{H_w}{N_w}\Big)^2(\widetilde{f}_{N_{w,t}} + 2\widetilde{H}_{w,t} - 2\widetilde{N}_{w,t}) + f_{N_w}\frac{e_p}{2}\Big(\frac{H_p}{N_p}\Big)^2(\widetilde{f}_{N_{w,t}} + 2\widetilde{H}_{p,t} - 2\widetilde{N}_{p,t}) \end{aligned}$$

$$\begin{aligned} g_{N_p}\widetilde{g}_{N_{p,t}} =& -e_p\Big(\frac{H_p}{N_p}\Big)^2 \frac{1}{N_p} f_{int}\Big(2\widetilde{H}_{p,t} - 2\widetilde{N}_{p,t} - \widetilde{N}_{p,t} + \widetilde{f}_{int,t}\Big) + \\ & f_{N_p}\frac{e_w}{2}\Big(\frac{H_w}{N_w}\Big)^2(\widetilde{f}_{N_{p,t}} + 2\widetilde{H}_{w,t} - 2\widetilde{N}_{w,t}) + f_{N_p}\frac{e_p}{2}\Big(\frac{H_p}{N_p}\Big)^2(\widetilde{f}_{N_{p,t}} + 2\widetilde{H}_{p,t} - 2\widetilde{N}_{p,t}) \end{aligned}$$

$$\widetilde{g}_{H_{w,t}} = \widetilde{H}_{w,t} - 2\widetilde{N}_{w,t} + \widetilde{f}_{int,t}$$

$$\widetilde{g}_{H_{p,t}} = \widetilde{H}_{p,t} - 2\widetilde{N}_{p,t} + \widetilde{f}_{int,t}$$

$$g_K\widetilde{g}_{K,t} = e_w\Big(\frac{H_w}{N_w}\Big)^2 f_K\Big(\widetilde{H}_{w,t} - \widetilde{N}_{w,t} + \frac{1}{2}\widetilde{f}_{K,t}\Big) + e_p\Big(\frac{H_p}{N_p}\Big)^2 f_K\Big(\widetilde{H}_{p,t} - \widetilde{N}_{p,t} + \frac{1}{2}\widetilde{f}_{K,t}\Big)$$

A.4.5 Final goods firms

1. The New Keynesian Phillips Curve:

$$\widetilde{\pi}_t = \frac{(\epsilon - 1)}{\phi_p}\widetilde{mc}^r_t + \beta\,\mathbb{E}_t\,\widetilde{\pi}_{t+1}$$

2. Real profit of the final good firms:

$$\widetilde{\Pi}_t^r = \widetilde{Y}_t - mc^r(\widetilde{mc}_t^r + \widetilde{Y}_t)$$

A.4.6 Monetary and fiscal policies

1. Monetary Policy Rule:

$$\widetilde{R}_t = \theta_r \widetilde{R}_{t-1} + (1-\theta_r)\left[\theta_\pi \widetilde{\pi}_t + \theta_y \widetilde{Y}_t\right]$$

2. Fiscal Policy Rule:

$$\widetilde{T}_t = \phi_B \widetilde{B}_{t-1} + \phi_{BG} \widetilde{G}_t$$

3. The real government budget constraint:

$$\widetilde{T}_t + \widetilde{B}_t = \frac{RB}{\pi Y}(\widetilde{R}_{t-1} + \frac{Y}{B}\widetilde{B}_{t-1} - \widetilde{\pi}_t) + \widetilde{G}_t$$

4. The distribution of lump-sum taxes:

$$\widetilde{T}_t = s_w \widetilde{t}_{w,t} + s_p \widetilde{t}_{p,t}$$

A.4.7 Aggregate resource constraint

$$Y\widetilde{Y}_t = C\widetilde{C}_t + I\widetilde{I}_t + Y\widetilde{G}_t$$

A.4.8 The exogenous process

1. Government spending:

$$\widetilde{G}_t = \phi_g \widetilde{G}_{t-1} + \epsilon_t^g$$

A.4.9 Aggregate variables

1. Aggregate consumption:

$$C\widetilde{C}_t = C_w \widetilde{C}_{w,t} + C_p \widetilde{C}_{p,t} = s_w c_w \widetilde{c}_{w,t} + s_p c_p \widetilde{c}_{p,t}$$

2. Labor supply of the wealthy:

$$\widetilde{N}_{w,t} = \widetilde{n}_{w,t}$$

3. Labor supply of the poor:

$$\widetilde{N}_{p,t} = \widetilde{n}_{p,t}$$

4. Aggregate capital stock:

$$\widetilde{K}_t = \widetilde{k}_{t-1}$$

5. Aggregate bonds:

$$\widetilde{B}_t = s_w \widetilde{b}_t$$

A.5 TANKrep model

The head of the household maximises discounted lifetime household utility choosing $\{c_t, i_t, k_t, b_t, \ell_{w,t}, \ell_{p,t}, n_{w,t}, n_{p,t}, u_{w,t}, u_{p,t}\}$

$$\max \ \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Big\{ \frac{1}{1-\sigma_c}(c_\tau - hC_{\tau-1})^{1-\sigma_c} - \varphi_{n,w} \frac{(s_w \ell_{w,\tau})^{1+\eta}}{1+\eta} - \varphi_{n,p} \frac{(s_p \ell_{p,\tau})^{1+\eta}}{1+\eta} \Big\}$$

The real budget constraint of a wealthy household in every period t is:

$$c_t + t_t + i_t + b_t \leq s_w w_{w,t} n_{w,t} + s_p w_{p,t} n_{p,t} + r_t^k k_{t-1} + \frac{R_{t-1} b_{t-1}}{\pi_t} + \Pi_t^{int} + \Pi_t^r$$

and the employment law of motion:

$$n_{w,t} = (1-\sigma_w) n_{w,t-1} + \frac{\mu_{w,t}}{1-\mu_{w,t}} u_{w,t} (= h_{w,t})$$

$$n_{p,t} = (1-\sigma_p) n_{p,t-1} + \frac{\mu_{p,t}}{1-\mu_{p,t}} u_{p,t} (= h_{p,t})$$

and the law of motion of physical capital:

$$i_t = k_t - (1-\delta_k) k_{t-1} + \frac{\phi_k}{2} \Big(\frac{k_t}{k_{t-1}} - 1 \Big)^2 k_{t-1}$$

Abstrakt

Tento článek rozvíjí model s kvalifikovanými a nekvalifikovanými pracovníky, aby ukázal expanzivní efekty vládních výdajů navzdory vysokým nákladům na školení pro nové zaměstnance. Hlavní myšlenkou je, že fiskální stimul vyvolává změny ve složení pracovní síly podmíněné rozsahem tlaku agregátní poptávky. Po období vysokého tlaku agregátní poptávky následuje vysoká hodnota ušlého výstupu, protože školicí činnost způsobuje narušení výroby. V tomto období se firmy rozhodují najímat více pracovníků s nízkou kvalifikací, kteří představují levnější část pracovní síly. Když se tlak agregátní poptávky sníží, firmy přejdou na najímání více vysoce kvalifikovaných pracovníků. Současná literatura však bere v úvahu pouze vysoce kvalifikované pracovníky, kteří mají tendenci zvyšovat úspory ve státních dluhopisech, aby se chránili před špatnými pracovními vyhlídkami. V tomto případě kombinace slabých pracovních vyhlídek a vytěsňovacích účinků vyšších jednorázových daní, a vládního dluhu na soukromou spotřebu a kapitálové investice vede k recesním efektům. Naproti tomu tento článek poskytuje model s realističtější strukturou pracovního a finančního trhu a naznačuje, že proticyklické vládní výdaje ve formě vládní spotřeby, a zejména vládních investic, lze použít k potlačení recese.

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